Heat Transport in High-Temperature Superconductors

by L. Taillefer and R.W. Hill

INTRODUCTION

Understanding the cuprates is one of the most challenging problems facing physicists today because the rich, complex and highly unusual behaviour of electrons in these materials is forcing us to re-examine the cornerstones of solid state theory. A remarkable property of the materials is the fact that a simple tuning of their chemistry can take any given compound all the way from insulator to superconductor, by doping electrons or holes into the CuO_2 planes which stack up to form the crystal structure. In attempting to unravel the mysterious ways of these doped carriers, scientists may be witnessing the breakdown of two hugely successful theories of 20th century physics: the Fermi-liquid theory of electrons in metals and the Bardeen-Cooper-Schrieffer (BCS) theory of electron pairing and condensation in superconductors. The concept of an electron as the basic particle carrying both spin and charge in the metallic state is being threathened. The notion of superconductivity as a phase transition at which both electron pairing and long-range phase coherence occur simultaneously is under siege. "Spin-charge separation" and "preformed pairs" are only two amongst several hotly debated issues. Others include the unification of magnetism and superconductivity and the microscopic nature of the vortex state produced in the presence of a magnetic field. Numerous experimental techniques have been brought to bear on this vast subject [1]. In this article, we outline what has been learnt from studies of heat transport. The most powerful applications of this technique – the thermal Hall effect and conduction at very low temperature – have only recently been developed to the point where they can lead to penetrating insights, and the first findings are the subject of this article. As we shall see, nothing in this will force us to go outside the framework of Fermi-liquid or BCS theory in their general form. However, note that much of the territory still lies ahead unexplored – for example, little has been done yet on the fascinating ("underdoped") regime between the superconducting state and the antiferromagnetic (insulating) state.

HEAT TRANSPORT

Superconductors are perfect conductors of electric charge but very bad conductors of heat. This is because electric currents are carried by the Cooper pairs that form the superconducting condensate, which has zero entropy, while heat (or entropy) is only carried by the elementary excitations out of that ground state, or *quasiparticles.* A measurement of heat transport therefore probes the nature of the superconducting state via its quasiparticle energy spectrum, by giving access to the *gap function*, its defining characteristic.

The measurement involves passing a heat current through a sample and detecting the temperature gradient that develops along it. The ratio of power $\dot{Q}$ generated by a heater fixed at one end over the temperature difference ΔT between two points separated by a distance L is the thermal conductivity:

$$\kappa = \frac{\dot{Q}}{\Delta T} \; \frac{L}{A} \tag{1}$$

where A is the sample cross-section. The technique is straightforward as long as one has good control over the thermometry and ensures that no heat flows through secondary channels. Fig. 1 shows the thermal conductivity of $YBa_2Cu_3O_7$, for a carrier concentration slightly higher than "optimal" doping (*i.e.* maximal $T_c \simeq 93$ K). The large peak below T_c, whose magnitude increases with increasing

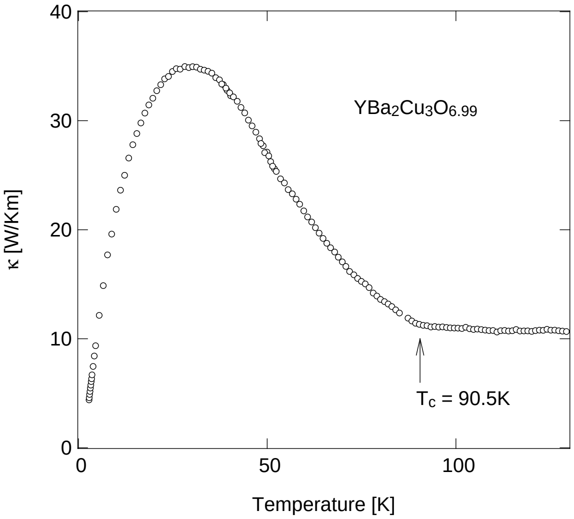

Figure 1: Temperature dependence of the thermal conductivity of $YBa_2Cu_3O_{6.99}$, for a heat current along the a axis of the orthorhombic crystal. The arrow marks the transition temperature observed in resistivity. [2]

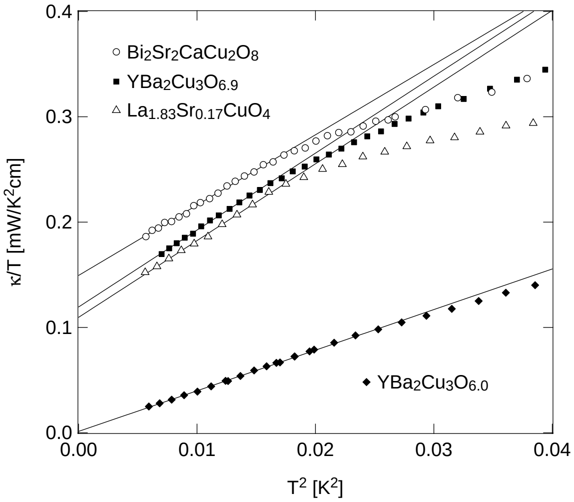

Figure 2: Thermal conductivity of optimally-doped $Bi_2Sr_2CaCu_2O_8$, $YBa_2Cu_3O_{6.9}$ and $La_{1.83}Sr_{0.17}CuO_4$ at very low temperature. Insulating (deoxygenated) $YBa_2Cu_3O_{6.0}$ is also shown for comparison. The lines are linear fits to the data below 130 mK. [6]

sample purity, is due to the tremendous growth of the electronic mean free path as the temperature is decreased below T_c, a result of the precipitous suppression of the strong electron-electron scattering present in the normal state as electrons condense into Cooper pairs. This understanding first came from electrodynamic studies at microwave frequencies (see article by Bonn and Hardy).

ELECTRONS AND PHONONS

The main difficulty with the interpretation of thermal conductivity data is that both electrons and phonons carry heat. For example in the data of Fig. 1, the conductivity above T_c is predominantly due to phonons. Two ways have been devised to accurately isolate the electronic contribution. The first is the thermal analog of the Hall effect, whereby a transverse magnetic field (normal to the CuO_2 planes) deflects the electrons but not the phonons. Measurements of the purely electronic thermal Hall conductivity $\kappa_{xy}(T)$ have been used to show that electrons are in large part responsible for the peak below T_c, while accounting for only 10% or so of the longitudinal κ_{xx} in the normal state [3]. As this novel technique further develops, a detailed comparison of κ_{xy} and σ_1 measured at microwave frequencies is expected to yield insight into the nature of electronic carriers and their mutual interaction. The second way of separating electron and phonon contributions is from their temperature dependences as $T \to 0$. Heat conduction is the product of specific heat, carrier velocity and mean free path:

$$\kappa = \frac{1}{3}\, c\, v\, \ell \tag{2}$$

At very low temperatures the mean free paths of electrons and phonons are independent of temperature; the electrons limited by impurity scattering and the phonons by the boundaries of the sample. In this case, temperature dependence is given entirely by the specific heat, which is linear in T for electrons and cubic in T for phonons. Therefore,

$$T \to 0 \ : \quad \frac{\kappa}{T} = A + BT^2 \tag{3}$$

In the remainder of this article, we concentrate on the residual linear term A, the magnitude of which is directly related to the quasiparticle energy spectrum. The ability to probe the electron system at $T \to 0$ has two advantages: the strongly correlated electrons have settled into their simplest configuration, free from much of the complexity that develops at higher temperature, and theoretical results are more accurate and robust at $T = 0$.

LOW-ENERGY QUASIPARTICLES

The thermal conductivity of $YBa_2Cu_3O_x$ (YBCO) at temperatures below 200 mK is shown in Fig. 2, for the two extreme states of carrier concentration: 1) the insulating state ($x = 6.0$), with no mobile holes, and 2) the superconducting state near optimal doping ($x = 6.9$), with mobile holes in the CuO_2 planes. In the insulator, $A = 0$ in Eq. 3 and all conduction is due to phonons. Exactly the same result is found for a standard superconductor, characterised by a finite gap for all directions of electron motion (s-wave symmetry): the number of thermally excited quasiparticles below $T_c/10$ is exponentially small. As holes are added to the CuO_2 planes in YBCO via oxygenation, a sizable linear term is seen to develop, with a value $A = 0.14 \pm 0.03$ mW K^{-2} cm^{-1} [4]. A similar behaviour is found for the other two holed-doped cuprate superconductors $Bi_2Sr_2CaCu_2O_8$ (BSCCO) [5, 6] and $La_{1.83}Sr_{0.17}CuO_4$ (LSCO) [6]. The observation of electronic conduction in a superconductor down to $T_c/1000$ is unprecedented, and it points unequivocally to the presence of itinerant fermionic excitations of zero energy. We now show that this "residual normal fluid" is due to quasiparticles in a gapped spectrum with d-wave symmetry.

In BCS theory, the excitation spectrum in the superconducting state is given by

$$E(\mathbf{k}) = \sqrt{\epsilon^2(\mathbf{k}) + \Delta^2(\mathbf{k})} \qquad (4)$$

where $\mathbf{k}$ is the momentum vector, $\epsilon(\mathbf{k})$ is the electronic energy in the metallic state (relative to the Fermi energy) and $\Delta(\mathbf{k})$ is the gap function. In a pairing state with $d_{x^2-y^2}$ symmetry, the quasiparticles are distinct from those in conventional superconductors in that they have a unique energy versus momentum relation as a result of their unconventional gap structure. This takes the approximate form $\Delta_d(\mathbf{k}) = \Delta_0\cos(2\phi)$, where ϕ is the azimuthal angle in the (k_x, k_y) plane measured relative to the k_x axis. In comparison, a standard superconductor with isotropic gap has $\Delta_s(\mathbf{k}) = \Delta_0$. Note that $\Delta_d(\mathbf{k})$ vanishes along the lines $\phi = \pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$

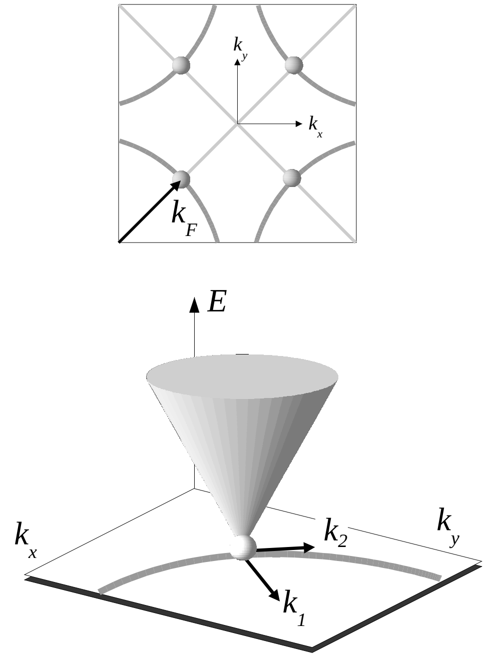

Figure 3: TOP PANEL: Planar section of the Fermi surface of a typical cuprate, made of cylinders centered on the corners of the Brillouin zone. The two diagonal lines correspond to directions along which $\Delta(\mathbf{k})$ vanishes for $d_{x^2-y^2}$ symmetry. Nodes in the energy spectrum are found where these lines cross the Fermi surface, shown as four dots. BOTTOM PANEL: Energy vs momentum relation for the nodal quasiparticles.

(or $k_x = \pm k_y$). As shown schematically in Fig. 3, this gives rise to four nodes on the Fermi surface of hole-doped cuprates, which in essence consists of a cylinder centered on each corner of the Brillouin zone at $(\pm\frac{\pi}{a}, \pm\frac{\pi}{a})$, where a is the lattice constant. Near each node, the energy is given by

$$E(\mathbf{k}) = \hbar\sqrt{v_F^2 k_1^2 + v_2^2 k_2^2} \qquad (5)$$

where $v_2 = (d\Delta_d/d\phi)/(\hbar k_F)$ is the slope of the gap at the node, v_F and k_F are the Fermi velocity and momentum, and k_1 and k_2 are the components of $\mathbf{k}$ normal and parallel to the local Fermi surface. Eq. 5 describes a conelike spectrum (see Fig. 3), with a dispersion perpendicular and parallel to the Fermi surface given respectively by v_F and v_2. The density of states of "nodal" quasiparticles grows linearly with energy at low energy, a property which governs all low-temperature properties, for example the linear temperature dependence of the penetration depth (see article by Bonn and Hardy). In the next section, we will show how the heat conduction measured at $T \to 0$ is clear evidence for these d-wave nodal quasiparticles and how it provides a direct measure of their dispersion.

UNIVERSAL CONDUCTION

In a crystal of $YBa_2Cu_3O_{6.9}$, the replacement by a Zn atom of 1 or 2 out of every 100 Cu atoms in the CuO_2 plane causes a major change in the transport properties. The electrical resistivity acquires a sizable constant term and the peak in the thermal and microwave conductivities below T_c disappears almost entirely, as the elastic scattering rate is increased by a factor of 10 to 100. Remarkably, such an increase in scattering rate was found to have no impact on the ability of the residual normal fluid to conduct heat [7]. This can arise in an unconventional superconductor because of the two-fold effect of impurity scattering: not only does it limit the mean-free path, it also generates a finite density of quasiparticles at zero energy, roughly speaking by broadening the apex of the cone in Fig. 3. In the case of a d-wave gap, those two effects compensate exactly at $T = 0$, resulting in a *universal* conductivity, independent of impurity concentration [8]:

$$T \to 0 \;:\quad \frac{\kappa}{T} = \frac{k_B^2}{3\hbar}\; n\; \left(\frac{v_F}{v_2} + \frac{v_2}{v_F}\right) \qquad (6)$$

where n is the number of CuO_2 planes per meter stacked along the c-axis. A measurement of the residual linear term in $\kappa(T)$ is thus seen to be a direct measure of the ratio of quasiparticle velocities, v_F and v_2. Using the data in Fig. 2 one gets $\frac{v_F}{v_2} = 14$, 19 and 12 for YBCO [4], BSCCO [5] and LSCO, respectively. In the case of BSCCO, the dispersion at the node was measured spectroscopically by angle-resolved photoemission, giving $\frac{v_F}{v_2} = 20$ for optimal doping [9]. This excellent agreement shows that the residual linear term in the heat transport is entirely due to d-wave quasiparticles, paving the way to a systematic study of the ground states of cuprates. Its observation in YBCO and LSCO, where the gap function has not been resolved via photoemission, confirms d-wave symmetry in these materials and gives us the dispersion.

Once a value for v_F/v_2 is obtained, it can be used to compute various properties within the Fermi-liquid framework. For example, it is of fundamental interest to understand by what mechanism the superfluid density $n_s(T)$ is suppressed with increasing temperature (see Fig. 4 in article by Bonn and Hardy). Is the suppression due to fluctuations in the *amplitude* of the order parameter (*i.e.* thermally excited quasiparticles) or fluctuations in the *phase*? The quasiparticle contribution to $n_s(T)$ is proportional to v_F/v_2 [8], and a comparison of heat transport and penetration depth data for YBCO and BSCCO reveals that there are enough quasiparticles at low temperatures to fully account for the drop in superfluid density, showing that phase fluctuations need not be invoked [5].

THE VORTEX STATE

An applied magnetic field H penetrates a superconductor in the form of vortices, *i.e.* lines at the centre of a rotating superfluid flow field. In the presence of this superfluid moving at velocity $\mathbf{v_s}$, the quasiparticle energy is modified according to

$$E(\mathbf{k}) \to E(\mathbf{k}) + \hbar\;\mathbf{k}\cdot\mathbf{v_s} \qquad (7)$$

where $\mathbf{v_s} = \mathbf{v_s}(\mathbf{r})$ varies in magnitude and direction throughout the material, being largest close to the vortex centre. Therefore, depending on the local direction of $\mathbf{v_s}$, the cone of excitation shown in Fig. 3 will either move up or down in energy. On average, this will induce a finite density of states at the Fermi energy proportional to $\sqrt{H}$, corresponding to those cones of excitation that have moved down in energy. The ideal way of probing these field-induced excitations is to look at the heat transport.

This is done in Fig. 4 for YBCO. The application of a modest magnetic field (compared to the field needed to destroy superconductivity) clearly leads to an increase in the electronic conduction [4, 10]. The field dependence of the residual linear term was reproduced quantitatively by semi-classical calculations based on the Doppler shift defined in Eq. 7 [11], assuming reasonable values for v_F, v_2 and the impurity scattering rate. It is far from obvious why such calculations should work, as the usual basis for the description of electron states in a magnetic field may not hold for d-wave quasiparticles in the vortex state. Several issues need be explored, including field-induced phase transitions [12] and the absence of Landau quantization.

In conclusion, the picture that emerges from studies of heat transport in cuprates at $T = 0$ and optimal doping is completely in agreement with Fermi-liquid and BCS theory (generalized for d-wave symmetry). The question is: does this continue to hold true at high temperatures or in the underdoped regime, where other evidence of breakdown is rife and dramatic? The future will reveal whether we have a revolution on our hands or if the old guard of Landau and Fermi will hold their ground.

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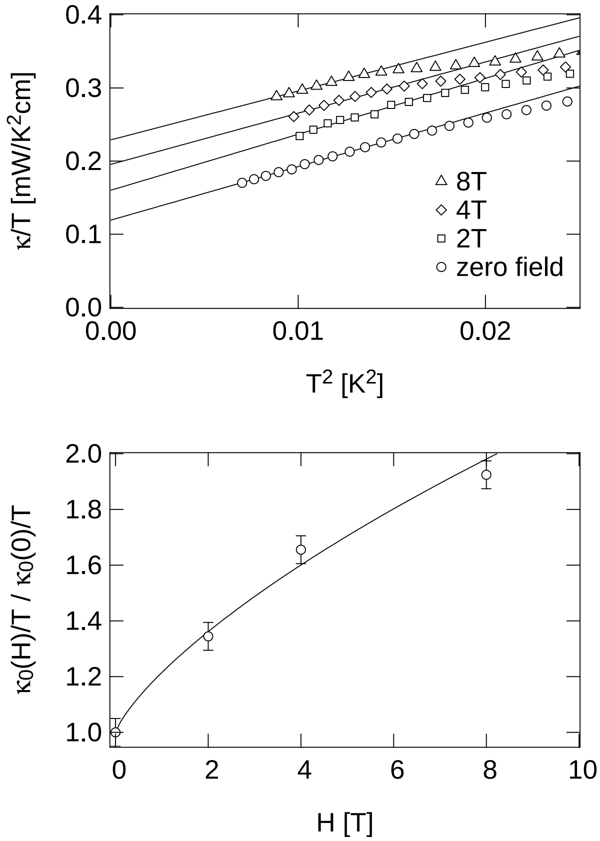

Figure 4: TOP PANEL: Thermal conductivity of $YBa_2Cu_3O_{6.9}$ at very low temperature in a magnetic field along the c-axis, up to 8 Tesla. BOTTOM PANEL: Residual linear term as a function of field. The line is a semi-classical calculation [11]. From [4].

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