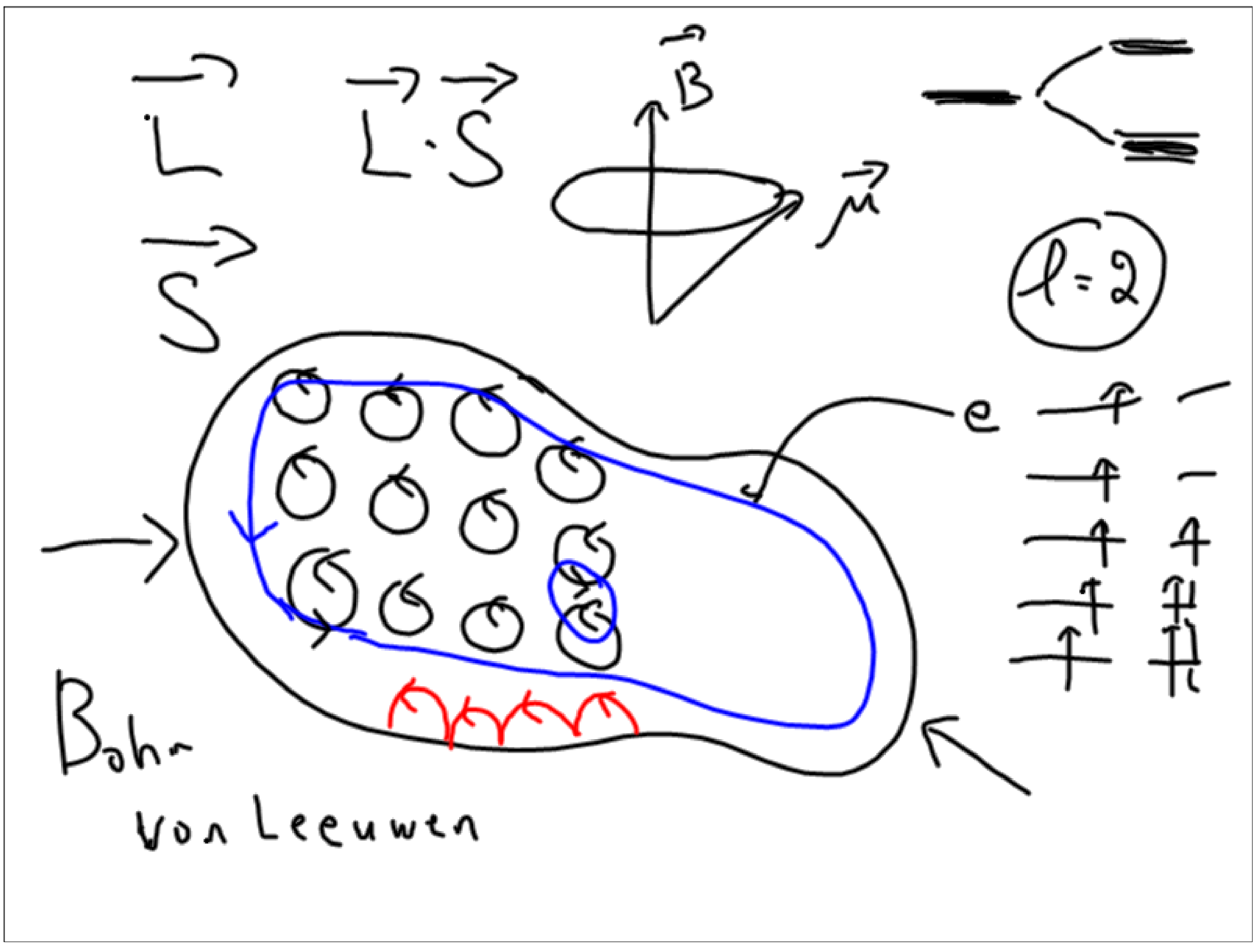

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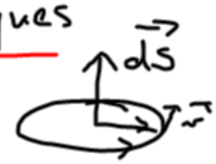
2.1 Moments magnétiques

$$\vec{\mu} = I\,d\vec{S}$$



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times q\left(\vec{v} \times \vec{B}\right)$$

$$\frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B} \qquad \vec{\mu} = \gamma \vec{L}$$

γ = rapport gyromagnétique.

$$\left[\frac{d\vec{L}}{dt} = \gamma \vec{L} \times \vec{B}\right.$$

$$\omega = \gamma B$$

Microscopique:

$$\mu \approx IS \sim \pi r^2 \left(\frac{-e}{T}\right) \sim \pi r^2 \frac{(-e)}{\frac{2\pi r}{v}}$$

$$\mu \sim \frac{r(-e)\,v\,m}{2m} \sim -\frac{e}{2m}\hbar$$

Magnéton de Bohr $\sim \gamma L \quad \gamma = \frac{-e}{2m}$

$$\boxed{\mu_B = \frac{e}{2m}\hbar} \quad 9.274\ \frac{A}{m^2}$$

Champ et aimantation

$$B = \mu_0 H \qquad \text{S.I}$$

T — induction magnétique ou $\frac{\text{flux}}{m^2} \sim \frac{\text{Weber}}{m^2}$

$$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$$

→ Force du champ magnétique

Unités $\frac{A}{m}$

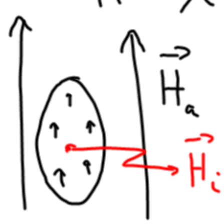
Dans un matériau

$$B = \mu_0 (\vec{H} + \vec{M})$$

M est l'aimantation

$$\vec{M} = \chi \vec{H}$$

χ = susceptibilité magnétique.



$$\boxed{H_i = H_a - NM}$$

N = facteur de démagnétisation

Sphère $N = \frac{1}{3}$

$$\vec{B}_i = \mu_0 (\vec{H}_i + \vec{M})$$

$$= \mu_0 (\vec{H}_a - N\vec{M} + \vec{M})$$

$$\vec{B}_i = \vec{B}_a + (1-N)\mu_0 M$$

$$\boxed{\chi = \frac{M}{H_a} = \frac{M}{H_i + NM} = \frac{M/H_i}{1 + N\,M/H_i}}$$

$$\chi_{intrinsèque} = \frac{M}{H_i} \qquad = \frac{\chi_{int.}}{1 + N\chi_{int}}$$

2.2. Moment magnétique et méc. classique

Moment canonique

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)}$$

$$\boxed{\begin{aligned}\vec{p} &= m\vec{v} + q\vec{A}\\ \vec{p} &= \frac{\partial L}{\partial \dot{r}}\end{aligned}}$$

$$L = \frac{1}{2}m\dot{r}^2 + q\dot{r}\cdot\vec{A}(\vec{r},t) - qV$$

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial r_i} = 0}$$

$$\boxed{\begin{aligned}&\boxed{\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}}\\ &\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\\ &\boxed{\vec{B} = \vec{\nabla}\times\vec{A}}\\ &\vec{\nabla}\cdot\vec{B} = 0\end{aligned}}$$

$$\begin{aligned}\vec{F} &= q\left(\vec{E} + \vec{v}\times\vec{B}\right)\\ &= -q\vec{\nabla}V - q\frac{\partial\vec{A}}{\partial t}\\ &\quad + q\vec{v}\times\left(\vec{\nabla}\times\vec{A}\right)\end{aligned}$$

$$\cancel{m\frac{d\vec{v}}{dt}} = -q\vec{\nabla}V - \cancel{q\frac{\partial\vec{A}}{\partial t}} + q\vec{\nabla}(\vec{v}\cdot\vec{A}) - \cancel{q(\vec{v}\cdot\vec{\nabla})\vec{A}}$$

$$\boxed{\frac{d}{dt}\left[m\vec{v} + q\vec{A}\right] = -q\vec{\nabla}\left(\underbrace{V - \vec{v}\cdot\vec{A}}\right)}$$

$$\frac{d\vec{A}}{dt} = \frac{\partial\vec{A}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{A} \qquad \vec{A}(\vec{r}(t), t)$$

$$\dot{r}_i = v_i$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_i}\right) = \frac{\partial L}{\partial r_i}$$

$$\boxed{L = \frac{1}{2}mv^2 + q\underline{\vec{v}\cdot\vec{A}} - qV}$$

$$\begin{aligned}H &= \vec{p}\cdot\vec{v} - L\\ &= (m\vec{v} + q\vec{A})\cdot\vec{v} - L\end{aligned}$$

$$\boxed{H = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + qV} \qquad \boxed{H = \frac{1}{2}mv^2 + qV}$$

$$O \rightarrow \frac{\partial L}{\partial \dot{O}} \qquad X \rightarrow \left(\frac{\partial L}{\partial \dot{x}} = p\right)$$

$$[X, p] = i\hbar \qquad \left[O, \frac{\partial L}{\partial \dot{O}}\right] = i\hbar$$

$$\boxed{\vec{p} = \frac{\hbar}{i}\vec{\nabla}}$$

En présence de $\vec{A}$

$$\vec{p} \longrightarrow \vec{p} - q\vec{A}$$

Couplage minimal

Bohr van Leeuwen

$$Z = \int \frac{d^3r_1 \ldots d^3r_N}{\hbar^{3N}} \int_{-\infty}^{\infty} \frac{d^3p_1 \ldots d^3p_N}{N!}$$

$$\exp\left[-\beta\left(\sum_{i=1}^{N} \frac{(\vec{p}_i - q\vec{A})^2}{2m} + V(r_1 \ldots r_N)\right)\right.$$

2.3 Mécanique quantique du spin

1. $[J_a, J_b] = i\hbar\, \epsilon_{abc} J_c$

$\epsilon_{abc} = 0$ si 2 indices id.

$= \begin{cases} 1 & \text{per. cycl.} \\ -1 & \text{per. anticycl.} \end{cases}$

Levi-Civita

$\vec{A} \cdot \vec{B} \times \vec{C} = A_a B_b C_c \epsilon_{abc}$

2.

$J_3 \, |j,m\rangle = \hbar m \, |j,m\rangle$

$J^2 \, |j,m\rangle = \hbar^2 j(j+1) \, |j,m\rangle$

$-j < m < j \quad (2j+1)$

3. Exemple $j = \frac{1}{2}$

$U = e^{-i \frac{\hat{n} \cdot \vec{J}}{\hbar} \theta}$ à prouver

---

$\vec{L} = \vec{R} \times \vec{P} \qquad [L_a, P_b]$

$[R_a, P_b] = i\hbar\, \delta_{ab}$

$\epsilon_{abc} [R_b P_c, P_e]$

$[AB, C] = A[B,C] + [A,C]B$

$ABC - CAB = ABC - \cancel{ACB} + \cancel{ACB} - CAB$

$\epsilon_{abc} [R_b, P_e] P_c$

$\epsilon_{abc} \delta_{be} i\hbar P_c$

$\epsilon_{aec} i\hbar P_c = [L_a, P_e]$

$$\boxed{\begin{aligned} [L_a, P_b] &= i\hbar\, \epsilon_{abc} P_c \\ [L_a, R_b] &= i\hbar\, \epsilon_{abc} R_c \\ [L_a, L_b] &= i\hbar\, \epsilon_{abc} L_c \end{aligned}}$$

Gén. inf. des rotations

$\epsilon_{abc} \epsilon_{ade} = \delta_{bd} \delta_{ce} - \delta_{be} \delta_{cd}$

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$$[J_a, J_b] = i\hbar\,\epsilon_{abc} J_c$$

$$J_1,\ J_2,\ J_3 \qquad J^2 = J_1^2 + J_2^2 + J_3^2$$

$$[J^2, J_a] = 0$$

$$|j\,m\rangle \longleftrightarrow J^2,\ J_3$$

Théorème des commutateurs d'opérateurs d'échelle

Si $\boxed{[A, B] = \beta B}$ et $A|\alpha\rangle = \alpha|\alpha\rangle$

alors $B|\alpha\rangle$ est un état propre de A de valeur propre $\alpha + \beta$

Preuve

$$(AB - BA)|\alpha\rangle = \beta B|\alpha\rangle$$

$$AB|\alpha\rangle - \alpha B|\alpha\rangle = \beta B|\alpha\rangle$$

$$A(B|\alpha\rangle) = (\alpha + \beta)(B|\alpha\rangle)$$

Pour J

$$\left[J_3, (J_1 \pm iJ_2)\right] = \pm \hbar \left[J_1 \pm iJ_2\right]$$

$$[J_3, J_1] = + iJ_2$$

$$\boxed{J_\pm = J_1 \pm iJ_2}$$

$$[J_3, J_\pm] = \pm \hbar J_\pm$$

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$$\langle j\,\bar m|\, \underbrace{J_+\ J_-}\,|j\,\bar m\rangle = 0$$

$$\rightarrow\ J^2 = J_3^2 + \frac{1}{2}\left(J_+J_- + \underbrace{J_-J_+}\right)$$

$J_+J_- - J_-J_+ = 2\hbar J_3$ $[J_+, J_-] = 2\hbar J_3$

$$J^2 = J_3^2 + \left(J_+J_- - \hbar J_3\right)$$

$$\langle j\,\bar m|\, J^2 - J_3^2 + \hbar J_3\,|j\,\bar m\rangle$$

$$J_3\,|j\,\bar m\rangle = \bar m\hbar\,|j\,\bar m\rangle$$

$$\langle j\,\bar m|\, J^2 - \hbar^2\bar m^2 + \hbar^2\bar m\,|j\,\bar m\rangle = 0$$

$$\langle j\,\bar m|\, J^2\,|j\,\bar m\rangle = \hbar^2(\bar m^2 - \bar m)$$

$$\bar m \equiv -j$$

$$\Longrightarrow\ J^2\,|j\,\bar m\rangle = \hbar^2\, j(j+1)\,|j\,\bar m\rangle$$

$$[J^2, J_\pm] = 0$$

$$\langle j\,m|\, J_-\,J_+\,|j\,m\rangle = |C|^2\langle j\,m{+}1|j\,m{+}1\rangle$$

$$|C|^2 = \Big(j(j+1) - m(m+1)\Big)\hbar^2$$

$$-j < m < j \qquad 2j = \text{entier}$$

$$(2j+1)$$

$$J_+\,|j\,m\rangle = C\,|j\ m{+}1\rangle$$

$$= \hbar\sqrt{j(j+1) - m(m+1)}\,|j\ m{+}1\rangle$$