$|j,m\rangle$

$J_+|j,m\rangle = \hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle$

$J_-|j,m\rangle = \hbar\sqrt{j(j+1)-m(m-1)}\,|j,m\rangle$

Spin $\frac{1}{2}$

$\langle \frac{1}{2}\frac{1}{2}|J_3|\frac{1}{2},\frac{1}{2}\rangle = \frac{\hbar}{2}$ $\quad \cdot J_3 = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$\langle \frac{1}{2}\frac{1}{2}|J_3|\frac{1}{2},-\frac{1}{2}\rangle = 0$

$\langle \frac{1}{2}\frac{1}{2}|J_+|\frac{1}{2},-\frac{1}{2}\rangle = \hbar\sqrt{\frac{3}{4}+\frac{1}{2}\frac{1}{2}}$ $\quad \cdot J_+ = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$J_\pm = J_1 \pm iJ_2$ $\quad \cdot J_- = \hbar\begin{pmatrix}0&0\\1&0\end{pmatrix}$

$J_1 = \frac{J_+ + J_-}{2}$ $\quad J_2 = -i\frac{(J_+ - J_-)}{2}$

$J_3 = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\quad J_2 = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ $\quad J_1 = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$

$\vec{J} = \frac{\hbar}{2}\vec{\sigma}$

$\mathrm{Tr}\,\vec{\sigma} = 0 \qquad \vec{\sigma}^+ = \sigma$

$a_a b_b \quad \boxed{\sigma_a\sigma_b = I\delta_{ab} + i\epsilon_{abc}\sigma_c}$

$J_1 J_2 = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}i&0\\0&-i\end{pmatrix} = i\sigma_3$

$\vec{\sigma} = \sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z}$

$$\boxed{(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = \vec{a}\cdot\vec{b}\,I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}}$$

janv. 26-08:56

Matrice de rotation:

À prouver: $\boxed{U = e^{-i\theta\hat{n}\cdot\vec{J}/\hbar} \quad U_{spin\frac{1}{2}} = e^{-i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}}}$

Symétrie: $|\psi\rangle \to U|\psi\rangle = |\psi'\rangle$

$\langle\psi'|\psi'\rangle = \langle\psi|U^+U|\psi\rangle \qquad U^+U = I$

En général $\boxed{U = e^{-iK}}$ où $K = K^+$

$\boxed{\lambda_i = e^{-ia_i}}$ où $a_i$ est réel

Dans base diagonale $\lambda_i$ = valeur propre

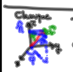
$U^+U = I$ s'écrit $\lambda_i^*\lambda_i = 1 \quad \forall i$

Si $\lambda = e^{-i(a_{réel} + i a_{im})}$

$= e^{a_{im}} e^{-i a_{réel}}$

$\lambda^*\lambda = e^{2a_{im}}$

Classique



$\vec{r}' = \vec{r} + d\vec{r}$

$d\vec{r} = d\varphi\,\hat{n}\times\vec{r}$

$\vec{R}' = \vec{R} + d\varphi\,\hat{n}\times\vec{R}$ ←

→ $|\psi'\rangle = U|\psi\rangle$

→ $\langle\psi'|\vec{R}|\psi'\rangle = \langle\psi|U^+\vec{R}U|\psi\rangle$

→ $U^+\vec{R}U = \vec{R} + \delta\varphi\,\hat{n}\times\vec{R}$

$U = e^{-iK}$

$(I+iK)\vec{R}(I-iK) = \vec{R} + \delta\varphi\,\hat{n}\times\vec{R}$

$\boxed{i[K,\vec{R}] = \delta\varphi\,\hat{n}\times\vec{R}}$ $\vec{L}$ est le générateur infinitésimal des rotations

$K = \frac{\delta\varphi}{\hbar}\hat{n}\cdot\vec{L}$ hypothèse.

Preuve: $i\cancel{\frac{\delta\varphi}{\hbar}}n_a[L_a,R_b] = \cancel{\delta\varphi}\,\epsilon_{bac}n_a R_c$

$\boxed{(\hat{n}\times\vec{R})_b = \epsilon_{bcd}n_c R_d}$

$[L_a,R_b] = i\hbar\,\epsilon_{abc}R_c$

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$$\boxed{\begin{aligned} U &= e^{-i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}} \\ U &= \cos\frac{\theta}{2}\,I - i\,\hat{n}\cdot\vec{\sigma}\sin\frac{\theta}{2} \end{aligned}}$$

$(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = \vec{a}\cdot\vec{b}\,I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

$(\hat{n}\cdot\vec{\sigma})^2 = I$

$e^{i\theta} = \cos\theta + i\sin\theta$

janv. 26-10:31

2.2 Magnétisme classique ✓

2.3 " quantique ✓

2.4 " dans les atomes ✓

· Dirac eq. de Pauli

· Couplage S.O.

2.5 Paramagnétisme, dia . . .

2.6 Structure fine

2.7 Environnement cristallin

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Forme finale de H en présence de $\vec{B}$ uniforme

paramagnétisme

$\vec{S} = \frac{\vec{\sigma}}{2}$

$g = 2$

$$H = H_0 + \mu_B(\vec{L} + g\vec{S})\cdot\vec{B} + \frac{e^2}{8m}\sum_{i=1}^{Z}(\vec{B}\times\vec{r}_i)^2 + \lambda\sum_{i=1}^{Z}\vec{L}_i\cdot\vec{S}_i$$

Diamagnétisme ✓ Spin orbite

janv. 26-10:31

En présence de $\vec{A}$ et $\varphi$ — Couplage minimal

$i\hbar\frac{\partial}{\partial t} \to i\hbar\frac{\partial}{\partial t} - q\varphi$

$\frac{\hbar}{i}\vec{\nabla} \to \frac{\hbar}{i}\vec{\nabla} - q\vec{A}$

$H = \sum_i \frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} + e\vec{A}\right)^2 + qV$

$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$

Pour $\vec{B}$ = cte $\quad \boxed{\vec{A}(\vec{r}) = \frac{1}{2}\vec{B}\times\vec{r}}$

$\vec{\nabla}\times\vec{A} = \vec{B}$

$[\nabla_i, A_i] = 0 \qquad \vec{\nabla}\cdot\vec{A} = 0$

$\frac{e^2A^2}{2m} = \frac{e^2}{2m}\frac{1}{4}(\vec{B}\times\vec{r})^2$

$\frac{e}{2m}2\vec{A}\cdot\frac{\hbar}{i}\vec{\nabla} = \frac{e}{m}\frac{1}{2}(\vec{B}\times\vec{r})\cdot\frac{\hbar}{i}\vec{\nabla}$

$= \left(\frac{e\hbar}{2m}\right)\vec{B}\cdot\left(\vec{r}\times\frac{\vec{\nabla}}{i}\right)$

$= \mu_B\,\vec{B}\cdot\vec{L}$

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Équation de Dirac

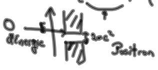
$i\hbar \frac{\partial \psi}{\partial t} = \left( c\vec{\alpha} \cdot \vec{p} + \beta mc^2 I \right) \psi$

Pour être cohérent avec transfo. de Lorentz

$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \alpha = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$

$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \quad \vec{p} = \frac{\hbar}{i}\vec{\nabla} + e\vec{A} \equiv \vec{\pi}$

$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = \left[ \begin{pmatrix} 0 & c\vec{\sigma}\cdot\vec{\pi} \\ c\vec{\sigma}\cdot\vec{\pi} & 0 \end{pmatrix} + \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} mc^2 \right] \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$




$i\hbar \frac{\partial}{\partial t} \gg 2mc^2 \chi$

$\begin{pmatrix} i\hbar \frac{\partial}{\partial t}\varphi \\ 2mc^2 \chi \end{pmatrix} = \begin{pmatrix} 0 & c\vec{\sigma}\cdot\vec{\pi} \\ c\vec{\sigma}\cdot\vec{\pi} & 0 \end{pmatrix} \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$

$\chi \sim \frac{c\vec{\sigma}\cdot\vec{\pi}}{2mc^2}\varphi$

$i\hbar \frac{\partial}{\partial t}\varphi = \frac{(\vec{\sigma}\cdot\vec{\pi})^2}{2m}\varphi \neq \frac{\vec{\pi}^2}{2m}\varphi + \frac{e\hbar}{2m}\vec{B}\cdot\vec{\sigma}$ $\mu_B$

$(\vec{\sigma}\cdot\vec{\pi})^2 = \vec{\pi}^2 I$

$(\sigma\cdot\pi)(\sigma\cdot\pi) = \sigma_a \pi_a \sigma_b \pi_b$

$= (I\delta_{ab} + i\epsilon_{abc}\sigma_c)\pi_a\pi_b$

$= I\vec{\pi}\cdot\vec{\pi} + i\epsilon_{abc}\sigma_c \left(\frac{\hbar}{i}\nabla_a + eA_a\right)\left(\frac{\hbar}{i}\nabla_b + eA_b\right)$

$= \vec{\pi}\cdot\vec{\pi} I + i\epsilon_{abc}\sigma_c \left(eA_a\nabla_b + e\nabla_a A_b\right)\frac{\hbar}{i}$

$= \vec{\pi}\cdot\vec{\pi} I + e\hbar \vec{B}\cdot\vec{\sigma}$

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$g = 2.002\ 319\ 304\ 36\ 22$
(15)

Perturbations

$g = C_1\left(\frac{\alpha}{\pi}\right) + \dots\dots \left(\frac{\alpha}{\pi}\right)^4$

$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137.035\ 999\ 07\ (9)}$

janv. 26-11:04

$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E\begin{pmatrix} X \\ Y \end{pmatrix}$

— y

= x

$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E\begin{pmatrix} X \\ \cancel{Y} \end{pmatrix}$

$Y = -\frac{H_{21}}{H_{22}} X$

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$i\hbar \frac{\partial \tilde{\varphi}}{\partial t} = c(\vec{\sigma}\cdot\vec{\pi})\tilde{\chi}$ $\quad \boxed{\tilde{\chi} = \frac{c(\vec{\sigma}\cdot\vec{\pi})\tilde{\varphi}}{2mc^2}}$

$\cancel{i\hbar\frac{\partial\tilde{\chi}}{\partial t}} + mc^2\tilde{\chi} = c(\vec{\sigma}\cdot\vec{\pi})\tilde{\varphi} - mc^2\tilde{\chi}$

$\varphi \rightarrow e^{-imc^2 t/\hbar}\tilde{\varphi}$

$\chi \rightarrow e^{-imc^2 t/\hbar}\tilde{\chi} \qquad e^{-i\frac{(H+\epsilon_0)t}{\hbar}}$

janv. 26-11:23