5.1 Perturbations stationnaires

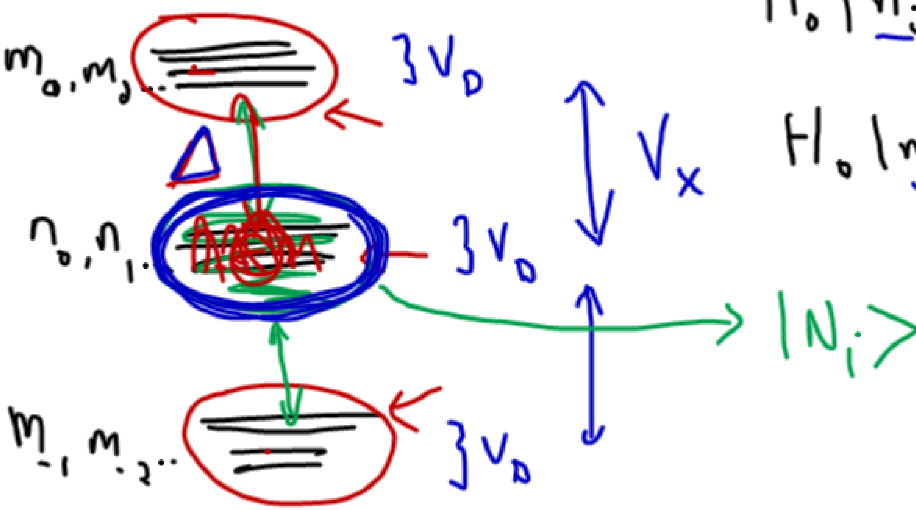
Brillouin-Wigner

$$H = \boxed{H_0 + V_D} + V_x = \tilde{H}_0 + V_x$$

$$\boxed{V = 0}$$

$$H_0 |n_i\rangle = E_{n_i} |n_i\rangle$$

$$H_0 |m_i\rangle = E_{m_i} |m_i\rangle$$



Schrödinger:

$$Q\left(E_i - \tilde{H}_0 - V_x\right)|N_i\rangle = 0$$

$$|N_i\rangle = P|N_i\rangle + \underline{Q|N_i\rangle}$$

$$P = \sum_i |n_i\rangle\langle n_i|$$

$$Q = \sum_i |m_i\rangle\langle m_i|$$

$$P + Q = 1$$

mars 1-08:47

$$(E_i - \tilde{H}_0) Q |N_i\rangle = Q V_x |N_i\rangle$$

$$Q|N_i\rangle = \frac{1}{E_i - \tilde{H}_0} Q V_x |N_i\rangle$$

$$\boxed{|N_i\rangle = |\tilde{n}_i\rangle + \frac{1}{E_i - \tilde{H}_0} Q V_x |N_i\rangle}$$

$$|\tilde{n}_i\rangle = \mathcal{J} |n_i\rangle$$

$$\boxed{|N_i\rangle = |\tilde{n}_i\rangle + \frac{1}{E_i - \tilde{H}_0} Q V_x |\tilde{n}_i\rangle + \frac{1}{E_i - \tilde{H}_0} Q V_x \frac{1}{E_i - \tilde{H}_0} Q V_x |\tilde{n}_i\rangle + \dots}$$

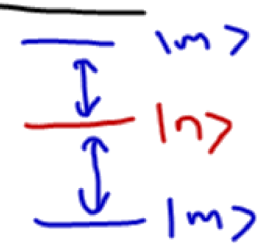
$$\Delta$$

Ce qu'il reste à faire. $\qquad \dfrac{V_x}{\Delta}$

$$\boxed{\begin{gathered}\langle N_i | H_0 + V_x + V_D | N_j\rangle = \delta_{ij} E_j \\ \langle \tilde{n}_i | \left[H_{eff} \right] | \tilde{n}_j\rangle = \delta_{ij} E_j\end{gathered}}$$

Cas particulier : 1 niveau

$$|N\rangle = |n\rangle + \frac{1}{E_N - H_0} Q V |n\rangle$$



$$+ \frac{1}{E_N - H_0} Q V \frac{1}{E_N - H_0} Q V |n\rangle + \dots$$

$$\boxed{\begin{aligned} |N\rangle = |n\rangle &+ \sum_i \frac{1}{E_N - \epsilon_{m_i}} |m_i\rangle\langle m_i| V |n\rangle \\ &+ \sum_{ij} \frac{1}{E_N - \epsilon_{m_i}} |m_i\rangle\langle m_i| V \frac{|m_j\rangle\langle m_j|}{E_N - \epsilon_{m_j}} V |n\rangle \end{aligned}}$$

$$\langle n | H_0 + V | N \rangle = E_N \langle n | N \rangle$$

pour évaluer l'énergie.

$$E_N \langle n | N \rangle = \epsilon_n \langle n | N \rangle + \langle n | V | N \rangle$$

$$E_N = \epsilon_n + \langle n | V | N \rangle \qquad \langle n | N \rangle = 1$$

Observable

$$\frac{\langle N | O | N \rangle}{\langle N | N \rangle}$$

$$\boxed{E_N = \underline{\epsilon_n + \langle n | V | n \rangle} + \sum_i \frac{|\langle n | V | m_i \rangle|^2}{\underline{E_N} - \epsilon_{m_i}}}$$

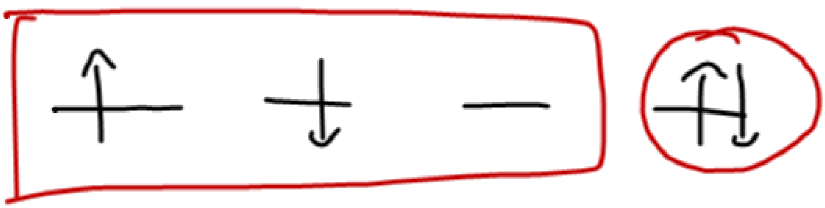
$$|m_i\rangle \neq |n\rangle$$

Rayleigh-Schrödinger

E_N à développer

$$E_N = \epsilon_n + \langle n | V | n \rangle + \sum_i \frac{|\langle n | V | m_i \rangle|^2}{\epsilon_n - \epsilon_{m_i}}$$

$$\boxed{\langle N | N \rangle = \langle n | n \rangle + \sum_i \frac{|\langle n | V | m_i \rangle|^2}{(\epsilon_n - \epsilon_{m_i})^2}}$$



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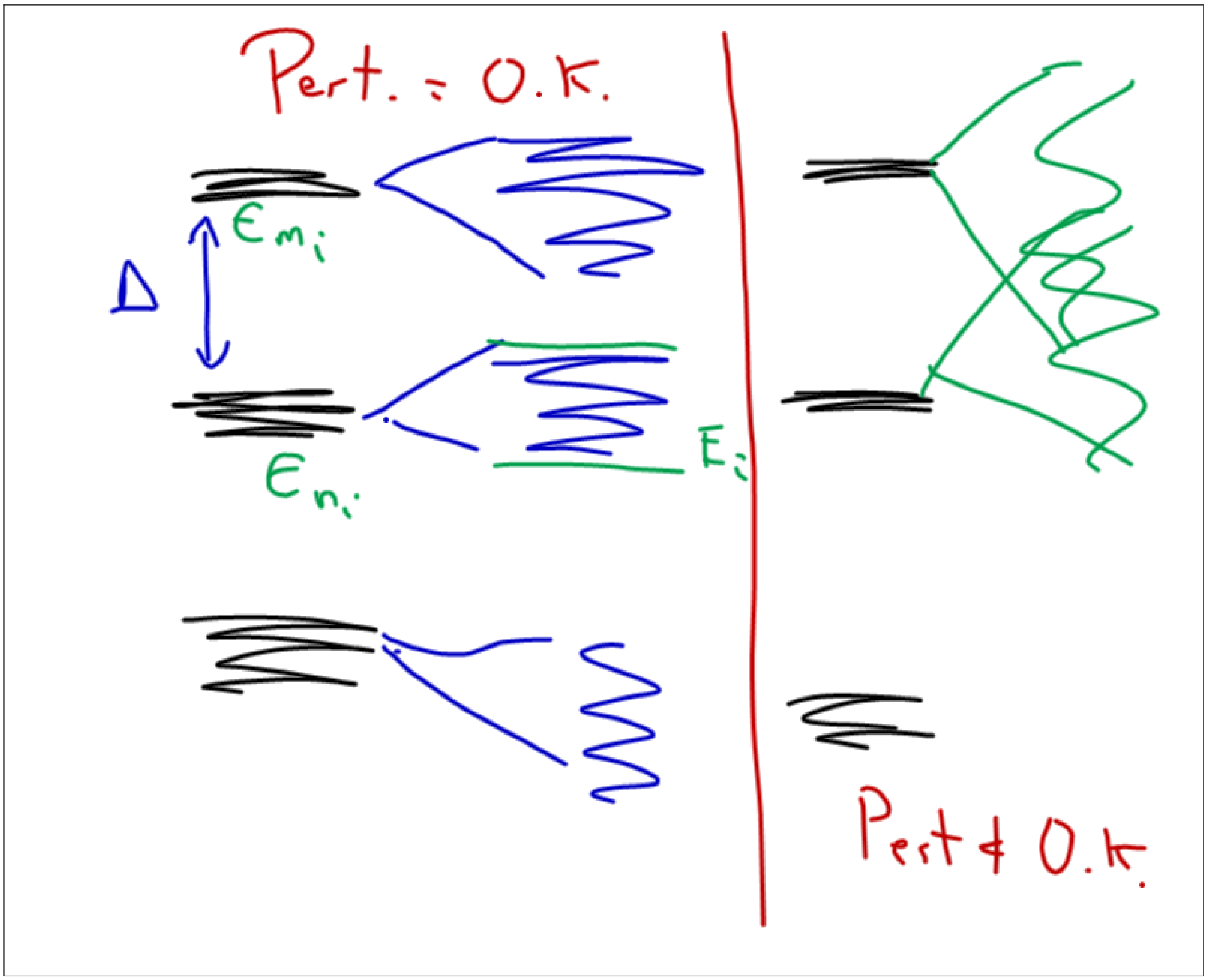
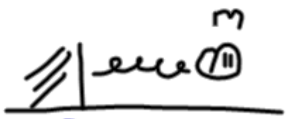
N sites 4^N

N sites 3^N


Pert. = O.K.
Δ
ϵ_{m_i}
ϵ_{n_i}
E_i
Pert ≠ O.K.

5.2 Oscillateur harmonique.

Sénéchal. Chap. 1.



$$\boxed{\mathcal{H} = \frac{P^2}{2m} + \frac{1}{2} m \omega^2 X^2} \qquad [X, P] = i\hbar$$

Opérateurs d'échelle: $\omega^2 = k/m$

$$a^+ = \sqrt{\frac{m\omega}{2\hbar}} \left(X - \frac{iP}{m\omega} \right)$$

$$a = \sqrt{\frac{\omega m}{2\hbar}} \left(X + \frac{iP}{m\omega} \right)$$

$$\boxed{\mathcal{H} = \frac{1}{2} \hbar \omega (a a^+ + a^+ a)}$$

Preuve: $$\frac{1}{2} \cancel{\hbar} \omega \left(\frac{m\omega}{2\cancel{\hbar}} \right) \left(\cancel{X^2} + \cancel{\frac{i}{m\omega} P X} - \cancel{\frac{i}{m\omega} X P} + \cancel{\frac{P^2}{m^2\omega^2}} + \cancel{X^2} - \cancel{\frac{i}{m\omega} P X} + \cancel{\frac{i}{m\omega} X P} + \cancel{\frac{P^2}{m^2\omega^2}} \right)$$

$$= \frac{1}{4} m \omega^2 \left(2X^2 + \frac{2P^2}{m^2\omega^2} \right)$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{iP}{m\omega}\right) \qquad a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(X - \frac{iP}{m\omega}\right)$$

$$[a, a] = [a^+, a^+] = 0$$

$$[a, a^+] = \frac{m\omega}{2\hbar}\left(\cancel{[X, X]}^{0} + \frac{i}{m\omega}[P, X] - \frac{i}{m\omega}[X, P] + \frac{1}{(m\omega)^2}\cancel{[P, P]}^{0} \right)$$

$$= \frac{m\omega}{2\hbar}\frac{2i}{m\omega}[P, X] = \frac{\hbar}{\hbar} = 1$$

$$\boxed{[a, a^+] = 1} \qquad a a^+ = 1 + a^+ a$$

$$H = \frac{1}{2}\hbar\omega\left(a a^+ + a^+ a\right)$$

$$\boxed{H = \hbar\omega\left(\frac{1}{2} + a^+ a\right)}$$

$$[a^+ a, a] = a^+ \cancel{[a, a]}^{0} + \cancel{[a^+, a]}^{-1} a$$

$$\boxed{[AB, C] = A[B, C] + [A, C]B}$$

$$(ABC - ACB) + (ACB - CAB)$$

$$\boxed{\begin{array}{l} [a^+ a, a] = -a \\ [a^+ a, a^+] = +a^+ \end{array}} \qquad [a^+, a^+ a] = -a^+$$

"Théorème" sur les commutateurs d'opérateurs d'échelle.

Si $\boxed{[A, B] = \beta B}$

et $A|\alpha\rangle = \alpha|\alpha\rangle$

alors $A\left(B|\alpha\rangle\right) = (\alpha + \beta)\left(B|\alpha\rangle\right)$

Preuve:

$$(AB - B\underbrace{A)|\alpha\rangle}_{\alpha|\alpha\rangle} = \beta\left(B|\alpha\rangle\right)$$

Solution de l'O.H.

$$\boxed{H = \hbar\omega\left(N + \tfrac{1}{2}\right)}$$ n "phonons"

où $N \equiv a^+ a$ opérateur "nombre"

$[N, a] = -a \quad [N, a^+] = a^+$

Donc si $|n\rangle$ est état propre $N|n\rangle = n|n\rangle \Rightarrow$

Quels sont les autres états propres ?

$$\begin{cases} a^+ |n\rangle = C\,|n+1\rangle \\ a\,|n\rangle = C'\,|n-1\rangle \end{cases}$$

Normalisation:

$$\left(\langle n|\, a\right)\left(a^+|n\rangle\right) = \langle n|\, 1 + a^+ a\,|n\rangle$$

$$= \langle n+1|n+1\rangle |C|^2 = 1 + n$$

$$|C|^2 = 1 + n$$

$$C = \sqrt{1+n}$$

$$\left(\langle n|\, a^+\right)\left(a\,|n\rangle\right) = |C'|^2 \langle n-1|n-1\rangle$$

$$n\langle n|n\rangle = |C'|^2 \langle n-1|n-1\rangle$$

$$|C'|^2 = n$$

$$C' = \sqrt{n}$$

Donc n est entier positif et prend donc les valeurs : $0, 1, 2, \ldots \infty$

États propres : $a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

Fondamental $|0\rangle$

$|1\rangle = a^+|0\rangle =$

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$$(a^+)^2 |0\rangle = a^+ |1\rangle$$

$$= \sqrt{2}\, |2\rangle$$

$$(a^+)^3 |0\rangle = a^+ \left(\sqrt{2}\, |2\rangle\right)$$

$$= \sqrt{3}\sqrt{2}\sqrt{1}\, |3\rangle$$

$$\boxed{|n\rangle = \frac{(a^+)^n}{\sqrt{n!}} |0\rangle}$$

Remarques

- $|n\rangle$ base des nombres d'occupation
- $\forall A \quad A^\dagger A$ a des valeurs propres ≥ 0

$$\langle\psi|A^\dagger A|\psi\rangle = \langle A\psi|A\psi\rangle \geq 0$$

- $[H, a] = -\hbar\omega\, a$

$$\dot{a} = \frac{i}{\hbar}[H, a] = -i\omega a$$

$$a(t) = a(0)\, e^{-i\omega t}$$

- Fondamental est tel que

$$\boxed{a|0\rangle = 0} \leftarrow$$

- Où sont les polynômes d'Hermite ?

$$\psi(x) = \langle x|\psi\rangle \qquad X|x\rangle = x|x\rangle$$

$$\boxed{\psi_0(x) = \langle x|0\rangle}$$

$$\langle x|a|0\rangle = 0$$

$$\langle x|\sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{iP}{m\omega}\right)|0\rangle = 0$$

$$= \sqrt{\frac{m\omega}{2\hbar}}\left(x\langle x|0\rangle + \frac{i}{m\omega}\frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|0\rangle\right) = 0$$

$$\frac{\partial\psi_0(x)}{\partial x} = -\frac{m\omega}{\hbar}x\,\psi_0(x)$$

$$\frac{\partial \ln\psi_0(x)}{\partial x} = -\frac{m\omega}{\hbar}x$$

$$\ln\psi_0(x) = -\frac{m\omega}{\hbar}\frac{x^2}{2} + \text{cte}$$

$$\psi_0(x) = C\, e^{-\frac{m\omega x^2}{2\hbar}}$$

États excités :

$$\langle x | 1 \rangle = \Psi_1(x)$$

$$\langle x | a^+ | 0 \rangle = \Psi_1(x)$$

$$\langle x | \sqrt{\frac{m\omega}{2\hbar}} \left(X - i \frac{P}{m\omega} \right) | 0 \rangle = \Psi_1(x)$$

$$\Psi_1(x) = \sqrt{\frac{m\omega}{2\hbar}} \left(x \langle x | 0 \rangle - \frac{\hbar}{i m\omega} \frac{\partial}{\partial x} \langle x | 0 \rangle \right)$$

$$= \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{\hbar}{i m\omega} \frac{\partial}{\partial x} \right) \Psi_0(x)$$

Fermions

$$\boxed{\left(\Psi^+ , \Psi \right)}$$