I. INTRODUCTION

The observation of a small electron pocket in the Fermi surface of underdoped YBa$_2$Cu$_3$O$_{y}$ (YBCO) [1,2] and HgBa$_2$CuO$_{4+\delta}$ (Hg1201) [3,4], in sharp contrast with the large holelike Fermi surface of the overdoped regime [5], shows that the Fermi surface of hole-doped cuprates undergoes a profound transformation with underdoping [6]. In the cuprate La$_{1-x}$Eu$_{x}$Sr$_x$CuO$_4$, the similar Fermi-surface reconstruction (FSR) [6–9] is clearly linked to the onset of charge-stripe order detected by x-ray diffraction [7,10].

In YBCO, the recent detection of charge-density-wave (CDW) modulations [11–14] in the same doping range where the electron pocket prevails [15] points here also to a scenario where CDW order causes the FSR. Moreover, recent evidence for an additional small holelike pocket in the Fermi surface of YBCO [16] is consistent with calculations of FSR by the observed CDW order [17]. Note that CDW order competes with superconductivity [12–14], causing a suppression of the latter that is directly visible in the upper critical field $H_c^u$ measured as a function of doping [18], which exhibits a local minimum where the CDW is strongest [19–21].

CDW modulations have also been seen in Hg$_{1201}$ [22], Bi$_2$Sr$_2$La$_{1-x}$CuO$_{4+\delta}$ (Bi-2201) [23], and La$_{2−x}$Sr$_x$CuO$_4$ (LSCO) [24], clear evidence that they are universal to hole-doped cuprates. This naturally begs the following question: what is the nature of the normal state of underdoped cuprates, in particular at low temperature, when superconductivity is suppressed by a magnetic field? Is it a dual state where charge order and superconductivity are intertwined, as proposed by certain theories [17,25–27], or is it a metal without any superconducting component? More generally, does this metal depart from standard Fermi-liquid behavior?

The sharp suppression of the longitudinal thermal conductivity $\kappa_{xx}$ with decreasing magnetic field recently observed in YBCO has been attributed to the onset of vortex scrambling, making it a direct measure of $H_c^2$ [18]. As shown in Fig. 1, $H_c^2$ and the vortex-melting field $H_m$ were found to be equal at $T → 0$, consistent with the absence of a vortex liquid at $T = 0$. Yet specific heat [28] and magnetization [29] data have been interpreted in terms of superconductivity persisting well above $H_m$, in the form of a vortex-liquid state. In [29], the anomaly in $\kappa_{xx}$ was interpreted as a transition to a pair-density-wave phase.

To help resolve this debate, and more generally shed light on the nature of the normal state, we have turned to the Wiedemann-Franz law, a fundamental law of electrons in metals. It states that the conduction of heat and charge are equal in the limit of $T = 0$, where all scattering is elastic, so $\kappa / T = (\pi^2/3)(k_B/e)^2\sigma$, is satisfied for fields immediately above the vortex-melting field $H_m$. This rules out the existence of a vortex liquid at $T = 0$ and it puts a clear constraint on the nature of the normal state in underdoped cuprates, in a region of the doping phase diagram where charge-density-wave order is known to exist. As the temperature is raised, the Lorenz ratio, $L_{xy} = \kappa_{xy}/(\sigma_{xy}T)$, decreases rapidly, indicating that strong small-$q$ scattering processes are involved.

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the longitudinal resistivities along the $a$ and $b$ axes of the orthorhombic structure, respectively, and $\rho_{xy}$ is the transverse (Hall) resistivity.

A. Samples

Our comparative study of heat and charge transport in YBCO was performed by measuring the electrical Hall conductivity $\sigma_{xy}$ and the thermal Hall conductivity $\kappa_{xy}$ on the same sample, using the same contacts. This sample was a detwinned single crystal of YBa$_2$Cu$_3$O$_{y}$ with oxygen content $y = 6.54$ and a high degree of ortho-II oxygen order [31], yielding large quantum oscillations, proof of a long electronic mean free path at low temperature. The hole concentration (doping) $p$ is obtained from the superconducting $T_c$ [32], defined as the temperature where the electrical resistance goes to zero. Our sample has $T_c = 61$ K, giving $p = 0.11$. At this particular doping, the upper critical field is at a local minimum with $H_{c2} = 25$ T [18], making it ideal for testing the Wiedemann-Franz law since available fields of 28 to 35 T are sufficient to access the normal state at $T_c$. The sample is in the shape of a rectangular platelet, with a width $w = 0.6$ mm (along the $b$ axis) and a thickness $t = 0.1$ mm (along the $c$ axis). Six contacts were applied in the standard geometry, using contacts that covered the ends of the sample to ensure uniformity. The longitudinal electrical resistivity $\rho_{xx} = \rho_z$ and the longitudinal thermal gradient $dT_z$ were both measured using the same two contacts on one side of the sample, separated by a distance $L = 0.8$ mm (along the $a$ axis). The transverse electrical resistivity $\rho_{xy}$ and the transverse thermal gradient $dT_y$ were both measured using the same two contacts on opposite sides of the sample, separated by a distance $w = 0.6$ mm (along the $b$ axis).

B. Electrical transport coefficients

The transverse Hall conductivity $\sigma_{xy}$ of our orthorhombic crystal is given by $\sigma_{xy} = \rho_{xy}/(\rho_{xx}\rho_{yy} + \rho_{xy})$, where $\rho_{xx}$ and $\rho_{yy}$ are the longitudinal resistivities along the $x$ and $y$ directions, i.e., the $a$ and $b$ axis, and $\rho_{xy}$ and $\rho_{yx}$ are the transverse resistivities. We take the latter to be equal, namely, $\rho_{xy} = \rho_{yx}$, or $\sigma_{xy} = \sigma_{yx}$, consistent with $\kappa_{ab} = \kappa_{ba}$ (see Appendix B). We also assume that $\rho_{yy} = \rho_{xx}$, i.e., $\rho_z = \rho_0$, as observed just above $T_c$ in similar YBCO crystals [1].

The latter assumption has no impact on our test of the Wiedemann-Franz law, since at high $H$ and low $T$ we observe $\rho_{yy}^{2} \gg \rho_{xx}^{2} \sim \rho_{yx} \rho_{yy}$ (Fig. 2).

The coefficients $\rho_{xx}(= \rho_0)$ and $\rho_{xy}$ were measured in magnetic fields up to 35 T at the National High Magnetic Field Laboratory (NHMFL) in Tallahassee, using an ac four-terminal method and applying the usual symmetrization $\{R_{ax} = [V_x(+/H) + V_x(+/H)]/2I_x\}$ and antisymmetrization $\{R_{xy} = [V_y(+/H) - V_y(-/H)]/2I_y\}$ procedures with respect to field direction. The electrical current ($I_x$) was applied along the $a$ axis. The resulting data are displayed in Fig. 2.

C. Thermal transport coefficients

The thermal Hall conductivity $\kappa_{xy}$ of our two YBCO samples was measured at the Laboratoire National des Champs Magnétiques Intenses in Grenoble up to 28 T at temperatures below 1 K and at the NHMFL in Tallahassee up to 35 T at temperatures from 4 to 68 K.

A constant heat current $Q_x$ was sent in the basal plane of the single crystal, generating a longitudinal temperature difference $dT_x$ and, in a magnetic field applied along the $c$ axis, a transverse temperature difference $dT_y$. The thermal Hall conductivity is defined as $\kappa_{xy} = \kappa_{yx}(dT_x/dT_c)(L/w)$, where $\kappa_{yx}$ is the longitudinal thermal conductivity along the $y$ axis (perpendicular to the $x$ axis). At all temperatures, we employed a one-heater-two-thermometers steady-state method, using Cernox sensors calibrated in situ as a function of temperature and magnetic field to measure $dT_x$. Below 4 K, $dT_y$ was measured with a calibrated Cernox sensor. At 4 K and above, $dT_y$ was measured using a differential type-E thermocouple known to have a weak magnetic field dependence. At $T = 10$ K, data obtained using the thermocouple were compared to data obtained using a Cernox sensor, in otherwise identical conditions, and the agreement was excellent. For further details, see Appendix C.

D. Error bars

The error on the magnitude of $\sigma_{xy}$ comes from the uncertainty in determining the geometric factor associated
with sample dimensions and contact separation, estimated to be ±10%. The error on the magnitude of $\kappa_{xy}$ includes a similar uncertainty on the geometric factor, to which is added an uncertainty of ±10% associated with thermometry, for a total of ±20%.

### III. RESULTS

In Fig. 3(a), we show the Hall coefficient $R_H = \rho_{xy}/H$ measured in a single crystal of YBCO with a doping $p = 0.11$ ($T_c = 61$ K), plotted as a function of field $H$ up to 35 T, at different temperatures $T$. Note that $R_H(H)$ at $T = 4$ K is flat above $H = 24$ T. This is a first strong evidence that there is no flux flow, and hence no long-lived vortices above $H_{c2} = 24$ T. In Fig. 3(b), we see that the low-$T$ isotherms of $\sigma_{xy}(H)$ collapse onto a single curve for $H > 24$ T, given by $\sigma_{xy} = 1/\rho_{xy} = 1/(R_H H)$, with $R_H = -11.5$ mm$^3$/C (dashed blue line), the value of the Hall coefficient at high $H$ and low $T$ [Fig. 3(a)]. The collapse of the various isotherms at high $H$ provides a convenient way to detect superconductivity as $H$ is reduced. Indeed, superconductivity is expected to produce strongly $T$
and $H$ dependent deviations in $\sigma_{xy}$, as indeed it does below $\sim 24$ T [Fig. 3(b)].

The thermal Hall conductivity $\kappa_{xy}$ was measured on the same single crystal on which $\sigma_{xy}$ was measured (see Methods). The various isotherms of $\kappa_{xy}$ are displayed in Fig. 4. Looking at the lowest isotherm, at $T = 0.7$ K, we see that $\kappa_{xy}$ is large and negative above 20 T, consistent with the negative electrical Hall and Seebeck coefficients, all showing that a high-mobility electron pocket dominates the transport properties of YBCO [2,6,8,9,15]. With decreasing $T$, $\sigma_{xy}$ becomes negligible below 20 T or so. We attribute this decrease to a loss of heat-carrying quasiparticles [33] and the onset of vortex scattering [18]. The onset field for this decrease is $H = 24 \pm 1$ T [Fig. 5(a)], in excellent agreement with prior estimates of $H_{c2}$ from $\kappa_{xx}$ measurements on similar YBCO samples [18] (Fig. 1).

IV. DISCUSSION

In Fig. 5(a), we compare the isotherm at $T = 0.7$ K, plotted as $\kappa_{xy}/T$ vs $H$, with its electrical counterpart, plotted as $L_0\sigma_{xy}$ vs $H$. Here $\sigma_{xy}$ is simply the common normal-state curve observed at low temperature [Fig. 3(b)]. In Fig. 5(b), we plot the ratio of the two, namely, the normalized Lorenz ratio $L_{xy}/L_0$. We see that the Wiedemann-Franz law is satisfied for $H > H_{c2}$, within error bars. (Note that the law was only tested at $p = 0.11$, and therefore, strictly speaking, it is only established for the field-induced CDW state [34–36].) This has two important implications for the normal state of underdoped cuprates. First, it shows that quasiparticles conduct heat and charge just as they do in a normal Fermi liquid. This is consistent with other signatures of Fermi-liquid behavior in YBCO, such as the temperature dependence of quantum oscillations [37] and the $T^2$ electrical resistivity at low temperature [15,38]. In general, it puts a clear and robust constraint on the nature of the low-energy excitations in the pseudogap phase of underdoped cuprates.

Second, it excludes the possibility of a vortex liquid above $H_{c1}$ at $T \to 0$. This means that the interpretation of the specific heat [28] and magnetization [29] of underdoped YBCO must be reexamined. In fact, recent specific-heat data [39] now
suggest a saturation at high magnetic fields, consistent with having no significant superconducting contribution, as our transport data show. The effect of FSR on the normal-state contributions to $\kappa_{xy}$, $\rho_{xx}$, $\rho_{xy}$ or $\rho_{yy}$.

Having established that $\kappa_{xy}/T = L_0\sigma_{xy}$ in the $T = 0$ limit, we now examine how $\kappa_{xy}/T$ and $L_0\sigma_{xy}$ separately evolve with increasing $T$, as a result of inelastic scattering. In Fig. 6(a), we plot $\kappa_{xy}/T$ and $L_0\sigma_{xy}$ vs $T$ at $H = 27$ T, above $H_c2$. As noted earlier, $L_0\sigma_{xy}$ remains constant up to 30 K. In sharp contrast, over the same $T$ interval, $\kappa_{xy}/T$ decreases in magnitude by a factor of 10. An electrical current is more effectively degraded by a large momentum transfer, while a heat current can also be diminished by an energy loss at small momentum transfer $q$. Consequently, the combination of a constant $L_0\sigma_{xy}$ and a rapidly decreasing $|\kappa_{xy}/T|$ between $T = 0$ and 30 K is an indication that the dominant inelastic scattering involves small-$q$ processes. We speculate that a possible candidate for a small $q$ vector in the reconstructed Fermi surface of YBCO is one that connects the tip of the square-shaped electron pocket and the tip of the hololike ellipse where the two nearly touch [16,17], at the CDW hot spot. Inelastic scattering at this small $q$ vector would affect precisely those regions of the Fermi surface that are responsible for the large negative Hall signal [Fig. 3(a)], namely, the tips of the electron pocket. This process is therefore expected to rapidly make the Hall signal less negative, as observed in the thermal channel.

Another potential scenario for small-$q$ inelastic scattering at low $T$ is fluctuations near a nematic quantum critical point [44].

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the test was done on longitudinal conductivities (κ) optimally doped and underdoped Bi-2201 [50]. In all cases, which with fully metallic behavior and transition. Our YBCO samples are in a completely different regime, from the Onsager reciprocity relation. The dashed line is a linear fit to the steepest part of κxy vs H. We define Hc2 as the field above which |κxy| departs from that linear rise (vertical gray band), giving Hc2 = 25 ± 1 T, as plotted in Fig. 1.

FIG. 7. Thermal Hall conductivity of our two samples of YBCO with p = 0.11 (y = 6.54), at T = 8 K, plotted as κxy/T vs H. In one sample, the heat current flows along the a axis of the orthorhombic crystal structure (red, J || a), while in the second it flows along the b axis (blue, J || b) (Methods). Within error bars (±20 %), we see that both samples yield the same curve, so that κab = κba, as expected from the Onsager reciprocity relation. The dashed line is a linear fit to the steepest part of |κxy| vs H. We define Hc2 as the field above which |κxy| departs from that linear rise (vertical gray band), giving Hc2 = 25 ± 1 T, as plotted in Fig. 1.

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APPENDIX A: PRIOR TESTS OF THE WIEDEMANN-FRANZ LAW IN CUPRATES

The Wiedemann-Franz law, κ/T = L0σ at T = 0, was investigated for cuprates in six prior studies: in optimally doped Pr2−xCe,CuO4 (PCCO) [45], in overdoped Tl2Ba2CuO6+δ [46], in overdoped LSCO [47,48], and in overdoped [49], optimally doped and underdoped Bi-2201 [50]. In all cases, the test was done on longitudinal conductivities (κxx and σxx). Because of the large phonon term in κxx, extracting the electronic term is done by extrapolating κxx/T to T = 0. In the study on PCCO, this procedure failed because of electron-phonon decoupling [51]. For all overdoped samples, the Wiedemann-Franz law was found to be valid in the field induced normal state, to within a few percent. In the only prior study on an underdoped cuprate (Bi-2201), the Lorenz ratio was found to be larger than expected: L/L0 > 1.0 [50]. However, values of L/L0 exceeding 1.0 were observed only in samples the normal state resistivity ρ0(T) of which showed an upturn at low T, achieving residual values ρ0 > 200 μΩ cm [50]. The violation was attributed to a metal-insulator transition. Our YBCO samples are in a completely different regime, with fully metallic behavior and ρ0 = 4 μΩ cm at H > Hc2 (Fig. 2).

APPENDIX B: ONSAGER RELATION

Because YBCO has an orthorhombic crystal structure, measurements on two samples are necessary to obtain κxy:

APPENDIX C: THERMOMETRY

The longitudinal temperature difference dTc = T_hot − T_cold was measured using Cernox resistive sensors positioned on one side of the sample near the hot (T_hot) and cold (T_cold) ends. In zero field, the sensors are calibrated in situ against a reference calibrated Cernox sensor. At T < 15 K or so, Cernox sensors show a pronounced (negative) magnetoresistance. In order to properly determine T_hot and T_cold in a finite field, the hot and cold Cernox sensors were calibrated by performing field sweeps at different closely spaced temperatures between 0.5 and 15 K. The probe temperature was kept constant when sweeping the magnetic field by using a strain gauge with a field-independent resistance as the temperature regulator of our probe. That the temperature was indeed kept constant was checked against a Cernox sensor independently calibrated in magnetic fields up to 27 T and down to 1.5 K, and also against a RuOx sensor known to have a weak and linear magnetoresistance. Below 1.0 K, the probe temperature was

FIG. 8. Thermal Hall conductivity κxy as a function of field at T = 10 K obtained from two different measurements of the transverse temperature difference dTc, using (1) a Cernox sensor (blue curve) and (2) a type-E thermocouple (red dots). In both cases, the longitudinal temperature difference dTc is measured with two Cernox sensors. The two measurements of κxy show excellent agreement, confirming that our thermometry in high fields is reliable. The dashed line is a linear fit to the steepest part of |κxy| vs H. We define as Hc2 the field above which |κxy| departs from that linear rise (vertical gray band), giving Hc2 = 25 ± 1 T (Fig. 1). The dotted line shows the linear behavior (κxy ∼ H) expected of a metal when κxy becomes comparable to or smaller than κxx.
kept constant against the vapor pressure of a helium-3 bath. Field sweeps going up or down gave identical traces. Above 4 K, the transverse temperature difference $dT_y$ was measured with a type-E constantan-chromel-constantan differential thermocouple known to have a weak-field dependence. Below 4 K, $dT_y$ was measured using Cernox sensors calibrated as for the $dT_x$ measurement. In a field $H$, $T_{hot}$ contains a contribution from the transverse gradient $dT_y$. $T_{hot}(\pm H) = T_{hot}(SYM) \pm dT_y/2$. By antisymmetrizing $T_{hot}$, we get the transverse thermal gradient $dT_y$ with a single sensor measurement. Quantitative agreement between the two methods used to measure $dT_y$ is demonstrated in Fig. 8. The excellent agreement demonstrates that our in-field thermometry is accurate and reliable. Data are systematically taken at positive and negative fields, and $dT_x$ and $dT_y$ are associated with the symmetric and antisymmetric traces, respectively. The magnetic field was swept at a rate of 1 T/min, well below the level at which thermal hysteretic effects are observed.