Retour sur l'examen

$$\vec{A} = -B\,\cancel{y\hat{x}} + B\left(\cancel{y\hat{x}} + x\hat{y}\right)$$

$$\vec{A} = Bx\hat{y}$$

$$\boxed{\vec{A} \to \vec{A} - \vec{\nabla}\xi}$$

$$\boxed{\xi = -Bxy}$$

$$\left(\frac{\hbar}{i}\nabla - qA + \nabla\xi\right)\psi'$$

$$\boxed{\psi' = e^{-\frac{i}{\hbar}\xi q}\psi}$$

Landau:

$$\boxed{F = \frac{1}{2} a M^2 + \frac{1}{4} b M^4 + \frac{1}{6} c M^6}$$

$b^2 > 4ac$

① Équilibre: $\frac{\partial F}{\partial M} = 0$

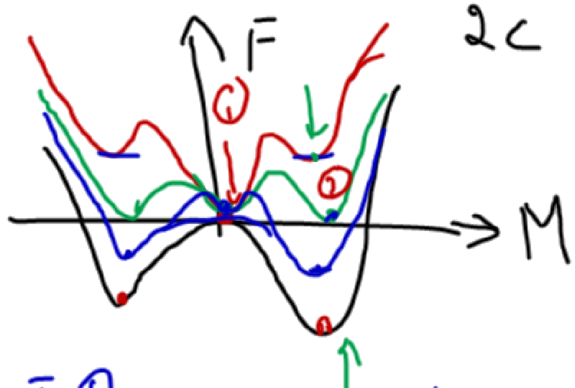
$$aM + bM^3 + cM^5 = 0$$

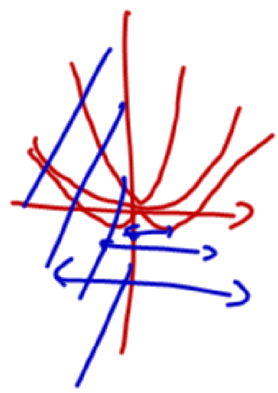
$$M\left(a + bM^2 + cM^4\right) = 0$$

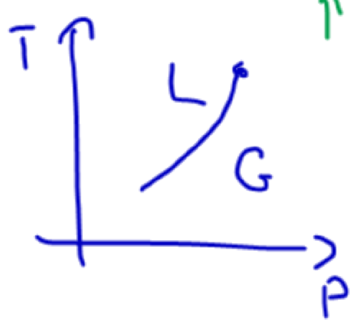
$\longrightarrow M = 0$

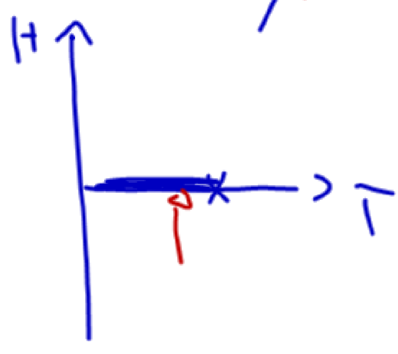
$\longrightarrow M^2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2c}$

②









$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} E & A & B & C \end{array}$$

$$\begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \quad \end{pmatrix}$$

$$\begin{pmatrix} \quad \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$

Résumé cours précédent

Bosons Fermion

$$\psi^{+}(x_1)\,\psi^{+}(x_2)\,|0\rangle$$

$$= \frac{1}{\sqrt{2}}\Big(|x_1\rangle|x_2\rangle \pm |x_2\rangle|x_1\rangle\Big)$$

$$\boxed{|x_1\, x_2 \,\ldots\ldots\, x_n\rangle = \frac{1}{\sqrt{n!}}\sum_{P} \epsilon_P\, |x_{p(1)}\rangle|x_{p(2)}\rangle \ldots\ldots |x_{p(n)}\rangle}$$

Bosons

$$[\psi(x), \psi^{+}(y)] = \delta(x-y)$$

$$\{\psi(x), \psi^{+}(y)\} = \delta(x-y)$$

<u>Fermions:</u>

$$\psi^{+}(x_i)\ \psi^{+}\ \psi^{+}\ \psi^{+}\ \psi^{+}(x_j)|0\rangle$$

$$\longrightarrow \{\psi^{+}(x_i), \psi^{+}(x_j)\} = 0$$

$$\boxed{\psi^{+}(x_i)\,\psi^{+}(x_j)}\ \psi^{+}\,\psi^{+}\,\psi^{+}\ |0\rangle$$

$$-\ \psi^{+}(x_j)\,\psi^{+}(x_i)\ \psi^{+}\,\psi^{+}\,\psi^{+}|0\rangle$$

Changements de base
ex. de fct. d'onde
Opérateurs à 1 corps en
seconde quant.

Changements de base

$$|k\rangle = \int dx\, |x\rangle \langle x|k\rangle$$

$$\langle x|k\rangle = e^{ikx}$$

$$\boxed{\begin{aligned} \psi^+(k) &= \int dx\, \psi^+(x) \langle x|k\rangle \\ \psi(k) &= \int dx\, \langle x|k\rangle^* \psi(x) \\ &= \int dx\, \langle k|x\rangle \psi(x) \end{aligned}}$$

$$[\psi(k), \psi^+(k')]$$

$$= \left[\int dx \langle k|x\rangle \psi(x), \int dx' \psi^+(x') \langle x'|k'\rangle \right]$$

$$= \int dx\,dx' \langle k|x\rangle \left[\underbrace{\psi(x), \psi^+(x')}_{\delta(x-x')} \right] \langle x'|k'\rangle$$

$$= \int dx \langle k|x\rangle \langle x|k'\rangle$$

$$= \langle k|k'\rangle = (2\pi)\, \delta(k-k')$$

Si la matrice de changement de base est unitaire, la transf. est canonique

Canonique ≡ préserve les rel. de comm. ou anticomm.

Bases discrètes

$$\delta(x-y) \to \delta_{r,s} \qquad \int dx \longrightarrow \sum_r$$

Exemple pour la normalisation.

$$\langle r_1\, r_2 \dots r_n | s_1\, s_2 \dots s_n \rangle$$

$$= \sum_p \eta_p \, \delta_{r_1 s_{p(1)}} \delta_{r_2 s_{p(2)}} \cdots \delta_{r_n s_{p(n)}}$$

$$\eta_p = \begin{cases} 1 & \text{bosons} \\ \epsilon_p & \text{fermions} \end{cases}$$

$$\langle \underbrace{r_1 \dots r_1}_{n_1} \; \underbrace{r_2 \dots r_2}_{n_2} \dots \underbrace{r_k\, r_k \dots r_k}_{n_k} |$$

$$| \underbrace{r_1 \dots r_1}_{n_1} \; \underbrace{r_2 \dots r_2}_{n_2} \dots \underbrace{r_k \dots r_k}_{n_k} \rangle$$

$$= n_1! \dots n_k!$$

$$|r_1 \dots r_1\rangle = \left(\psi^+_{r_1}\right)^{n_1} |0\rangle$$

État normalisé

$$|n_1\rangle = \frac{(a^+)^{n_1}}{\sqrt{n_1!}} |0\rangle$$

États de bosons normalisés.

$$|n_1, n_2 \dots\rangle = \frac{\left(\psi^+_{r_1}\right)^{n_1}}{\sqrt{n_1!}} \frac{\left(\psi^+_{r_2}\right)^{n_2}}{\sqrt{n_2!}} \dots |0\rangle$$

$$\underset{\text{Fond}}{|\Omega\rangle} = \frac{\left(\psi^+_0\right)^N}{\sqrt{N!}} |0\rangle$$

Fermions :

État "simple"

$$|\Omega\rangle = a^{+}_{\varphi_1}\, a^{+}_{\varphi_2} \ldots\ldots a^{+}_{\varphi_N} |0\rangle$$

$$\boxed{\langle x | \varphi_1 \rangle = \varphi_1(x)}$$

$$\langle x_1\, x_2 \ldots x_N | \varphi_1 \ldots\ldots \varphi_N \rangle$$

$$= \frac{1}{n!} \sum_{p,q \in S_n} \epsilon_p \epsilon_q \langle x_{p(1)} | \varphi_{q(1)} \rangle \langle x_{p(i)} | \varphi_{q(i)} \rangle \ldots \langle x_{p(N)} | \varphi_{q(N)} \rangle$$

$m \qquad i = q^{-1}(m)$

$$= \cancel{\frac{1}{n!}} \sum_{p,\cancel{q}} \epsilon_p \epsilon_q \langle x_{pq^{-1}(1)} | \varphi_1 \rangle \ldots\ldots \langle x_{pq^{-1}(m)} | \varphi_m \rangle \ldots \langle x_{pq^{-1}(N)} | \varphi_N \rangle$$

$$\epsilon_{q^{-1}} = \epsilon_q \qquad \epsilon_p \epsilon_{q^{-1}} = \epsilon_{pq^{-1}}$$

$$= \sum_{p'} \epsilon_{p'} \langle x_{p'(1)} | \varphi_1 \rangle \ldots\ldots \langle x_{p'(m)} | \varphi_m \rangle \ldots \langle x_{p'(N)} | \varphi_N \rangle$$

$$= \det \begin{bmatrix} \varphi_1(x_1) & \varphi_1(x_2) & \ldots & \varphi_1(x_N) \\ \varphi_2(x_1) & \varphi_2(x_2) & \ldots & \varphi_2(x_N) \\ \vdots & \vdots & & \vdots \\ \varphi_N(x_1) & \varphi_N(x_2) & \ldots & \varphi_N(x_N) \end{bmatrix}$$

déterminant de Slater

mars 20-09:41

Opérateur à 1 corps en seconde quantification.

Opérateur énergie cin. en première quant.: $n = \#$ part.

$$\sum_{i=1}^{n} -\frac{\hbar^2}{2m}\nabla_i^2$$

$$\langle x_1 \ldots x_n | \sum_{i=1}^{n} O_i | y_1 \ldots y_n\rangle$$

Travaillons dans la base diagonale

Soit $O|\alpha\rangle = \alpha|\alpha\rangle$

la base diagonale

"Negele-Orland" (Many-Particle physics)

$$\boxed{\langle\alpha_i|O|\alpha_j\rangle = \langle\alpha_i|O|\alpha_i\rangle\langle\alpha_i|\alpha_j\rangle}$$

Étape int. pour n particules

$$\langle\alpha_1'|\langle\alpha_2'|\ldots\langle\alpha_n'|\sum_{i=1}^{n} O_i$$

$$|\alpha_1\rangle|\alpha_2\rangle\ldots|\alpha_n\rangle$$

$$= \langle\alpha_1'|\langle\alpha_2'|\ldots\langle\alpha_N'|$$

$$\Big(O_1|\alpha_1\rangle|\alpha_2\rangle\ldots|\alpha_N\rangle$$

$$+ |\alpha_1\rangle O_2|\alpha_2\rangle\ldots|\alpha_N\rangle$$

$$+ \ldots$$

$$+ |\alpha_1\rangle|\alpha_2\rangle\ldots O_N|\alpha_N\rangle\Big)$$

$$= \boxed{\sum_{i=1}^{N}\langle\alpha_i|O|\alpha_i\rangle}\Big(\langle\alpha_1'|\ldots\langle\alpha_N'|\,|\alpha_1\rangle\ldots|\alpha_N\rangle\Big)$$

$$\Rightarrow$$

$$\langle\alpha_1'\ldots\alpha_N'|\sum_{i=1}^{N} O_i|\alpha_1\ldots\alpha_N\rangle$$

$$= \sum_{i=1}^{N} \langle \alpha_i | 0 | \alpha_i \rangle \left(\langle \alpha_1' \ldots \alpha_N' | \alpha_1 \ldots \alpha_N \rangle \right)$$

En seconde quant.:

$$= \langle \alpha_1' \alpha_2' \ldots \alpha_N' | \left(\sum_\alpha \langle \alpha | 0 | \alpha \rangle c_\alpha^+ c_\alpha \right)$$

$$| \alpha_1 \ldots \alpha_N \rangle$$

$$c_{\alpha_1}^+ | 0 \rangle$$

$$\sum_\alpha c_\alpha^+ c_\alpha \left(c_{\alpha_1}^+ | 0 \rangle \right) = c_{\alpha_1}^+ c_{\alpha_1} c_{\alpha_1}^+ | 0 \rangle$$

$$= c_{\alpha_1}^+ [1 - \underbrace{c_{\alpha_1}^+ c_{\alpha_1}] | 0 \rangle}_{0}$$

Fermions

$$= c_{\alpha_1}^+ | 0 \rangle$$

$$\sum_\alpha c_\alpha^+ c_\alpha \left(c_{\alpha_2}^+ c_{\alpha_3}^+ \ldots | 0 \rangle \right)$$

$$[c_\alpha^+ c_\alpha , c_\beta^+] = + c_\beta^+$$

$$[c_\alpha^+ c_\alpha , c_\beta] = - c_\beta$$

Prouver les relations de commutation précédentes pour les fermions.

$$[AB, C] = ABC + ACB - ACB - CAB$$

$$= A\{B, C\} - \{A, C\}B$$

$$[c^+ c, c^+]$$

$$= c^+ \{c, c^+\} - \{c^+, c^+\} c$$

$$= -c$$

$$= c^+$$

$$[c^\dagger c, c] = -c$$

$$[c^\dagger c, c]^\dagger = -c^\dagger$$

$$[c^\dagger, c^\dagger c] = -c^\dagger$$

$$[c^\dagger c, c^\dagger] = c^\dagger$$

$$|\alpha\rangle = \int dx \; |x\rangle\langle x|\alpha\rangle$$

$$c^\dagger_\alpha = \int dx \; \psi^\dagger(x) \langle x|\alpha\rangle$$

$$\boxed{\sum_\alpha c^\dagger_\alpha c_\alpha = \sum_\alpha \int dx \; \psi^\dagger(x) \langle x|\alpha\rangle \int dx' \langle \alpha|x'\rangle \psi(x')}$$

$$c_\alpha = \int dx' \langle \alpha|x'\rangle \psi(x')$$

$$= \int dx\,dx' \; \psi^\dagger(x) \langle x|x'\rangle \psi(x')$$

$$= \boxed{\int dx \; \psi^\dagger(x) \psi(x)}$$

e.g. énergie cinétique

$$= \int dx \; \psi^\dagger(x) \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \psi(x)$$