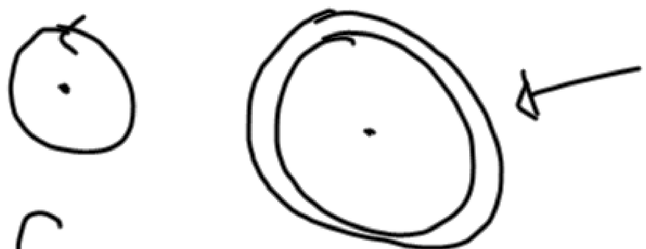
$$\int d^3r \left(\vec{\nabla}\theta + \frac{2e}{\hbar}\vec{A}\right)^2$$

$\vec{\nabla}\theta$ $\vec{A}$ parellèles ?

$$\vec{\nabla}\times(\vec{\nabla}\theta) = 0$$

$$\vec{\nabla}\times\vec{A} \neq 0$$



$\oint \vec{\nabla}\theta \cdot \vec{dl} = 0$ gradient

$\oint \vec{\nabla}\theta \cdot \vec{dl} = 2\pi n$ anneau

Pb: $\lambda_L \sim 305$ Å

$\xi_0 \approx 960$ Å Type I

8.3 Bardeen-Cooper-Schrieffer (1957)

1950 Frölich
"Phonons"
Maxwell-Reynolds
exp.: effet isotopique

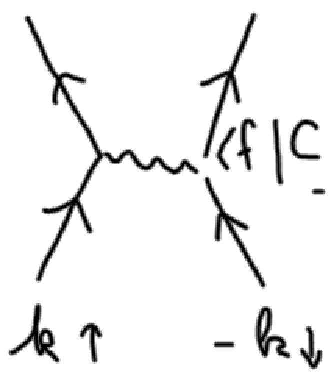
$$T_c \sim M^{-\alpha} \qquad \alpha = 1/2$$

BCS : 3 facteurs importants :

1) Attraction
2) Formation de pairs
3) "Cohérence"

Mécanisme d'attraction

$$H_I = \frac{1}{2V} \sum_{\sigma\sigma'} \sum_{k k' q} \boxed{V(q)}\, c^+_{k\sigma} c^+_{k'\sigma'} c_{k'-q\,\sigma'} c_{k+q\,\sigma}$$



$$\frac{\langle f | c_{-k'\downarrow} c_{k'\uparrow} | n \rangle \langle n | c^+_{k\uparrow} c^+_{k\downarrow} | 0 \rangle}{E_n - E_i}$$

$U(q)$ macroscopiquement

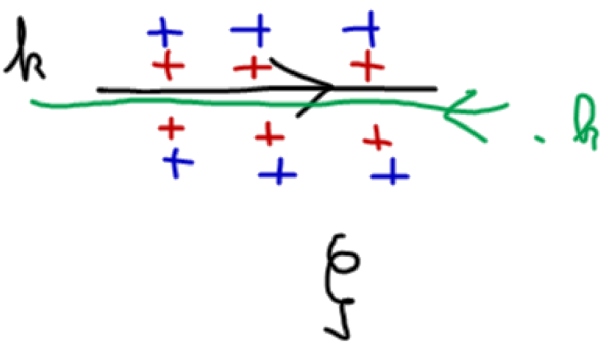
$$U(q) \approx \frac{e^2}{\epsilon_0 q^2}$$

En présence d'autres électrons

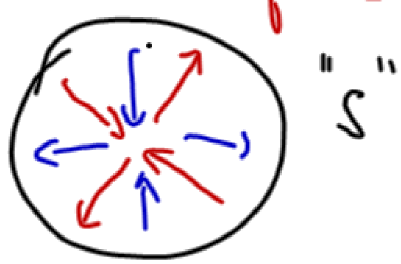
$$\sim \frac{e^2}{\epsilon_0 (q^2 + q_{TF}^2)}$$

À démontrer :

$$U(q) \sim \frac{e^2}{\epsilon_0} \frac{1}{q^2 + q_{TF}^2} \left(1 + \frac{\omega_q^2}{\omega^2 - \omega_q^2} \right)$$



$\xleftrightarrow{\ \xi\ }$ $\omega_q = c_s q$



$T = 0 \qquad \xi_0 \sim \frac{\hbar v_F}{k_B T_c}$

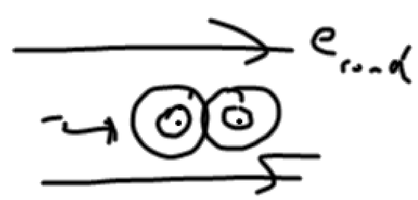
Modèle: "jellium"

Densité uniforme d'électrons
n de charge $-e$
de masse m

Densité uniforme d'ions positifs
de charge Ze

"Coulomb"

- Répulsion Pauli



- Phonons transversaux

a) Équation de Poisson.

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho$$

$$\vec{E} = -\vec{\nabla} V$$

$$\boxed{\nabla^2 V = -\frac{1}{\epsilon_0}\left(\rho_i + \rho_e + \delta\rho\right)}$$

Charge externe

Interaction:

$$U(q) = \frac{e^2}{4\pi\epsilon_0\,\epsilon(q,\omega)\,q^2}$$

$$\boxed{\nabla^2 V = -\frac{1}{\epsilon_0\epsilon}\delta\rho}$$

Déf. de ϵ

$$\epsilon = \frac{\delta\rho}{\rho_i + \rho_e + \delta\rho}$$

b) Pour les ions :

$$\nabla\cdot\left(M\frac{\partial \vec{j}_i}{\partial t}\right)=\nabla\cdot\left(nZe^2\vec{E}\right)$$

$$\dot{j}_i=\frac{n}{Z}e\vec{v}Z$$

$$\frac{\partial\rho_i}{\partial t}+\vec{\nabla}\cdot\vec{j}_i=0$$

$$-\frac{\partial^2\rho_i}{\partial t^2}M=nZe^2\left(-\nabla^2 V\right)$$

$$\frac{\partial^2\rho_i}{\partial t^2}=-\frac{nZe^2}{M\epsilon_0}\left(\rho_i+\rho_e+\delta\rho\right)$$

$$\boxed{\rho_i(q,\omega)=\frac{\omega_i^2}{\omega^2}\left(\rho_i(q,\omega)+\rho_e(q,\omega)+\delta\rho(q,\omega)\right)}$$

$$\omega_i^2=\frac{nZe^2}{M\epsilon_0}$$

avr. 24-08:58

c) Réaction de ρ_e à la présence des autres charges

$$\boxed{E_F = \frac{\hbar^2 k_F^2(\vec{r})}{2m} - eV(\vec{r})}$$

Potentiel chimique constant

$$n = 2\int \frac{d^3k}{(2\pi)^3} f(\epsilon_k - \mu)$$

$$= \frac{2}{(2\pi)^3} \frac{4\pi}{3} k_F^3 \qquad T = 0$$

avr. 24-09:05

$$k_F^2(\vec{r}) = \frac{2m}{\hbar}\left(\underline{E_F} + eV(r)\right)$$

$$= k_F^2 + \frac{2m}{\hbar} eV(r)$$

$$\rightarrow k_F^2(\vec{r}) = k_F^2\left(1 + \frac{eV(\vec{r})}{E_F}\right)$$

$$eV(r) \ll E_F$$

$$k_F^3(r) = k_F^3\left(1 + \frac{3}{2}\frac{eV(r)}{E_F}\right)$$

$$\rho_e(\vec{r}) = -en\left(\underset{?}{1} + \frac{3}{2}\frac{eV(r)}{E_F}\right)$$

$$\rho_e(q,\omega) = -\frac{3}{2}\frac{e^2 n}{E_F} V(q,\omega)$$

$$\boxed{\rho_e(q,\omega) = -\boxed{\frac{3}{2}\frac{e^2 n}{E_F}\frac{1}{\epsilon_0}}\, q^2\left(\rho_i(q,\omega) + \rho_e(q,\omega) + \delta\rho(q,\omega)\right)}$$

$$q_{TF}^2 = \frac{3}{2}\frac{ne^2}{E_F\epsilon_0}$$

$$\rho_i(q,\omega)+\rho_e(q,\omega)$$

$$=\left(\frac{\omega_i^2}{\omega^2}-\frac{q_{TF}^2}{q^2}\right)\left(\underline{\rho_i(q,\omega)+\rho_e(q,\omega)}+\delta\rho(q,\omega)\right)$$

$$\epsilon(q,\omega)=\frac{\delta\rho}{\rho_i+\rho_e+\delta\rho}$$

$$\delta\rho(q,\omega)=\frac{1}{\frac{\omega_i^2}{\omega^2}-\frac{q_{TF}^2}{q^2}}\left[1-\frac{\omega_i^2}{\omega^2}+\frac{q_{TF}^2}{q^2}\right]\left(\rho_e(q,\omega)+\rho_i(q,\omega)\right)$$

$$\frac{\delta\rho}{\rho_i+\rho_e+\delta\rho}=\frac{\left[1-\frac{\omega_i^2}{\omega^2}+\frac{q_{TF}^2}{q^2}\right]\cancel{(\rho_e+\rho_i)}}{\underbrace{\cancel{\left(\frac{\omega_i^2}{\omega^2}-\frac{q_{TF}^2}{q^2}\right)}\left[1+\frac{\cancel{\left[1-\frac{\omega_i^2}{\omega^2}+\frac{q_{TF}^2}{q^2}\right]}\cancel{(\rho_e+\rho_i)}}{\cancel{\frac{\omega_i^2}{\omega^2}-\frac{q_{TF}^2}{q^2}}}\right]}}$$

$$\boxed{\epsilon(q,\omega)=\left[1-\frac{\omega_i^2}{\omega^2}+\frac{q_{TF}^2}{q^2}\right]}$$

Fréquence propre des ions : $\delta\rho=0$

$$\epsilon(q,\omega)=0$$

$$\omega^2q^2-\omega_i^2q^2+q_{TF}^2\omega^2=0$$

$$\omega^2\left(q^2+q_{TF}^2\right)=\omega_i^2q^2$$

$$\omega^2=\left(\frac{\omega_i^2}{q^2+q_{TF}^2}\right)q^2$$

Grande longueur d'onde

$$\boxed{\omega_q=\left(\frac{\omega_i}{q_{TF}}\right)q}\qquad\boxed{C_s=\frac{\omega_i}{q_{TF}}}$$

Bohm-Staver

Z

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"Interaction"

$$\frac{e^2}{4\pi\epsilon_0\,\epsilon(q,\omega)\,q^2} = \frac{e^2}{4\pi\epsilon_0 q^2}\,\frac{1}{\left[1-\frac{\omega_i^2}{\omega^2}+\frac{q_{TF}^2}{q^2}\right]}$$

$$= \frac{e^2}{4\pi\epsilon_0 \cancel{q^2}}\,\frac{\omega^2 \cancel{q^2}}{\left[\omega^2 \cancel{q^2} - \omega_i^2 q^2 + q_{TF}^2\,\omega^2\right]}$$

$$= \frac{e^2}{4\pi\epsilon_0}\,\frac{1}{q^2+q_{TF}^2}\left[\frac{\omega^2}{\omega^2 - \frac{\omega_i^2 q^2}{q^2+q_{TF}^2}}\right]$$

$$= \frac{e^2}{4\pi\epsilon_0}\,\frac{1}{q^2+q_{TF}^2}\left[\frac{\omega^2}{\omega^2-\omega_q^2}\right]$$

$$\boxed{= \frac{e^2}{4\pi\epsilon_0}\,\frac{1}{q^2+q_{TF}^2}\left[1+\frac{\omega_q^2}{\omega^2-\omega_q^2}\right]}$$

3

$$\omega < \omega_p$$

BCS

Hamiltonien réduit

$$H_I = \frac{1}{2V}\sum_{\sigma\sigma'}\sum_{k\cancel{k'}\, q} U(q)\, c^+_{k\sigma} c^+_{k'\sigma'}\, c_{\cancel{k'}-q\,\sigma'}\, c_{k+q\,\sigma}$$

$k' = -k$ $\quad -k \quad p'$

"Paires de Cooper" $\boxed{\langle c_{-p\downarrow} c_{p\uparrow}\rangle}$ $\quad \Delta$

$$c_{-p\downarrow} c_{p\uparrow} \longleftrightarrow b$$

$$\langle b \rangle \neq 0$$

Rôle de Ψ, G.L.

$$= \frac{1}{2V}\sum_{\sigma\sigma'}\sum_{pp'} U(|p-p'|)\; c^+_{p\sigma} c^+_{-p\sigma'}\, c_{-p'\sigma'}\, c_{p'\sigma}$$

$$= \frac{1}{V}\sum_{pp'} U(|p-p'|)\, c^+_{p\uparrow} c^+_{-p\downarrow}\, c_{-p'\downarrow}\, c_{p'\uparrow}$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \mp |\downarrow\uparrow\rangle\right)$$

$$H_E - \mu N = H_0 - \mu N$$

$$+ \frac{1}{V}\sum_{pp'} U(p-p') \langle c^\dagger_{p\uparrow} c^\dagger_{-p\downarrow} \rangle c_{-p'\downarrow} c_{p'\uparrow}$$

$$+ \boxed{\frac{1}{V}\sum_{pp'} U(p-p')} \boxed{c^\dagger_{p\uparrow} c^\dagger_{-p\downarrow}} \boxed{\langle c_{-p'\downarrow} c_{p'\uparrow} \rangle}$$

$$\boxed{= H_0 - \mu N + \sum_p \left(\Delta^*_p \, c_{-p\downarrow} c_{p\uparrow} + \boxed{\Delta_p} c^\dagger_{p\uparrow} c^\dagger_{-p\downarrow} \right)}$$

$$\longrightarrow \boxed{\Delta_p = \frac{1}{V}\sum_{p'} U(p-p') \langle c_{-p'\downarrow} c_{p'\uparrow} \rangle}$$

$$H_0 - \mu N = \sum_{p\sigma} \epsilon_p c^\dagger_{p\sigma} c_{p\sigma} - \mu \sum_p c^\dagger_{p\sigma} c_{p\sigma}$$

$$= \sum_{p\sigma} \underline{\xi_p c^\dagger_{p\sigma} c_{p\sigma}}$$

$$\xi_p = \xi_{-p} = \epsilon_p - \mu = \frac{p^2}{2m} - \mu$$

$$\boxed{H_E - \mu N = \sum_p \begin{pmatrix} c^+_{p\uparrow} & c_{-p\downarrow} \end{pmatrix} \begin{pmatrix} \xi_p & \Delta_p \\ \Delta^*_p & -\xi_{-p} \end{pmatrix} \begin{pmatrix} c_{p\uparrow} \\ c^+_{-p\downarrow} \end{pmatrix}}$$

$$= \sum_p \begin{pmatrix} c^+_{p\uparrow} & \underline{c_{-p\downarrow}} \end{pmatrix} \begin{pmatrix} \cancel{\xi_p} c_{p\uparrow} + \cancel{\Delta_p} c^+_{-p\downarrow} \\ \underline{\Delta^*_p c_{p\uparrow} - \xi_{-p}} c^+_{-p\downarrow} \end{pmatrix}$$

$$= \sum_p \Big(\underline{c^+_{p\uparrow} c_{p\uparrow}} \xi_p + \underline{c^+_{p\uparrow} c^+_{-p\downarrow}} \Delta_p + \underline{c_{-p\downarrow} c_{p\uparrow} \Delta^*_p} - \xi_p \underline{c_{-p\downarrow} c^+_{-p\downarrow}} \Big)$$

$$\Psi_p \equiv \begin{pmatrix} c_{p\uparrow} \\ c^+_{-p\downarrow} \end{pmatrix} \begin{array}{l} i=1 \\ i=2 \end{array}$$

Spineur de Nambu

$$\boxed{\left\{ \Psi_{pi}, \Psi^+_{p'j} \right\} = \delta_{pp'} \, \delta_{ij}}$$

$$\left\{ \overset{i=1}{C_{p\uparrow}}, \overset{j=2}{C_{-p\downarrow}} \right\} \qquad \left\{ \right.$$

$$\Psi^+_p = \left(c^+_{p\uparrow} \; C_{-p\downarrow} \right)$$

$$\sum_{i,j} U^+_{li} \left\{ \Psi_{pi}, \Psi^+_{pj} \right\} U_{jk}$$

$$= \sum_{i,j} U^+_{li} \, \delta_{ij} \, U_{jk} = \sum_i U^+_{li} U_{ik}$$

$$= \delta_{lk}$$

U = matrice unitaire

$$U^+U = UU^+ = I$$

avr. 24-10:12

$$\left(c^{+}_{p\downarrow} \; c_{-p\downarrow}\right) U \; U^{+} \begin{pmatrix} \xi_p & \Delta_p \\ \Delta^{*}_p & -\xi_p \end{pmatrix} U \; U^{+} \begin{pmatrix} c_{p\uparrow} \\ c^{+}_{-p\downarrow} \end{pmatrix}$$

$$\left(\alpha^{+}_{\uparrow} \; \alpha_{\downarrow}\right) \boxed{= \begin{pmatrix} E_p & 0 \\ 0 & -E_p \end{pmatrix}} \begin{pmatrix} \alpha_{p\uparrow} \\ \alpha^{+}_{\downarrow} \end{pmatrix}$$

$$E_p \, \alpha^{+}_{p\uparrow} \alpha_{p\downarrow} - E_p \, \alpha_{-p\downarrow} \alpha^{+}_{-p\downarrow}$$

Diagonalisation:

$$\lambda_p I - \begin{pmatrix} \xi_p & \Delta_p \\ \Delta_p^* & -\xi_p \end{pmatrix}$$

$$\det \begin{bmatrix} \lambda_p - \xi_p & -\Delta_p \\ -\Delta_p^* & \lambda_p + \xi_p \end{bmatrix} = (\lambda_p - \xi_p)(\lambda_p + \xi_p) - |\Delta_p|^2 = 0$$

$$\lambda_p^2 = \xi_p^2 + |\Delta_p|^2$$

$$\lambda_p = \pm E_p$$

où $E_p = \sqrt{\xi_p^2 + |\Delta_p|^2}$

$$\begin{pmatrix} \pm E_p - \xi_p & -\Delta_p \\ -\Delta_p^* & \pm E_p + \xi_p \end{pmatrix} \begin{pmatrix} a_{1p} \\ a_{2p} \end{pmatrix} = 0$$

$\rightarrow$ $\boxed{(\pm E_p - \xi_p)\, a_{1p} = \Delta_p\, a_{2p}}$ $\leftarrow$

$\rightarrow$ $|a_{1p}|^2 + |a_{2p}|^2 = 1$ $\leftarrow$

$$\left[\frac{|\Delta_p|^2}{(\pm E_p - \xi_p)^2} + 1\right] |a_{2p}|^2 = 1$$

$$\left[\frac{|\Delta_p|^2 + (E_p^2 \mp 2E_p\xi_p + \xi_p^2)}{(\pm E_p - \xi_p)^2}\right] |a_{2p}|^2 = 1$$

$$\left[\frac{2E_p^2 \mp 2E_p\xi_p}{(\pm E_p - \xi_p)^2}\right] |a_{2p}|^2 = 1$$

$$2E_p \left[\frac{E_p \mp \xi_p}{(E_p \mp \xi_p)^2}\right] |a_{2p}|^2 = 1$$

$$|a_{2p}|^2 = \frac{1}{2E_p}(E_p \mp \xi_p)$$

$$= \frac{1}{2}\left(1 \mp \frac{\xi_p}{E_p}\right)$$

$$|a_{1p}|^2 = \frac{1}{2}\left(1 \pm \frac{\xi_p}{E_p}\right)$$

avr. 24-10:20

Soit

$$u_p \equiv \frac{1}{\sqrt{2}}\left(1+\frac{\xi_p}{E_p}\right)^{1/2} e^{-i\phi_{1p}}$$

$$v_p \equiv \frac{1}{\sqrt{2}}\left(1-\frac{\xi_p}{E_p}\right)^{1/2} e^{-i\phi_{2p}}$$

Vecteur propre pour $+E_p$

$$\begin{pmatrix} a_{1p} \\ a_{2p} \end{pmatrix} = \begin{pmatrix} u_p \\ v_p^* \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \left(1+\frac{\xi_p}{E_p}\right)^{1/2} e^{-i\phi_{1p}} \\ \left(1-\frac{\xi_p}{E_p}\right)^{1/2} e^{+i\phi_{2p}} \end{pmatrix}$$

$$(+E_p - \xi_p)\, a_{1p} = \Delta_p\, a_{2p}$$

$$(E_p - \xi_p)\cancel{\frac{1}{\sqrt{2}}}\left(1+\frac{\xi_p}{E_p}\right)^{1/2} e^{-i\phi_{1p}} = \frac{\Delta_p}{\cancel{\sqrt{2}}}\left(1-\frac{\xi_p}{E_p}\right)^{1/2} e^{i\phi_{2p}}$$

$$(E-\xi_p)\left(1-\frac{\xi_p^2}{E_p^2}\right)^{1/2} e^{-i\phi_{1p}} = \Delta_p\left(1-\frac{\xi_p}{E_p}\right)e^{i\phi_{2p}}$$

$$E_p^2 = \xi_p^2 + |\Delta_p|^2$$

$$(E-\xi_p)\left(\frac{|\Delta_p|^2}{E_p^2}\right)^{1/2} e^{-i\phi_{1p}} = \Delta_p\left(1-\frac{\xi_p}{E_p}\right)e^{i\phi_{2p}}$$

$$\cancel{(E-\xi_p)}\frac{|\Delta_p|}{\cancel{E_p}} e^{-i\phi_{1p}} = \Delta_p \cancel{\left(1-\frac{\xi_p}{E_p}\right)} e^{i\phi_{2p}}$$

$$\boxed{\Delta_p = |\Delta_p|\, e^{-i\phi_{1p} - i\phi_{2p}}}$$

$\boxed{-E_p}$

$$\begin{pmatrix} a_{1p} \\ a_{2p} \end{pmatrix} = \begin{pmatrix} -v_p \\ u_p^* \end{pmatrix}$$

Donc

$$U = \begin{pmatrix} u_p & -v_p \\ v_p^* & u_p^* \end{pmatrix} \quad \begin{pmatrix} -v_p \\ u_p^* \end{pmatrix} \rightsquigarrow e^{i\phi}$$

$$\det U = |u_p|^2 + |v_p|^2 = 1$$

$$\begin{pmatrix} \alpha_{p\uparrow} \\ \alpha^+_{-p\downarrow} \end{pmatrix} = U^+ \begin{pmatrix} c_{p\uparrow} \\ c^+_{-p\downarrow} \end{pmatrix}$$

$$\boxed{\alpha_{p\sigma} |BCS\rangle = 0}$$

$$H_{\bar{F}} - \mu N = \sum_{p\sigma} E_p \, \alpha^+_{p\sigma} \alpha_{p\sigma}$$

avr. 24-10:31