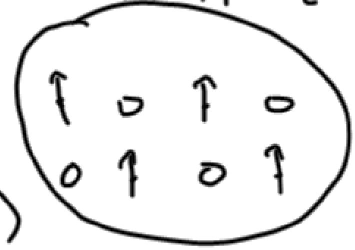
$$\mathcal{H} = J \sum_{\langle R, R' \rangle} \vec{S}_{\vec{R}} \cdot \vec{S}_{R'} = J \sum_{\vec{R} \in \Gamma_1} \vec{S}_{\vec{R}} \cdot \sum_{\vec{e}} \vec{S}_{\vec{R}+\vec{e}}$$



$$\textcircled{\uparrow} \begin{cases} S^z = s - a^+ a \\ S^+ \simeq \sqrt{2s}\, a \quad (\text{d.v. } \tfrac{1}{s}) \\ S^- \simeq a^+ \sqrt{2s} \end{cases}$$

$$\textcircled{\downarrow} \begin{cases} S^z = -s + b^+ b \\ S^+ \simeq \sqrt{2s}\, b^+ \\ S^- \simeq \sqrt{2s}\, b \end{cases}$$

$$H = J \sum_{\vec{R} \in \Gamma_1} \sum_{\vec{e}} \left(s - a^+_{\vec{R}} a_{\vec{R}}\right)\left(-s + b^+_{\vec{R}+\vec{e}} b_{\vec{R}+\vec{e}}\right)$$

$$+ \left(\frac{1}{2} 2s \left(a^+_{\vec{R}} b^+_{\vec{R}+\vec{e}} + a_{\vec{R}} b_{\vec{R}+\vec{e}}\right)\right)$$

T.F.

$$\boxed{\mathcal{H} = sJz \int_{ZBM} \frac{d^3k}{(2\pi)^3} \left(a^+(k) \;\; b(k)\right) \begin{pmatrix} 1 & \gamma(k) \\ \gamma(k) & 1 \end{pmatrix} \begin{pmatrix} a(k) \\ + \\ b(k) \end{pmatrix}}$$

$$(a^+ \;\; b)\boxed{\underbrace{M^+ D M}_{\underbrace{\begin{pmatrix} 1 & \gamma(k) \\ \gamma(k) & 1 \end{pmatrix}}_{L}}}\begin{pmatrix} a \\ b^+ \end{pmatrix}$$

$$L = M^+ D M$$

$$(M^+)^{-1} L M^{-1} = D$$

$$\begin{pmatrix} \alpha \\ \beta^+ \end{pmatrix} = M \begin{pmatrix} a \\ b^+ \end{pmatrix} \qquad [\alpha, \alpha^+] = 1 \qquad [\beta, \beta^+] = 1$$

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = M \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \boxed{d_i = M_{ik} c_k} \qquad \boxed{d_i^+ = c_k^+ M_{ki}^+}$$

$$[c_i, c_j^+] = (\sigma_3)_{ij}$$

$$[c_1, c_1^+] = [a, a^+] = 1$$

$$[c_2, c_2^+] = [b^+, b] = -1$$

$$\boxed{[d_i, d_j^+] = (\sigma_3)_{ij}} \qquad M_{ik}[c_k, c_l^+] M_{lj}^+ = (\sigma_3)_{ij}$$

$$\boxed{M_{ik}(\sigma_3)_{kl} M_{lj}^+ = (\sigma_3)_{ij}}$$

$$(dx \;\; dt)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} dx \\ dt \end{pmatrix} = dx^2 - dt^2$$

$$\left[M(\theta) = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \right] \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M\sigma_3 M^+ = \begin{pmatrix} \cosh\theta & -\sinh\theta \\ \sinh\theta & -\cosh\theta \end{pmatrix}\begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cosh^2\theta - \sinh^2\theta & 0 \\ 0 & -\cosh^2\theta + \sinh^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_3$$

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$$\left[M \, \underline{e^{i[\varphi I + \varphi' \sigma_3]}} \right] \sigma_3 \left[\underline{e^{-i[\varphi I + \varphi' \sigma_3]}} \, M^{+} \right]$$

$$= \sigma_3$$

$$\boxed{M(\theta_1)\, M(\theta_2) = M(\theta_1 + \theta_2)}$$

$$\begin{pmatrix} \cosh\theta_1 & \sinh\theta_1 \\ \sinh\theta_1 & \cosh\theta_1 \end{pmatrix} \begin{pmatrix} \cosh\theta_2 & \sinh\theta_2 \\ \sinh\theta_2 & \cosh\theta_2 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\theta_1 \cosh\theta_2 + \sinh\theta_1 \sinh\theta_2 & X \\ \sinh\theta_1 \cosh\theta_2 + \cosh\theta_1 \sinh\theta_2 & X \end{pmatrix}$$

$$D = (M^{+})^{-1} \, L \, (M^{-1})$$

$$L = \begin{pmatrix} 1 & \gamma(k) \\ \gamma(k) & 1 \end{pmatrix} = \sqrt{1-\gamma^2(k)} \; M(\eta_k)$$

$$= \sqrt{1-\gamma^2(k)} \begin{pmatrix} \frac{1}{\sqrt{}} & \frac{\gamma(k)}{\sqrt{}} \\ \frac{\gamma(k)}{\sqrt{}} & \frac{1}{\sqrt{}} \end{pmatrix}$$

$$\cosh \eta_k = \frac{1}{\sqrt{1-\gamma^2(k)}}$$

$$\sinh \eta_k = \frac{\gamma(k)}{\sqrt{1-\gamma^2(k)}}$$

$$\boxed{\cosh^2 \eta_k - \sinh^2 \eta_k = 1}$$

$$D(k) = \left(M^{+}(\theta_k)\right)^{-1} \underbrace{\sqrt{1-\gamma^2(k)} \; M(\eta_k)}_{L}$$

$$M(\theta_k)^{-1}$$

$$M = M^{+}$$

$$M^{-1}(\theta_k) = M(-\theta_k)$$

$$M^{-1} M = 1 \qquad M^{-1}(\theta) M(\theta) = M(0)$$

$$D(k) = M(-\theta_k) M(\eta_k) M(-\theta_k) \sqrt{1-\gamma^2(k)}$$

$$\eta_k - 2\theta_k = 0$$

$$= I \sqrt{1-\gamma^2(k)}$$

$$\mathcal{H} = sJz \int \frac{d^3k}{(2\pi)^3} \begin{pmatrix} \alpha^\dagger(k) & \beta(k) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha(k) \\ \beta^\dagger(k) \end{pmatrix}$$

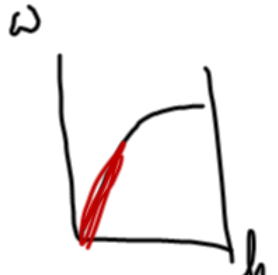
$$\times \sqrt{1-\gamma^2(k)}$$

$$= \int \frac{d^3k}{(2\pi)^3}\, sJz \sqrt{1-\gamma^2(k)} \left(\alpha^\dagger(k)\alpha(k) + \beta^\dagger(k)\beta(k) \right)$$

$$\boxed{\hbar\omega(k) = sJz \sqrt{1-\gamma^2(k)}}$$

$$\gamma(k) = \frac{1}{z} \sum_{\vec{e}} \cos \vec{k}\cdot\vec{e}$$

$$= \frac{2}{z} \left(\cos k_x a + \cos k_y a + \cos k_z a \right)$$



$$\frac{\hbar\omega(k)}{sJz} = \sqrt{(1-\gamma(k))(1+\gamma(k))}$$

$$\sim \sqrt{2}\sqrt{1-\gamma(k)}$$

$$\sim \cancel{\sqrt{2}} \sqrt{\frac{1}{3}\left[\frac{k_x^2 a^2}{\cancel{2}} + \frac{k_y^2 a^2}{2} + \frac{k_z^2 a^2}{2} \right]}$$

$$= \frac{1}{\sqrt{3}} \sqrt{k^2 a^2} = \frac{1}{\sqrt{3}} ka$$

$$S^z_{tot} = -\int_{ZBM} \frac{d^3k}{(2\pi)^3} \left[\alpha^+(k)\alpha(k) - \beta^+(k)\beta(k)\right]$$

Fondamental : $\alpha(k)|\Omega\rangle = 0$

$\beta(k)|\Omega\rangle = 0$

$$\begin{pmatrix}\alpha_k \\ \beta^+_k\end{pmatrix} = \begin{pmatrix}\overset{u}{\cosh\theta_k} & \overset{v}{\sinh\theta_k} \\ \overset{v}{\sinh\theta_k} & \overset{u}{\cosh\theta_k}\end{pmatrix}\begin{pmatrix}a_k \\ b^+_k\end{pmatrix}$$

$$\boxed{\alpha_k = u_k a_k + v_k b^+_k}$$

$$\boxed{\begin{pmatrix}a_k \\ b^+_k\end{pmatrix} = \begin{pmatrix}u_k & -v_k \\ -v_k & u_k\end{pmatrix}\begin{pmatrix}\alpha_k \\ \beta^+_k\end{pmatrix}}$$

$\langle\Omega| a^+_k a_k |\Omega\rangle \neq 0 \qquad a_k|0\rangle = 0$

$b_k|0\rangle = 0$

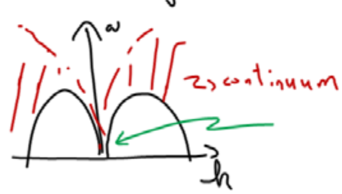
$$S^z_i = \langle\Omega| S - a^+_i a_i |\Omega\rangle$$

$$\langle S^z\rangle \to \infty$$

Exact: $d=1$ (Bethe)

$$\epsilon(k) = \frac{J}{2}\pi \sin k \qquad 0<k<\pi$$

Fadeev + Takhtajan 81



Excitations élémentaires

→ Spinon $\boxed{S=1/2}$ $\boxed{q=0}$

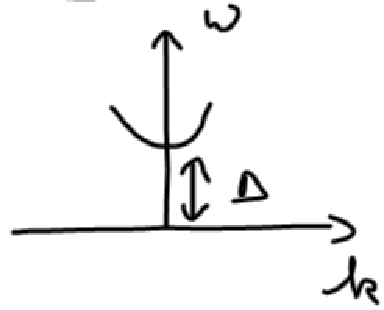
fractionalisation → Holon $\boxed{S=0}$ $\boxed{q=1}$

↓ ↑ ↓ — ↓ ↑ ↓

↓ ↑ ↓ ⊗ ↓ 0 ↑ ↓

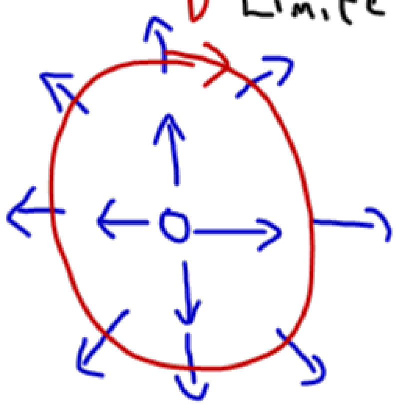
↓ ⊗ ↓ ↑ ↓ ↑ 0 ↓

$\boxed{S = 1}$

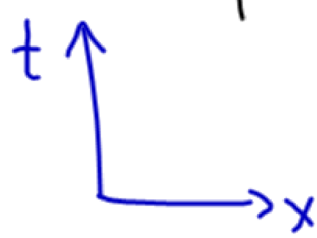


Gap de Haldane

→ Limite du continu



Termes topologiques



$$O_4 \quad |x_{p(1)}\rangle\, |x_{p(2)}\rangle\, |x_{p(3)}\rangle\, |x_{p(4)}\rangle\, |x_{p(5)}\rangle$$

$$|x_{p(1)}\rangle\, |x_{p(2)}\rangle\, |x_{p(3)}\rangle\, O_4\, |x_{p(4)}\rangle\, |x_{p(5)}\rangle$$

$$O_4 |x_4\rangle = \boxed{\int dy\, |y\rangle \langle y| O_4 |x_4\rangle}$$

Etats cohérents

$|\Omega\rangle$ relation $|0\rangle$

$\alpha(k)|\Omega\rangle = 0 \qquad a(k)|0\rangle = 0$

$\beta(k)|\Omega\rangle = 0 \qquad b(k)|0\rangle = 0$

$$\alpha_k = u_k a_k - v_k b^+_k$$

$$(u_k a_k - v_k b^+_k)\left(\prod_q e^{\frac{v_q}{u_q} a^+_q b^+_q}|0\rangle\right) = 0$$

$$(u_k b_k - v_k a^+_k)\left(\prod_q e^{\frac{v_q}{u_q} a^+_q b^+_q}|0\rangle\right) = 0$$

$$\beta_k = (u_k b_k - v_k a^+_k)$$

$$a\left(e^{z a^+}|0\rangle\right) = z\left(e^{z a^+}|0\rangle\right)$$

Définition des états cohérents :

à prouver

$$\boxed{a|z\rangle = z|z\rangle} \qquad \boxed{\langle z|z\rangle = 1}$$

où

$$|z\rangle = e^{-|z|^2/2}\, e^{z a^\dagger}\, |0\rangle$$

$$|z\rangle = e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{(z a^\dagger)^n}{n!}\, |0\rangle$$

(m) $$\boxed{|z\rangle = e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}}\, |n\rangle}$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}\, |0\rangle \qquad z = |z|\, e^{i\varphi}$$

Laser

$$\langle z|N^2|z\rangle \longrightarrow \neq |\langle z|N|z\rangle|^2$$

Antenne

Preuve que $a|z\rangle = z|z\rangle$

$$[a, e^{za^+}] = z e^{za^+}$$

$$a|0\rangle = 0$$

$\left(e^{za^+}|0\rangle\right) \longrightarrow$ état propre de a de valeur propre

$$(0+z)$$

$$[a, (a^+)^n] = n (a^+)^{n-1}$$

Preuve par induction

$n = 1 \qquad [a, a^+] = 1$

$$[a, (a^+)^n] = [a, a^+ (a^+)^{n-1}]$$

$$= [a, a^+] (a^+)^{n-1} + a^+ [a, (a^+)^{n-1}]$$

$$= (a^+)^{n-1} + a^+ (n-1) (a^+)^{n-2}$$

$$= n (a^+)^{n-1}$$

$$[a, f(a^+)] = \frac{\partial f}{\partial a^+}$$ Formel

$$[a, e^{z a^+}] = z e^{z a^+}$$

$$\langle z|z\rangle = e^{-|z|^2/2}\langle 0|e^{z^*\underline{a}}\,|\underline{z}\rangle$$

$$\langle z| = \langle 0|\, e^{z^* a}\, e^{-|z|^2/2}$$

$$= e^{-|z|^2/2}\, e^{|z|^2}\,\langle 0|z\rangle$$

$$= e^{-|z|^2/2}\, e^{|z|^2}\, e^{-|z|^2/2} = 1$$

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$$\langle w | z \rangle \neq 0$$

$$\boxed{I = \frac{1}{2\pi i}\int dz\,dz^* \; |z\rangle\langle z|}$$

David Sénéchal
Chap. 1

$$\langle m | \left[\frac{1}{2\pi i}\int dz\,dz^* |z\rangle\langle z|\right] | n \rangle = \delta_{m,n}$$

$$= \frac{1}{\sqrt{m!}}\frac{1}{\sqrt{n!}}\int \frac{dz\,dz^*}{2\pi i} e^{-|z|^2} z^{*m} z^n$$

$z = x + iy$
$z^* = x - iy$

$$\boxed{dz\,dz^* = 2i\,dx\,dy}$$

$$\left|\begin{matrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \\ \frac{\partial z^*}{\partial x} & \frac{\partial z^*}{\partial y} \end{matrix}\right| = \left|\begin{matrix} 1 & i \\ 1 & -i \end{matrix}\right| = (-2i)$$

$$= \frac{1}{\sqrt{m!\,n!}}\int_0^\infty \frac{r\,dr}{\pi}\int_0^{2\pi} d\varphi\; e^{-r^2} r^{m+n} e^{-im\varphi} e^{in\varphi}$$

$$= \frac{\delta_{nm}\,\cancel{2\pi}}{n!}\frac{1}{\cancel{2\pi}}\int_0^\infty d(r^2)\,e^{-r^2} r^{2n}$$

$$= \frac{\delta_{n,m}}{n!}\int_0^\infty dx\,e^{-x} x^n = \delta_{n,m}$$