^{7}Li, ^{23}Na, ^{87}Rb

Spin nucléaire $I = 3/2$

électron de valence $S = 1/2$

$$F = 2, 1$$

$$g \mu_B S^z B^z = \text{couplage}$$

$$g = 2$$

$$\boxed{\langle F, M_F | 2 \mu_B S^z B | F, M_F \rangle}$$

$$|F, M_F\rangle = \sum_{I,S} C_{I,S} |M_I, M_S\rangle$$

$$\rightarrow \quad |F=2, M_F=2\rangle = |\tfrac{3}{2}\rangle \otimes |\tfrac{1}{2}\rangle$$

$$|F=2, M_F=1\rangle$$

$$\boxed{\hat{F}^- = \hat{I}^- + \hat{S}^-} \qquad \sqrt{\tfrac{3}{2}\tfrac{5}{2} - \tfrac{3}{2}\tfrac{1}{2}}\; |\tfrac{1}{2}\rangle$$

$$\rightarrow \quad \hat{S}^- |m\rangle = \sqrt{s(s+1) - m(m-1)}\, |m-1\rangle$$

$$|F=2, M_F=1\rangle = \frac{1}{2}\left(\sqrt{3}\, |\tfrac{1}{2}\rangle |\tfrac{1}{2}\rangle + |\tfrac{3}{2}\rangle |-\tfrac{1}{2}\rangle\right)$$

$$|F=2, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(|-\tfrac{1}{2}\rangle |\tfrac{1}{2}\rangle + |\tfrac{1}{2}\rangle |-\tfrac{1}{2}\rangle\right)$$

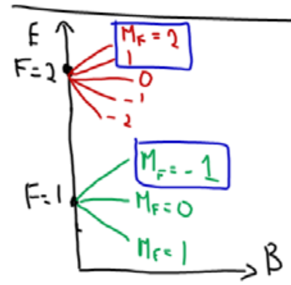
$$|F=2, M_F=-1\rangle = \frac{1}{2}\left(\sqrt{3}\, |-\tfrac{1}{2}\rangle |-\tfrac{1}{2}\rangle + |-\tfrac{3}{2}\rangle |\tfrac{1}{2}\rangle\right)$$

$$|F=2, M_F=-2\rangle = |-\tfrac{3}{2}\rangle |-\tfrac{1}{2}\rangle$$

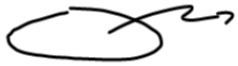
$$|F=1, M_F=1\rangle = \frac{1}{2}\left(|\tfrac{1}{2}\rangle |\tfrac{1}{2}\rangle - \sqrt{3}\, |\tfrac{3}{2}\rangle |-\tfrac{1}{2}\rangle\right)$$

$$|F=1, M_F=0\rangle = \frac{1}{\sqrt{2}}\left(-|-\tfrac{1}{2}\rangle |\tfrac{1}{2}\rangle + |\tfrac{1}{2}\rangle |-\tfrac{1}{2}\rangle\right)$$

$$|F=1, M_F=-1\rangle = \frac{1}{2}\left(|-\tfrac{1}{2}\rangle |-\tfrac{1}{2}\rangle - \sqrt{3}\, |-\tfrac{3}{2}\rangle |\tfrac{1}{2}\rangle\right)$$



Équation de Gros-Pitaevskii

 → Piège

$$|\Omega_F\rangle = a_1^+ a_2^+ \dots a_N^+ |0\rangle$$

$$\to |\Omega_{\text{Bosons}}\rangle = \frac{(a_0^+)^{N_0}}{\sqrt{N_0!}} a_1^+ a_2^+ \dots |0\rangle$$

$$\Psi^+(\vec{r}) = \sum_i \psi_i(\vec{r})\, a_i^+$$

$$\boxed{\left(-\frac{\hbar^2\nabla^2}{2m} + V_{opt}(\vec{r}) + V_H(\vec{r})\right)\psi_i = \epsilon_i \psi_i}$$

Gros-Pitaevskii

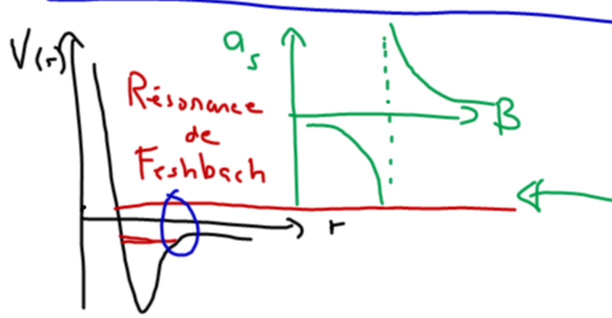
$$V_H(\vec{r}) = \int d^3r'\, U(r-r')\, n(r')$$

$$U(\vec{r}-\vec{r}') = \frac{4\pi a_s \hbar^2}{m}\, \delta(\vec{r}-\vec{r}')$$

$$\equiv g\, \delta(\vec{r}-\vec{r}')$$

$$\boxed{V_H(\vec{r}) = g\, n(\vec{r})}$$

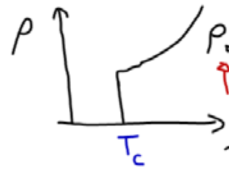
$$\boxed{n(\vec{r}) = \sum_i \frac{1}{e^{\beta(\epsilon_i-\mu)}-1} |\psi_i(\vec{r})|^2}$$



8. Supraconductivité

Phénoménologie

(1) ρ



$\rho_0 + \beta T^5$

1911

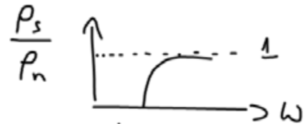
(2) Effet Meissner-Ochsenfeld

Diamagnétisme parfait

$B = \mu_0 (H + M)$

$M = -H$

(3) Résistance AC

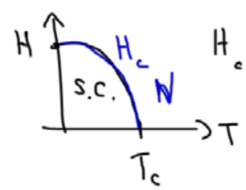


$\hbar\omega_g \sim 3.5 k_B T$

(4) Courants persistants

$\tau > 10^{15}$ ans

(5)



$H_c = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

Gorter-Casimir

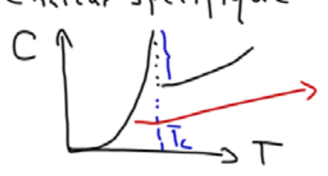
(6) $F_s(T) + \mu_0 \frac{H_c^2(T)}{2} = F_N(T)$

(7) Effet isotopique

$T_c \sim M^{-1/2}$

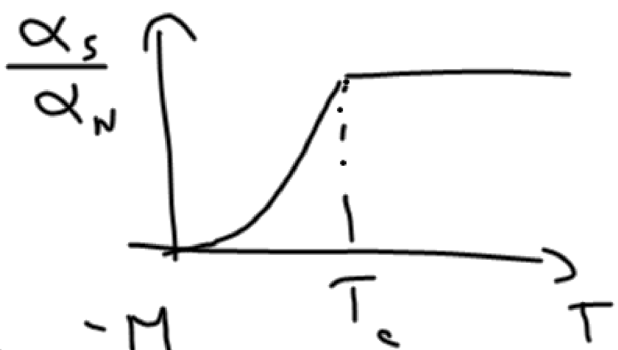
Frölich (1950)

(8) Chaleur spécifique

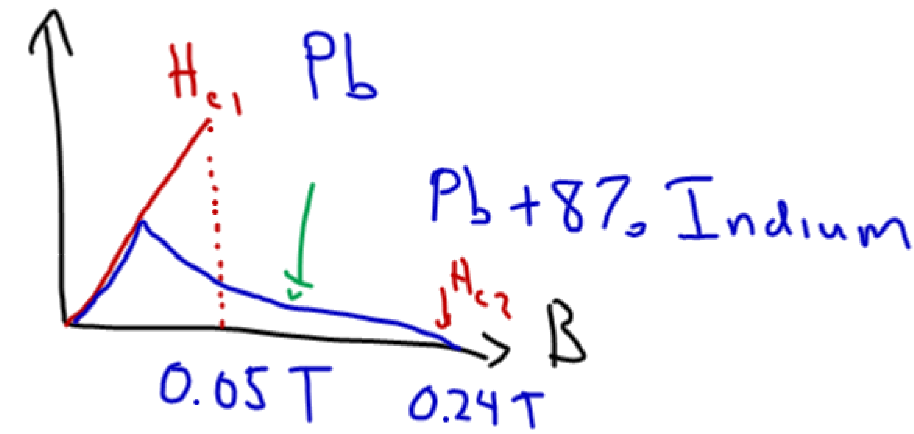


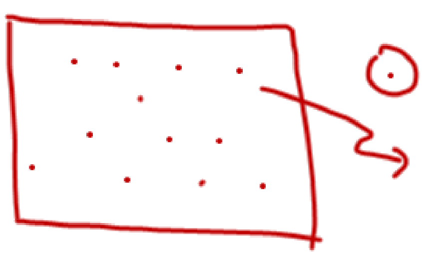
$e^{-E_g/k_B T}$

(9) Atténuation ultrasonore



(10)



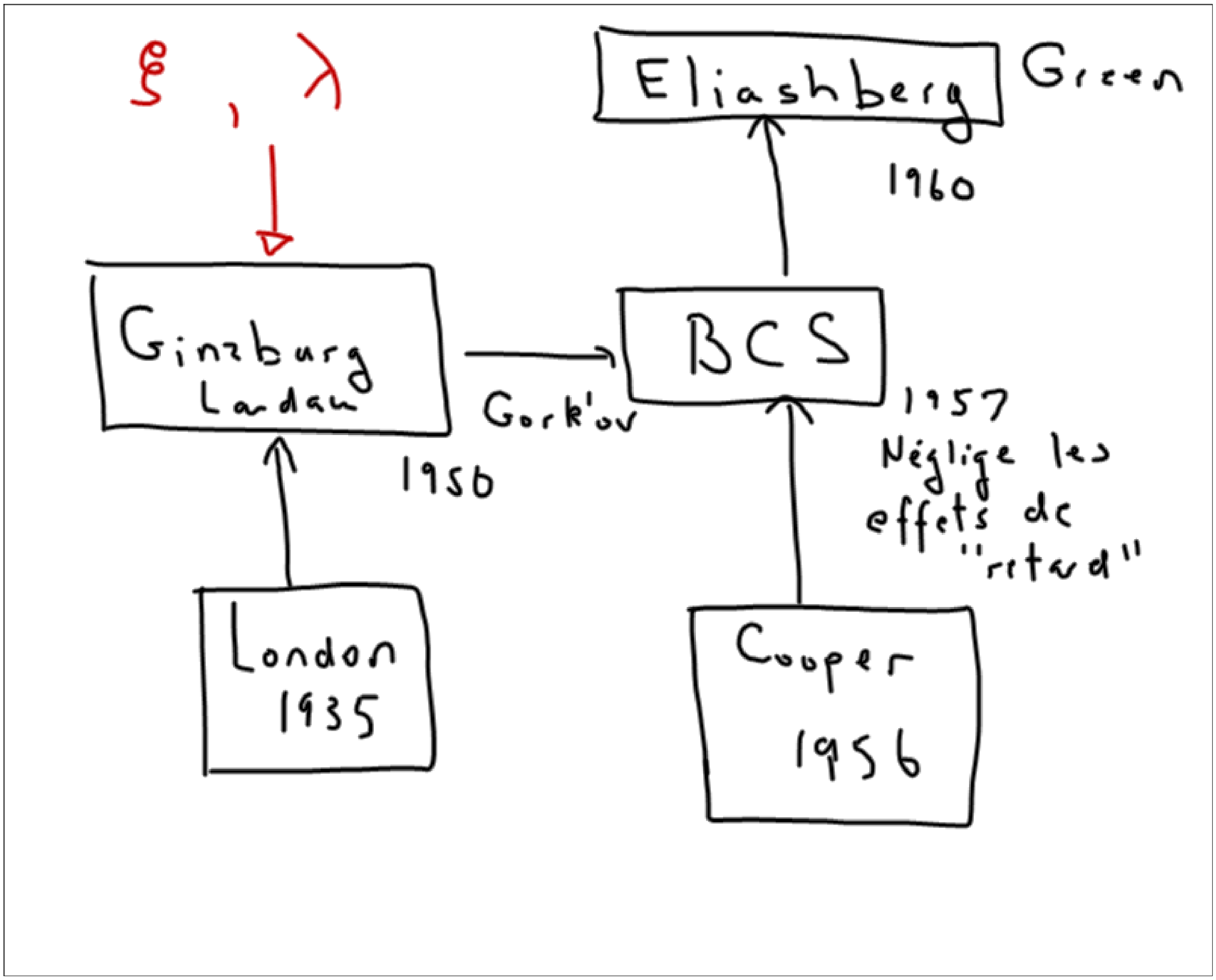


$$\Phi = \frac{h}{2e}$$

Problème de Cooper 1956

Force attractive

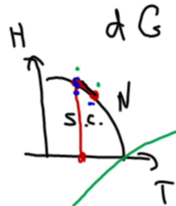
État lié quel que soit la force de l'interaction


ξ , λ
Eliashberg
Green
1960
Ginzburg
Landau
BCS
Gork'ov
1957
Néglige les effets de "retard"
1950
London
1935
Cooper
1956

Énergie de condensation (T.D)

à T = cte

$$dG = -\mu_0 V \, \vec{M} \cdot d\vec{H}$$



$$G_s(T, H_c) - G_s(T, 0) = -\mu_0 V \int_0^{H_c} \vec{M} \cdot d\vec{H}$$

$$= \mu_0 V \frac{H_c^2}{2} \qquad (\vec{M} = -\vec{H})$$

$$G_s(T, H_c) = G_n(T, H_c) \checkmark$$

$$= G_n(T, 0) \checkmark$$

$$\boxed{G_s(T,0) - G_n(T,0) = -\mu_0 \frac{H_c^2}{2} V}$$

H_c,

Énergie de condensation

Relation de Clausius-Clapeyron.

$$\rightarrow dG = -S\,dT - \mu_0 V \, \vec{M} \cdot d\vec{H}$$

$$dG_s = dG_n$$

$$-S_s \, dT - \mu_0 V \, \vec{M}_s \cdot d\vec{H} = -S_n \, dT - \mu_0 V \, \vec{M}_n \cdot d\vec{H}$$

$$\vec{M}_n \sim 0$$

$$(S_n - S_s)\, dT = \mu_0 V \, \vec{M}_s \cdot d\vec{H}$$

$$= -\mu_0 V \, \vec{H}_c \cdot d\vec{H}$$

$$S_n(T_c, H_c) - S_s(T_c, H_c)$$

$$= -\mu_0 V \, \vec{H}_c \cdot \frac{d\vec{H}_c}{dT_c}$$

Approche Ginzburg-Landau

$$f_s(T) = f_n(T) + a(T)|\Psi|^2 + \frac{1}{2} b |\Psi|^4$$

où Ψ est un paramètre d'orde <u>complexe</u>

$$\Downarrow$$

$$a(T) = a'(T - T_c)$$

$$(T < T_c) \quad f_s(T) - f_n(T) = -\frac{a'^2 (T - T_c)^2}{2b} = -\mu_0 \frac{H_c^2}{2}$$

$$\boxed{H_c = \frac{a'}{(\mu_0 b)^{1/2}} (T_c - T)}$$

Situations inhomogènes:

$$f_s(T,\vec{r}) = f_n(T)$$

$$+ \frac{\hbar^2}{2m^*}|\nabla\psi|^2 + a(T)|\psi(\vec{r})|^2 + \frac{1}{2}b|\psi(r)|^4$$

$$F_s = F_n + \int d^3r \left[\frac{\hbar^2}{2m^*}|\nabla\psi|^2 + a(T)|\psi(\vec{r})|^2 + \frac{1}{2}b|\psi(\vec{r})|^4\right]$$

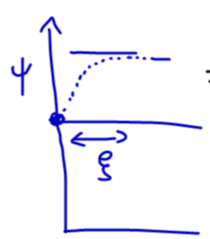
$$\frac{\delta \boxed{\int f_s(r)\, d^3r}}{\delta\psi(r')} = 0 \qquad \frac{\delta\psi(r)}{\delta\psi(r')} = \delta(\vec{r}-\vec{r}')$$

$$\frac{\delta\int f_s(r)\, d^3r}{\delta\psi(r')} = a(T)\,\psi^*(r') + \underbrace{b\,\psi(r)\,\psi^{*2}(r)}_{b\,\psi^*(r')|\psi(r)|^2}$$

$$+ \frac{\hbar^2}{2m^*}\frac{\delta}{\delta\psi(r')}\int \nabla\psi^*(r)\nabla\psi(r)\, d^3r$$

$$\int \nabla\psi^*\nabla\psi \ldots = \underbrace{\cancel{\nabla\psi^*(r)\,\psi(r)}}_{0} - \int \nabla^2\psi^*\,\psi(r)\, d^3r$$

$$\frac{\delta}{\delta\psi(r')}\int f_s(r)\, d^3r = a(T)\,\psi^*(r') + b\,|\psi(r')|^2\,\psi^*(r') - \frac{\hbar^2}{2m^*}\nabla^2\psi^*(r') = 0$$



$$a(T)\,\psi(r) = \frac{\hbar^2}{2m^*}\nabla^2\psi(r)$$

$$\psi(x) = e^{\pm\left[\frac{a(T)\,2m^*}{\hbar^2}\right]^{1/2} x}$$

$$\xi = \left[\frac{a(T)\,2m^*}{\hbar^2}\right]^{-1/2}$$

$$\frac{\hbar}{i}\vec{\nabla} \longrightarrow \frac{\hbar}{i}\nabla - q\vec{A}$$

$$q = -2e$$

$$m^* = 2m_e$$