SUPERCONDUCTIVITY AND QUANTUM CRITICALITY

BY LOUIS TAILLEFER

The current era of research on superconductivity might be said to have started in 1986, with the discovery of superconductivity in the cuprates, oxides made of stacks of CuO_2 planes, in which the critical temperature T_c can be as high as 164 K – seven times higher than the previous record and halfway to room temperature. The discovery sparked huge interest both for reasons of technology – a room-temperature superconductor would be a revolution – and reasons of fundamental science – what new microscopic mechanisms might be at play? It is widely believed that the force – or glue – that binds electrons into Cooper pairs is not the usual attraction mediated by the ionic lattice and its vibrations. The force would come from within the electron system itself.

However, it may be useful to view this major event not in isolation but as a landmark in an era that actually started earlier, with the discovery of heavy-fermion and organic superconductors in 1979. Although in both families of materials – the former, *f*-electron metals, and the latter, stacks of organic molecules – T_c is rarely higher than 10 K, the case for a purely electronic pairing is just as strong as in the cuprates. In fact, here there is little doubt that the glue for pairing is magnetic in origin. The basic idea is that the pairing interaction which relies on the polarizability of the surrounding ionic lattice in conventional superconductors (like mercury, aluminium or lead) would now rely instead on the magnetic susceptibility of the surrounding electrons[1]. And this susceptibility is enhanced near the quantum critical point (QCP) for spontaneous magnetic order. (A QCP is a quantum phase transition, a phase transition at $T = 0$ induced by tuning some parameter like pressure, composition, doping or magnetic field.) This makes quantum criticality a fertile ground for the emergence of novel states of matter quite generally, in particular unconventional superconductivity[2].

SUMMARY

In several families of materials, including the recently discovered iron pnictides, superconductivity is found near the quantum critical point where a magnetic phase ends, pointing to a magnetic glue as the source of electron pairing, distinct from the usual phononic glue. In the copper oxides where superconductivity persists to the highest temperatures, the nature and role of an elusive quantum critical point are the subject of much debate. Is the glue again magnetic? If so, why is it so strong in this case?

Because of its relative simplicity – a single-band quasi-one-dimensional metal, with neither Kondo effect nor

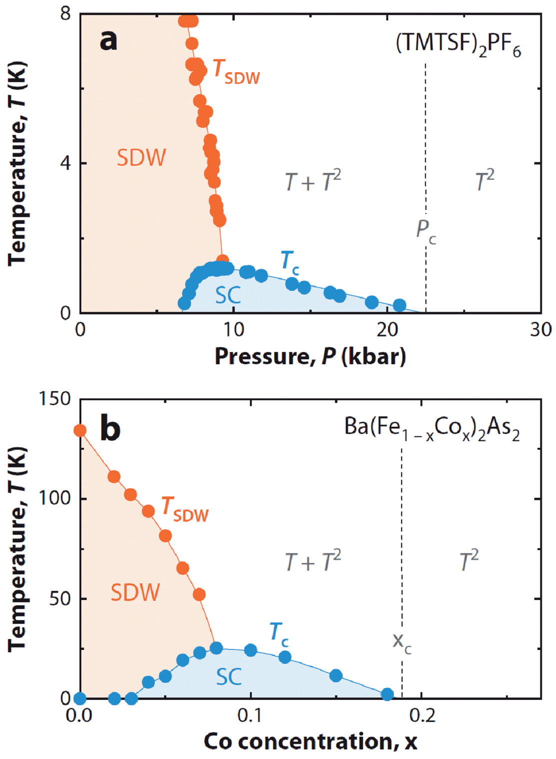
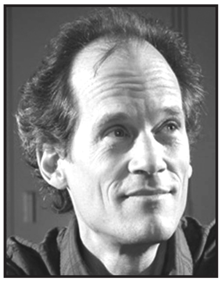


Fig. 1 **Phase diagram of organic and pnictide superconductors.** a) Temperature-pressure phase diagram of $(TMTSF)_2PF_6$, showing a spin-density-wave (SDW) phase below T_{SDW} (orange dots) and superconductivity (SC) below T_c (blue dots). The latter phase ends at the critical pressure P_c. b) Temperature-doping phase diagram of the iron-pnictide superconductor $Ba(Fe_{1-x}Co_x)_2As_2$, as a function of nominal Co concentration x, showing a metallic SDW phase below T_{SDW} and superconductivity below a T_c which ends at the critical doping x_c. In both panels the vertical dashed line separates a regime where the resistivity $\rho(T)$ grows as T^2 (on the right) from a regime where it grows as $T + T^2$ (on the left). From [4].



L. Taillefer
<Louis.Taillefer@USherbrooke.ca>,
Director, Quantum Materials Program, Canadian Institute for Advanced Research,
Toronto, ON,
M5G 1Z8;
Regroupement Québécois sur les Matériaux de Pointe, Département de physique, Université de Sherbrooke,
Sherbrooke, QC,
J1K 2R1

Mott physics – the Bechgaard salt $(TMTSF)_2PF_6$ may be considered the archetype of magnetic pairing [3]. Its pressure-temperature phase diagram is shown in Figure 1, where the superconducting phase is seen to reside on the boundary of an antiferromagnetic (or spin-density-wave; SDW) phase. Renormalisation group calculations reproduce this phase diagram, as well as the detailed evolution of the antiferromagnetic spin fluctuations with pressure and temperature. The calculations show that those fluctuations are the cause of superconductivity [3].

Each of the three phases in the phase diagram (Figure 1) has some key characteristic signatures. 1) Because interactions from antiferromagnetic correlations are intrinsically anisotropic, the resulting superconducting state is also anisotropic. In the Bechgaard salts, calculations predict a *d*-wave state, with a gap that changes sign along the Fermi surface [3] – in contrast with conventional superconductors where the *s*-wave gap is isotropic. 2) Because it introduces a new periodicity, thereby breaking the translational symmetry of the crystal lattice, the SDW phase causes a reconstruction of the Fermi surface. This leads to a sharp drop in carrier density and a concomitant drop in conductivity, or rise in resistivity, as shown in Figure 2. 3) Near the QCP, the spin fluctuations that cause pairing below T_c also cause scattering above T_c. This scattering leads to deviations from the standard Fermi-liquid behaviour of metals, and its typical signature is a linear (as opposed to quadratic) temperature dependence of the resistivity as $T \to 0$. A linear resistivity was recently observed in $(TMTSF)_2PF_6$ at the QCP (see Figure 2) and an interesting empirical correlation was discovered [3,4]: the pressure needed to restore the Fermi-liquid behaviour (by moving away from the QCP) is the same pressure that is needed to suppress superconductivity. In other words, non-Fermi-liquid resistivity and *d*-wave superconductivity are intimately linked: superconductivity and quantum criticality coexist.

The magnetic pairing mechanism, which only causes a T_c of order 1 K in the Bechgaard salts (and not much higher in most heavy-fermion metals), acquired new significance with the discovery of iron pnictides, whose T_c can be as high as 55 K [5]. The phase diagram of pnictides is strikingly similar to that of organics – see Figure 1 – except that temperature scales (magnetic and superconducting) are now up to two orders of magnitude higher. And here again there is evidence of linear resistivity at the antiferromagnetic QCP (see Figure 2) and of the same empirical correlation between scattering and pairing [4]. An interesting difference is the multi-band Fermi surface, which offers the possibility of inter-band scattering (not present in the Bechgaard salts), believed to play a key role in the pairing, as the SDW wavevector now connects different sheets of the Fermi surface. That could offer more flexibility in the design and search for a stronger interaction and a higher T_c.

This brings us to the cuprates: in what ways are they similar to pnictide and organic superconductors? What are the important differences? Is quantum criticality relevant? Is the pairing glue magnetic? The doping-temperature phase diagram (Figure 3) reveals some of the differences: 1) the antiferromagnetic phase at low doping is an insulator, rather than a metal; 2) the QCP where it ends lies either before or at the onset of superconductivity, rather than at optimal doping (where T_c is maximal); 3) there is an additional region called the "pseudogap phase" which sets in below a crossover temperature T^*, above T_c in the underdoped regime [6]. There is no consensus on the nature of the pseudogap phase. There are

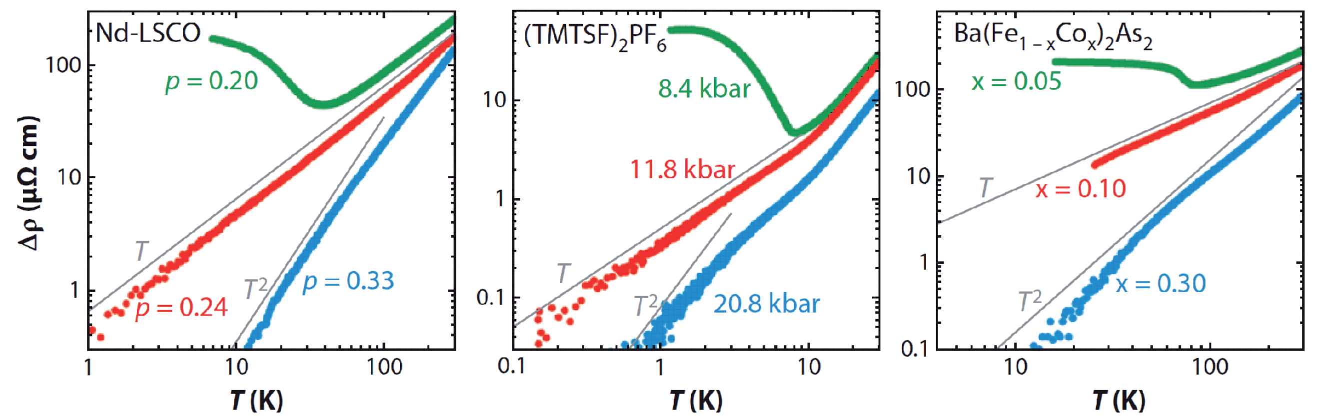


Fig. 2 **Quantum criticality in the resistivity of cuprate, organic and pnictide superconductors.** Temperature dependent part of the in-plane normal-state resistivity of materials in three families of superconductors, plotted as $\rho(T) - \rho_0$ vs T on a log-log scale. Three values of the relevant tuning parameter were chosen: below, at and above their respective quantum critical points (QCPs). Left panel: data on hole-doped cuprates Nd-LSCO at $p = 0.20$ and $p = 0.24$ and LSCO at $p = 0.33$; the QCP at a hole doping $p^* \approx 0.24$ marks the end of the stripe-ordered phase in Nd-LSCO. Middle panel: data on the organic Bechgaard salt $(TMTSF)_2PF_6$; the QCP at a pressure $P^* \approx 10$ kbar marks the end of the SDW phase. Right panel: data on the pnictide $Ba(Fe_{1-x}Co_x)_2As_2$; the QCP at a Co concentration $x^* \approx 0.10$ marks the end of the SDW phase. Note in all cases: a linear dependence as $T \to 0$ at the QCP; a Fermi-liquid T^2 dependence above the QCP (beyond the superconducting phase); an upturn caused by Fermi-surface reconstruction upon entry into the ordered phase below the QCP. From [4].

basically three viewpoints: 1) it is a real phase, with broken symmetry, and a number of order parameters have been proposed (*e.g.* d-density-wave, nematic, circulating currents); 2) it is a precursor to some ordered phase which sets in at lower temperature, or which would set in if superconductivity didn't intervene; 3) it is a precursor to superconductivity, where Cooper pairs are pre-formed without long-range coherence. Having a wide precursor region of fluctuating order is compatible with the strong 2D character of cuprates. Scenarios 1) and 2), but not 3), require the existence of a QCP at $T = 0$, located at some critical doping p^* (in the absence of superconductivity) below which a symmetry is broken.

There is one cuprate material where the picture is fairly clear: $La_{1.6-x}Sr_xNd_{0.4}CuO_4$ (Nd-LSCO)[4]. In the absence of superconductivity, suppressed by application of a magnetic field, there is a QCP at $p^* = 0.24$, below which the Fermi surface undergoes a reconstruction, as revealed by a pronounced rise in the resistivity (see Figure 2). This QCP is where "stripe order" ends. This is a unidirectional modulation of both the spin and charge densities, which break the rotational and translational symmetries of the lattice. So this is a kind of SDW order, with an associated charge-density-wave (CDW). The standard signature of quantum criticality is observed: the resistivity is perfectly linear as $T \to 0$. At high doping, far away from the Mott insulator, it is legitimate to compare this cuprate to the Bechgaard salt $(TMTSF)_2PF_6$, even though its T_c is 20 times higher: both are good single-band metals with strong 2D character (at low temperature), both are *d*-wave superconductors, both have a QCP at which SDW order ends, and both show a linear resistivity at that QCP. It therefore seems reasonable to invoke a similar pairing mechanism for the cuprate, associated with antiferromagnetic (SDW) spin fluctuations.

Two questions arise: 1) is Nd-LSCO representative of other cuprates, in particular those with a higher T_c? 2) What is the relation between pseudogap phase and stripe order? Let us take these in turn. There is growing evidence that the basic mechanisms at play in Nd-LSCO are also active in $YBa_2Cu_3O_y$ (YBCO), one of the archetypal cuprate superconductors, with a maximal T_c = 94 K. Since the discovery of quantum oscillations in 2007[7], it is clear that YBCO also undergoes a Fermi-surface reconstruction, so that it too must have a QCP somewhere in the phase diagram. Recent studies show this reconstruction to be very similar to that of Nd-LSCO[4], such that stripe order is also the likely cause of broken translational symmetry in YBCO. Now all cuprates, including YBCO, show a linear resistivity near optimal doping, of universal slope. This means that the same scattering is responsible for the linear resistivity in YBCO and in Nd-LSCO, the latter being associated with the stripe QCP. Moreover, the onset of non-Fermi-liquid behaviour in overdoped cuprates is known to set in precisely at the onset of superconductivity, so as in the organic $(TMTSF)_2PF_6$ and the pnictide $Ba(Fe_{1-x}Co_x)_2As_2$ (Figure 1), scattering and pairing are again intimately linked[4].

If stripe order sets in at low temperature, one expects, in a strongly 2D metal, a precursor regime of stripe fluctuations at temperatures above that onset[8], and this could be the pseudogap phase going up to T^* – an example of scenario 2) above. Supporting evidence for this came recently with the discovery that rotational symmetry in YBCO is broken at T^*[9], consistent with the onset of stripe correlations, well before the onset of stripe order at $T \sim T^*/2$. Interestingly, this same sequence of broken symmetries upon cooling – first rotational, then translational – is also found in the iron pnictides[10], associated with their stripe-like SDW order. The wider regime of "nematic order" (as a state of broken rotational symmetry may be called[11]) seen in the cuprates may come from their stronger 2D character.

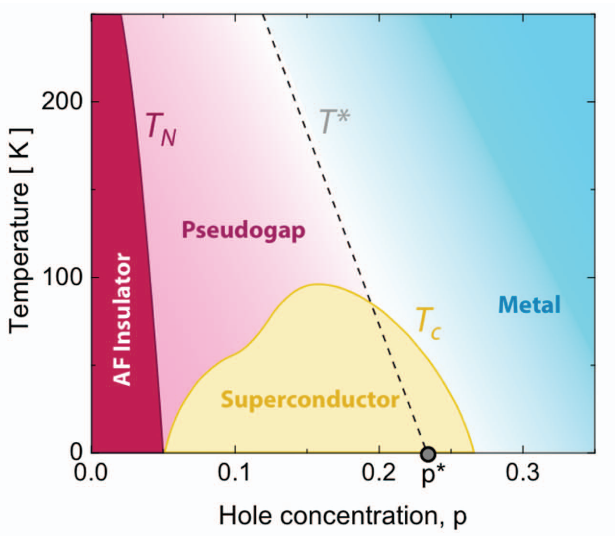


Fig. 3 **Phase diagram of cuprate superconductors.** Schematic phase diagram of cuprate superconductors as a function of hole concentration (doping) p. The Mott insulator at $p = 0$ shows antiferromagnetic (AF) order below T_N, which vanishes rapidly with doping. At high doping, the metallic state shows all the signs of a conventional Fermi liquid. At the critical doping p_c, two events happen simultaneously: superconductivity appears (below a critical temperature T_c) and the resistivity deviates from its Fermi-liquid behaviour, acquiring a linear temperature dependence. The simultaneous onset of T_c and linear resistivity reveals a link between pairing and scattering, between superconductivity and quantum criticality. The evolution from metal to insulator is interrupted by the onset of the "pseudogap phase" which sets in below a crossover temperature T^*, which goes to zero at a quantum critical point (QCP) located at p^* in the absence of superconductivity (removed for example by application of a large magnetic field). The existence, nature and location of such a QCP are a major focus of current research and debate. From [4].

In summary, if we start from the metallic state on the far right of the phase diagrams (Figures 1 and 3), sit at low temperature, and tune to the left, we find that in all three families of materials – organics, pnictides and cuprates – superconductivity appears precisely where quantum criticality is first being felt. T_c then increases steadily as the QCP is approached. So proximity to SDW or stripe order is good for superconductivity, the glue to be found in the fluctuations of those orders. If we go further, past the QCP, and enter the SDW or stripe phase then T_c drops. So coexistence with SDW or stripe order is bad for superconductivity: the two phases compete. This suggests two guiding principles for a high T_c: 1) the magnetic energy scale should be high; 2) magnetic fluctuations should be strong, but magnetic order weak. The first principle is illustrated in Figure 1: even though T_c in the pnictide is 20 times higher than in the organic salt, the ratio of maximal T_{SDW} to maximal T_c is nevertheless comparable (6 in the former, 10 in the latter). In the cuprate Nd-LSCO, the maximal onset temperature for stripe order is 4 times larger (and the maximal T^* 10 times larger) than maximal T_c, again comparable. The second principle is illustrated by comparing Nd-LSCO and YBCO. They have comparable T^* values (only 1.5 times larger in the latter [4]), but the maximal T_c is 5 times lower in Nd-LSCO, a material whose crystal structure stabilizes stripe order, to the detriment of superconductivity. Our second principle argues for strong 2D character, which favours fluctuations over order. It also argues for frustration of the underlying magnetic state, and hole-doped cuprates may be an example of this, with the commensurate antiferromagnetic phase at low doping becoming unstable with respect to the incommensurate stripe phase (with spin and charge order) at intermediate doping. This instability in the magnetic ordering is absent in the electron-doped cuprates [12], where the commensurate antiferromagnetic phase persists up to higher doping, without any sign of incommensurate order or charge order. This might explain why T_c is lower in the electron-doped cuprates.

In the last 3-4 years, the current era of superconductivity research has come to a point of convergence: in a wide range of materials, superconductivity is found to live in the wake of quantum criticality. A major organizing principle is emerging. The task is now to elucidate its workings. And to find out how the quantum-critical environment can be optimized to enhance T_c. An intriguing avenue is the role of a nematic tendency, found in both pnictides and hole-doped cuprates, the two families with the highest T_c. Is the 1D character of spin fluctuations a good thing? What about a tendency towards charge-density-wave order – is that a booster for superconductivity? The second century is off to an exciting start.

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