A brief history

• What happens to Ohm’s law at $T=0$?

• Electrons stop (Kelvin)
• They travel without resistance (Dewar)

• Kamerlingh Onnes in 1911 has just liquified Helium so that he can reach $T = 4K$
Perfect conductivity

Kamerlingh Onnes 1911

Temperature (°K)
$\chi = -1$

Perfect diamagnetism (Shielding of magnetic field)

(Meissner effect)
«No harm was done to animals during this experiment»
« Experimentalists could have been harmed if this experiment had failed. »
For a material to be superconducting, it must possess the following *two* properties:

1. Zero resistance
2. Perfect diamagnetism
The old

• Looking for an explanation: a series of failures and a triumph

- During 46 years, from 1911 to 1957, superconductivity remains unexplained.

- In 1950 it is the most important problem in theoretical Physics.

- Richard Feynman: « No one is smart enough to explain it »
Feynman

Heisenberg

Bohr

Einstein

Fail : F

Failure: Neils Bohr (1885-1962) introduced the idea that the electron moved about the nucleus in well-defined orbits. This photograph was made in 1922, nine years after the publication of his paper.
The old

- BCS theory (1957)

Quantum behavior at the macroscopic scale

Leon Cooper

Nobel Prize : 1972

John Bardeen*

Robert Schrieffer

- John Bardeen :
- The only person to receive two Nobel prizes in Physics !!!
Invention: TRANSISTOR!

W. Shockley, J. Bardeen, W.H. Brattain

Marie Curie:
1903 Physics with H.A. Becquerel
1911 Chemistry (alone)
Resistance of a normal metal
Ingredient #1
Attraction and formation of Cooper pairs
Cooper pairs
or
Prelude to supraconductivity
Cooper form "bosons" that all "condense" in the same quantum state. Their wave functions all have the same quantum phase. Electrons are in a single coherent state (Like in a big atom!).

A spectacular application of coherence:

Josephson effect

\[ I = I_0 \sin (2\pi \phi) \]
\[ \frac{d\phi}{dt} = \frac{2eV}{h} \]

N.B. - Factor 2

- Standard for the Volt
**Detection of weak magnetic fields**

A SQUID (Superconducting Quantum Interference Device) is the most sensitive type of detector known to science. Consisting of a superconducting loop with two Josephson junctions, SQUIDs are used to measure magnetic fields.

**SQUID "Superconducting Quantum Interference Device"**

A magnetic field modifies the phase of matter waves. By detecting current oscillations, one can detect very weak magnetic fields.
Detection of weak magnetic fields: Scanning SQUID microscope
Scanning SQUID microscope
Transmission lines

BiSrCaCuO

Ag
MAGLEV
Magnetic resonance imaging (MRI)

Research:
1) Higher fields
2) New detectors (SQUID)
By the end of 1984 the three manufacturing plants had between them produced more than 400 magnets for clinical MR imaging and spectroscopy applications at permanent hospital sites, many times more than the total of all other producers worldwide. Oxford is far ahead as the only manufacturer to have succeeded in making superconducting magnets in any quantity and this in-depth experience helps to maintain the three companies' world market share at an extremely high level.

The customers for Oxford's MR imaging magnets include all the world's major manufacturers involved in the development of whole-body diagnostic scanners. They are international companies who are the leaders in medical
“Psychological barrier” : 77K

Liquid Nitrogen
(The most abundant molecule in air!)

![Graph showing transition temperatures for various materials, with MgB$_2$ highlighted.](image-url)
The old

- Vanishing act

- As early as 1970 many researchers had left the field

- Superconductivity is one of the best understood phenomena in all of Physics!

- In 1969, R.D. Parks edits a two volume treatise entitled "Superconductivity"

- One of the authors writing about the book: «It is the last nail in the coffin of Superconductivity"
January 1986:

Alex Müller and Georg Bednorz at IBM Zurich discover indications of superconductivity in the system $Ba-La-Cu-O$

September 1986 issue of de "Zeitschrift für Physik"
-(submitted April 17, 86)

"Possibility of high $T_c$ superconductivity in the $Ba-La-Cu-O$ system"

- This article is ignored... for good reasons:
  - transition towards $R=0$ is not sharp
  - The Meissner effect is not verified
1986: Discovery of superconductivity in the La-Ba-Cu-O system

$T_c \sim 30-40K$

P. Chu’s group (Houston)

Under high pressure: 50K!!!
- Boston, "Materials Research Society"
  December 1986
  Présentation of Koitchi Kitazawa and Shoji Tanaka from Tokyo convinces everyone.

  Madness! These materials are easy to make. In China, India, everywhere everyone is trying.

  Loss of sleep. The Nitrogen barrier has become much closer.

- 16 February 1987, Houston:
  Paul Chu and his group call a press conference to announce the discovery of $\text{Y-Ba-Cu-O}$

    \[ T_c = 93 \, \text{K} \]

  The Nitrogen barrier has been crossed?
1987: $\text{YBa}_2\text{Cu}_3\text{O}_7 \quad 93\text{K}$
--- $\text{LN}_2 = 77\text{K}$ ---
Paul Chu: U. of Houston

![Graph showing temperature vs. resistivity for different compounds, including $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$.](image)
New York: March meeting of the American Physical Society.

- President Neil Ashcroft, (Cornell) hesitates

18 March 1987...

- New York Times title the next day: "The Woodstock of Physics"

- 3000 people until 3 AM

"They began lining up outside the New York Hilton Sutton Ballroom at 5:30PM for an evening session that would last until 3:00 AM"
The "Woodstock of physics." On March 18, 1987, thousands of physicists crammed a ballroom at the New York Hilton to celebrate the coming of the age of superconductivity.

AMERICAN INSTITUTE OF PHYSICS

(right) Alex Müller, Paul Chu, and Shoji Tanaka, answering questions at the "Woodstock" meeting. Tanaka and Koichi Kitazawa were the first to confirm Bednorz and Müller's discovery, launching a worldwide race to find still better superconductors.

AMERICAN INSTITUTE OF PHYSICS
Understanding the mechanism --- Where are we?

Type $s$ ($n = 1$)

Type $p$ ($n = 2$)

Type $d$ ($n = 3$)

n=3
n=2
n=1
Understanding the mechanism --- Where are we?

d-wave (n = 3)

n=1
n=2
n=3
FIG. 1. The structures of superconductors (A) Copper oxide plane, (B) copper oxide ladder, and (C) $C_{60}$ molecule. Ladders of copper and oxygen atoms, as shown in (B), form spontaneously in some compounds.

FIG. 2. Head-to-head race, $T_c$ versus year of discovery for some superconducting materials. Orange, representative low-$T_c$ compounds, which held the $T_c$ record before cuprates and fullerenes were discovered. Light blue, representative planar cuprates. Magenta, representative fullerenes, with the highest $T_c$ to date reported in [4].

E. Dagotto, cond-mat/0110190
A more microscopic view
FIG. 1 Phase diagram of $n$ and $p$-type superconductors.

Optimal doping

$n$, electron density

The normal state \((T > T_c)\) of high temperature superconductors cannot be explained in the context of the band theory of metals or any of its extensions.

**Two great mysteries:**

1. The normal state (pseudogap).

2. The origin of the attraction leading to superconductivity (magnetic instead of phononic?)
Pseudogap
  • New Ising character phase?
    • RVB
    • Preformed pairs
    • Flux phase
    • D Density Wave
  • Fluctuations?
    • SC, AFM, singlet...

\[ d = 3 \ Néel \ T \]

« Mott » Physics explains decreasing \( T_c \)
(Small \( \rho_s \): Phase fluctuations)

Quantum critical points
Magnetic mechanism

t = 0

t = T
I. Standard paradigm

II. Why is there a problem with standard approaches?

III. Microscopic model

IV. Theoretical difficulties
   (a) Straight numerical approaches
   (b) Inadequacies of mean-field theory in low dimension
   (c) Approaching from weak coupling
   (d) Approaching from strong coupling
   (e) Phonons
   (f) Inhomogeneities

V. Conclusion
I. Standard Paradigm

Theory of solids \( H = \text{Kinetic} + \text{Coulomb} \)

- Many new ideas and concepts needed for progress
  (Born-Oppenheimer, H-F, Bands...)

- Successful program
  - Semiconductors, metals \textit{and superconductors}
  - Magnets

- Is there anything left to do?
  - Unexplained materials: High Tc, Organics, heavy fermions...
  - Strong correlations:
    strong interactions, low dimension
The standard approaches:

A. Quasiparticles, Fermi surface and Fermi liquids
   - LDA (Nobel prize 1998)

La$_2$CuO$_4$

The standard approaches:

- Matrix elements of $H$ in LDA basis
  - In practice, from general considerations:
    - Short range
    - Single Slater determinant not eigenstate
  - Phase space + Pauli restricts possible scatterings:
    - Quasiparticles $m^*$, effective fields,

- «See» the quasiparticles with ARPES:

\[
\frac{k^2}{2m} = E_{ph} + \omega + \mu - W
\]
$$\frac{k^2}{2m} = E_{ph} + \omega + \mu - W$$
FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cut-off. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the binding energy (a), temperature (b), and hydrogen exposure (c) is shown.

FIG. 1. ARPES intensity plot of the Mo(110) surface recorded along the $\overline{\Gamma} - \overline{N}$ line of the SBZ at 70 K. Shown in the inset is the spectrum of the region around $k_F$ taken with special attention to the surface cleanliness.

T. Valla, A. V. Fedorov, P. D. Johnson, and S. L. Hulbert
The standard approaches:

B. Thermodynamics and phase transitions

- Thermodynamics of Fermi liquids
  - particle-hole excitations
- Phase transitions
  \[ \chi \sim \frac{N(0)}{1 + F_0 a} \]
- Superconducting transition

C. Heisenberg model and related models

- Band for s-p
- Localized (often) for d-f
  Only spin degrees of freedom
  Use symmetry to write \( H \)
II. Failure of standard paradigm

\( n = 1, \)

Metal according to band
AFM insulator in reality
Optimally doped BISCCO

III. Establishing a microscopic model

$YBa_2Cu_3O_{7-\delta}$
Atomic Physics $CuO_2$ plane.
- $Cu$ is $3d^{10}4s^1$.
- In high $T_c$, $Cu^{++}$
  - $3d_{x^2-y^2}$ has one hole
  - Four other levels are filled

Zhang-Rice singlet

Charge transfer insulator

Insulator

$Cu$

$O$

Hole doped

Electron doped

$\mu$
The « Hubbard model »

Simplest microscopic model for Cu O planes.

- Screened interaction $U$
- $U, T, n$
- $a = 1, t = 1, h = 1$

• Potential problems
  • Poor screening
  • Phonons

$$H = - \sum_{<ij>\sigma} t_{i,j} \left( c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) + U \sum_i n_i \uparrow n_i \downarrow$$
\[ A(k, \omega) = \frac{-U}{2} \]
\[ \omega \]
\[ t = 0 \]
\[ A(k, \omega) \]

\[ U = 0 \]
\[ \mu \]
\[ +U/2 \]
\[ -U/2 \]

\[ t = 0 \]
\[ A(k, \omega) \]

\[ \omega \]
\[ k \]
\[ -\pi/a \]
\[ \pi/a \]

\[ -U/2 \]
\[ U/2 \]

\[ \omega \]
\[ k \]
\[ -\pi/a \]
\[ \pi/a \]
Weak vs strong coupling, $n=1$

$A(k_F, \omega)$

$U \sim 1.5W$  ($W = 8t$)

Mott transition

Effective model, Heisenberg: $J = 4t^2 / U$
\textit{t-J}

\begin{itemize}
  \item Number of states: $3^N$
\end{itemize}
Upper and lower Hubbard band

Strong on-site repulsion (Mott transition)

Insulator

Doped system: Doping $\delta$

- UHB loses $\delta$ states
- LHB gains $2\delta$ states

Observable experimentally (XPS)
IV. Theoretical difficulties

(a) Straight numerical approaches

- $2^{30} \sim 1$ GB. Compare with $4^{16} = 2^{32}$ for 16 site lattice!

- Exact diagonalizations of $t$-$J$ for one hole suggest:
  - One peak in $A(k, \omega)$ plus incoherent background
  - Peak disperses with width of $J$. Thus
    - Number of carriers small.
    - Effective mass finite.

- Cluster Perturbation Theory (New method)
  • AFM at $n = 1$. Away from half-filling problems with BC
  • Pseudogap in weak to intermediate coupling
  • Temperature not low enough to establish $d$-wave superconductivity
  • Useful as a benchmark for analytical approaches.

• Main drawback: « fermion sign problem » and instabilities

• Density Matrix Renormalization Group
  • People are working on $d = 2$ generalizations.
(b) Inadequacy of mean field in low dimension

• Fluctuations dominate the physics

  • $d = 1$  Spin-Charge separation (*Luttinger* liquid Behavior) (Quantum fluctuations at $T = 0$)

  • $d = 2$  Thermal fluctuations:
    • *Hohenberg-Coleman-Mermin-Wagner* theorem.

  And the strong interactions complicate all that.
C. Bourbonnais et al. (cond-mat/9903101).

- \( d=1 \) spin-charge separation
• Straight perturbation theory for nested \( d=2 \) Fermi surface gives non-trivial results (F. Lemay PhD thesis, 2000)

• Thermal and quantum fluctuations in \( d = 2 \)

\[
(\nabla \theta)^2 \rightarrow q^2 \theta_q \theta_{-q}
\]

\[
\langle \theta^2 \rangle \propto \int d^2 q \frac{kT}{q^2} \rightarrow \infty
\]

\( d = 1 \) : R.G., Bosonization, Conformal Field Theory...

\( d = 2 \) : Slave bosons, R.G., strong coupling p.t., TPSC
(c) Approaching from weak coupling

- Perturbation theory (Wick’s theorem)
- Finding « effective interactions »
  - RPA with long range interaction vs Hubbard
- Notion of « channel »
- RG and « interference »

- Problem with Pauli principle (Parquet, Bickers)
- Self-consistent treatments
- Migdal’s theorem vs theory of « ordinary » superconductors

- Alternative: satisfy sum-rules
  - Conservation laws
  - Pauli principle
  - Mermin-Wagner theorem

Fluctuation-induced pseudogap?
• Perturbation theory:
\[
\frac{1}{(X + Y)} = \frac{1}{X} - \frac{1}{X} \frac{Y}{X + Y}.
\]

• Effective interaction (Screening in usual solids)

• Problem with « short-range »
• Notion of « channel »

- Particle – particle channel: $k + k'$

- «Peierls» p-h channel: $k - k' + q$

- «Landau» p-h channel: $q$
• Renormalization group : « interference »


• Problem with Pauli principle when summing infinite sets of diagrams
  • Need to include all crossed diagrams. In practice, impossible
  • Can be done approximately with Parquet approximation (Bickers) (with self-consistency, unsatisfactory)

• « Self-consistency » and conservation laws

\[ \sum = \sum + \sum \]

\[ \sum = \sum + \sum + \sum \]
Self-consistency in conventional « Eliashberg » superconductivity

- Take phonons as « given »

\[ \Sigma = \frac{\delta F}{\delta G} \]
\[ \Gamma_{\text{irr}} = \frac{\delta \Sigma}{\delta G} \]

Migdal’s theorem:
- can be dropped because \((m/M)^{1/2} \ll 1\)
- Not so for spin-fluctuation exchange

Conservation laws, but:
- no Pauli, infinite number of theories, assumes Migdal
Non perturbative but from weak coupling:
  • Pauli
  • Conservation laws
  • Mermin-Wagner

• Main result: Fluctuation-induced pseudogap
\[ E_k A(\omega) \]

\[ A(\omega) \]

\[ T = 0 \]

\[ T_x \]

\[ n_c \]
(d) Approaching from strong coupling

• Straight perturbative treatment is difficult:
  • No Wick’s theorem
  • Non-causality

• Slave bosons and slave fermions
  • \( c^+ (1 - n_\downarrow) \rightarrow f b^+ \) or \( f b^+ \)
  • Constraint: \( \sum_\sigma f_\sigma^+ f_\sigma^+ + b^+ b = 1 \)
    • Mean field: constraint with Lagrange multiplier

• Gauge theory
  • \( f^+ \rightarrow e^{i\theta} f^+ \)
  • \( b \rightarrow e^{-i\theta} b \)
  • \( \lambda \rightarrow \lambda + \delta \theta / \delta \tau \)
• Gauge theory:
  • Break Gauge symmetry
  • Look for mass of gauge field (stable if mass)
  • Find topological excitations
    • (charge carriers prop. to $\delta$)

• There are ambiguities (Slave-fermions vs slave-bosons)
  • e.g. limit $J = 0$ (Nagaoka)

(Daniel Boies, F. Jackson and A.-M.S. T. Int. J. Mod. Phys. 9, 1001 (1995))

• Spin-charge separation
Figure 1. ARPES data showing a kink in a heavily overdoped Bi2212 sample, after ref. Cuk, et al. Stanford, cond-mat/0403743
(f) Inhomogeneities

• Disorder is important in underdoped
  • Even without interactions, complicated: localization
  • Main theoretical tools: impurity averaging, Replica trick

• Instabilities to inhomogeneous ground states:
  • \( S_Q \cdot S_Q \rho_{-2Q} \)
  • \( n = 1 \) magnetic « anti-stripes »
  • \( n = 0.5 \) « charged stripes »

• Using impurities as a « diagnostic tool » for superconducting state
  • Complicated: Kondo effect etc…
E.W. Hudson et al. cond-mat/0104237

See also: C. Howald, P. Fournier, A. Kapitulnik cond-mat/0101251
(g) Quantum critical points….

Would help decide « What type » of fluctuations are important. (See later)

Loram and Tallon, cond-mat/0005063

Fig. 1. Two scenarios for the “phase diagram” for HTS cuprates. In (a) T* represents an energy scale which falls abruptly to zero at a critical doping, p=0.19. In (b) T* merges with T_c on the overdoped side and often a lower T*2 associated with a small pseudogap or a spin gap is invoked. T_N is the Neel temperature for the 3-D AF state.
V. Conclusion

• The pseudogap summarizes anomalous « normal state» properties
  • It is ill-understood
  • Need to understand it to understand the phase diagram and superconductivity.

• It is the motivation for a vast body of work in many directions

• Methods have to be developed at the same time

• Sociology
- How can we understand electronic systems that show both localized and extended character?
- Why do both organic and high-temperature superconductors show broken-symmetry states where mean-field-like quasiparticles seem to reappear?
- Why is the condensate fraction in this case smaller than what would be expected from the shape of the would-be Fermi surface in the normal state?
- Are there new elementary excitations that could summarize and explain in a simple way the anomalous properties of these systems?
- Do quantum critical points play an important role in the Physics of these systems?
- Are there new types of broken symmetries?
- How do we build a theoretical approach that can include both strong-coupling and $d = 2$ fluctuation effects?
- What is the origin of d-wave superconductivity in the high-temperature superconductors?
C'est fini... enfin