Résumé :

1. Phén.

E de Condensation $\longrightarrow H_{c_1}$

2. Équ. de Landau-Ginzburg

• $B = 0$

$$f_s(T) = f_n(T) + a(T)|\Psi|^2 + b|\Psi|^4$$

$$a(T) = a'(T - T_c)$$

•
$$\boxed{f_s(T,r) = f_n(T,\vec{r}) + \frac{\hbar^2}{2m^*}|\underline{\nabla\Psi}|^2 + a(T)|\Psi|^2 + b|\Psi|^4}$$

$$\frac{\delta \int_{\underline{d^3r}} f_s(T,\vec{r})\, d^3r}{\delta\Psi(\vec{r})} = 0$$

$$-\frac{\hbar^2}{2m^*}\nabla^2\Psi + a(T)\Psi(\vec{r}) + b\Psi(\vec{r})|\Psi(r)|^2 = 0$$

$$\xi = \left(\frac{\hbar^2}{2m^* a(T)}\right)^{1/2} \sim \frac{1}{(T_c - T)^{1/2}}$$

• G.L. en présence de $\vec{B}$

$$\frac{\hbar}{i}\vec{\nabla} \longrightarrow \frac{\hbar}{i}\vec{\nabla} - q\vec{A}$$

$-2e$ $\quad 2e$

$$-\frac{\hbar^2}{2m^*}\left(\nabla + \frac{2ei}{\hbar}A\right)^2 \Psi(\vec{r})$$

$$+ a(T)\Psi(\vec{r}) + b\Psi(\vec{r})|\Psi|^2 = 0$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j}_s}$$

Quel est $\vec{j}_s$?

$$\Psi^* \frac{\partial \Psi}{\partial t} = \frac{\Psi^*}{i\hbar}\left[-\frac{\hbar^2}{2m^*}\nabla^2\Psi + \cancel{a\Psi} + \cancel{b\Psi|\Psi|^2}\right]$$

$$\Psi \frac{\partial \Psi^*}{\partial t} = -\frac{\Psi}{i\hbar}\left[-\frac{\hbar^2}{2m^*}\nabla^2\Psi^* + \cancel{a\Psi^*} + \cancel{b\Psi^*|\Psi|^2}\right]$$

Equ. de continuité :

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot j_s = 0$$

$$\rho = -2e|\Psi|^2$$

$$\frac{\partial \rho}{\partial t} = -2e\left(\frac{\partial \Psi^*}{\partial t}\Psi + \Psi^*\frac{\partial \Psi}{\partial t}\right)$$

$$= -2e\left[\frac{\Psi^*}{i\hbar}\left(-\frac{\hbar^2}{2m^*}\nabla^2\Psi\right) + \frac{\Psi}{i\hbar}\left(\frac{\hbar^2}{2m^*}\nabla^2\Psi^*\right)\right]$$

$$= -\frac{e\hbar}{im^*}\left[\Psi\nabla^2\Psi^* - \Psi^*\nabla^2\Psi\right]$$

$$= -\vec{\nabla}\cdot\left[\frac{e\hbar}{im^*}\left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi\right)\right]$$

$$= -\vec{\nabla}\cdot j_s$$

$$\boxed{\vec{j}_s = \frac{e\hbar}{im^*}\left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi\right)}$$

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$$j_s = \frac{e\hbar}{m^* i}\left[\psi \nabla \psi^* - \psi^* \nabla \psi\right]$$

$$\frac{\hbar}{i}\vec{\nabla} \rightarrow \frac{\hbar}{i}\vec{\nabla} + 2e\vec{A}$$

$$\vec{\nabla} \rightarrow \vec{\nabla} + \frac{2ei}{\hbar}\vec{A}$$

$$\vec{J}_s = \frac{e\hbar}{m^* i}\left[\psi \nabla \psi^* - \psi^* \nabla \psi\right] - \frac{4ei}{\hbar}\frac{e\hbar}{m^* i}\psi^*\psi\vec{A}$$

$$\boxed{j_s = \frac{e\hbar}{m^* i}\left[\psi \nabla \psi^* - \psi^* \nabla \psi\right] - \frac{(2e)^2}{m^*}|\psi|^2\vec{A}}$$

Énergie libre :

$$F_s(T) = F_n(T, \vec{A}=0)$$
$$+ \int d^3r \left[\frac{\hbar^2}{2m^*} \left| \left(\nabla + \frac{2ei\vec{A}}{\hbar} \right) \psi(r) \right|^2 \right.$$
$$\left. + a(T) |\psi(\vec{r})|^2 + \frac{1}{2} b |\psi(\vec{r})|^4 \right]$$
$$+ \frac{1}{2\mu_0} \int d^3r \, B^2(\vec{r})$$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j}_s + j_p)$$

- Rigidité superfluide
(Brisure de symétrie)

Soit $|\psi|^2 =$ constante

$$\boxed{\psi(\vec{r}) = |\psi| e^{i\theta(\vec{r})}}$$

$$\boxed{\begin{aligned} F_s(T,\vec{A}) &= F_s(T,\vec{A}=0) \\ &+ \int d^3r \frac{\hbar^2}{2m^*} \left|\nabla\theta + \frac{2e}{\hbar}\vec{A}\right|^2 |\psi|^2 \\ &+ \frac{1}{2\mu_0}\int d^3r\, B^2(\vec{r}) \end{aligned}}$$

$$\frac{1}{2}\rho_s \int d^3r \left|\left(\vec{\nabla}\theta + \frac{2e}{\hbar}\vec{A}\right)\right|^2$$

$$\boxed{\rho_s \equiv \frac{\hbar^2}{m^*}|\psi|^2}$$ Rigidité superfluide

Fixer la jauge. $\vec{\nabla}\cdot\vec{A} = 0$ jauge de London

Ensuite, l'état d'équibre correspond à $\vec{\nabla}\theta = 0$!

"Cohérence de phase"

Reste une phase "globale"

θ indép. de $\vec{r}$

$B\hat{z}$ $\boxed{\vec{A} = Bx\,\hat{y}}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & Bx & 0 \end{vmatrix} = \hat{z}B$$

- Longueur de pénétration de London λ_L et effet Meissner-Ochsenfeld

Cas où $|\Psi|^2 = cte.$

$$j_s = \frac{e\hbar}{m^* i}\left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi\right) - \frac{(2e)^2}{m^*}|\Psi|^2 A$$

$$\Psi = |\Psi| e^{i\theta(r)}$$

$$= -\frac{2e\hbar}{m^* i}\, i\nabla\theta\, |\Psi|^2 - \frac{(2e)^2}{m^*}|\Psi|^2 A$$

$$= -\frac{2e}{\hbar}\rho_s\left(\vec{\nabla}\theta + \frac{2e}{\hbar}\vec{A}\right)$$

$$\rho_s \equiv \frac{\hbar^2}{m^*}|\Psi|^2$$

Minimum de F_s : $\nabla\theta = 0$

$$\boxed{\begin{aligned} j_s &= -\frac{(2e)^2}{m^*}|\Psi|^2 \vec{A} \\ &= -\frac{n_s e^2}{m_e}\vec{A} \end{aligned}}$$

$n_s \equiv$ densité superfluide.

$$m^* = 2m_e$$

$$\boxed{n_s = 2|\Psi|^2}$$

$\vec{\nabla}\times\vec{B} = \mu_0 \vec{j}_s$ pour trouver l'effet de $\vec{B}$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\, \vec{\nabla}\times\vec{j}_s$$

$$= -\mu_0 \frac{n_s e^2}{m_e}\vec{\nabla}\times\vec{A} = -\mu_0 \frac{n_s e^2}{m_e}\vec{B}$$

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$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\cdot\vec{B}}}_{0}) - \nabla^2\vec{B}$$

$$\boxed{-\nabla^2\vec{B} = -\mu_0\frac{n_s e^2}{m_e}\vec{B}}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0\frac{n_s e^2}{m_e}B_z$$

$$B_z = B_0 e^{-x/\lambda_L}$$

$$\boxed{\frac{1}{\lambda_L^2} = \frac{\mu_0 n_s e^2}{m_e}}$$

λ_L = Longueur de pénétration de London

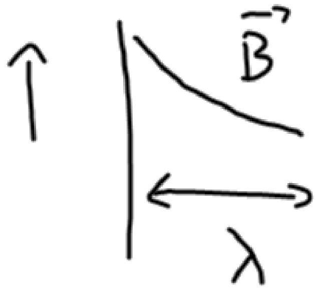
$$\frac{1}{\lambda_L^2} = \frac{2|\psi|^2 e^2}{m_e}\mu_0$$

$$\frac{1}{\lambda_L^2} = \frac{2e^2 a^2(T_c-T)}{m_e b}\mu_0$$

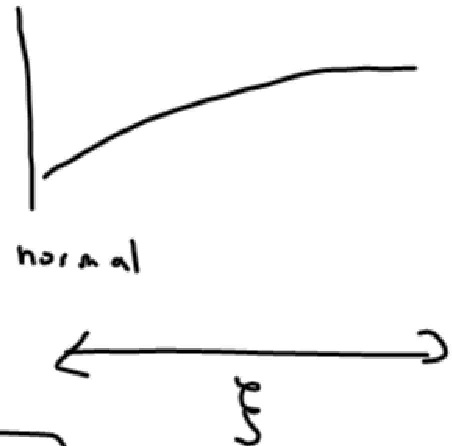
$$\lambda_L \sim \frac{1}{(T_c-T)^{1/2}}$$

$$\kappa \equiv \frac{\lambda_L(T)}{\xi(T)} = \frac{\left(\dfrac{\cancel{m_e}\,\cancel{b}}{\cancel{2}e^2\cancel{a^2}\cancel{(T_c-T)}\cancel{\mu_0}}\right)^{1/2}}{\left(\dfrac{\cancel{\hbar^2}}{\cancel{4}\cancel{m_e}\cancel{a^2}\cancel{(T_c-T)}}\right)}$$

$$\kappa = \left(\frac{4m_e^2 b}{2e^2\hbar^2\mu_0}\right)^{1/2}$$



$\boxed{\xi \gg \lambda}$



$\kappa = \frac{\lambda}{\xi}$

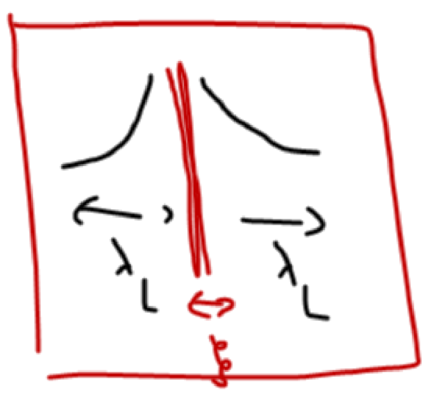
$\kappa < \frac{1}{\sqrt{2}}$

Type I

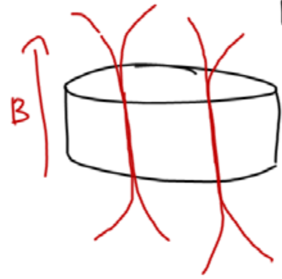
$\boxed{\xi \ll \lambda}$

$\kappa > \frac{1}{\sqrt{2}}$

Type II



• "Vortex" Tourbillon de London



Géométrie cylindrique.

$$\frac{\partial^2 B_z}{\partial r^2} + \frac{1}{r}\frac{\partial B_z}{\partial r} - \frac{B_z}{\lambda_L^2} = 0$$

$$\boxed{B_z(\vec{r}) = \frac{\Phi_0}{2\pi\lambda_L^2} K_0\left(\frac{r}{\lambda}\right)}$$

K_0 = fonction de Bessel modifiée

$$\Phi_0 \equiv \int_{-\infty}^{\infty} B_z(r)\, d^2 r$$

Pour $\boxed{r \ll \lambda_L}$ $\boxed{\lambda_L \gg \xi}$

$$\rightarrow B_z(r) = \frac{\Phi_0}{2\pi\lambda_L^2} \ln\left(\frac{\lambda_L}{r}\right)$$

$r = \xi$ est la valeur la plus petite de r permise.

$$\vec{\nabla}\times\vec{B} = \mu_0 j_s$$

$$j_s \sim \frac{\hat{e}_\varphi}{r}$$

De dessus

Si $r \gg \lambda_L$

$$B_z(\vec{r}) = \frac{\Phi_0}{2\pi\lambda_L^2}\sqrt{\frac{\pi\lambda_L}{2r}}\, e^{-r/\lambda_L}$$

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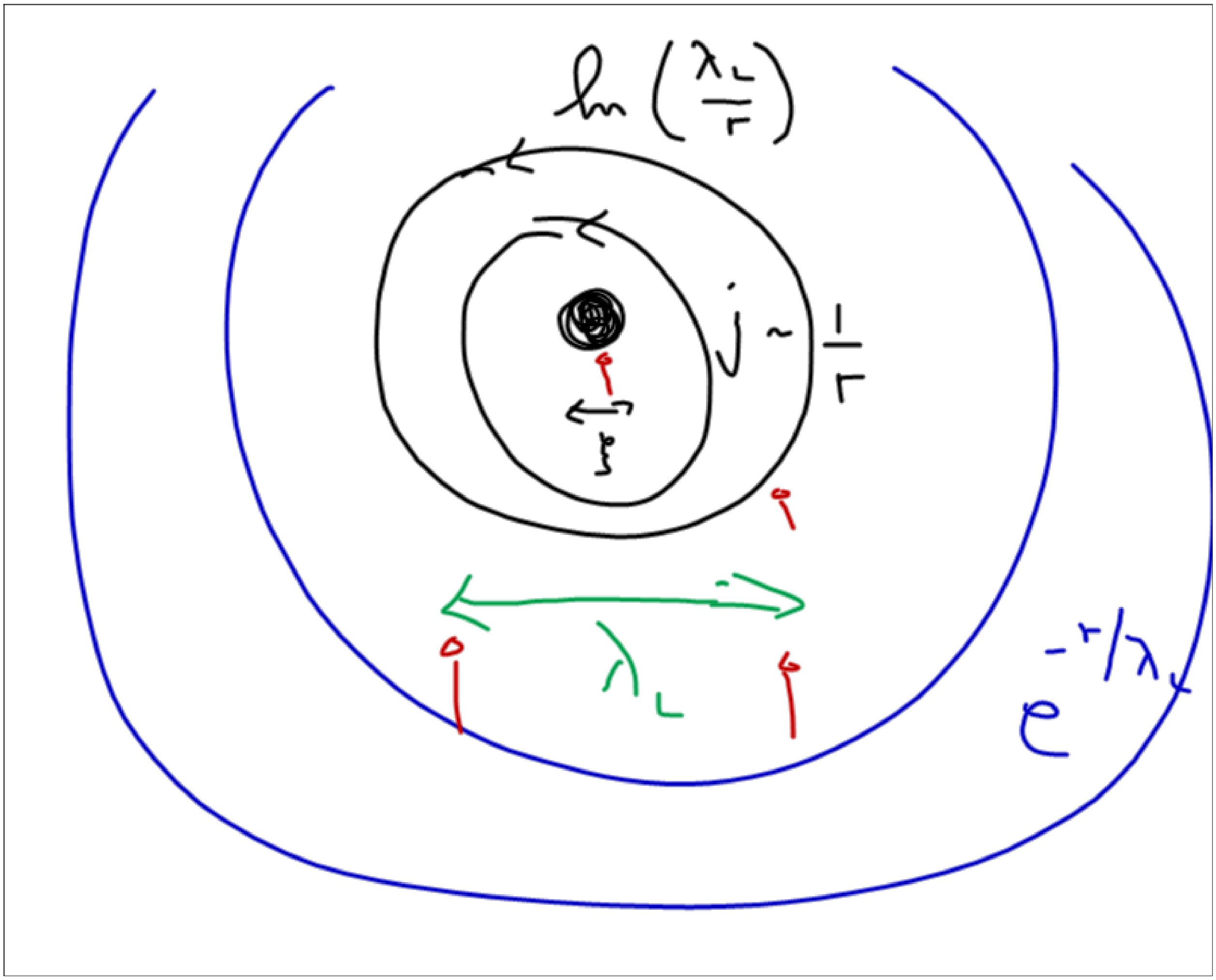
$\ln\left(\frac{\lambda_L}{r}\right)$
$j \sim \frac{1}{r}$
ξ
λ_L
e^{-r/λ_L}

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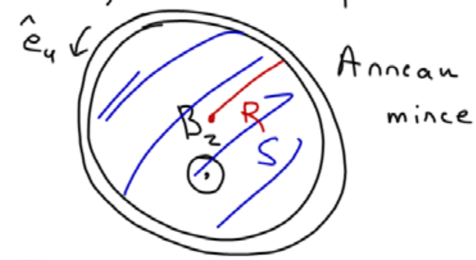
• Quantification du flux



$$\boxed{\vec{A} = \frac{\Phi}{2\pi R}\,\hat{e}_\varphi}$$

$$\Phi = \int \vec{B}\cdot d\vec{S} = \int \vec{\nabla}\times\vec{A}\cdot d\vec{S}$$

$$= \oint \vec{A}\cdot d\vec{\ell} = 2\pi R\, A_\varphi = \Phi$$

$$\Psi = \Psi_0\, e^{-in\varphi}$$

où n est entier

$$F_s(T,\vec{A}) = F_s(T,\vec{A}=0)$$

$$+\int d^3r \left[\frac{\hbar^2}{2m}\left|\frac{-in}{R} + \frac{2e}{\hbar}\, i\, \frac{\Phi}{2\pi R}\right|^2 |\Psi|^2\right]$$

$$+\frac{1}{2\mu_0}\int B^2(r)\, d^3r$$

$$\hookrightarrow \sim \Phi^2$$

$$n = \frac{2e\,\Phi}{2\pi\hbar}$$

$$\Phi = \frac{(2\pi\hbar)}{2e}\, n = \frac{h}{2e}\, n = \Phi_0\, n$$

$$\boxed{\Phi_0 = \frac{h}{2e}} = \text{Quantum de flux magnétique supraconducteur}$$

$$\Phi_0 = 2.07\times10^{-15}\ \text{Wb}$$

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