1- Condensation de Bose-Einstein
T finie.

2- Effet des interactions ?

3- OHDLP – ODLRO
état cohérents

4- Superfluidité

5- Pièges magnétiques + optiques

6- Equ. Gros - Pitaevskii

1. B. E.

$$|\Omega\rangle = \frac{\left(a^{+}_{k=0}\right)^{N}}{\sqrt{N!}}|0\rangle \leftarrow$$

$$n_k = \frac{1}{e^{\beta(\epsilon_k-\mu)}-1} = \langle a^{+}_k a_k\rangle$$

$$\epsilon_k = \frac{\hbar^2 k^2}{2m} \leftarrow$$

$$N = \sum_k n_k = \sum_k \frac{1}{e^{\beta(\epsilon_k-\mu)}-1}$$

$N_0 \equiv$ # dans fond.

$$N_0 = \frac{1}{e^{-\beta\mu}-1} = \frac{1}{-\beta\mu}$$

$$\mu = -\frac{k_B T}{N_0} \longrightarrow 0$$

$$\epsilon_{k_1} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2$$

$$n_{k_1} = \frac{1}{e^{-\beta\left(\frac{\hbar^2}{2m}\frac{(2\pi)^2}{L^2} - \frac{k_B T}{L^3 n_0}\right)}-1}$$

$$N = N_0 + \sum_{k \neq 0} \frac{1}{e^{\beta(\epsilon_k - \mu)} - 1}$$

$T > T_c$ ($N_0 = 0$)

$$N = N_0 + V \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta \epsilon_k} - 1}$$

$$= N_0 + V \frac{4\pi}{(2\pi)^3} \int_0^\infty k \, k\,dk \frac{1}{e^{\beta\epsilon_k} - 1}$$

$$\epsilon = \frac{\hbar^2 k^2}{2m} \qquad d\epsilon = \frac{\hbar^2 k \, dk}{m}$$

$$k = \sqrt{\frac{2m\epsilon}{\hbar^2}}$$

$$N = N_0 + V \frac{1}{\cancel{2\pi^2}} \int_0^\infty \frac{m}{\cancel{\hbar^2}} d\epsilon \sqrt{\frac{2m}{\cancel{\hbar^2}}\epsilon} \frac{1}{e^{\beta\epsilon} - 1}$$

$$n = n_0 + \left(\frac{m}{\hbar^2}\right)^{3/2} \frac{1}{\sqrt{2}\,\pi^2} \int_0^\infty \frac{\beta \epsilon^{1/2} d\epsilon}{\beta^{3/2}} \frac{1}{e^{\beta\epsilon} - 1}$$

$y \equiv \beta\epsilon$

$$= n_0 + \left(\frac{m k_B T}{\hbar^2 2\pi}\right)^{3/2} \qquad \zeta\left(\frac{3}{2}\right)$$

$$\int_0^\infty dy \, y^{1/2} \frac{e^{-y}}{1 - e^{-y}}$$

$$= \int_0^\infty dy \, y^{1/2} e^{-y} \left(1 + e^{-y} + e^{-2y} + e^{-3y} + \dots\right)$$

$$= \int_0^\infty dy \, y^{1/2} e^{-y} \sum_{p=0}^\infty e^{-py} \qquad \frac{p^{1/2}}{p^{3/2}}$$

$$= \sum_{p=1}^\infty \frac{1}{p^{3/2}} \int_0^\infty dx \, x^{1/2} e^{-x}$$

$$= \zeta\left(\frac{3}{2}\right) \Gamma\left(\frac{3}{2}\right)$$

$$= 2.612 \frac{\sqrt{\pi}}{2}$$

$$n = n_0 + \left(\frac{k_B T m}{2\pi \hbar^2}\right)^{3/2} \zeta(3/2)$$

fixe

change avec T

$$\frac{n_0}{n} = 1 - \underbrace{\boxed{\frac{1}{n}\left(\frac{k_B T m}{2\pi \hbar^2}\right)^{3/2} \zeta(3/2)}}_{=1}$$

$$\frac{k_B T_c m}{2\pi \hbar^2} = \left(\frac{n}{\zeta(3/2)}\right)^{2/3}$$

$$\boxed{k_B T_c = \frac{2\pi \hbar^2}{m}\left(\frac{n}{\zeta(3/2)}\right)^{2/3}}$$

$$\boxed{\frac{n_0}{n} = 1 - \left(\frac{T}{T_c}\right)^{3/2}}$$

2. Stabilité sous l'effet des interactions

$$|\Omega_N\rangle = \frac{(a^+_{k=0})^N}{\sqrt{N!}}|0\rangle \qquad \text{CBE}$$

$$|\Omega'_{N/2,N/2}\rangle = \left(\frac{a^+_{k=0}}{\sqrt{\frac{N}{2}!}}\right)^{N/2}\left(\frac{a^+_{k=k_1}}{\sqrt{\frac{N}{2}!}}\right)^{N/2}|0\rangle$$

$$H_{int} = \frac{1}{2}\int d^3r\, d^3r'\, U(\vec r - \vec r')\, \psi^+(\vec r)\psi^+(\vec r')\psi(\vec r')\psi(\vec r)$$

$\langle\Omega_N|H_{int}|\Omega_N\rangle$

$\langle\Omega'_{N/2,N/2}|H_{int}|\Omega'_{N/2,N/2}\rangle$

$a|n\rangle = \sqrt{n}\,|n-1\rangle$

$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$$\boxed{\psi(\vec r) = \frac{1}{\sqrt{V}}\sum_k e^{i\vec k\cdot\vec r} a_k} \quad k_1$$

$$\psi(\vec r)|\Omega_N\rangle = \sqrt{\frac{N}{V}}\,|\Omega_{N-1}\rangle$$

$$\psi(r')\psi(r)|\Omega_N\rangle = n\,|\Omega_{N-2}\rangle$$

$$\langle\Omega_N|\psi^+\psi^+\psi\psi|\Omega_N\rangle = n^2$$

$$\boxed{\frac{1}{2}\int d^3r\, d^3r'\, U(r-r')}\, n^2 = Vg\, n^2$$

$$\psi(r)|\Omega'_{N/2,N/2}\rangle = \sqrt{\frac{n}{2}}\,|\Omega'_{\frac{N}{2}-1,\frac{N}{2}}\rangle + \sqrt{\frac{n}{2}}\,e^{ik_1\cdot r}|\Omega'_{\frac{N}{2},\frac{N}{2}-1}\rangle$$

$$\psi^+(r)\psi(r)|\Omega'_{\frac{N}{2},\frac{N}{2}}\rangle$$

$$= \frac{n}{2}|\Omega'_{\frac{N}{2},\frac{N}{2}}\rangle + \frac{n}{2}e^{-ik_1\cdot r}|\Omega'_{\frac{N}{2}-1,\frac{N}{2}+1}\rangle$$

$$\frac{n}{2}e^{ik_1\cdot r}|\Omega'_{\frac{N}{2}+1,\frac{N}{2}-1}\rangle + \frac{n}{2}e^{ik_1\cdot r - ik_1\cdot r}|\Omega'_{\frac{N}{2},\frac{N}{2}}\rangle$$

$$\left(\langle\Omega'_{\frac{N}{2},\frac{N}{2}}|\psi^+(r')\psi(r')\right)\left(\psi^+(r)\psi(r)|\Omega'_{\frac{N}{2},\frac{N}{2}}\rangle\right)$$

$$= n^2 + \left(\frac{n}{2}\right)^2 e^{ik_1\cdot(r'-r)} + \left(\frac{n}{2}\right)^2 e^{-ik_1\cdot(r'-r)}$$

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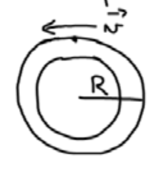
4. Superfluidité, vitesse critique



$$L = \frac{1}{2} I \omega^2$$

$$I = m R^2$$

Du pt de vue des parois ?

$$(\vec{p} - m\vec{v})^2 = (\vec{p}\,' - m\vec{v})^2$$

↑ particule dans le Lab.

$$p^2 - 2\vec{p}\cdot m\vec{v} + \cancel{(mv)^2} = p'^2 - 2\vec{p}\,'\cdot m\vec{v} + \cancel{(mv)^2}$$

$p = 0$ pour condensat.

$$\boxed{p'^2 = 2\vec{p}\,'\cdot m\vec{v}}$$

Pour obtenir un superfluide rajouter les interactions

$$\Psi(\vec{r}) = \underbrace{\Psi_0}_{\text{Condensat}} + \underbrace{\delta\Psi(\vec{r})}_{\text{Partie add.}}$$

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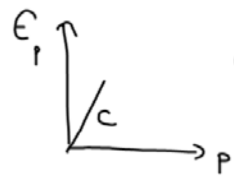
à l'ordre $(\delta\Psi)^2$ (e.g. $\Psi_0^2(\delta\Psi)^2$?)

$$a^+ a, \quad a^+ a^+, \quad a a$$

Transformation de Bogoliubov

→ on trouve



$$\epsilon_p = c|p|$$

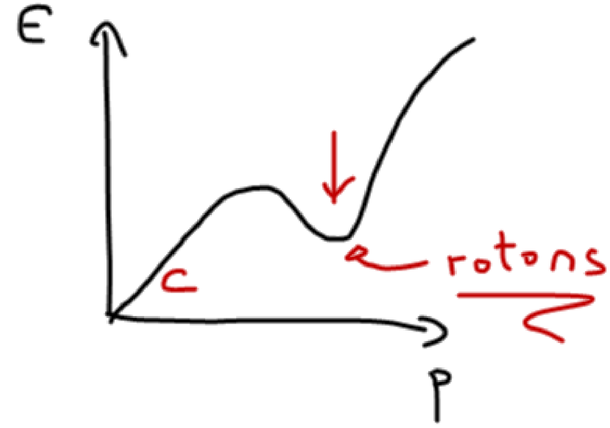
où $c = \left(\frac{\hbar^2 n_0 g}{m}\right)^{1/2}$ ←

$$U(r - r') = g\,\delta(r - r')$$

Vitesse critique $v_c \equiv c$

Pour $v < v_c$

^4He



3. Ordre hors diagonal à longue portée

Condensat de B.E.

$$\lim_{r-r'\to\infty} \langle \Omega_N | \psi^\dagger(\vec{r}) \psi(\vec{r}') | \Omega_N \rangle$$

$$\langle \Omega_N | \frac{1}{V} \sum_k \sum_{k'} e^{-ik\cdot r + ik'\cdot r'} a^\dagger_k a_{k'} | \Omega_N \rangle = n_0$$

Fluide normal :

$$\underbrace{\langle \Omega | \frac{1}{V} \sum_k e^{-i(r-r')\cdot k} a^\dagger_k a_k | \Omega \rangle}$$

$$= \frac{1}{V} \sum_k e^{-ik\cdot(r-r')} n_k = n(\vec{r}-\vec{r}') \underset{r-r'\to\infty}{=} 0$$

Dans l'état superfluide, $\psi(\vec{r})$ a une valeur moyenne non nulle

$$\boxed{\langle \Omega_{s,\varphi} | \psi(r) | \Omega_{s,\varphi} \rangle = \sqrt{n_0}\, e^{i\varphi}}$$

Avec ce paramètre d'ordre on brise la conservation du # total de particules

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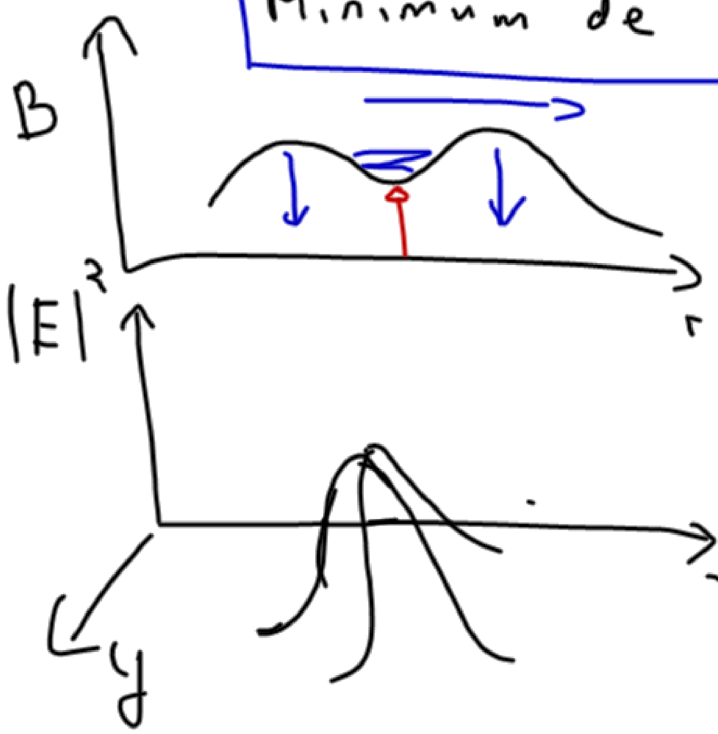
5. Pièges magnétiques et optiques pour les atomes froids et observation de la cond. de B.E.

$^4He \qquad T_c \approx 2.2K$

$^3He \qquad T_c \sim 3mK$

Pièges magnétique.

$$\boxed{\text{Minimum de } |B|}$$



$$\vec{\nabla} \cdot \vec{B} = 0$$

$$-\frac{1}{2}\alpha E^2$$