

## METHODS

**Nd-LSCO.** Single crystals of  $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$  (Nd-LSCO) were grown at the University of Texas using a travelling float zone technique. *ab*-plane single crystals were cut from boules with nominal Sr concentrations  $x = 0.20$  and  $x = 0.25$ . The actual doping  $p$  of each crystal was estimated from its  $T_c$  and  $\rho(250\text{ K})$  values compared with published data, giving  $p = 0.20 \pm 0.005$  and  $0.24 \pm 0.005$ , respectively.

**Eu-LSCO.** Single crystals of  $\text{La}_{1.8-x}\text{Eu}_{0.2}\text{Sr}_x\text{CuO}_4$  (Eu-LSCO) were grown at the University of Tokyo using a travelling float zone technique, with Sr concentrations  $x = 0.125$  and  $x = 0.16$ . The doping  $p$  is taken to equal the Sr content  $x$ , to within  $\pm 0.005$ . The physical dimensions of the *ab*-plane samples cut out of the single-crystal boules were measured using an optical microscope and are shown in Table 1. The length  $L$  is measured between the contacts used to measure the temperature difference or voltage drop along the current direction ( $x$ -axis).

**Table 1**

| Sample            | Length, $L$ [mm] | Width, $w$ [mm] | Thickness, $t$ [mm] |
|-------------------|------------------|-----------------|---------------------|
| Eu-LSCO $x=0.125$ | $0.94 \pm 0.10$  | $0.28 \pm 0.02$ | $0.19 \pm 0.02$     |
| Eu-LSCO $x=0.16$  | $0.45 \pm 0.10$  | $0.43 \pm 0.02$ | $0.23 \pm 0.02$     |
| Nd-LSCO $x=0.20$  | $1.51 \pm 0.05$  | $0.50 \pm 0.02$ | $0.64 \pm 0.02$     |
| Nd-LSCO $x=0.25$  | $2.50 \pm 0.05$  | $0.51 \pm 0.02$ | $0.51 \pm 0.02$     |

**Superconducting transition temperature  $T_c$ .** The superconducting transition temperature  $T_c$  of our Nd / Eu-LSCO samples was determined via resistivity measurements. In Table 2, we give  $T_c$  values for two different criteria: 1) the temperature where the resistivity goes to zero; 2) the midpoint of the transition.

**Table 2**

| Sample            | $T_c$ [K] ( $\rho = 0$ ) | $T_c$ [K] (midpoint $\rho$ ) |
|-------------------|--------------------------|------------------------------|
| Eu-LSCO $x=0.125$ | $5 \pm 2$                | $8 \pm 4$                    |
| Eu-LSCO $x=0.16$  | $16 \pm 3$               | $24 \pm 5$                   |
| Nd-LSCO $x=0.20$  | $20 \pm 1$               | $23 \pm 3$                   |
| Nd-LSCO $x=0.25$  | $17 \pm 1$               | $20 \pm 3$                   |

**Contacts.** Electrical contacts on the Eu / Nd-LSCO samples were made to the crystal surface using Epo-Tek H20E silver epoxy. This epoxy was cured for 5 min at 180 C then annealed at 500 C in flowing oxygen for 1 hr so that the silver diffused into the surface. This resulted in contact resistances of less than  $0.1 \Omega$  at room temperature. The longitudinal contacts were wrapped around all four sides of the sample. The current contacts covered the end faces. Nernst / Hall contacts were placed opposite each other in the middle of the samples, extending along the length of the  $c$ -axis, on the sides. The uncertainty in the quoted length  $L$  of the sample (between longitudinal contacts) reflects the width of the voltage / temperature contacts along the  $x$ -axis.

**Measurement of the Nernst coefficient.** The Nernst signal was measured by applying a steady heat current through the sample (along the  $x$ -axis). The longitudinal thermal gradient was measured using two uncalibrated Cernox chip thermometers (Lakeshore), referenced to a further calibrated Cernox. The transverse electric field was measured using nanovolt preamplifiers and a nanovoltmeter. The temperature of the experiment was stabilized at each point to within  $\pm 10$  mK. The temperature and voltage were measured with and without applied thermal gradient ( $\Delta T$ ) for calibration. The magnetic field  $B$ , applied along the  $c$ -axis ( $B \parallel z$ ), was then swept, with the heat on, from  $-10$  to  $+10$  T at  $0.35$  T / min, continuously taking data. The thermal gradient was monitored

continuously and remained constant during the course of a sweep. The Nernst coefficient ( $N$ ) was extracted from the part of the measured voltage antisymmetric with respect to magnetic field:

$$N = E_y / (\partial T / \partial x) = [ \Delta V_y(B) / \Delta T_x - \Delta V_y(-B) / \Delta T_x ] (L / 2 w) ,$$

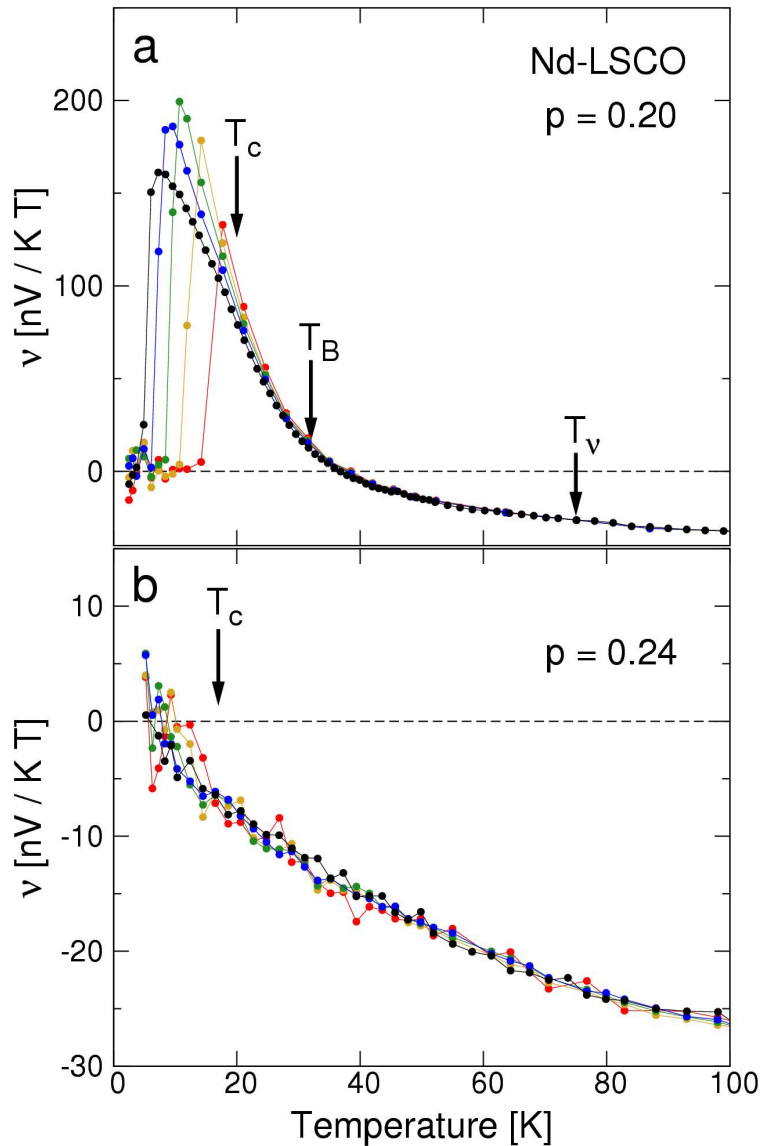
where  $\Delta V$  is the difference in the voltage measured with and without thermal gradient.  $L$  is the length (between contacts along the  $x$ -axis) and  $w$  the width (along the  $y$ -axis) of the sample. This anti-symmetrization procedure removes any thermoelectric contribution from the sample or from the rest of the measurement circuit.

**Extraction of  $T_v$ .** We define  $T_v$  as the point where  $\nu / T$  deviates from linearity at high temperature; see Figures S2 and S5. This criterion is based on the fact that  $\nu / T$  is linear in  $T$  at all  $T$  in Nd-LSCO at  $p = 0.24 > p^*$ , our reference sample where there is neither superconducting contribution to the Nernst signal nor any Fermi-surface reconstruction. This qualitative definition allows us to identify  $T_v$  unambiguously to within  $\pm 10$  K.

**Measurements of resistivity and Hall coefficient.** The resistivity  $\rho(T) \equiv R_{xx} w t / L$  and Hall coefficient  $R_H(T) \equiv R_{xy} t / B$  of each sample were measured using the standard six-terminal AC technique. A resistance bridge or a lock-in amplifier was used to measure the resistance. Field reversal was used to obtain the symmetric and anti-symmetric parts of the voltages, accounting for any misalignment of the contacts. Therefore, the longitudinal ( $R_{xx}$ ) and transverse ( $R_{xy}$ ) resistances were obtained as follows:

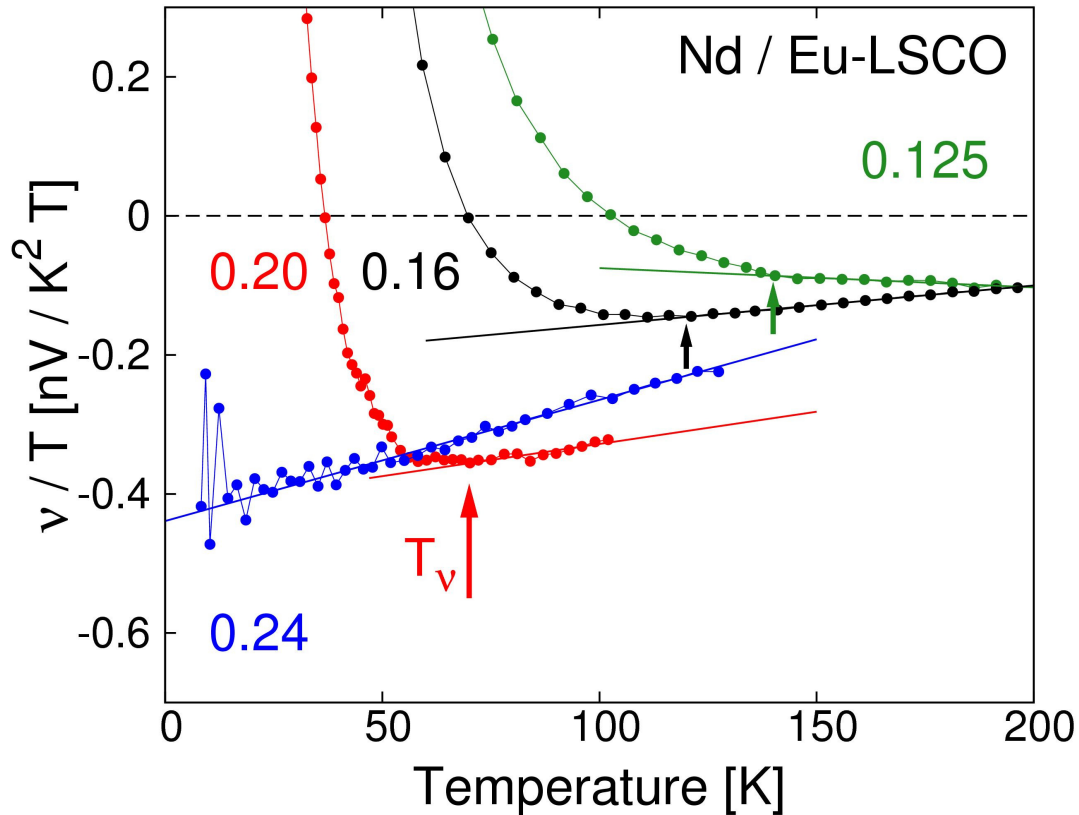
$$R_{xx} = ( R(B) + R(-B) ) / 2 \quad \text{and} \quad R_{xy} = ( R(B) - R(-B) ) / 2.$$

**Measurements of hard X-ray diffraction.** Hard X-ray diffraction measurements were performed with the BL19LXU beamline at RIKEN SPring-8. The photon energy was tuned to 24 keV. Q-scan profiles along the  $h$  direction revealed a broad superstructure reflection at  $(4-2\varepsilon, 0, 0.5)$  with  $2\varepsilon = 0.238(5)$  at low temperatures, indicative of stripe charge ordering. The peak was modelled with a Gaussian, assuming a linear background.



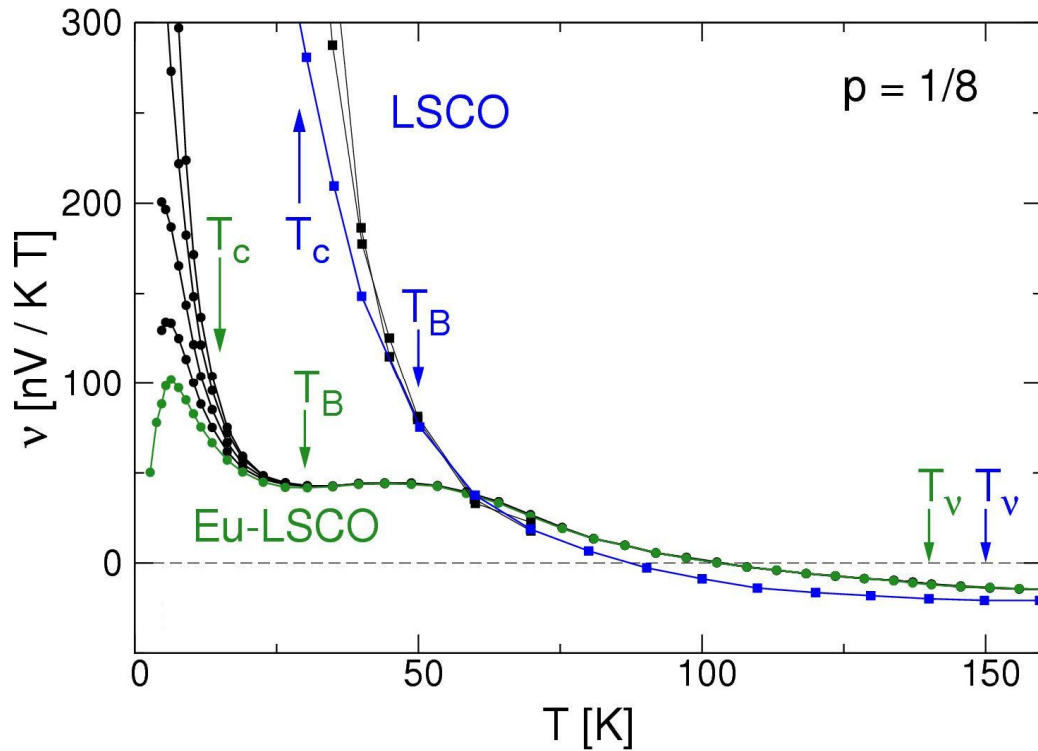
**Figure S1 | Effect of a magnetic field on the Nernst coefficient of Nd-LSCO.**

Nernst coefficient  $v$  as a function of temperature for Nd-LSCO at  $p = 0.20$  (upper panel) and  $p = 0.24$  (lower panel), for different magnetic field strengths:  $B = 2$  T (red), 4 T (yellow), 6 T (green), 8 T (blue), 10 T (black).  $T_c$  is the zero-field superconducting transition (where  $\rho = 0$ ). For  $p = 0.20$ , the onset of field dependence is labelled  $T_B$ . At higher temperature,  $v / T$  becomes linear in temperature above  $T_v$  (see Fig. S2). By contrast, for  $p = 0.24$ , the field dependence is within the noise of the measurement down to  $T_c$  and both  $T_B$  and  $T_v$  are indistinguishable from zero.



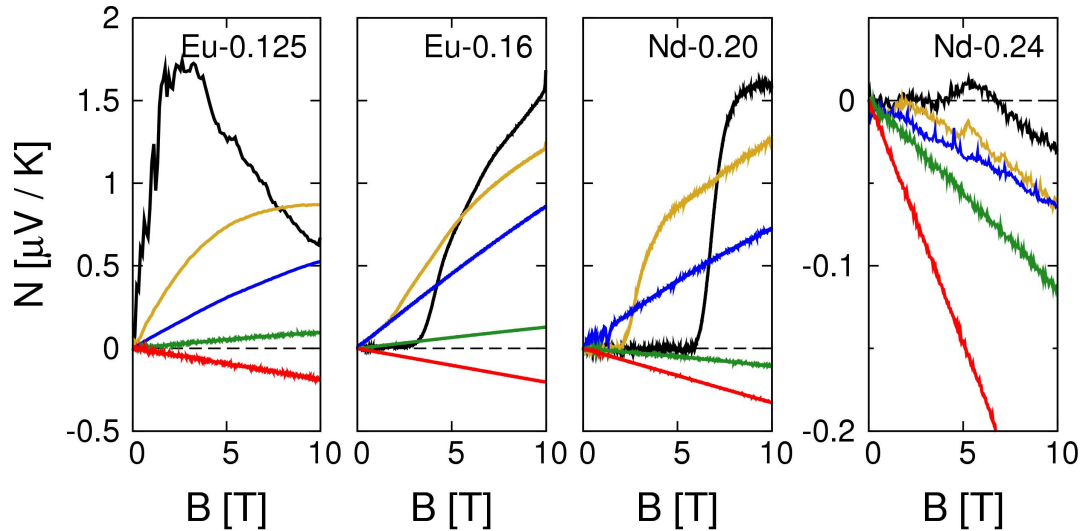
**Figure S2 | Onset of the positive upturn in the Nernst coefficient.**

Nernst coefficient  $v$  divided by temperature  $T$  for Eu-LSCO at  $p = 0.125$  (green) and  $p = 0.16$  (black), and for Nd-LSCO at  $p = 0.20$  (red) and  $p = 0.24$  (blue). All curves are taken in  $10 T$ . The onset temperature  $T_v$  (arrows) is defined as the deviation of  $v / T$  from a linear fit at high temperature. This yields  $T_v = 140 \pm 10$ ,  $120 \pm 10$ ,  $70 \pm 10$  and  $0$  K, respectively.



**Figure S3 | Comparison of Nernst coefficient in Eu-LSCO vs LSCO.**

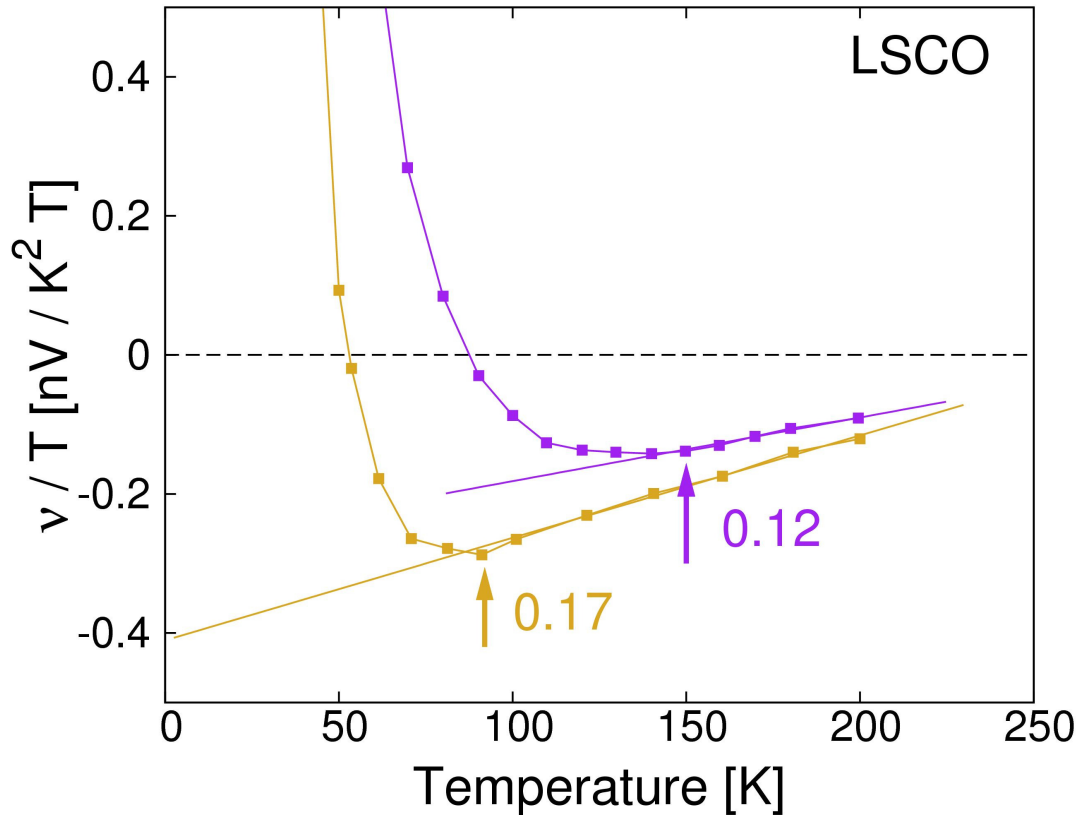
Temperature dependence of the Nernst coefficient  $\nu(T)$  for different magnetic fields in Eu-LSCO at  $p = 0.125$  [circles] and LSCO at  $p = 0.12$  [squares; from ref. 1]. Field strengths are 2, 4, 6, 8 and 10 T for Eu-LSCO (top to bottom), and 1, 6 and 14 T for LSCO (top to bottom).  $T_v$  marks the onset of the positive rise at high temperature, as defined in Supplementary Figs. S2 and S5.  $T_B$  marks the onset of a field dependence in  $\nu(T)$ , the expected signature of superconducting fluctuations.  $T_c$  marks the onset of the superconducting transition in the zero-field resistivity. Note how  $\nu(T)$  in Eu-LSCO exhibits two separate peaks, at 7 K and 45 K, which we attribute respectively to superconducting fluctuations (characterized by a strong field dependence) and quasiparticles (no field dependence), with respective onsets at  $T_B$  and  $T_v$ . In LSCO at the same doping, the rise in  $\nu(T)$  at high temperature is very similar, but the low-temperature field-dependent rise has moved up in temperature, with  $T_B$  tracking  $T_c$ .



**Figure S4 | Magnetic field dependence of the Nernst signal.**

Field dependence of the Nernst signal in the four samples of Eu / Nd-LSCO measured in this study, at several temperatures: above  $T_v$  [red]; between  $T_v$  and  $T_B$  (the onset of non linearity in  $N$  vs  $B$ ) [green]; between  $T_B$  and the midpoint of the zero-field superconducting transition,  $T_c$  [blue]; below  $T_c$  [yellow and black]. The temperature of each curve is, respectively : 184, 83.6, 17.6, 9.7, 3.4 K ( $p = 0.125$ ); 196, 59.2, 32.0, 18.6, 7.2 K ( $p = 0.16$ ); 106, 45.4, 21.1, 14.2, 8.4 K ( $p = 0.20$ ); 132, 28.9, 16.5, 12.4, 8.3 K ( $p = 0.24$ ).





**Figure S5 | Onset of the positive upturn in the Nernst coefficient in LSCO.**

Nernst coefficient  $v$  divided by temperature  $T$  for LSCO at  $p = 0.12$  (purple) and  $p = 0.17$  (yellow) (from refs. 1, 2, and 3). Both curves show the zero field limit; there is no evidence of field dependence above 60 K. The onset temperature  $T_v$  is defined as the deviation of  $v/T$  from a linear fit at high temperature, giving  $T_v = 150 \pm 10$  and  $90 \pm 10$  K, respectively.

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<sup>1</sup> Wang, Y., Li, P. & Ong, N.P. *Phys. Rev. B* **73**, 024510 (2006).

<sup>2</sup> Ong, N.P. *et al.*, *Ann. Phys. (Leipzig)* **13**, 9-14 (2004).

<sup>3</sup> Wang, Y. *et al.*, *Phys. Rev. Lett.* **88**, 257003 (2002).