$$Z \sim \int e^{-N f(x)} \, dx$$

$$\sim \int e^{-N\left[f(x_0) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x - x_0)^2\right]} \, dx$$

$$\ln Z \propto -N f(x_0)$$

$$+ \ln\left[\int \cdots \right]$$

févr. 22-08:28

$$\mathcal{H} = -\sum_{ij} S_i J_{ij} S_j - \sum_i S_i B_i$$

$$\boxed{Z = \mathrm{Tr}\left[e^{\beta \underline{S^T J' S} + \beta B^T S} \right]}$$

Transfo. de Hubbard-Stratonovich

$$\int_{-\infty}^{\infty} d\varphi \, e^{-\frac{\varphi^2}{2a} + \varphi S_i}$$

$$= \int d\varphi \, e^{-\frac{\varphi^2}{2a}} \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\varphi S_i)^n}$$

$$= \int_{-\infty}^{\infty} d\varphi \, e^{-\frac{1}{2a}(\varphi - a S_i)^2 + \frac{a^2 S_i^2}{2a}}$$

$$= \sqrt{2\pi a} \, e^{\boxed{\frac{a S_i^2}{2}}}$$

?

$$Z(\beta, B')$$

$$= \frac{1}{\sqrt{\left(\frac{\pi}{\beta}\right)^N \det J^{-1}}} \mathrm{Tr}_S \int_{-\infty}^{\infty} e^{-\phi^T \frac{J^{-1}}{4\beta} \phi} \, e^{(\phi+\beta B')^T S} \, D\varphi$$

$$D\varphi = \prod_{i=1}^{N} d\varphi_i$$

$$\propto \int_{-\infty}^{\infty} e^{-\phi^T \frac{J^{-1}}{4\beta} \phi} \prod_{i=1}^{N} \left(2\cosh(\phi_i + \beta B')\right) D\varphi$$

$$\propto \int e^{-\phi^T \frac{J^{-1}}{\beta} \phi + \sum_i \ln\left(2\cosh(\phi_i + \beta B')\right)} D\varphi$$

Phase désordonnée, $T > T_c$

$$Z(\beta, B = 0)$$

$$= Z'_0 \int_{-\infty}^{\infty} e^{-\frac{\beta}{2} \sum_q G^{-1}(q)\, M(q)\, M(-q)} \mathcal{D}M$$

$$(G^{-1})_{ij} = -2J'_{ij} + \frac{1}{\beta}$$

$$\ln Z = \text{cte} + \sum_q \ln \frac{1}{\underset{\xi}{a[T]} + cq^2}$$

$$a[T] \propto (T - T_c)$$

$$C_v \propto -T \frac{\partial^2 F}{\partial T^2} \qquad S = -\frac{\partial F}{\partial T}$$

$$F = -k_B T \ln Z$$

$$\frac{\partial^2 F}{\partial T^2} \propto \sum_q \frac{d\,a(T)/dT}{a(T) + cq^2}$$

$$\propto \sum_q \frac{(da/dT)^2}{[a(T) + cq^2]^2}$$

$$\propto \int d^d q \frac{1}{(a(T) + cq^2)^2}$$

$$\propto \xi^{4} \int \frac{d^d q'}{\xi^{d}} \frac{1}{(1 + q'^2)^2}$$

$$q' = q\xi$$

$$\rightarrow \xi^2 = \frac{c}{a(T)} = \frac{c}{a'(T - T_c)}$$

$$\nu = 1/2$$

$$C \propto \xi^{4-d}$$

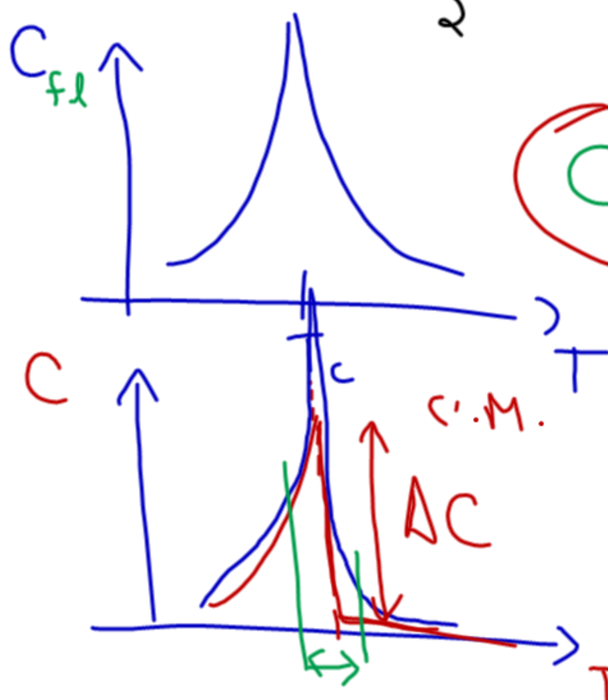
$\Rightarrow$ Si $d > 4$ pas de divergence, Landau = O.K.

Si $d < 4 \Rightarrow$

$$C \propto (T - T_c)^{-(4-d)/2}$$

$$C \sim (T - T_c)^{-\alpha}$$

$$\alpha = 2 - \frac{d}{2}$$



$C_{fl} > \Delta C$

Lois d'échelle (4.7)

$$M = \chi B \qquad t \equiv \frac{T - T_c}{T_c}$$

$$M_{ind} \sim |t|^{-\gamma} B \sim |t|^{\beta} \sim M_{sp}$$

"varie comme"

Soit B suffisamment grand pour que $M_{ind} \sim M_{spontané}$

$$\boxed{B \sim |t|^{\beta+\gamma}} \checkmark$$

0 $$\left\{ \begin{array}{l} VBM \sim t^2 C \sim |t|^{2-\alpha} \\ \text{énergie magn.} \sim \text{énergie thermique} \end{array} \right.$$

$$B \sim |t|^{2-\alpha-\beta}$$

$$|t|^{2-\alpha-\beta} \sim |t|^{\beta+\gamma}$$

Essam Fisher 63 $$\boxed{2 = \alpha + 2\beta + \gamma}$$

Si champ est fort

$$M \propto B^{1/\delta} \checkmark$$

$$\boxed{B \propto M_{sp}^{\delta} \sim |t|^{\beta\delta}}$$

Widom 64 $$\boxed{\beta\delta = \beta + \gamma'}$$

$$\gamma = \gamma'$$

$$\int \frac{d^d r_1\, d^d r_2}{V^2} \langle M(r_1) M(r_2) \rangle$$

$$\frac{1}{V}\int d^d r\, G(r) = \frac{k_B T_c}{V}\, \chi \sim |t|^{-\gamma}$$

$$\boxed{\begin{aligned} \langle M(r_1) M(r_2)\rangle &= G(r_1 - r_2) \\ &= \frac{1}{r^{d-2+\eta}} e^{-r/\xi} \\ r = r_1 - r_2 & \end{aligned}}$$

$$\frac{1}{V}\frac{\xi^d}{\xi^{d-2+\eta}} \sim \xi^{2-\eta} \sim |t|^{-\gamma}$$

$$\sim |t|^{-\nu(2-\eta)} \sim |t|^{-\gamma}$$

$$\boxed{\gamma = (2-\eta)\nu}$$

Remarque

Posons:

$$M \sim B^{1/\delta}\, g\left(\frac{t}{B^{1/\delta\beta}}\right)$$

Si $t = 0$ ou B grand $\Big\}$ $g(0) = \text{cte}$.

$$M \sim B^{1/\delta}$$

Si $t < 0$, $B \to 0$

$$x = t/B^{1/\delta} \to -\infty$$

$$g(x) \to (-x)^{\beta} \quad \text{i.e.} \quad M \sim |t|^{\beta}$$

Si $t > 0$, $B \to 0$

$$x = t/B^{1/\delta} \to \infty$$

$$g(x) \to x^{-\gamma} \quad \text{i.e.}$$

$$M \sim B^{1/\delta}\, \frac{t^{-\gamma}}{B^{-\gamma/\delta\beta}}$$

$$\sim B^{\frac{1}{\delta}\left(1+\frac{\gamma}{\beta}\right)}\, t^{-\gamma}$$

La relation de Widom donne

$$\beta + \gamma = \beta\delta \quad \text{donc}$$

$$M \sim t^{-\gamma} B$$

Invariance d'échelle
quatrième relation
"Hyperscaling"

Soit r_0 la longueur où
où les fluct. du paramètre
d'ordre sont comparables
à la valeur spontanée.

févr. 22-09:53

$$\rightarrow \int \frac{d^3 r_1}{V_0} \int \frac{d^3 r_2}{V_0} \langle \Delta M(r_1) \Delta M(r_2) \rangle$$

$$= M_{sp}^2 \sim |t|^{2\beta}$$

$$G(r) \sim \frac{1}{r^{d-2+\eta}} e^{-r/\xi}$$

Si $\xi \gg r_0$ $\quad V_0 = r_0^3$

$$\rightarrow \frac{1}{r_0^{d-2+\eta}} \sim |t|^{2\beta}$$

$$\boxed{r_0 \sim |t|^{-2\beta/(d-2+\eta)}}$$

d = 3
Champ
moyen

$$r_0 \sim |t|^{-1/1} \sim |t|^{-1}$$

$$\xi \sim |t|^{-1/2}$$

Critère de Ginzburg

$$C_{fl} \sim \Delta C$$

$$\boxed{r_0 \sim \xi}$$

La "bonne" théorie

r_0 disparaît du problème

$$r_0 \sim \xi$$

$$|t|^{-2\beta/(d-2+\eta)} \sim |t|^{-\nu}$$

$$\nu(d-2+\eta) = 2\beta$$

$$\begin{cases} \nu d - (2-\eta)\nu = 2\beta \\ \nu d - \gamma = 2\beta \\ \nu d = 2\beta + \gamma \\ \boxed{\nu d = 2 - \alpha} \text{ hyperscaling} \end{cases}$$

févr. 22-09:55