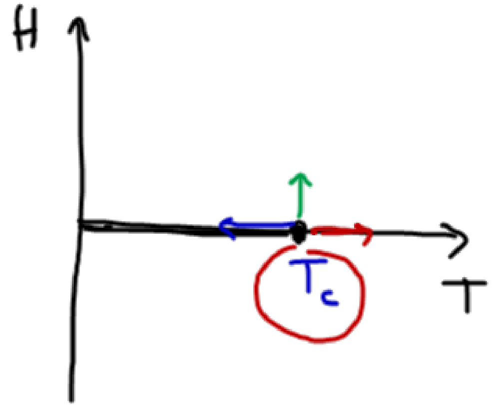

$\beta = 1/2 \qquad M \propto (T_c - T)^{\beta}$

$\gamma' = \gamma = 1 \qquad \chi = \frac{\partial M}{\partial B} \propto (T - T_c)^{-\gamma}$

$\delta = 3 \qquad M \propto B^{1/\delta}$

$$\left\{ \begin{array}{l} \langle M_i \, M_j \rangle \propto \dfrac{e^{-r/\xi}}{r^{d-2+\eta}} \qquad r = |r_i - r_j| \\ \longrightarrow \boxed{\xi \propto (T - T_c)^{-\nu}} \quad \nu = \dfrac{1}{2} \\ \eta = 0 \end{array} \right.$$

Rappel

$$\boxed{F_E + \langle H - H_E \rangle_E \geqslant F}$$

$$\boxed{H_E = -\sum_i x_i S_i}$$

$$H = -\sum_{ij} J'_{ij} S_i S_j - B' \sum_i S_i$$

$$\frac{\partial}{\partial x_i}\left[F_{CM} \equiv F_E + \langle H - H_E \rangle_E\right] = 0$$

$$\boxed{x_i^* = B'_i + 2\sum_j J'_{ij} \tanh \beta x_j^*}$$

$$\Gamma_{CM}[\beta, \bar{M}_i] = F_{CM} + \sum_i B'_i M_i$$

$$\boxed{\begin{aligned}\Gamma_{CM}[\beta, \bar{M}_i] &= -N k_B T \ln 2 \\ &- \sum_{\langle ij \rangle} J'_{ij} \bar{M}_i \bar{M}_j \\ &+ \frac{1}{\beta}\sum_i \left[\frac{(1+\bar{M}_i)}{2} \ln(1+\bar{M}_i) + \frac{(1-\bar{M}_i)}{2} \ln(1-\bar{M}_i)\right]\end{aligned}}$$

$$M_i = -\frac{\partial F_{CM}}{\partial B'_i} = \tanh \beta x_i$$

$$\frac{\partial}{\partial M}\left[-J' z N M^2 + \frac{N}{\beta} \left(\frac{1+M}{2}\right) \ln(1+M) + \frac{N}{\beta} \left(\frac{1-M}{2}\right) \ln(1-M)\right]$$

$$N\left[-2J' z M + \frac{1}{\beta} \frac{1}{2} \ln\left(\frac{1+M}{1-M}\right)\right] = 0$$

$$-2J' z M + \frac{1}{\beta} \operatorname{artanh} M$$

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Taylor

$$\Gamma_{cm}[\beta, M_i] = \Gamma_0 - \sum_{(ij)} J'_{ij}\,\bar{M}_i\,\bar{M}_j$$

$$+\frac{1}{\beta}\sum_i\left(\frac{\bar{M}_i^2}{2} + \frac{1}{12}\bar{M}_i^4\right)$$

Théorie de Landau

$\bar{M}_i \equiv M$ indép. de i

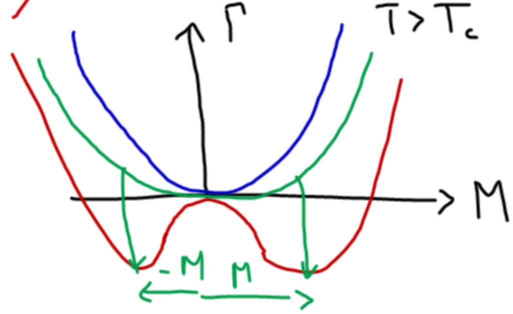
∃ dév. en puissances du param. d'ordre

$$-Nz J' M^2 + \frac{N}{\beta}\frac{M^2}{2}$$

$$-\frac{N}{2\beta}\left[2\beta J' z - 1\right] M^2$$

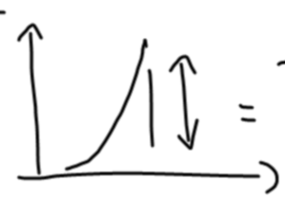
$$k_B T_c \equiv 2J'z$$

$$\boxed{\frac{\Gamma}{N}\ \alpha\ -\frac{k_B}{2}[T_c - T]M^2 + bM^4}$$



$$C = T\frac{\partial S}{\partial T} \qquad S = -\frac{\partial \Gamma}{\partial T}$$

$$= -T\frac{\partial^2 \Gamma}{\partial T^2}$$



$$= \frac{T_c a'^2}{2b}$$

$$C \propto (T - T_c)^{-\alpha}$$

$$\alpha = 0$$

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Corrélations spatiales

$$\boxed{\Gamma = \Gamma_0 - \sum_{\langle ij \rangle} J_{ij}^{\circ} \bar{M}_i \bar{M}_j + \frac{1}{\beta} \sum_i \left(\frac{1}{2} \bar{M}_i^2 + \frac{1}{12} \bar{M}_i^4 \right)}$$

Fonction de corrélation

$$\boxed{G_{ij} = \frac{\partial \langle M_i \rangle}{\partial B_j}} \propto \langle M_i M_j \rangle$$

$$B_j = \frac{\partial \Gamma}{\partial M_j} = \frac{\partial \Gamma}{\mu_B \partial \bar{M}_j} \qquad M_i = \mu_B \bar{M}_i$$

$$\langle M_i \rangle = -\frac{\partial F}{\partial B_i}$$

$$\Gamma = F + \sum_i M_i B_i$$

$$\frac{\partial \Gamma}{\partial M_i} = B_i$$

$$\frac{\partial^2 \Gamma}{\mu_B^2 \partial \bar{M}_i \partial \bar{M}_j} = \frac{\partial B_i}{\partial M_j} = \left(G^{-1} \right)_{ij}$$

$$(G^{-1})_{kl} = \frac{1}{\mu_B^2}\left[-2J'_{kl} + \frac{\delta_{kl}}{\beta}(1+M_0^2)\right]$$

M_0 à l'équilibre.

$$G^{-1}(q) = \sum_l e^{i\vec{q}\cdot(r_k - r_l)} (G^{-1})_{kl}$$

$$= \frac{1}{\mu_B^2}\left[-2\sum_l e^{i\vec{q}\cdot(r_k-r_l)} J'_{kl} + \frac{1}{\beta}(1+M_0^2)\right]$$

Premiers voisins $\Rightarrow$

$$-2J'\left[e^{iq_x d_0} + e^{-iq_x d_0} + \cdots\cdots\right]$$

$$-4J'(\cos q_x d_0 + \cos q_y d_0 + \cos q_z d_0)$$

$$G(q) = \frac{\mu_B^2\beta}{-4J'\beta(\cos q_x d_0 + \cdots) + 1}$$

$$G(q) = \frac{\mu_B^2\beta}{-4J'\beta\frac{z}{2}}$$

$$1 - \frac{q_x^2 d_0^2}{2}$$

q petit

$$-4J'\beta(\cos q_x d_0 \cdots - \frac{z}{2}$$

$$+1$$

$$G(q) = \frac{\mu_B^2\beta}{(1-2J'z\beta) + \frac{4J'\beta}{2} q^2 d_0^2}$$

$$\boxed{G(q) = \frac{\mu_B^2\beta_c}{\left(\frac{T-T_c}{T_c}\right) + (2J'\beta_c d_0^2)q^2}}$$

$$= \frac{\mu_B^2\beta_c}{\left(\frac{T-T_c}{T_c}\right)}\left[\frac{1}{1+q^2\xi^2}\right]$$

$\xi \propto (T-T_c)^{-\nu}$

$\nu = 1/2$

$$\boxed{\xi^2 = \frac{2Jd_0^2}{k_B T_c}\,\frac{T_c}{T-T_c} = \xi_0^2\left(\frac{T-T_c}{T_c}\right)^{-2\nu}}$$

$$G(r) \propto \frac{T_c}{T-T_c} \int d^d q \, \frac{e^{i\vec{q}\cdot\vec{r}}}{1+q^2\xi^2}$$

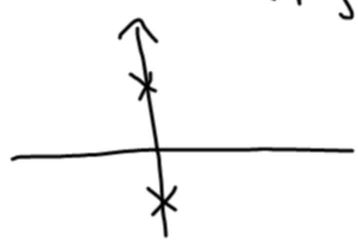
$$\propto \frac{T_c}{T-T_c} \int_0^{q_0=\pi/d_0} q^{d-1} dq \int_{-1}^{1} \frac{d(\cos\theta)\, e^{iqr\cos\theta}}{1+q^2\xi^2}$$

$$\propto \frac{T_c}{T-T_c} \int_0^{q_0} q^{d-1} \frac{dq}{iqr} \frac{e^{iqr}-e^{-iqr}}{1+q^2\xi^2}$$

$$q' \equiv q\xi$$

$$\propto \frac{T_c}{T-T_c} \int_0^{\infty} \frac{q'^{d-2}}{ir}\frac{dq'}{\xi^{d-1}} \frac{e^{iq'\frac{r}{\xi}}-e^{-iq'\frac{r}{\xi}}}{1+q'^2}$$

$$\propto \frac{T_c}{T-T_c} \int_{-\infty}^{\infty} \frac{q' dq'}{ir\,\xi^{d-1}} \frac{e^{iq'r/\xi}}{1+q'^2}$$



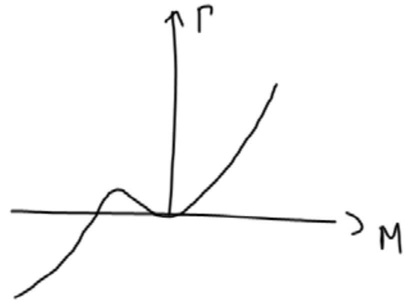
$$G(r) \propto \frac{1}{r} \, e^{-r/\xi}$$

$$\eta = 0$$

4.4 Symétrie et ordre de la transition dans la théorie de Landau

$$a(\vec{M}\cdot\vec{M}) + b(\vec{M}\cdot\vec{M})^2$$

$$M^2 + M^3$$



Cristal près de $T = T_c$.

$$\rho(\vec{r}) = \sum_{n,i} a_i^{(n)} \varphi_i^{(n)}(\vec{r})$$

Si à $T = T_c$ on a un groupe de symétrie G_0

$\varphi_i^{(n)}$ = module de la reps. irréductible (n)

$$\rho(r) = \rho_0 + \delta\rho$$

$$= \rho_0 + \sum_{n \neq 1, i} a_i^{(n)} \varphi_i^{(n)}(\vec{r})$$

Seule qui intervient à $T = T_c$

à $T = T_c$ $\qquad R\rho_0 = \rho_0$

$\delta\rho = 0$, tous $a_i^{(n)} = 0$ $\qquad$ élément de G_0

Condition nécessaire pour transition continue, est que G_i de phase i soit un sous-groupe de G_0

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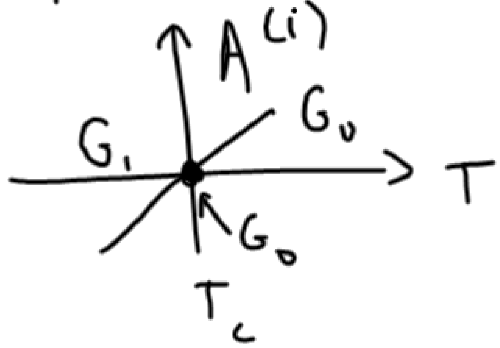
$$\Gamma = \Gamma_0 + \sum_{n \neq 1} A^{(n)} \sum_i \left[a_i^{(n)}\right]^2$$

analogue $\vec{M} \cdot \vec{M}$

$A^{(n)}(T, p) = 0$ à $T = T_c$



- Vérifier si $\exists$ invariant d'ordre 3
- Vérifier coeff. ordre 4 si > 0
- Pour les solides: $\vec{k}$
- Théorie plus générale que Landau inclue les "fluctuations"
  Peut modifier les considérations ci-dessus.

4.5 Théorie gaussienne autour du pt. critique.
(Limitations de la théorie de Landau)

$$\boxed{Z = Z_0 \int \mathcal{D}\bar{M}\, e^{-\beta \int d^3r \left(c(\vec{\nabla} M)^2 + a(T) M^2(r) + b(T) M^4(r) \right)}}$$

$$Z = \int e^{-f(x)N}\, dx$$

$$\simeq \int e^{-Nf(x_0) - \frac{1}{2} N \frac{\partial^2 f}{\partial x^2}(x-x_0)^2 + \dots}\, dx$$

$$\left.\frac{\partial f}{\partial x}\right|_{x_0} = 0$$ condition du col (de minimum)

$$\simeq e^{-Nf(x_0)} \int_{-\infty}^{\infty} e^{-\frac{1}{2} N \frac{\partial^2 f}{\partial x^2}(x-x_0)^2}\, dx$$

"inclure les fluct."

$$= e^{-Nf(x_0)} \sqrt{2\pi \frac{1}{N \frac{\partial^2 f}{\partial x^2}}}$$

$$F = -\frac{1}{\beta} \ln Z$$

$$= -\frac{1}{\beta}\left(-Nf(x_0)\right) - \cancel{\frac{1}{2\beta} \ln\left(\frac{2\pi}{N \frac{\partial^2 f}{\partial x^2}}\right)} \to 0$$

$$N! = \int dx\, x^N e^{-x}$$

$$= \int dx\, e^{(N \ln x - x)}$$

À démontrer

Théorie de champ moyen

= approximation du col

pour Z

À l'ordre gaussien

$$\alpha = 2 - \nu d = 2 - \frac{1}{2}3$$

$$= -\frac{1}{2}$$

Critère de Ginzburg

Champ moyen s'applique jusqu'à

$$\frac{T - T_c}{T_c} = \left(\xi_0^{-d} \right)^{\frac{2}{4-d}}$$

Si ξ_0 grand

C'est le cas pour supraconducteur

$d > 4$ $\beta, \gamma, \delta \ldots$

exacts pour $d > 4$