$F = |\uparrow\uparrow\uparrow....\rangle$ $\quad n=1$

$AF = |\uparrow\downarrow\underbrace{\uparrow}_{}\downarrow ....\rangle$ $\quad S_i^z S_j^z$ $\quad n=1$

$\boxed{\left(U\sum_i n_{i\uparrow}n_{i\downarrow}\right) = H_0}$ $\quad$ —— $E=U$

Interaction Coulomb
1^{er} voisin

$|\alpha\rangle$ —— $E=0$ $\quad 2^N$

$$\langle\alpha|\, H_0 + H_1\, |\alpha'\rangle$$

Heisenberg ferro: $\boxed{-|J|\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j}$

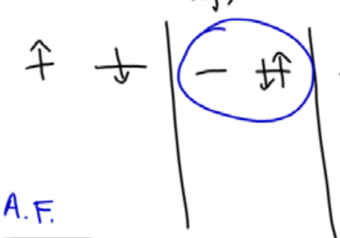
$$H_1 = +\sum_{\langle ij\rangle,\sigma} t_{ij}\left(c^{\dagger}_{i\sigma}c_{j\sigma} + c.h.\right)$$

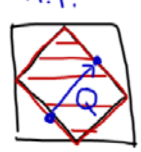
1^{er} voisins.

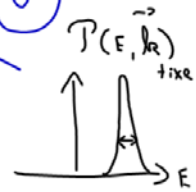
$$\boxed{|\psi\rangle = |\alpha\rangle + \sum_{m\neq\alpha}\frac{|m\rangle\langle m|H_1|\alpha\rangle}{E_\alpha - E_m}}$$

$$\langle\psi'|\, H_0 + H_1\, |\psi\rangle$$

$$J_{positif}\ \left(\frac{4t^2}{U}\right)\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$$







$8t$ $\quad d=2$

$12t$ $\quad d=3$

Heisenberg ferro:

Transfo. de Holstein-Primakov:

$$\boxed{\begin{aligned} S_i^z &= s - a_i^+ a_i \\ S_i^+ &= \sqrt{2s - a_i^+ a_i}\, a_i \checkmark \\ S_i^- &= a_i^+ \sqrt{2s - a_i^+ a_i} \end{aligned}}$$

$$[a_i, a_j^+] = \delta_{ij}$$
$$[a_i, a_j] = 0$$

Preuve: $[S^z, S^+] = S^+$

$$\boxed{\underbrace{(s - a^+a)\sqrt{2s - a^+a}}_{①}\; a} - \sqrt{2s - a^+a}\; a\,(s - a^+a)$$

$$(-a a^+) a = -a - a^+ a a$$

$$a a^+ - a^+ a = 1$$
$$a a^+ = 1 + a^+ a$$

$$\boxed{-\underbrace{\sqrt{2s - a^+a}\,(s - a^+a}_{①}{} - 1)\, a} = \sqrt{2s - a^+a}\; a$$
$$= S^+$$

$$[S^-, S^z] = S^- \rightarrow [S^z, S^-] = -S^-$$

$$\boxed{[S^+, S^-] = 2S^z} \qquad a a^+ = 1 + a^+ a$$

$$\sqrt{2s - a^+a}\,(a^+a + 1)\sqrt{2s - a^+a}$$

$$- a^+ \quad (2s - a^+a) \quad a$$

$$= (2s - a^+a) + \boxed{a^+ \quad (2s - a^+a - 1) \quad a}$$

$$- \boxed{a^+(2s - a^+a)a}$$

$$\left\{ a n = (n+1) a \right. \qquad \left\{ \begin{array}{l} [n, a^+] = a^+ \\ \boxed{\begin{array}{l} n a^+ = a^+ + a^+ n \\ \quad = a^+ (n+1) \end{array}} \end{array} \right.$$

$$= (2s - a^+a) - a^+a = 2(s - a^+a)$$

Développement S grand

(approx. semi-classique)

$$2s \gg a^+ a$$

$$S^z = s - a^+ a$$

$$S^+ \simeq \sqrt{2s}\, a$$

$$S^- \simeq \sqrt{2s}\, a^+$$

$$\mathcal{H} = -|J| \sum_{R_i R_j} \left[S^z_{R_i} S^z_{R_j} + \frac{1}{2}\left(S^+_{R_i} S^-_{R_j} + S^-_{R_i} S^+_{R_j} \right) \right.$$

$$= -|J| \sum_{\langle R_i R_j \rangle} \left[\left(s - a^\dagger_{R_i} a_{R_i}\right)\left(s - a^\dagger_{R_j} a_{R_j}\right) \right.$$

$$\left. + \frac{2s}{2}\left(a^\dagger_{R_i} a_{R_j} + a^\dagger_{R_j} a_{R_i} \right) \right]$$

$$\boxed{a_R = \sqrt{V_r} \int_{\text{Z.B.}} \frac{d^3k}{(2\pi)^3}\, e^{+i\vec{k}\cdot\vec{R}}\, a(\vec{k})}$$

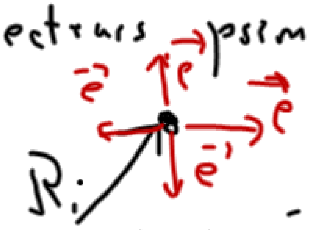
(d^3k)

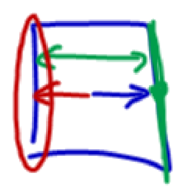
$$\sum_{\langle R_i R_j \rangle} a^+_{R_i} a_{R_j}$$

$$= V_r \sum_{\langle R_i R_j \rangle} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \, e^{-i\vec{k}\cdot\vec{R}_i + i\vec{k}'\cdot\vec{R}_j} \, a^+(k)\, a(k')$$

$$= \cancel{V_r} \, \frac{1}{2} \sum_{R_i} \sum_{\vec{e}} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \, e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_i + i\vec{k}'\cdot\vec{e}} \, a^+(k)\, a(k')$$

$\vec{e}$ = vecteurs premiers voisins.





$$\sum_{R_i} e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_i} = \frac{(2\pi)^3}{\cancel{V_r}} \sum_{\vec{K}} \delta(\vec{k}-\vec{k}'-\vec{K})$$

$$= \frac{1}{2} \sum_{\vec{e}} \int \frac{d^3k}{(2\pi)^3} \, e^{i\vec{k}\cdot\vec{e}} \, a^+(k)\, a(k)$$

$$H = -|J|\, s\, \frac{1}{2} \sum_{\vec{e}} \int \frac{d^3k}{(2\pi)^3} \left(e^{i\vec{k}\cdot\vec{e}} + e^{-i\vec{k}\cdot\vec{e}} \right) \left[a^+(k)\, a(k) \right)$$

$$-|J|(-)s \; 2 \; \frac{1}{2} \sum_{\vec{e}} \int \frac{d^3k}{(2\pi)^3} \, a^+(k)\, a(k) \leftarrow$$

$$\boxed{= |J|\, s\, Z \int \frac{d^3k}{(2\pi)^3} \left(1 - \gamma(k) \right) a^+(k)\, a(k)}$$

$$\gamma(\vec{k}) \equiv \frac{1}{Z} \sum_{\vec{e}} \cos \vec{k}\cdot\vec{e}$$

<u>Vide</u> : Fond : $a(k)|0\rangle = 0 \quad \forall k$

$$\hbar\omega(k) \propto k^2$$

$$H = \sum_n E_n\, a_n^+ a_n = \int \frac{d^3k}{(2\pi)^3} \left[\hbar\omega(k) \right] a^+(k)\, a(k)$$

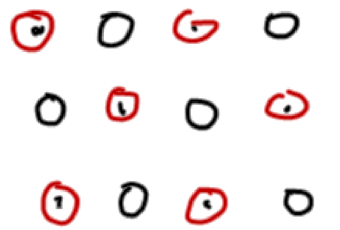
État excité : $\boxed{a^+(k)|0\rangle}$

$$\left[a(k), a^+(k') \right] = (2\pi)^3 \delta(k - k')$$

$$\boxed{H = \int \frac{d^3k'}{(2\pi)^3} a^+(k')\, a(k')\, \hbar\omega(k')}$$

Antiferroaimant :

Réseau bipartite :



Γ_1

$$\boxed{H = J \sum_{\vec{R}\in\Gamma_1} \vec{S}_{\vec{R}} \cdot \sum_{\vec{e}} \vec{S}_{\vec{R}+\vec{e}}}$$

Sous-réseau up (Γ_1) a, a^+ comme ferro

" " down (Γ_2)

$$\left.\begin{aligned} S^z_R &= -S + b^+_R b_R \\ S^+_R &= b^+_R \sqrt{2S - b^+_R b_R} \\ S^-_R &= \sqrt{2S - b^+_R b_R}\, b_R \end{aligned}\right\}$$

État de Néel :

$$\boxed{a_R |0\rangle = 0 \quad \Big| \quad b_R |0\rangle = 0}$$

$$H = sJz\left\{\sum_{R\in\Gamma_1} a^+_R a_R + \sum_{R\in\Gamma_2} b^+_R b_R\right.$$

$$\left. + sJ \sum_{R\in\Gamma_1} \sum_{\vec{e}} \left(a^+_{\vec{R}} b^+_{\vec{R}+\vec{e}} + a_{\vec{R}} b_{\vec{R}+\vec{e}} \right) \right\}$$

$$H|0\rangle \neq |0\rangle \, cte$$

$$b_R = \sqrt{V_\Gamma} \int_{ZBM} \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{R}} \, b(\vec{k})$$

$$H = sJ_z \int \frac{d^3k}{(2\pi)^3} \left[a^+(k)a(k) + b^+(k)b(k) \right.$$

$$z\beta M \qquad \left. + \gamma(k)\left[a^+(k)b^+(k) + a(k)b(k)\right] \right]$$

$$H = sJ_z \int \frac{d^3k}{(2\pi)^3} \begin{pmatrix} a^+(k) & b(k) \end{pmatrix} \boxed{\begin{pmatrix} 1 & \gamma(k) \\ \gamma(k) & 1 \end{pmatrix}} \begin{pmatrix} a(k) \\ b^+(k) \end{pmatrix}$$

$$= sJs \int \frac{d^3k}{(2\pi)^3} \left[a^+(k) \quad b(k)\right] \boxed{M^+(\theta_k)\, D(k)\, M(\theta_k)} \begin{pmatrix} a(k) \\ b^+(k) \end{pmatrix}$$

$$\begin{pmatrix} \alpha(k) \\ \beta^+(k) \end{pmatrix} = M(\theta_k) \begin{pmatrix} a(k) \\ b^+(k) \end{pmatrix}$$

$$\left[\alpha^+(k) \quad \beta(k)\right] D(k) \begin{bmatrix} \alpha(k) \\ \beta^+(k) \end{bmatrix}$$

$$\alpha^+(k)\alpha(k)\, D_{11}$$

$$+ \beta^+(k)\beta(k)\, D_{22}$$

$$\alpha(k)|\Omega\rangle = 0$$

$$\beta(k)|\Omega\rangle = 0$$

$$\boxed{\langle \Omega | a^+_k \, a_k \, . | \Omega \rangle} \qquad c.\langle \Omega | \beta \beta^+ | \Omega \rangle \neq 0$$

$$\langle 0 | a^+(k) a(k) | 0 \rangle = 0$$

$$\begin{pmatrix} a(k) \\ b^+(k) \end{pmatrix} \to \begin{pmatrix} c_1(k) \\ c_2(k) \end{pmatrix}$$

$$[c_i, c_j^+] = (\sigma_3)_{ij} \to \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

avr. 3-10:07

$$\begin{pmatrix} dx & d\tau \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} dx \\ d\tau \end{pmatrix}$$

$$dx^2 - d\tau^2$$