$$H_{BCS} = \sum_{p,\sigma} \xi_p \, c^{+}_{p\sigma} c_{p\sigma}$$

$$+ \sum_{p} \left(\Delta^{*}_{p} c^{+}_{p\uparrow} c^{+}_{-p\downarrow} + \Delta_p c_{-p\downarrow} c_{p\uparrow} \right)$$

$$= \sum_{p} \left(c^{+}_{p\uparrow} \; c_{-p\downarrow} \right) \begin{pmatrix} \xi_p & \Delta_p \\ \Delta^{*}_{p} & -\xi_{-p} \end{pmatrix} \begin{pmatrix} c_{p\uparrow} \\ c^{+}_{-p\downarrow} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{p\uparrow} \\ \alpha^{+}_{-p\downarrow} \end{pmatrix} = \begin{pmatrix} u^{*}_{p} & v_p \\ -v^{*}_{p} & u_p \end{pmatrix} \begin{pmatrix} c_{p\uparrow} \\ c^{+}_{-p\downarrow} \end{pmatrix}$$

$$\begin{cases} u_p = \dfrac{1}{\sqrt{2}} \left(1 + \dfrac{\xi_p}{E_p} \right)^{1/2} e^{-i\phi_{1p}} \\ v_p = \dfrac{1}{\sqrt{2}} \left(1 - \dfrac{\xi_p}{E_p} \right)^{1/2} e^{-i\phi_{2p}} \end{cases}$$

$$\Delta_p = |\Delta_p| \, e^{-i(\phi_{1p} + \phi_{2p})}$$

$$E_p = \sqrt{\xi_p^2 + |\Delta_p|^2}$$

$$\boxed{\Delta_p = \frac{1}{V} \sum_{p'} U(\vec{p} - \vec{p}') \langle c_{-p'\downarrow} c_{p'\uparrow} \rangle}$$

$$\alpha_{p\sigma} |BCS\rangle = 0$$

- Solutions de BCS
  - Cohérence
  - Singulet, Triplet, s, p, d, f.
- T_c, Δ_0 equ. G.L.

Effet Josephson.

- Ham. de Josephson
- Base θ, N
- Courant, effet Josephson

$$\langle \alpha^{+}_{p\sigma} \alpha_{p\sigma} \rangle = \frac{1}{e^{\beta E_p} + 1} \quad ?$$

$$Z = \mathrm{Tr}\, e^{-\beta \sum_{p\sigma} E_p \alpha^{+}_{p\sigma} \alpha_{p\sigma}}$$

$$\langle O \rangle = \frac{\mathrm{Tr}\, e^{-\beta H_{BCS}} O}{\mathrm{Tr}\, e^{-\beta H_{BCS}}}$$

$$\langle \alpha^{+}_{p\sigma} \alpha^{+}_{-p\sigma} \rangle = 0$$

$$\alpha^{+}_{p\sigma} \alpha^{+}_{p\sigma} |BCS\rangle$$

$$\alpha_{p\sigma} |BCS\rangle = 0$$

$$\boxed{\langle c_{-p'\downarrow}\, c_{p'\uparrow} \rangle} =$$

$$\langle (v_{p'}\alpha^+_{p'\uparrow} + u_{p'}\alpha_{-p'\downarrow})$$

$$(u_{p'}\alpha_{p'\uparrow} - v_{p'}\alpha^+_{-p'\downarrow}) \rangle$$

$$= v_{p'}u_{p'} \langle (\underbrace{\alpha^+_{p'\uparrow}\alpha_{p'\uparrow}} - \underbrace{\alpha_{-p'\downarrow}\alpha^+_{-p'\downarrow}}) \rangle$$

$$= \frac{1}{2}\left(1 - \frac{\xi^2_{p'}}{E^2_{p'}}\right)^{1/2} e^{-i\phi_{1p'} - i\phi_{2p'}} \left(2n(E_{p'}) - 1\right)$$

$$\boxed{= \frac{1}{2}\frac{\Delta_{p'}}{E_{p'}}(2n(E_{p'}) - 1)}$$

$$\Delta_p = -\frac{1}{V}\sum_{p'} U(p-p') \frac{\boxed{|\Delta_{p'}| e^{-i\phi_{1p'} - i\phi_{2p'}}}}{E_{p'}} (1 - 2n(E_{p'}))$$

$$\boxed{\Delta_p = -\frac{1}{V}\sum_{p'} \underbrace{U(p-p')}_{Y_p Y^*_{p'}} \frac{\Delta_{p'}}{E_{p'}}(1 - 2n(E_{p'}))}$$

Supra. de type s, d, p.... cohérence.

BCS $\quad U(p-p') = -|U_0|$

$$\Delta_p = |U_0| \int \frac{d^3p'}{(2\pi)^3} \frac{\Delta_{p'}}{E_{p'}}(1 - 2n(E_{p'}))$$

$$\boxed{\Delta = |U_0| \int \frac{d^3p'}{(2\pi)^3} \frac{\Delta}{E_{p'}}(1 - 2n(E_{p'}))}$$

$\Delta = |\Delta| e^{i\phi}$ où ϕ est indép. de $\vec{p}$

$$\prod_k \left(1 + \frac{v_k}{u^*_k} \boxed{c^+_{-k\downarrow} c^+_{k\uparrow}}\right)|0\rangle$$

$$e^{-i\phi_{2p} - i\phi_{1p}} = e^{i\phi}$$

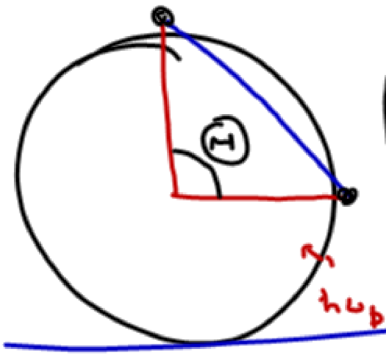
$$\langle c_{-p'\downarrow}\, c_{p'\uparrow} \rangle = \underset{P}{+} \langle c_{p'\downarrow}\, c_{-p'\uparrow} \rangle$$

$$= - \langle c_{-p'\uparrow}\, c_{p'\downarrow} \rangle$$

S spatial

Singulet de spin

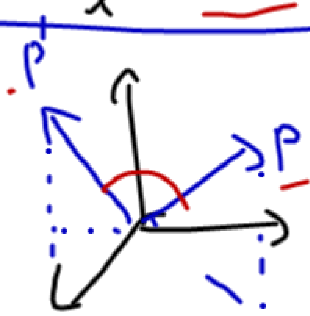
$$U(p-p') = U(\cos\Theta)$$



$$|p-p'| = \sqrt{p_F'^2 + p_F^2 - 2p_F p_F' \cos\Theta}$$

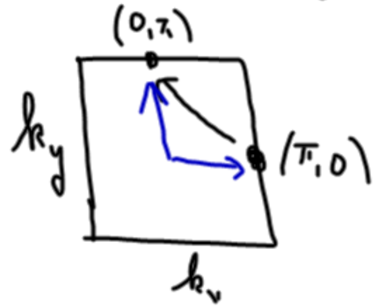
$$U(\cos\Theta) = \sum_{l=0}^{\infty} U_l P_l(\cos\Theta)$$

$$P_l(\cos\Theta) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\theta,\varphi)\, Y^*_{lm}(\theta',\varphi')$$



$$\Delta_{l,m} = -\int \frac{d^3p}{(2\pi)^3} U_l \frac{4\pi}{2l+1} \frac{\Delta_{lm}}{2E_{p'}} \left(1-2n(E_{p'})\right)$$

$$U(\cos(\phi-\phi')) = U_0 - V\cos^2(\phi-\phi')$$

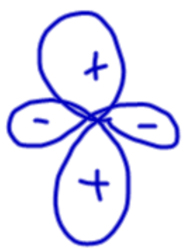


$a = 1$

$(\pi, -\pi)$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= U_0 - V\left(\frac{1+\cos 2\phi \cos 2\phi' + \sin 2\phi \sin 2\phi'}{2}\right)$$



$$\Delta_p = -\frac{1}{2}\int \frac{d^3p'}{(2\pi)^3} U(p-p') \frac{\Delta_{p'}}{E_{p'}} \left(1-2n(E_{p'})\right)$$

Solution BCS, T_c, Δ_0, L.G.

$$\int \frac{d^3p'}{(2\pi)^3} \Longrightarrow \int d\Omega \int_0^\infty \frac{p'^2 dp'}{(2\pi)^3}$$

$$= 4\pi \int_0^\infty \frac{p'^2 dp'}{(2\pi)^3} \rightarrow \int_{-\hbar\omega_D}^{\hbar\omega_D} N(\xi')\, d\xi'$$

$$\Delta = |U_0| \int_{-\hbar\omega_D}^{\hbar\omega_D} N(\xi') \frac{\Delta}{E_{p'}} \left(1 - 2n(E_{p'})\right) d\xi'$$

$$= |U_0|\, D(E_F) \int_0^{\hbar\omega_D} \frac{\Delta}{E_{p'}} \left(1 - 2n(E_{p'})\right) d\xi'$$

$D(E_F) = 2N(E_F)$

$$\boxed{\Delta = \Delta \frac{|U_0|\, D(E_F)}{2} \int_0^{\hbar\omega_D} d\xi \left\{ \frac{\tanh\left(\frac{1}{2}\beta\sqrt{\xi^2+\Delta^2}\right)}{\sqrt{\xi^2+\Delta^2}} \right.}$$

Près de T_c

$$\Delta = a''\Delta + b''\Delta|\Delta|^2$$

$$\boxed{(1-a'') = b\; |\Delta|^2}$$

$$1 = \frac{|U_0| D(E_F)}{2} \int_0^{\hbar\omega_D \beta/2} \frac{\tanh \beta_c \xi/2}{\beta \xi/2} \, d\frac{\xi\beta}{2}$$

$$= \frac{|U_0| D(E_F)}{2} \left(\ln\left(\frac{\beta_c \hbar\omega_D}{2}\right) - \int_0^{\infty} \frac{\ln x}{\cosh^2 x} \, dx \right)$$

$\beta_c \hbar\omega_D >> 1$

$$\boxed{\hbar\omega_D >> T_c}$$

$$1 = \frac{|U_0| D(E_F)}{2} \left(\ln\left(\frac{\beta_c \hbar\omega_D}{2}\right) - \ln\left(\frac{4\gamma}{\pi}\right) \right)$$

$$\boxed{k_B T_c = \frac{2\gamma}{\pi} \hbar\omega_D \, e^{-2/|U_0| D(E_F)}}$$

à $T = 0$

$$\Delta_0 = \Delta_0 \frac{|U_0| D(E_F)}{2} \int_0^{\hbar\omega_D} \frac{d\xi}{\sqrt{\xi^2 + |\Delta_0|^2}}$$

$$\xi = |\Delta_0| \sinh\theta$$

$$\frac{2}{|U_0| D(E_F)} = \sinh^{-1}\left(\frac{\hbar\omega_D}{|\Delta_0|}\right)$$

$$|\Delta_0| = \frac{\hbar\omega_D}{\sinh\left(\frac{2}{|U_0| D(E_F)}\right)}$$

$$\sinh x = \frac{e^x + \cancel{e^{-x}}}{2}$$

$$\boxed{|\Delta_0| = 2\hbar\omega_D \, e^{-2/|U_0| D(E_F)}}$$

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$$\boxed{\frac{2\Delta_0}{k_B T_c} = 3.53}$$

Al

3.37 ± 0.1

Cd

3.2 ± 0.1

Sn

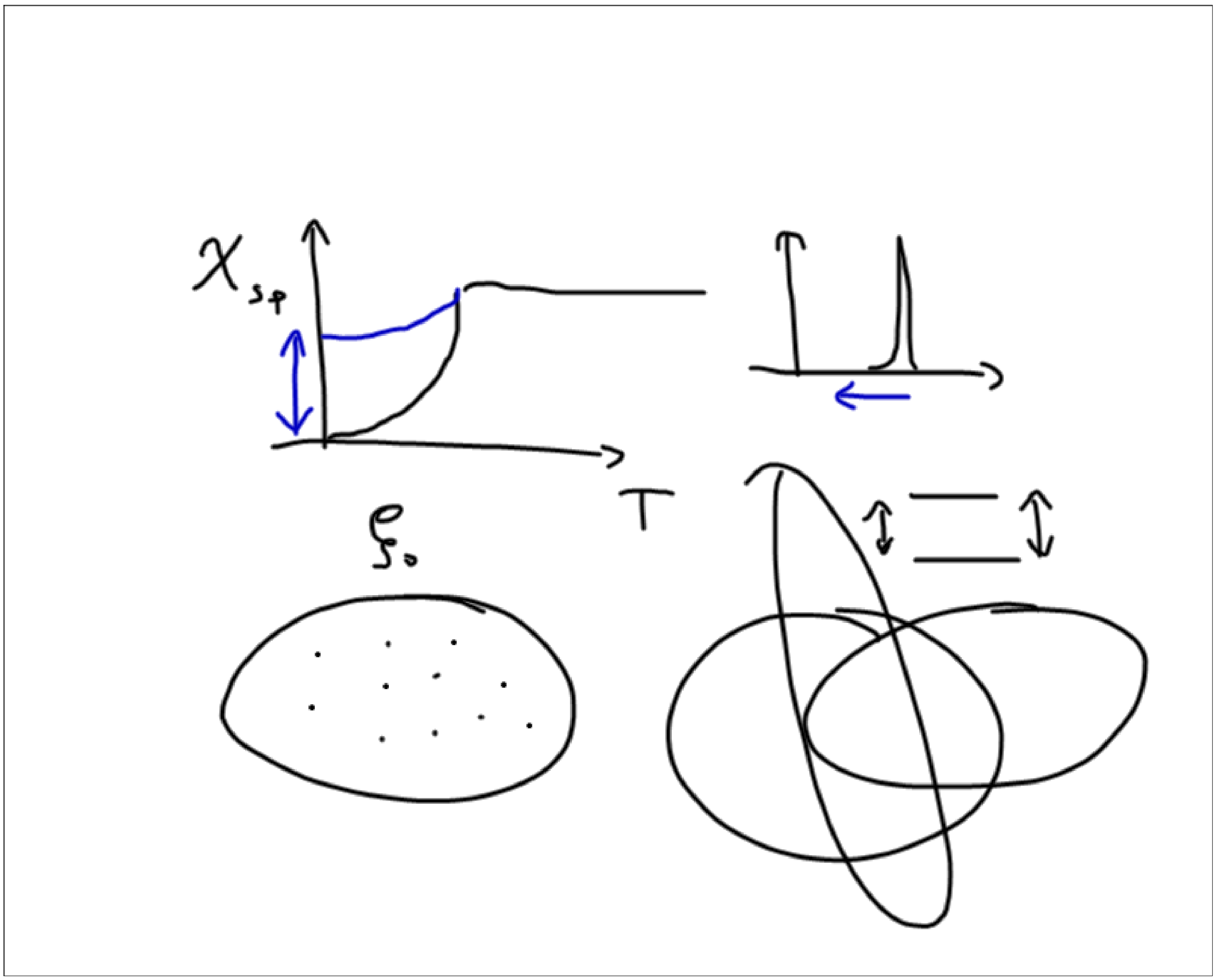
3.46 ± 0.1

Pb

4.29 ± .4

Hg

4.6



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Josephson

$$\phi_G \quad \phi_{\nu} \quad e^{-\frac{2ieV}{\hbar}t}$$

$$|BCS\rangle_G = \prod_k \left(1 + \left|\frac{v_k}{u_k}\right| e^{-i(\phi_{2k}+\phi_{1k})} c^+_{-k\downarrow} c^+_{k\uparrow}\right)|0\rangle$$

$$\left(1 + g(k)\, c^+_{-k\downarrow} c^+_{k\uparrow}\right)|0\rangle$$

$$\boxed{\Psi_\phi = \sum_{n=0}^{N} e^{-in\phi}\, \Psi_n}$$

$$\Psi_m = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{im\phi}\, \Psi_\phi$$

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{ip\phi} = \delta_{p,0}$$

$$\varphi_G \qquad \varphi_D$$

Base ou tunnel = 0

$$\Psi_{\phi_G} \Psi'_{\phi_D} = \sum_{n_G=0}^{N_G} e^{-in_G\phi_G}\, \Psi_{n_G} \times \sum_{n_D=0}^{N_D} e^{-in_D\varphi_D}\, \Psi'_{n_D}$$

$$n_G = \frac{N_T}{2} + n \qquad n_D = \frac{N_T}{2} - n$$

$$= \sum_{N_T=0}^{N_G+N_D} \sum_{n=-\frac{N_T}{2}}^{N_T/2} e^{-i\left(\frac{N_T}{2}+n\right)\phi_G}\, e^{-i\left(\frac{N_T}{2}-n\right)\phi_D}\, \Psi_{\frac{N_T}{2}+n} \Psi'_{\frac{N_T}{2}-n}$$

$$= \sum_{N_T=0}^{N_G+N_T} e^{-i\frac{N_T}{2}(\phi_G+\phi_D)} \boxed{\sum_{n=-\frac{N_T}{2}}^{N_T/2} e^{-in(\phi_G-\phi_D)}\, \Psi_{\frac{N_T}{2}+n} \Psi'_{\frac{N_T}{2}-n}}$$

θ

$$\boxed{|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle}$$

Base pour calcul

$$\rightarrow \hat{\hat{H}}_J = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Big(|n\rangle\langle n+1| + |n+1\rangle\langle n| \Big)$$

$$\rightarrow \boxed{|n\rangle = \int_0^{2\pi} e^{in\theta} |\theta\rangle \frac{d\theta}{2\pi}}$$

$$\rightarrow \sum_{n=-\infty}^{\infty} e^{-in\theta} = 2\pi\,\delta(\theta)$$

$$0 < \theta \leq 2\pi$$

$$\hat{H}_J = -\frac{E_J}{2} \sum_{n=-\infty}^{\infty} \Bigg[\int_0^{2\pi} e^{i\cancel{n}\theta} |\theta\rangle \frac{d\theta}{2\pi} \int_0^{2\pi} \langle\theta'| e^{-i(\cancel{n}+1)\theta'} \frac{d\theta'}{2\pi} + \text{c.h.} \Bigg]$$

$$= -\frac{E_J}{2} \int_0^{2\pi} \frac{d\theta}{2\pi} \Big(e^{-i\theta} |\theta\rangle\langle\theta| + \text{c.h.} \Big)$$

$$\boxed{\hat{H}_J = -E_J \int_0^{2\pi} \frac{d\theta}{2\pi} \cos\theta\, |\theta\rangle\langle\theta|}$$

$$\hat{H}_J |\theta'\rangle = -E_J \cos\theta' \,|\theta'\rangle$$

Base θ, N :

$$e^{-i\hat{\theta}} = \int_0^{2\pi} e^{-i\theta} |\theta\rangle\langle\theta| \frac{d\theta}{2\pi}$$

$$\hat{H}_J = -E_J \cos\hat{\theta}$$

$$\hat{N} = \sum_{n=-\infty}^{\infty} n |n\rangle\langle n|$$

$$\hat{N}|n'\rangle = n'|n'\rangle$$

$$e^{-i\hat{\theta}}|\theta'\rangle = e^{-i\theta'}|\theta'\rangle$$

$$e^{\pm i\hat{\theta}}|n\rangle = e^{\pm i\hat{\theta}} \int_0^{2\pi} \frac{d\theta}{2\pi} e^{in\theta}|\theta\rangle$$

$$= \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i(n\pm 1)\theta}|\theta\rangle$$

$$= |n\pm 1\rangle$$

$$\hat{N} e^{\pm i\hat{\theta}}|n\rangle = (n\pm 1)|n\pm 1\rangle$$

$$= (n\pm 1) e^{\pm i\hat{\theta}}|n\rangle$$

$$e^{\pm i\hat{\theta}}\hat{N}|n\rangle = n\,|n\pm 1\rangle$$

$$= n\, e^{\pm i\hat{\theta}}|n\rangle$$

$$\boxed{[\hat{N}, e^{\pm i\hat{\theta}}] = \pm e^{\pm i\hat{\theta}}}$$

$$[\hat{N}, \hat{\theta}] = -i$$

Courant et effet Josephson

$$\frac{d\hat{N}}{dt} = \frac{i}{\hbar}\left[\hat{H},\hat{N}\right]$$

$$= \frac{i}{\hbar}\left(-\frac{E_J}{2}\right)\left[e^{i\hat{\theta}}+e^{-i\hat{\theta}},\hat{N}\right]$$

$$= \frac{1}{\hbar}\left(+\frac{E_J}{2i}\right)\left[-e^{i\hat{\theta}}+e^{-i\hat{\theta}}\right]$$

$$= -\frac{E_J}{\hbar}\sin\hat{\theta}$$

$$\boxed{\hat{I} = -2e\frac{d\hat{N}}{dt} = \frac{2e}{\hbar}E_J\sin\hat{\theta}}$$

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Si on applique une diff. de potentiel $V = V_D - V_G$

$$\hat{H} = (-2eV)(-\hat{N}) + \hat{H}_J$$

$$\frac{d\hat{\theta}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{\theta}]$$

$$= \frac{i}{\hbar} 2eV [\hat{N}, \hat{\theta}] = \frac{2e}{\hbar} V$$

$$\boxed{\frac{d\hat{\theta}}{dt} = \frac{2eV}{\hbar} = \frac{2e}{\hbar} V}$$

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$$\frac{d c^+_{p,\sigma}}{dt} = \frac{i}{\hbar}\left[H, c^+_{p\sigma}\right]$$

$$-eV N_{ptcle} = -eV \sum_{p,\sigma} c^+_{p\sigma} c_{p\sigma}$$

$$= -eV \frac{i}{\hbar} c^+_{p\sigma}$$

$$c^+_{p\sigma}(t) = c^+_{p\sigma}(0)\, e^{-\frac{ieV}{\hbar}t}$$

$$\frac{\hbar}{i}\nabla \longrightarrow \frac{\hbar}{i}\nabla - eA$$

$$i\hbar\frac{\partial}{\partial t} \longrightarrow i\hbar\frac{\partial}{\partial t} + eV$$

$$|\psi| e^{i\phi}$$

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