

# Mott Physics in Strongly Correlated Superconductivity

A.-M. Tremblay

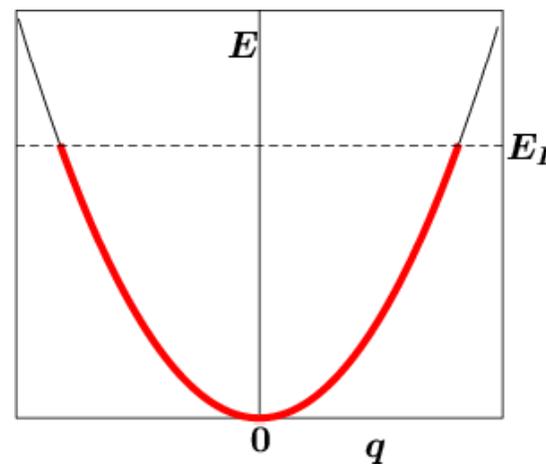
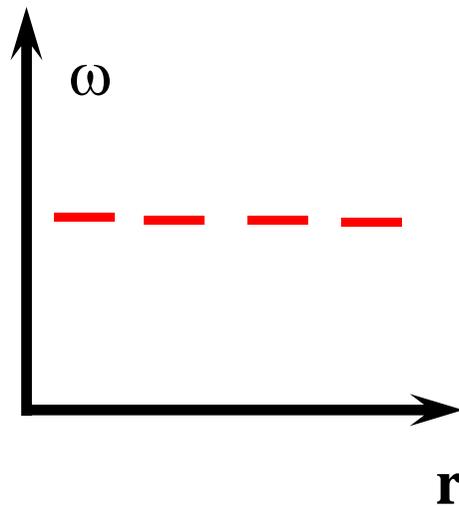
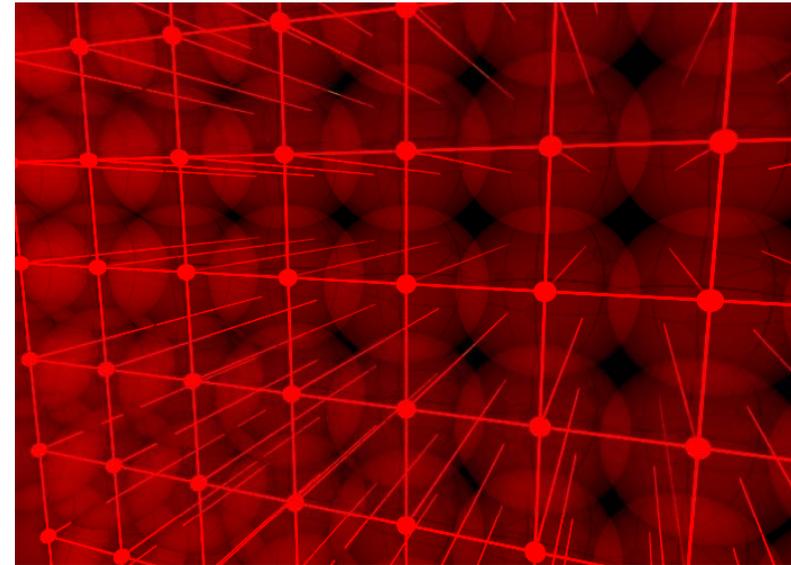
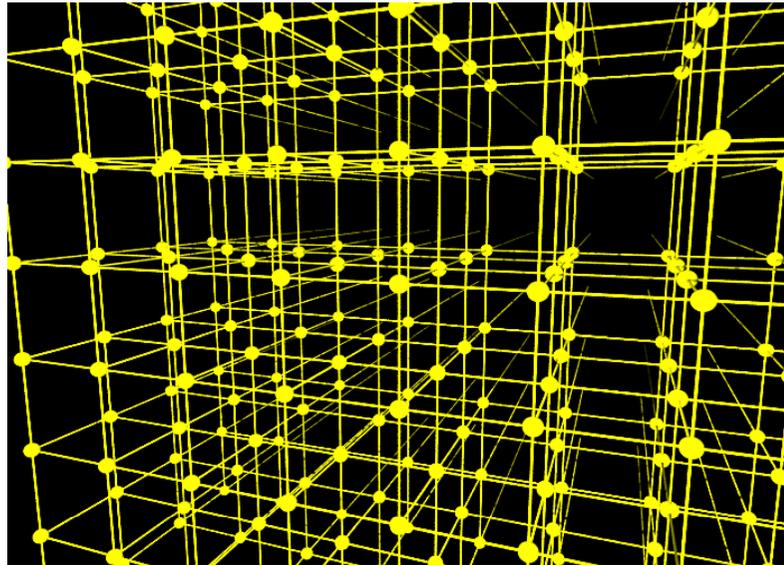
G. Sordi, D. Sénéchal, K. Haule,  
S. Okamoto, B. Kyung, M. Civelli



Los Alamos, 3 May 2011



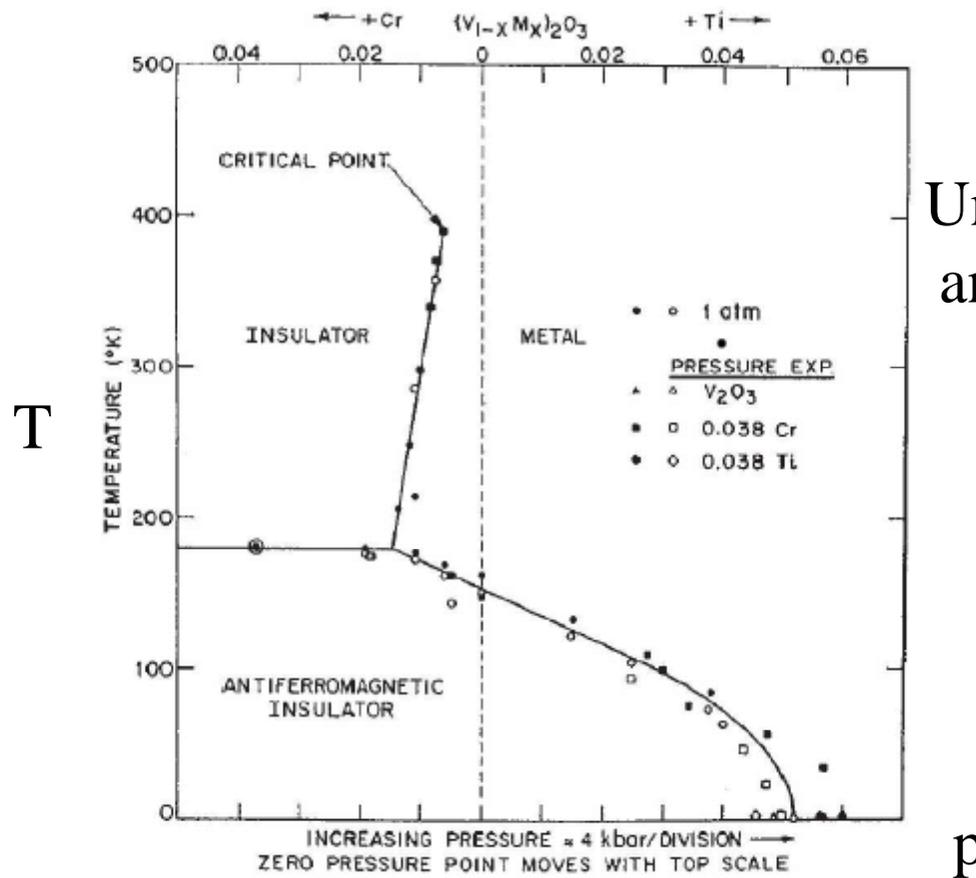
# How to make a metal



Courtesy, S. Julian



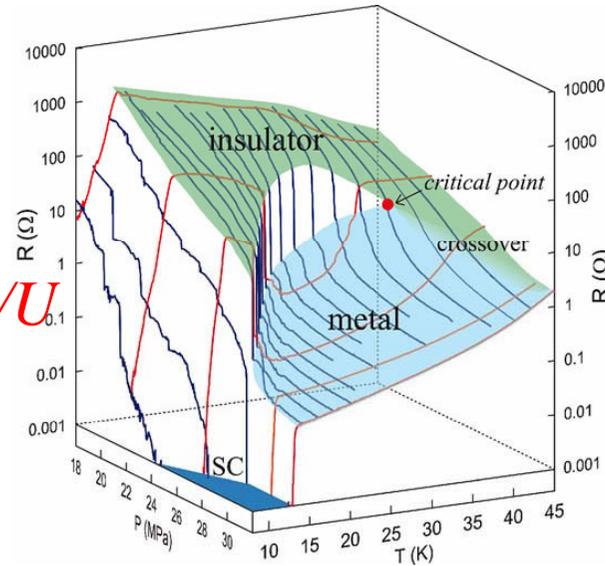
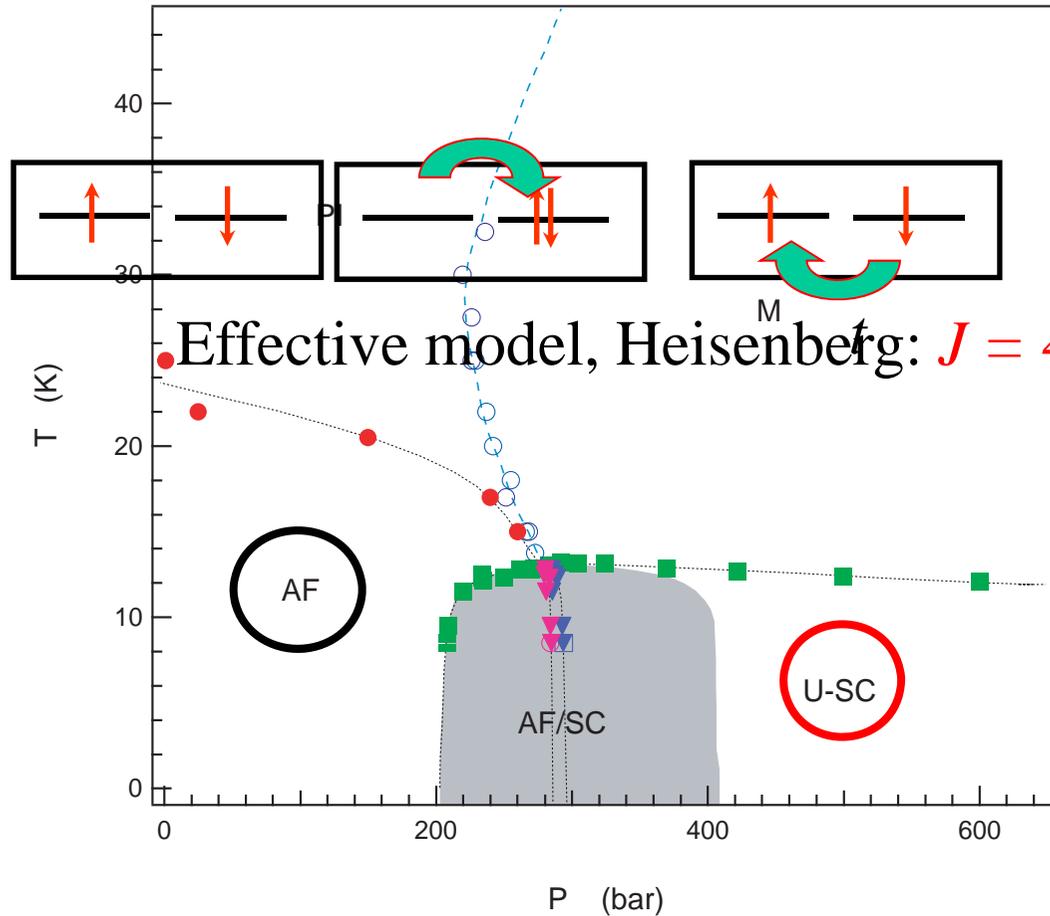
# « Conventional » Mott transition



Understood from Hubbard model  
and dynamical mean field theory

Figure: McWhan, PRB 1970; Limelette, Science 2003

# Bare Mott critical point in organics

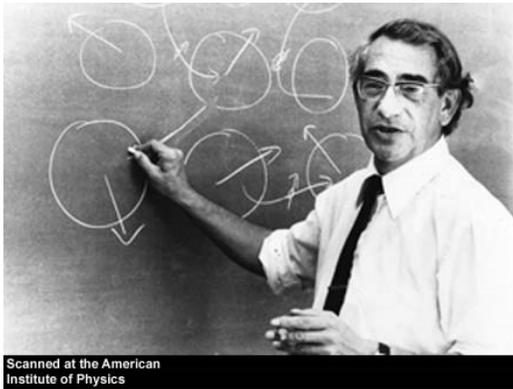


F. Kagawa, K. Miyagawa, + K. Kanoda  
 PRB **69** (2004) + Nature **436** (2005)

Phase diagram (X=Cu[N(CN)<sub>2</sub>]Cl)

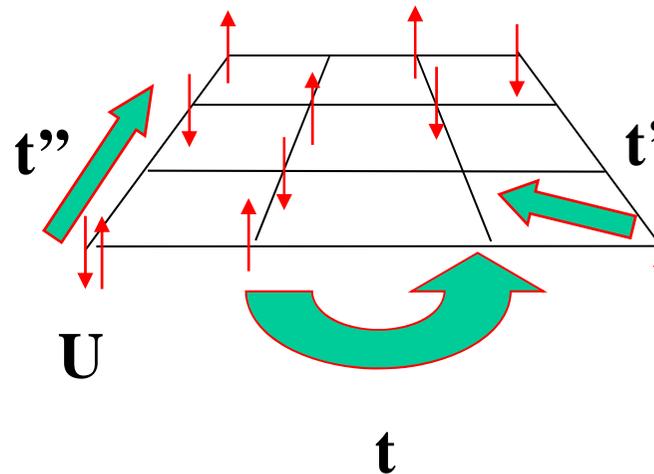
S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

# Hubbard model



1931-1980

$\mu$

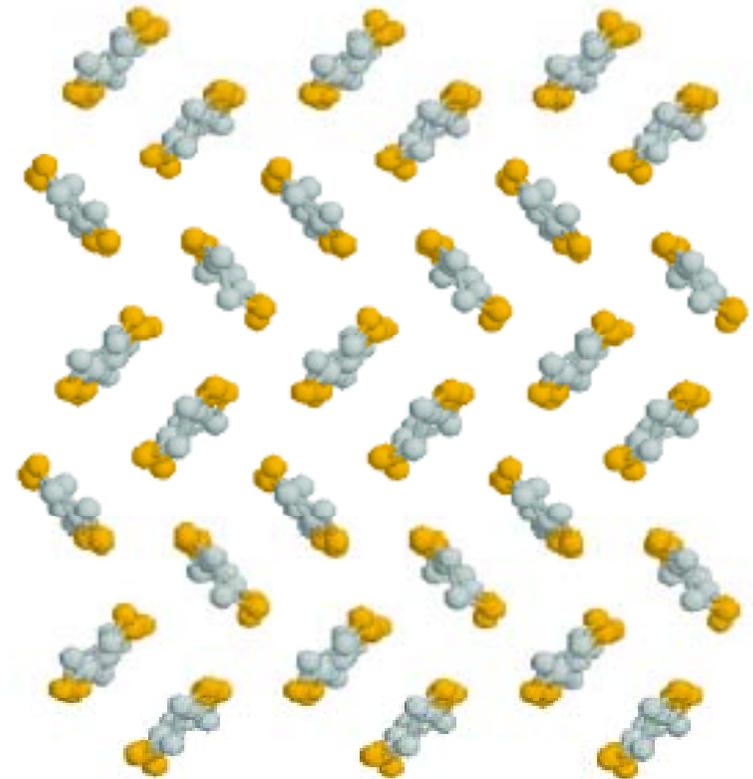
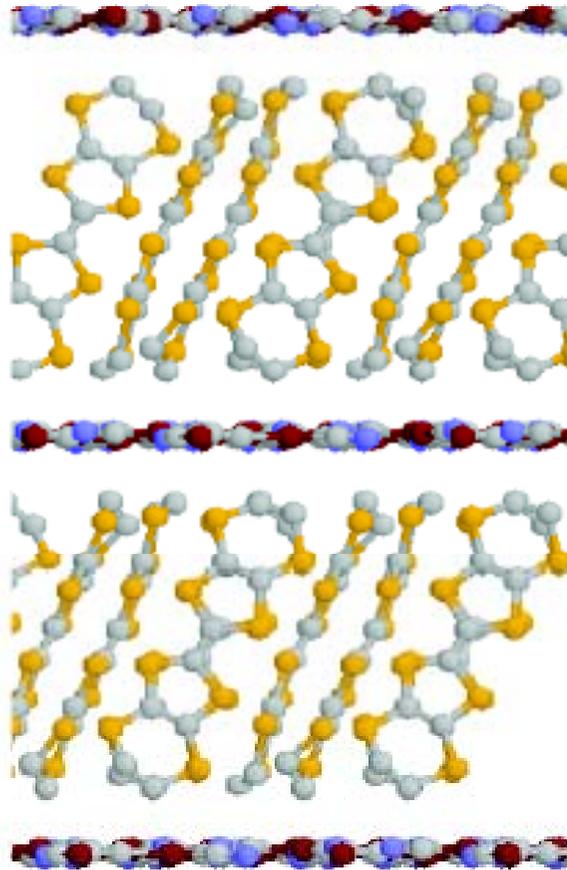


$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

# Layered organics ( $\kappa$ -BEDT-X family)

BEDT-TTF  
layer

Anion layer



$$t \approx 50 \text{ meV}$$

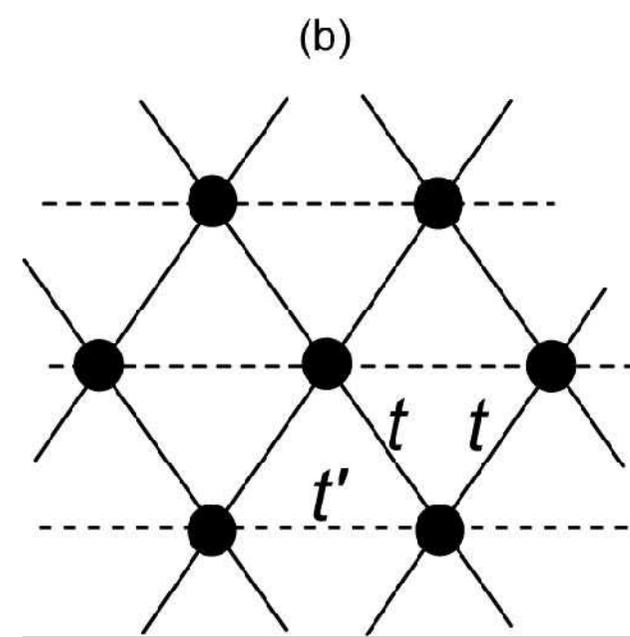
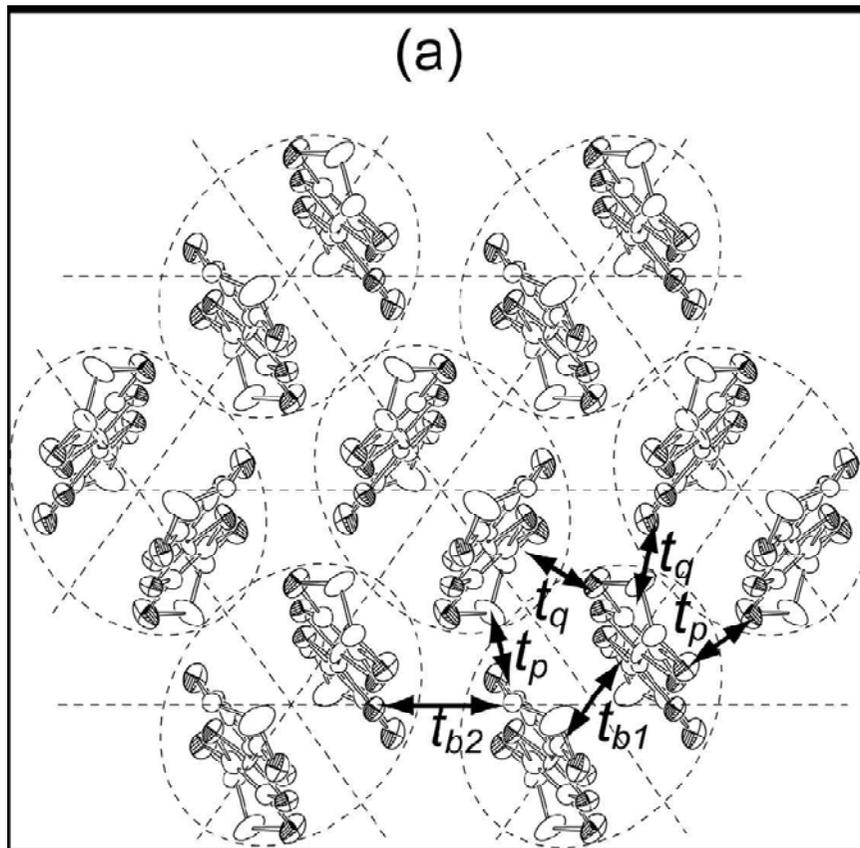
$$\Rightarrow U \approx 400 \text{ meV}$$

$$t'/t \sim 0.6 - 1.1$$



# One-band Hubbard model of BEDT organics

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),  
R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



$$t \approx 50 \text{ meV}$$

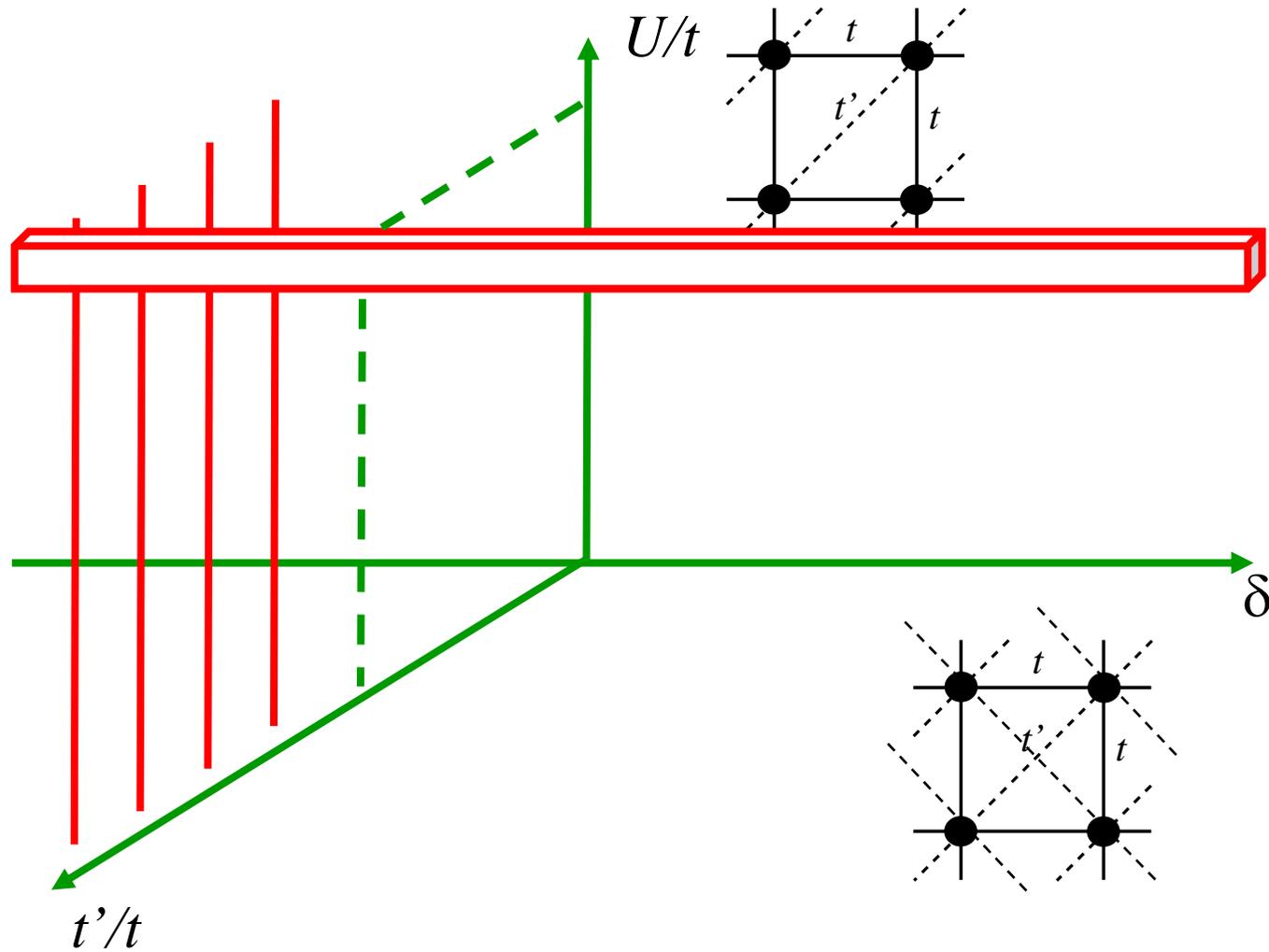
$$\Rightarrow U \approx 400 \text{ meV}$$

$$t'/t \sim 0.6 - 1.1$$

Y. Shimizu, et al. Phys. Rev. Lett. **91**,  
107001(2003)

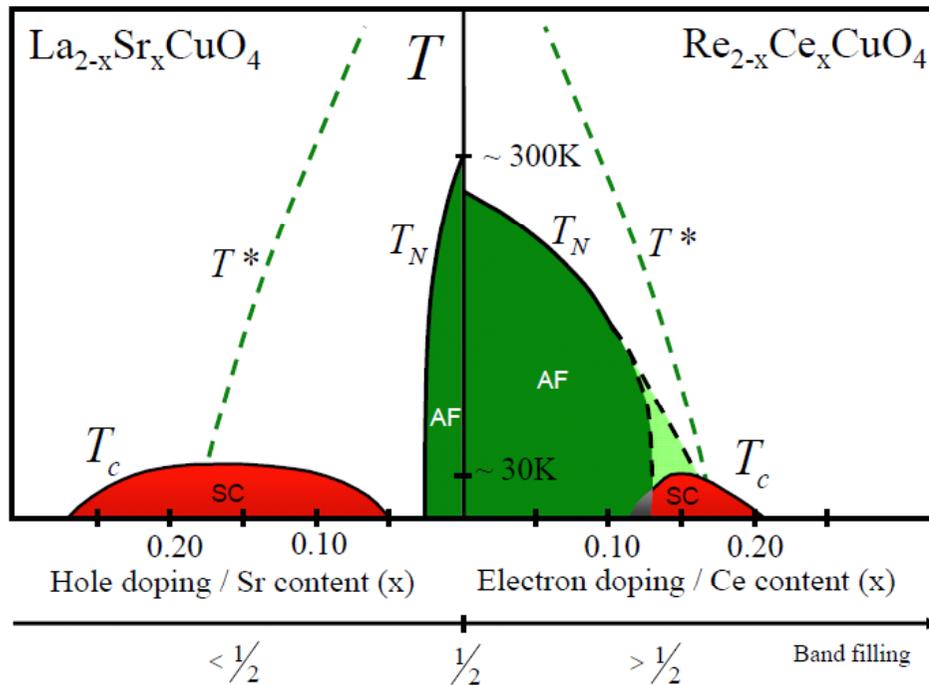


# Perspective



# Normal state of high-temperature superconductors

Armitage, Fournier, Greene, RMP (2009)



What is under the dome?  
Mott Physics away from  $n = 1$

## • Competing order

- Current loops: Varma, PRB **81**, 064515 (2010)
- Stripes or nematic: Kivelson et al. RMP **75** 1201(2003); J.C.Davis
- d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
- SDW: Sachdev PRB **80**, 155129 (2009) ...

## • Or Mott Physics?

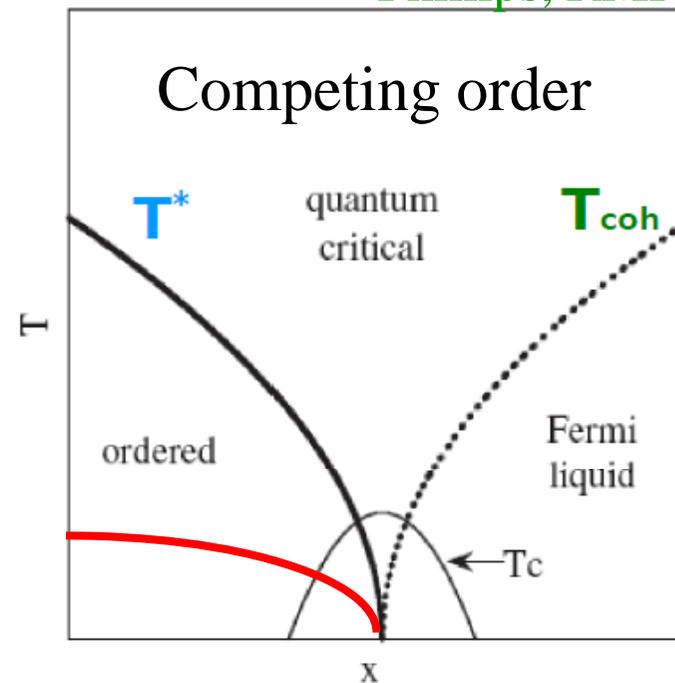
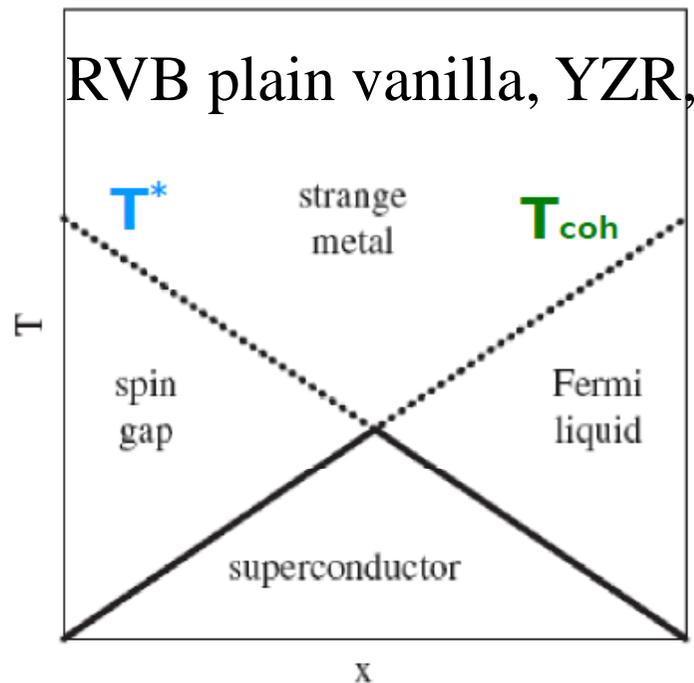
- RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

# Two views (caricature)

Norman, Adv. Phys. (2005)

Broun, Nat. Phys. (2006)

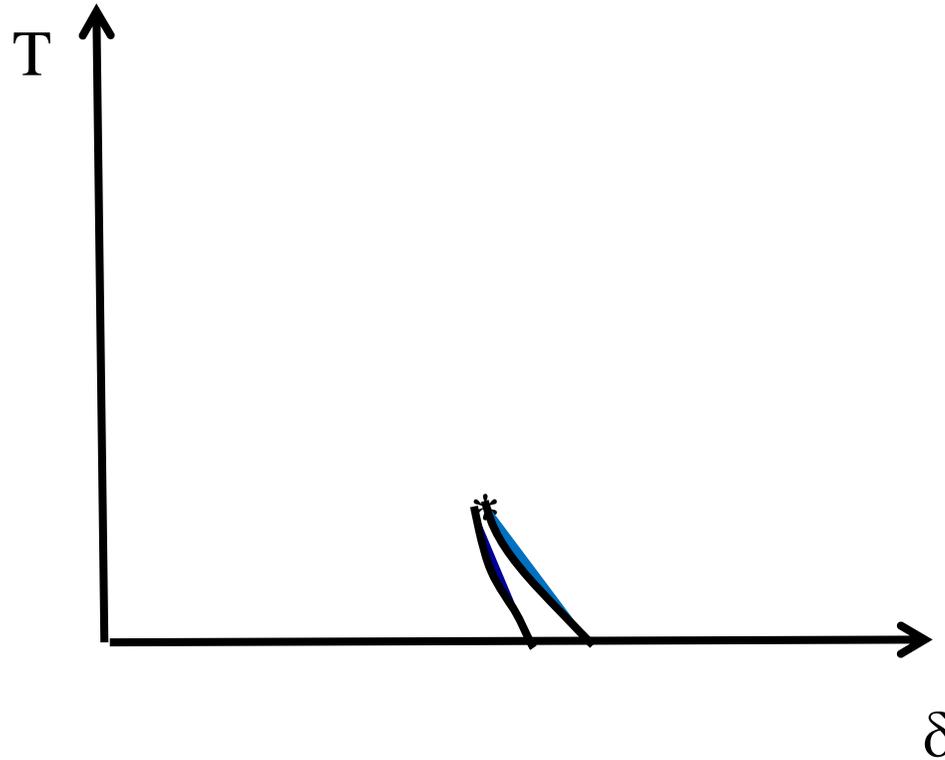
Phillips, RMP (2010)



Why  $T_c$  decreases?  
What is the origin of  $T^*$ ?  
What is the strange metal?

Long correlation length or not.  
What lies beneath the dome.  
Mott Physics away from  $n = 1$

# An alternate view (a bit of both)



G. Sordi, K. Haule, A.-M.S.T  
PRL, **104**, 226402 (2010)  
and  
arXiv:1102.0463



# Outline

- Method
- Normal state
- Superconducting state

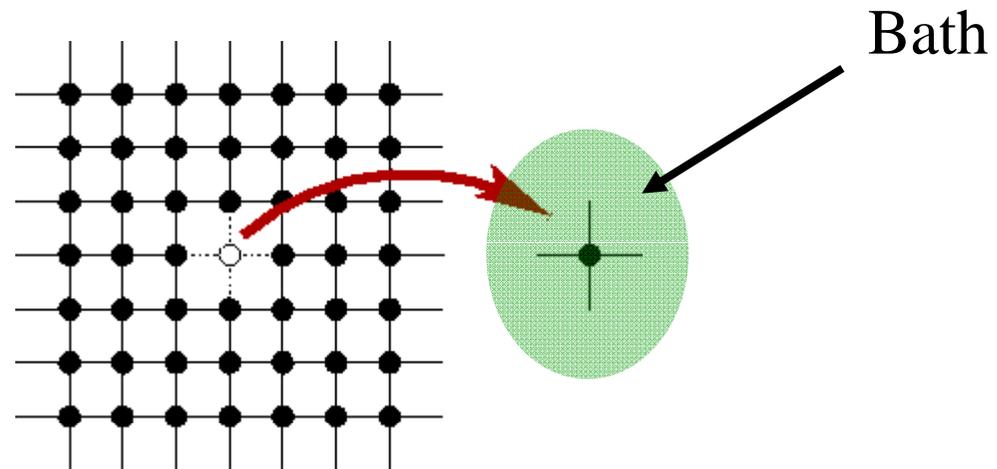


# Method

# Mott transition and Dynamical Mean-Field Theory.

## The beginnings in $d = \text{infinity}$

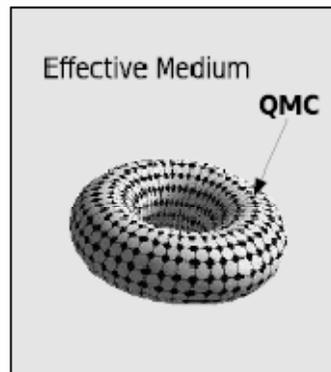
- Compute scattering rate (self-energy) of impurity problem.
- Use that self-energy ( $\omega$  dependent) for lattice.
- Project lattice on single-site and adjust bath so that single-site DOS obtained both ways be equal.



W. Metzner and D. Vollhardt, PRL (1989)  
A. Georges and G. Kotliar, PRB (1992)  
M. Jarrell PRB (1992)

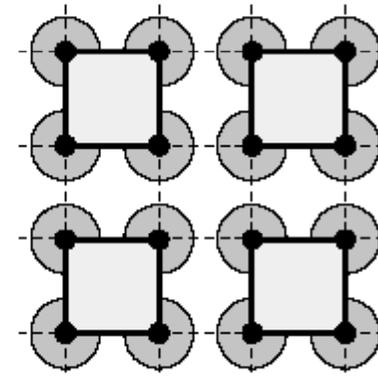
DMFT, ( $d = 3$ )

# 2d Hubbard: Quantum cluster method

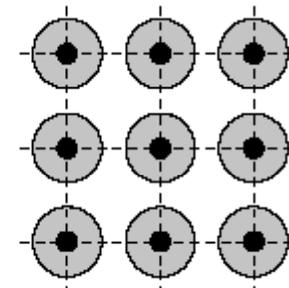


**DCA**

**C-DMFT**



**DMFT**



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

Maier, Jarrell et al., Rev. Mod. Phys. **77**, 1027 (2005)

# Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

- Fermi liquid
  - Start from Fermi sea
  - Self-energy analytical
  - One to one correspondence of elementary excitations
  - Landau parameters
- Mott insulator
  - Hubbard model
  - Atomic limit
  - Self-energy singular
  - DMFT
  - How many sites in the cluster determines how low in temperature your description of the normal state is valid.



# Mott insulator at finite $T$

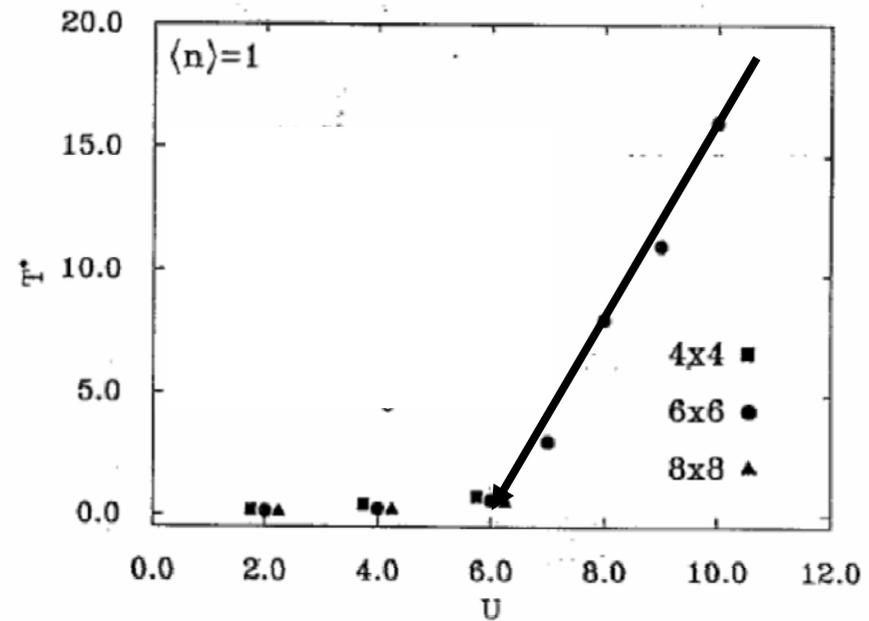
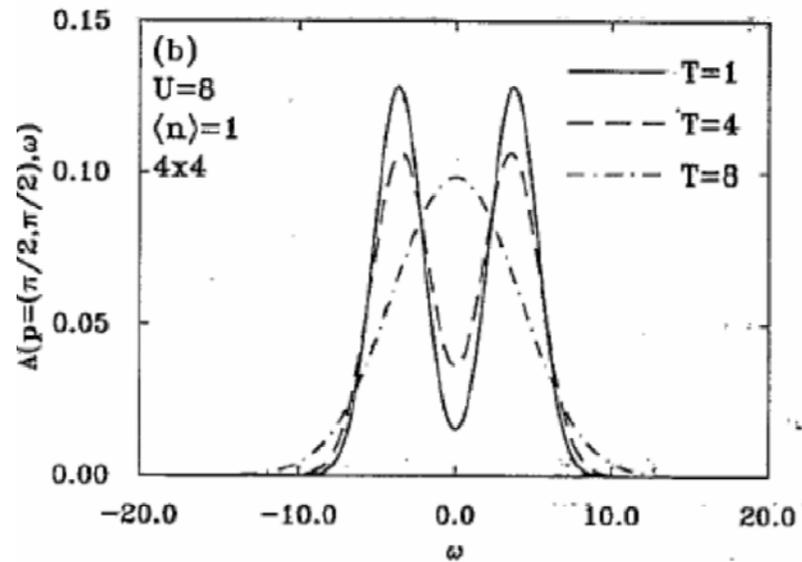


FIG. 5. The temperature  $T^*$  at which the gap develops vs  $U$  for  $4 \times 4$ ,  $6 \times 6$ , and  $8 \times 8$  lattices.

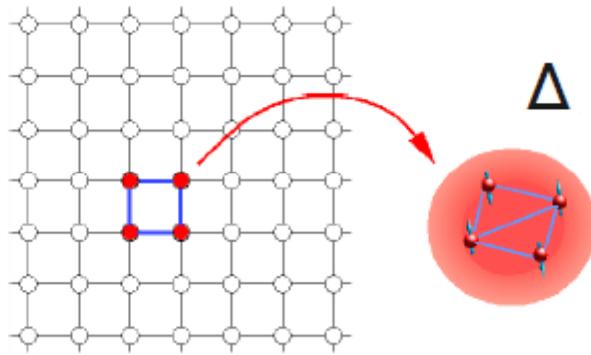
M. Vekic and S.R. White, PRB 47, 1160 (1993)

# Interaction-induced Mott transition, $n = 1$

Method	$U_{c1}$	$U_c$	$U_{c2}$	Ref.
VCA+ED 2 x 2 + 8b	5.25	5.5	6.37	Balzer et al. EPL (2009)
CDMFT+CTQMC+H 2 x 2	5.3		5.7	Park et al. PRL (2008)
DCA+CTQMC+H 8	5.7		6.4	Gull et al. cond-mat (2009)
DCA+CTQMC+H 4	!	~4.2	!	Gull et al. EPL (2008)
Dual fermions	!	~6.5	!	Hafermann et al. (2008)
CDMFT+ED 2 x 2 + 8b 15 parameters	?	~5.6	?	Liebsch, Merino... (2008)
CDMFT+ED 2,3,4		~4		Zhang et al. PRB (2007) (3d also)
QMC 6 x 6		6		Vekic et al. (1993)



# C-DMFT

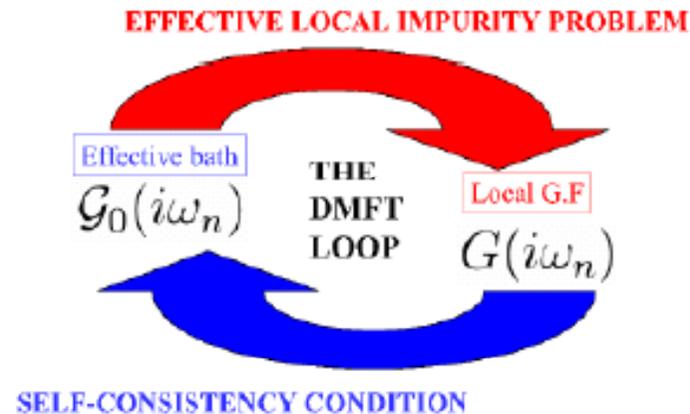


Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006  
 P. Werner, PRB 2007  
 K. Haule, PRB 2007

$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta_{\mathbf{k}}(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

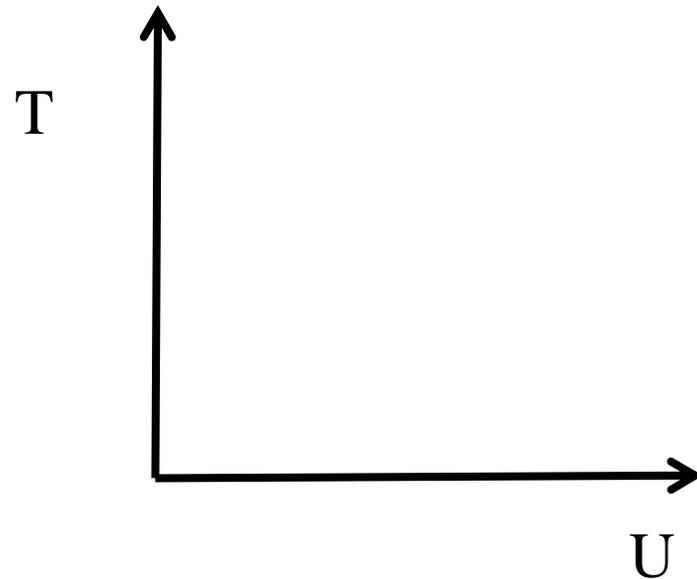
$$- \left[ \sum_{\tilde{\mathbf{k}}} \frac{1}{i\omega_n + \mu - t_c(\tilde{\mathbf{k}}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

# Solving cluster in a bath problem

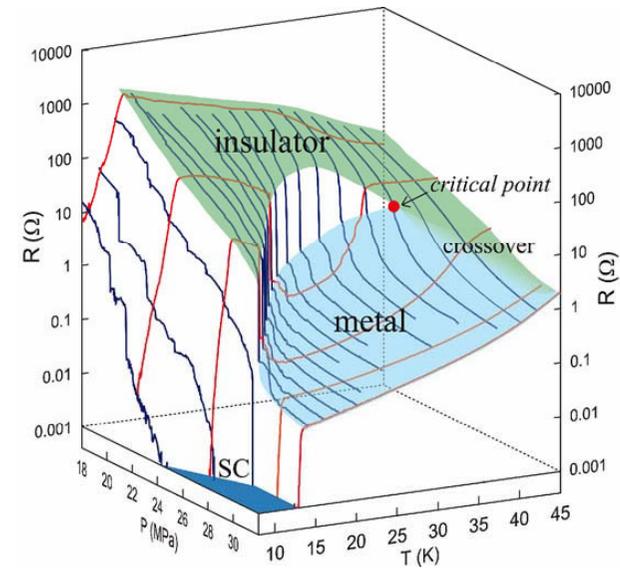
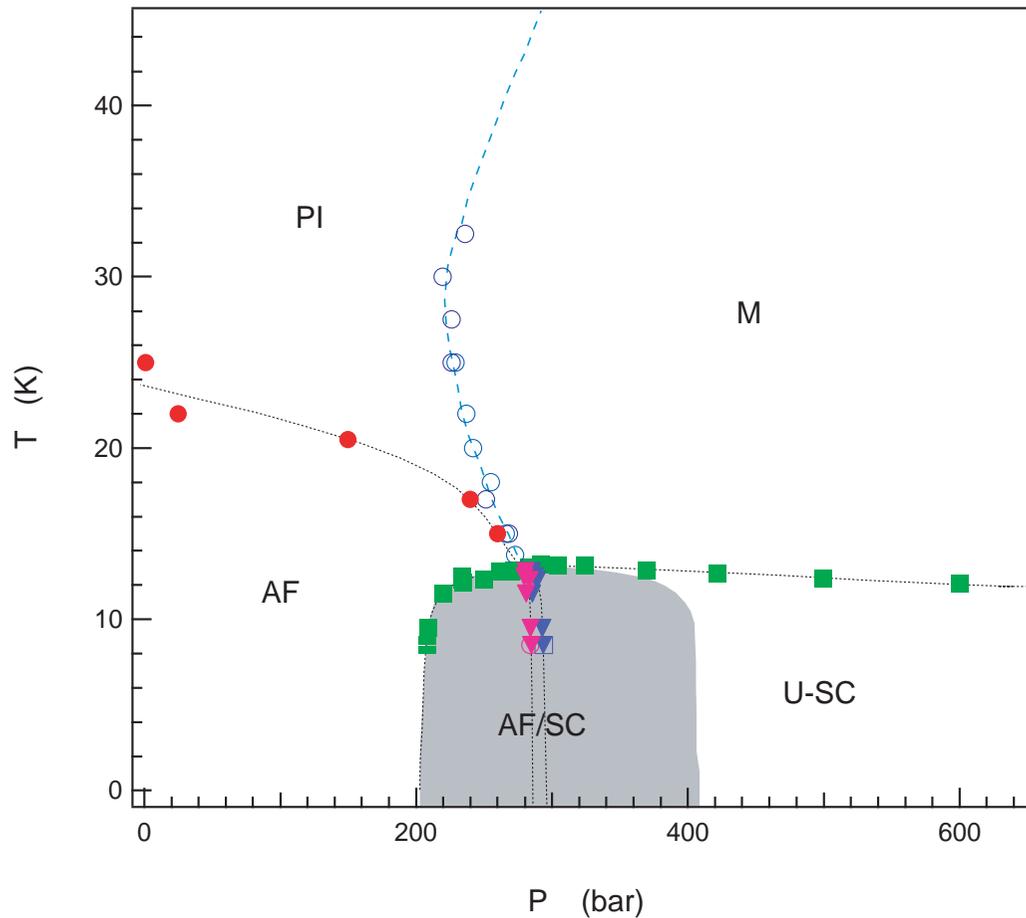
- Continuous-time Quantum Monte Carlo calculations to sum all diagrams generated from expansion in powers of hybridization.
  - P. Werner, A. Comanac, L. de' Medici, M. Troyer, and A. J. Millis, Phys. Rev. Lett. **97**, 076405 (2006).
  - K. Haule, Phys. Rev. B **75**, 155113 (2007).



# Interaction-induced Mott transition $d = 2$



# Bare Mott critical point in organics

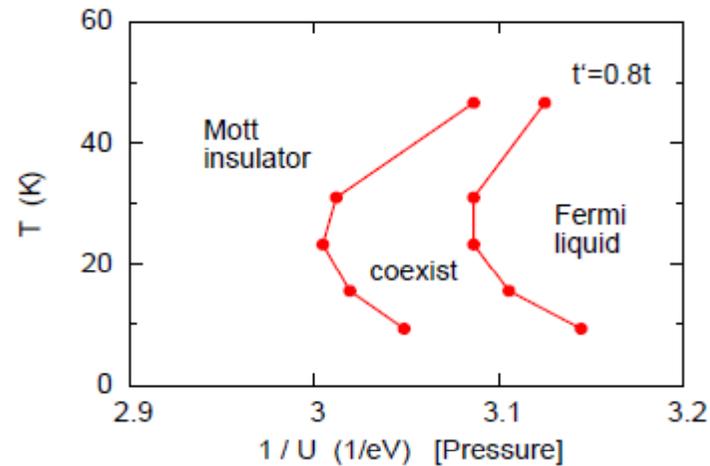


F. Kagawa, K. Miyagawa, + K. Kanoda  
PRB **69** (2004) + Nature **436** (2005)

Phase diagram ( $X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ )

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

# Interaction-induced Mott transition theory



$\kappa$ -BEDT-Cl

$\kappa$ -BEDT-CN

Liebsch Phys. Rev. B **79**, 195108 (2009)

See also: Ohashi et al. PRL **100**, 076402 (2008)

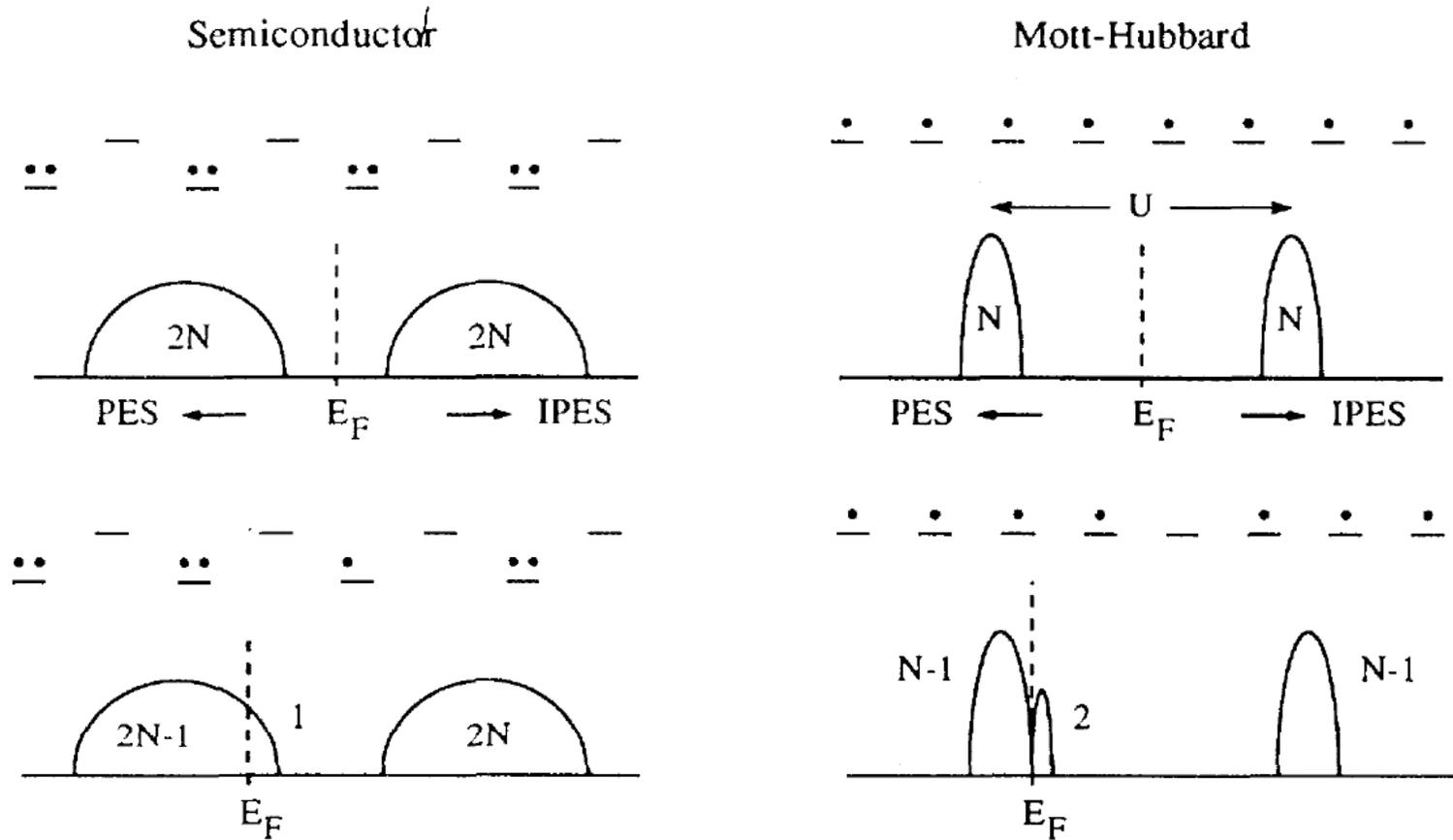


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# Cuprates as doped Mott insulators

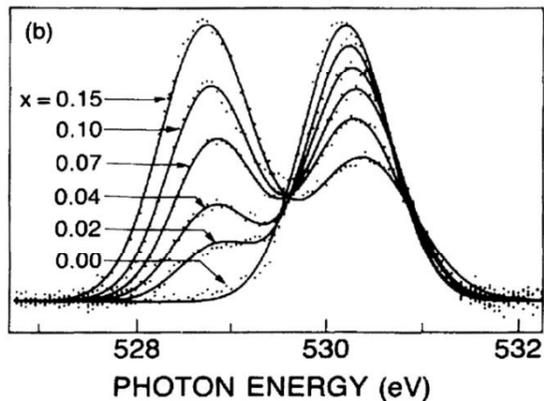


# Spectral weight transfer

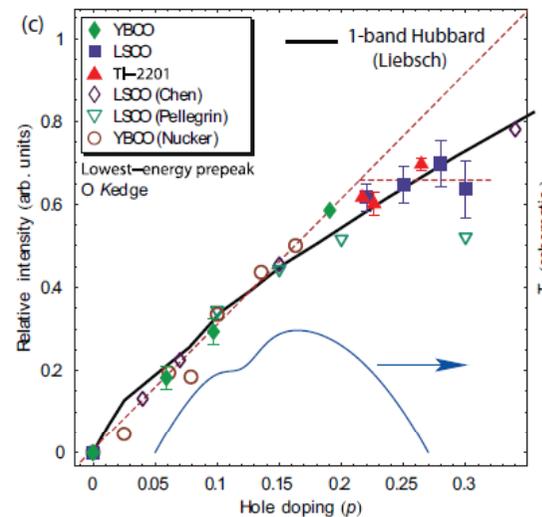


Meinders *et al.* PRB **48**, 3916 (1993)

# Experiment: X-Ray absorption



Chen et al. PRL **66**, 104 (1991)



Peets et al. PRL **103**, (2009), Phillips, Jarrell arXiv

Number of low energy states above  $\omega = 0$  scales as  $2x +$   
Not as  $1+x$  as in Fermi liquid

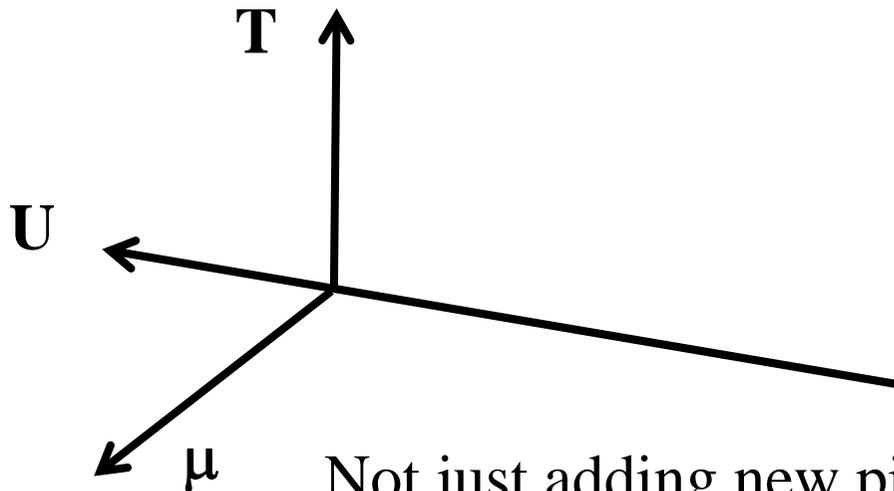
Meinders *et al.* PRB **48**, 3916 (1993)



Giovanni Sordi

G. Sordi, K. Haule, A.-M.S.T  
PRL, **104**, 226402 (2010)  
and  
arXiv:1102.0463

## Doping-induced Mott transition ( $t'=0$ )



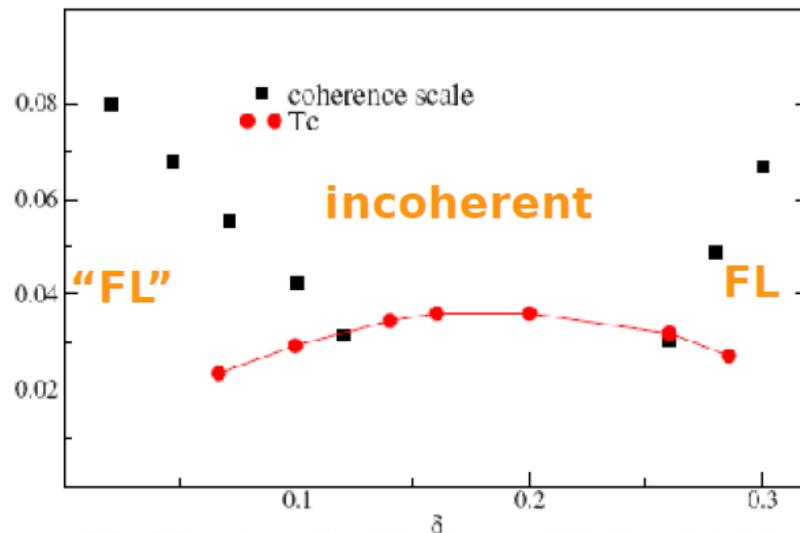
Kristjan Haule

Not just adding new piece:  
Lesson from DMFT, first order transition + critical  
point governs phase diagram

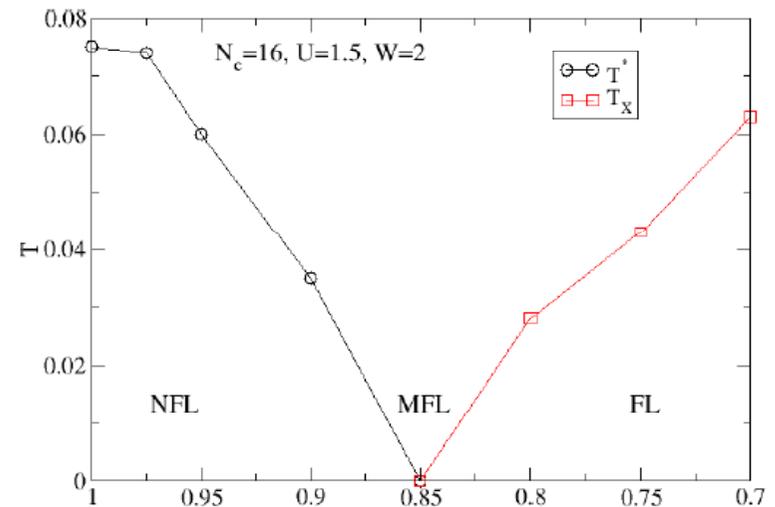


# Doping driven Mott transition, $t' = 0$

Method	$t'$	Orbital selective	U	Critical point	Ref.
D+C+H 8			7		Werner et al. cond-mat (2009)
D+C+H 4					Gull et al. EPL (2008)
	-0.3		10,6		Liebsch, Merino... (2008)
					Ferrero et al. PRB (2009)
D+C+H 8			7		Gull, et al. PRB (2009)

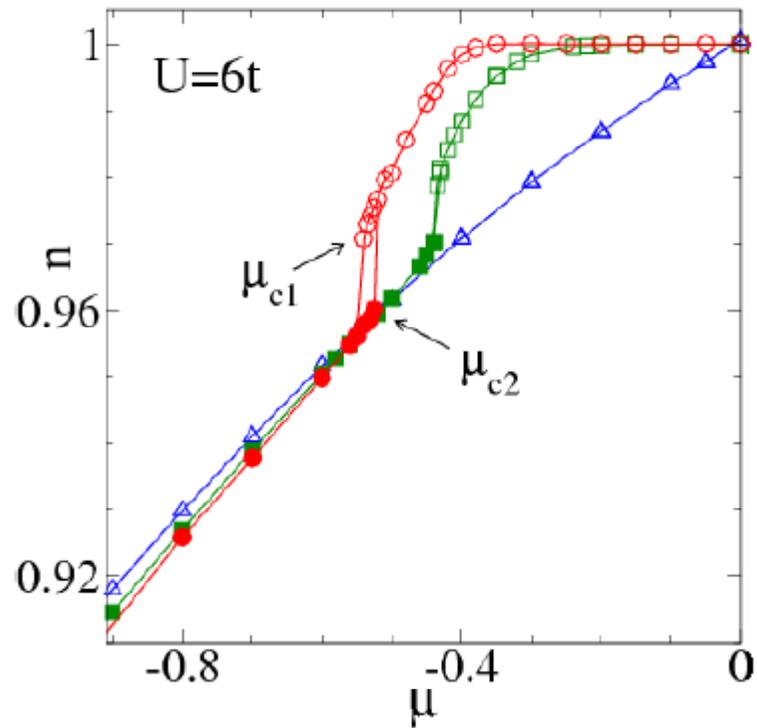


K. Haule, G. Kotliar, PRB (2008)



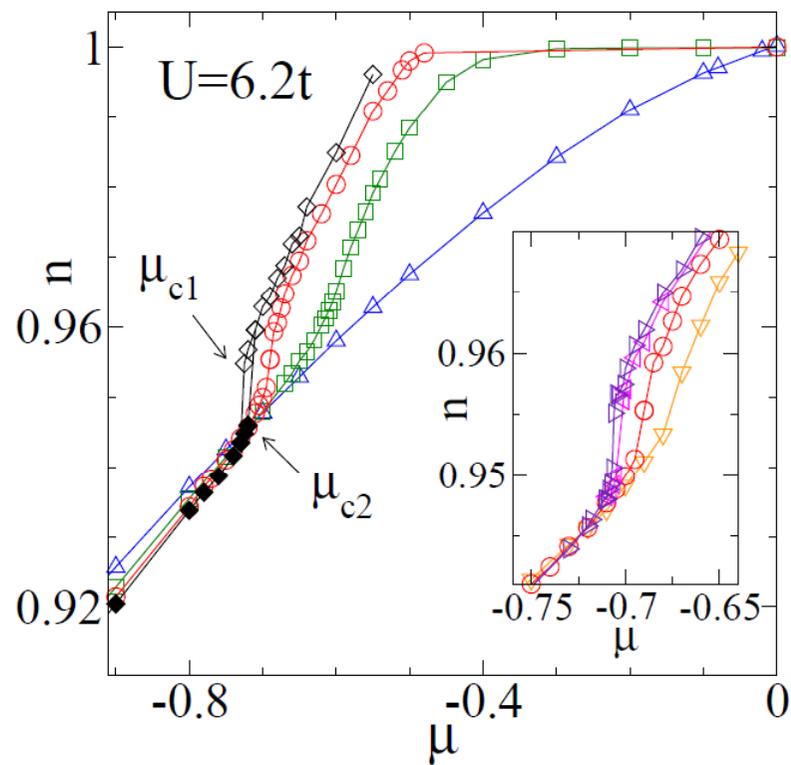
Vildhyadhiraja, PRL (2009)

# First order transition at finite doping

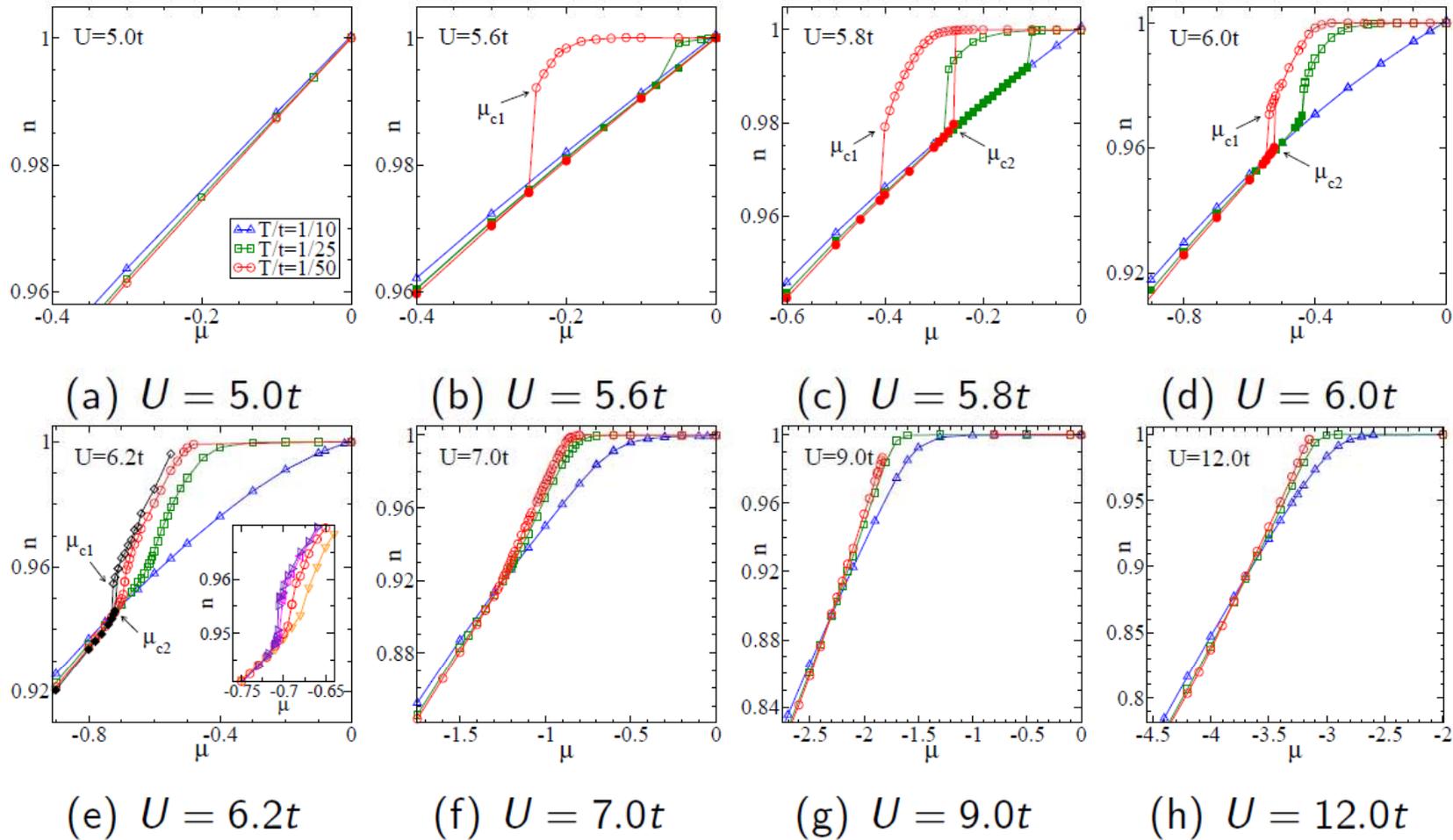


$n(\mu)$  for several temperatures:  
 $T/t = 1/10, 1/25, 1/50$

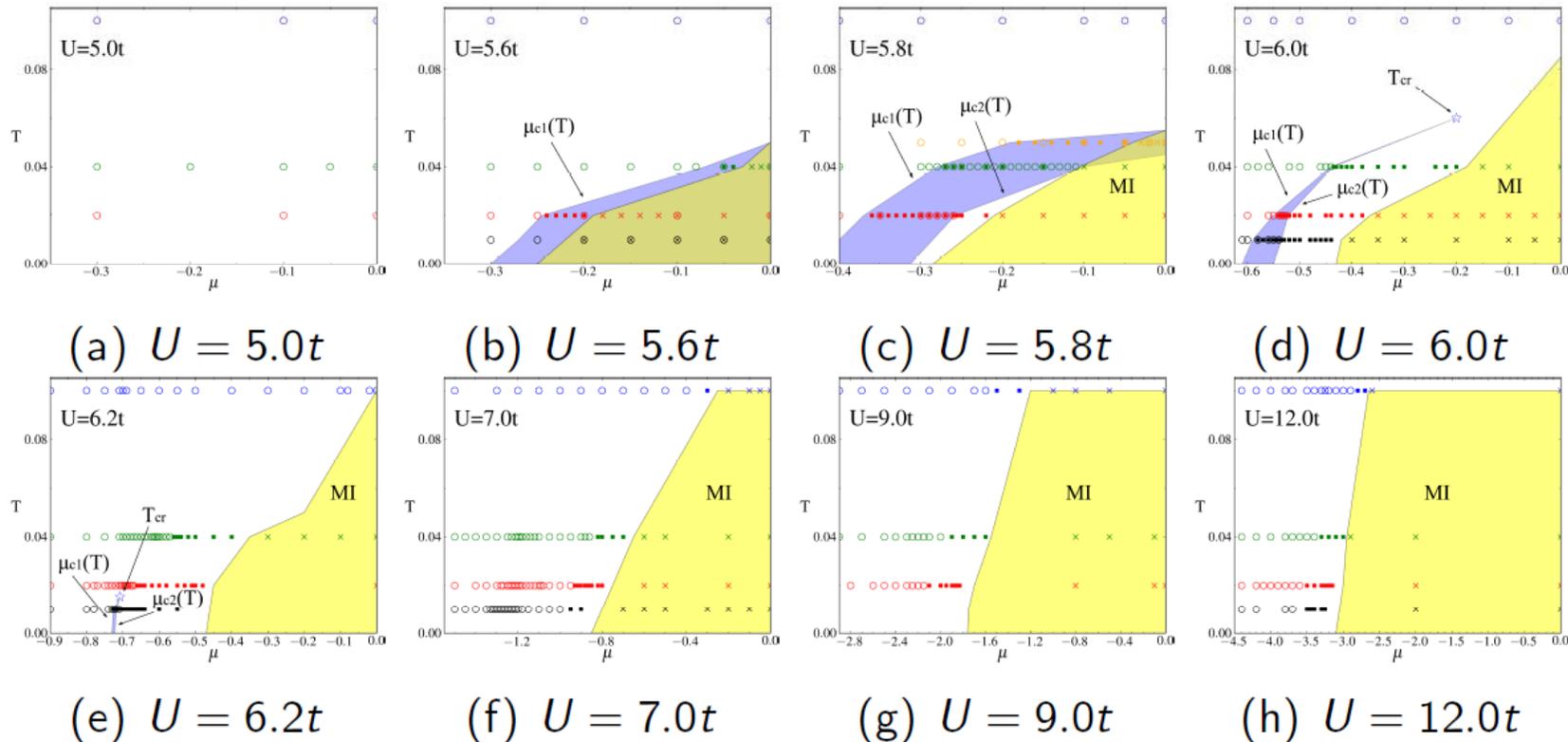
# The critical point



# Systematic exploration $n(\mu)$



# Systematic exploration of the phase diagram

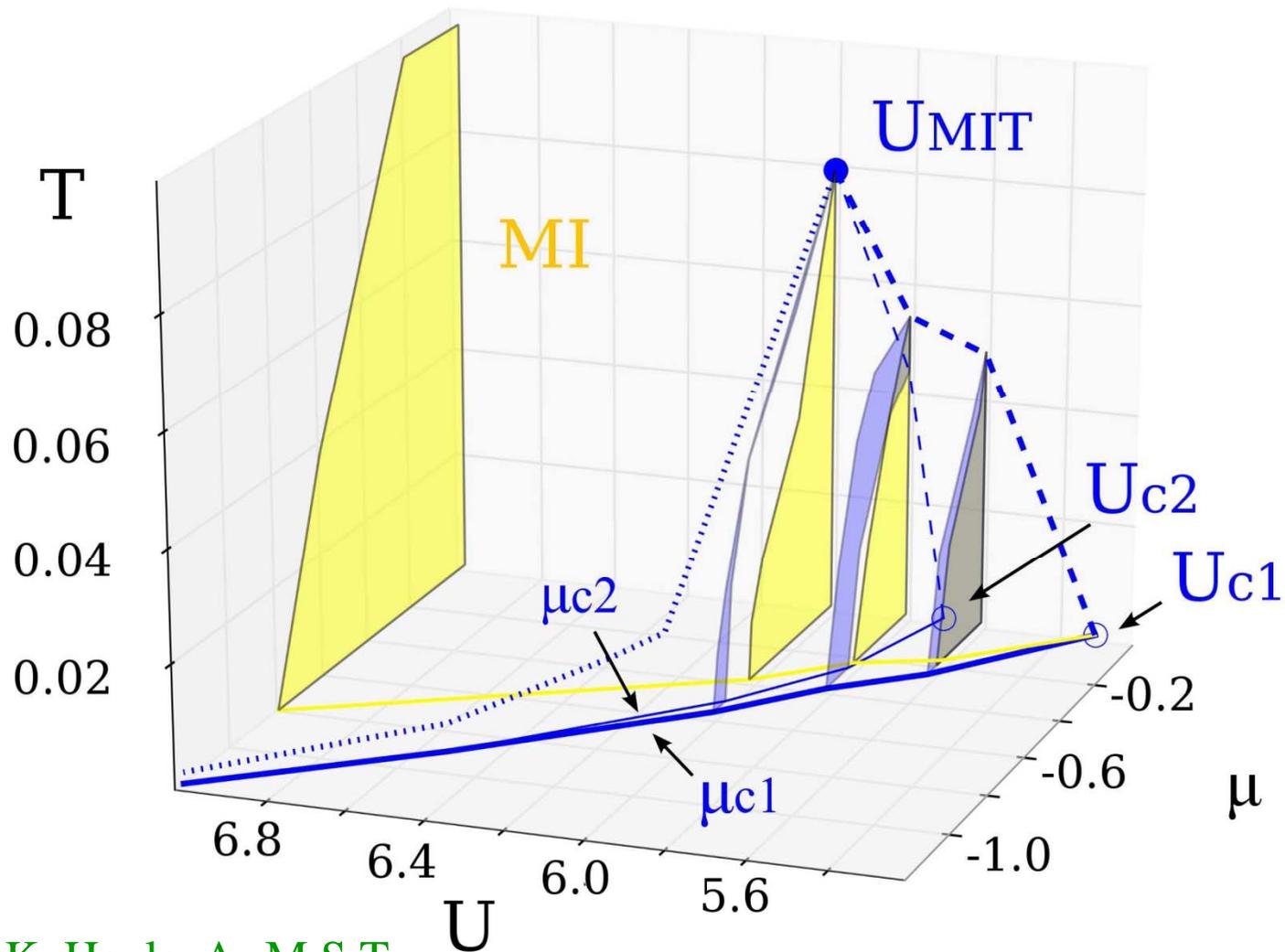


Yellow region: Mott insulator ( $n(\mu) = 1, dn/d\mu = 0$ )

White region: metal ( $dn/d\mu = 0$ )

Blue region: coexistence region

# Normal state phase diagram

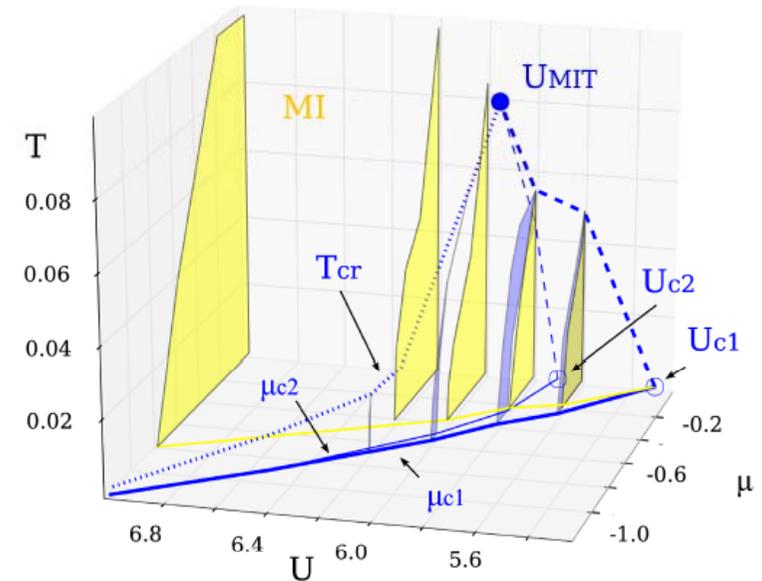
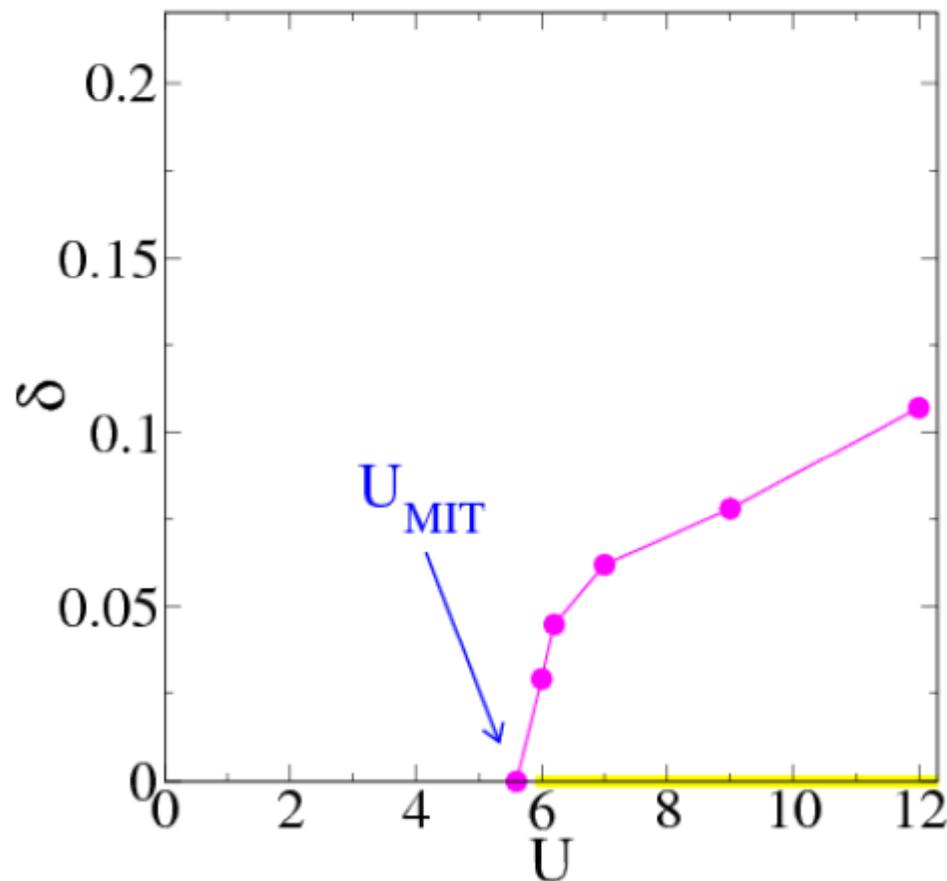


G. Sordi, K. Haule, A.-M.S.T  
PRL, **104**, 226402 (2010)

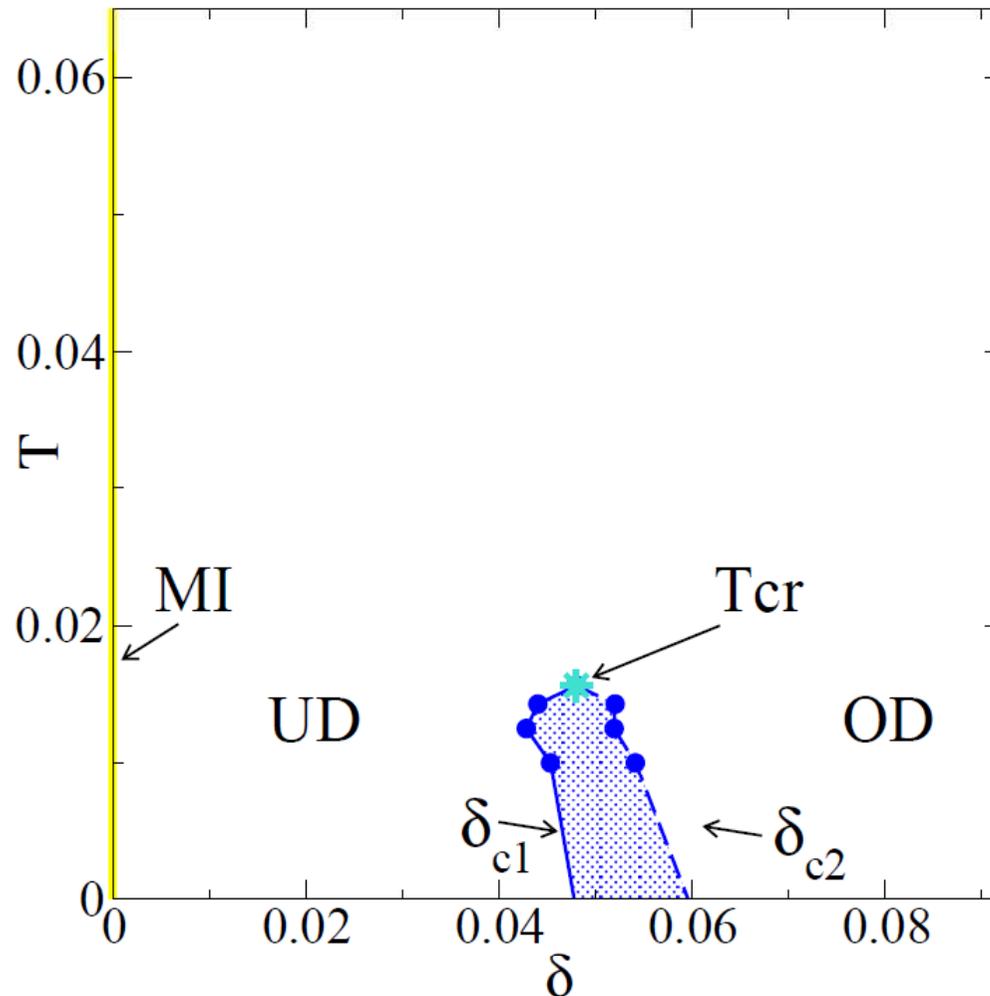


# Link to Mott transition up to optimal doping

Doping dependence of critical point as a function of  $U$



# Characterisation of the phases ( $U=6.2t$ )



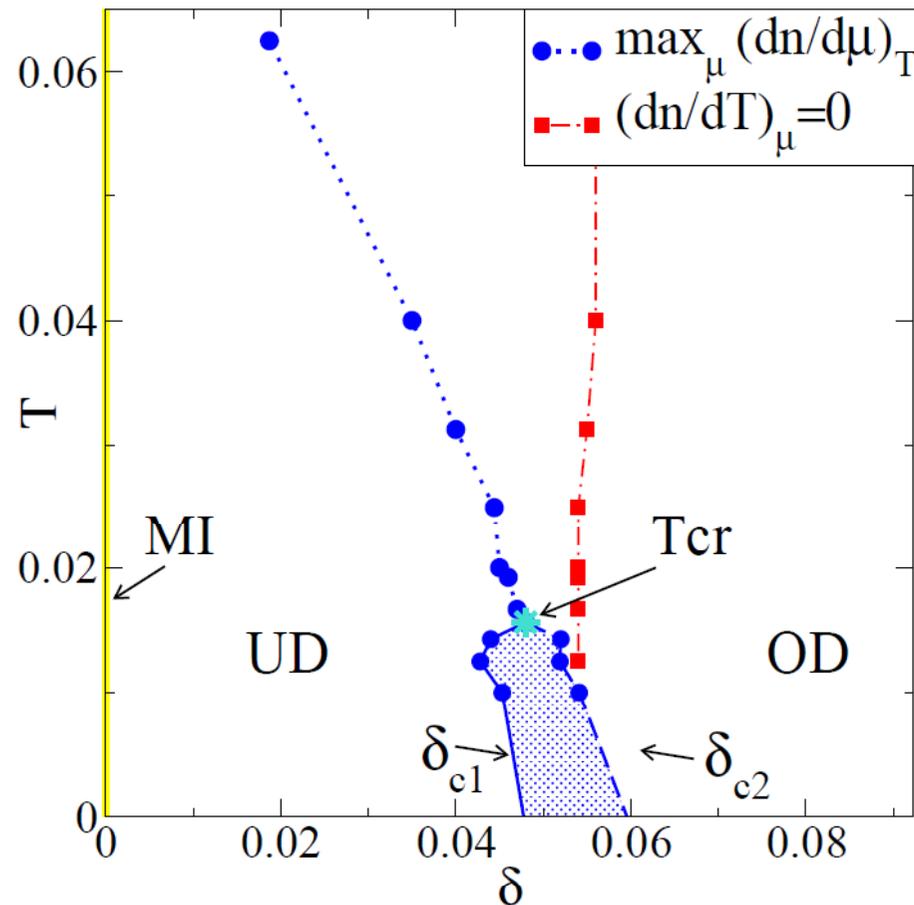
$U > U_{\text{MIT}}$ :

1. Mott insulator (MI)
2. Underdoped phase (UD):  
 $\delta < \delta_c$
3. Overdoped phase (OD):  
 $\delta > \delta_c$
4. Coexistence/forbidden region

Here “optimal doping”  $\delta_c =$   
doping at which the 1st order  
transition occurs

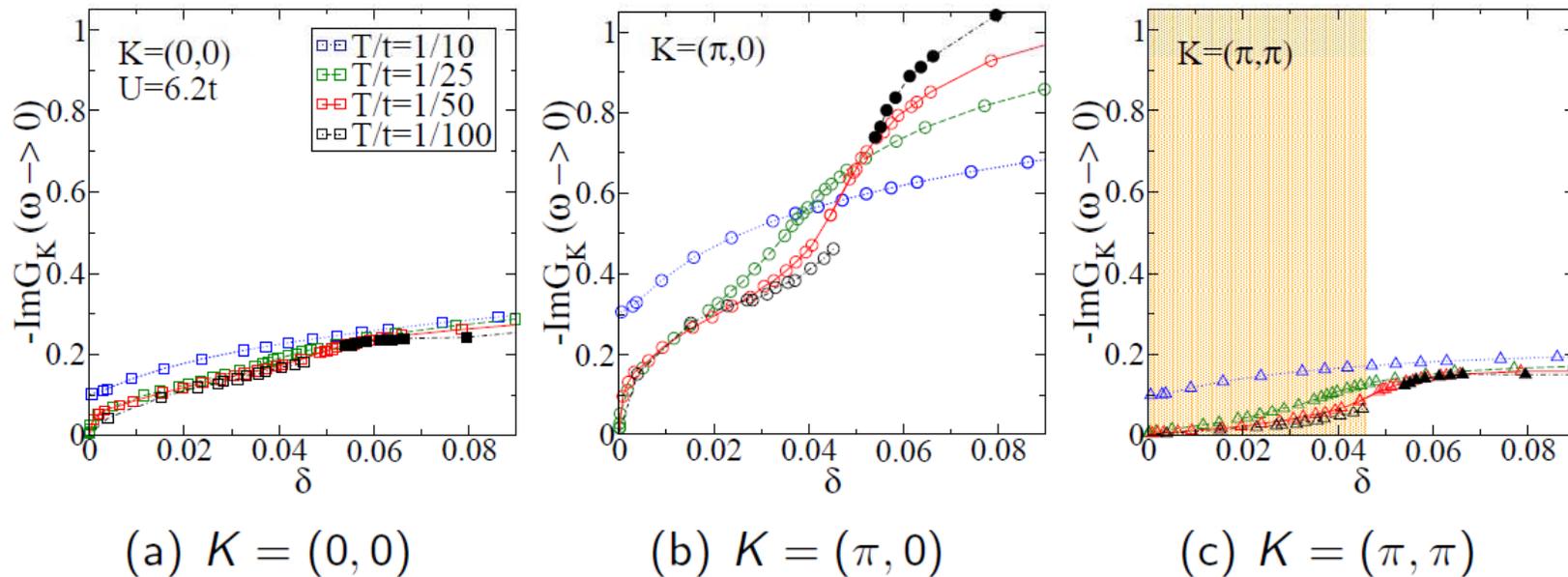
How does the UD phase differ  
from the OD phase?

# Phase characterisation: thermodynamics



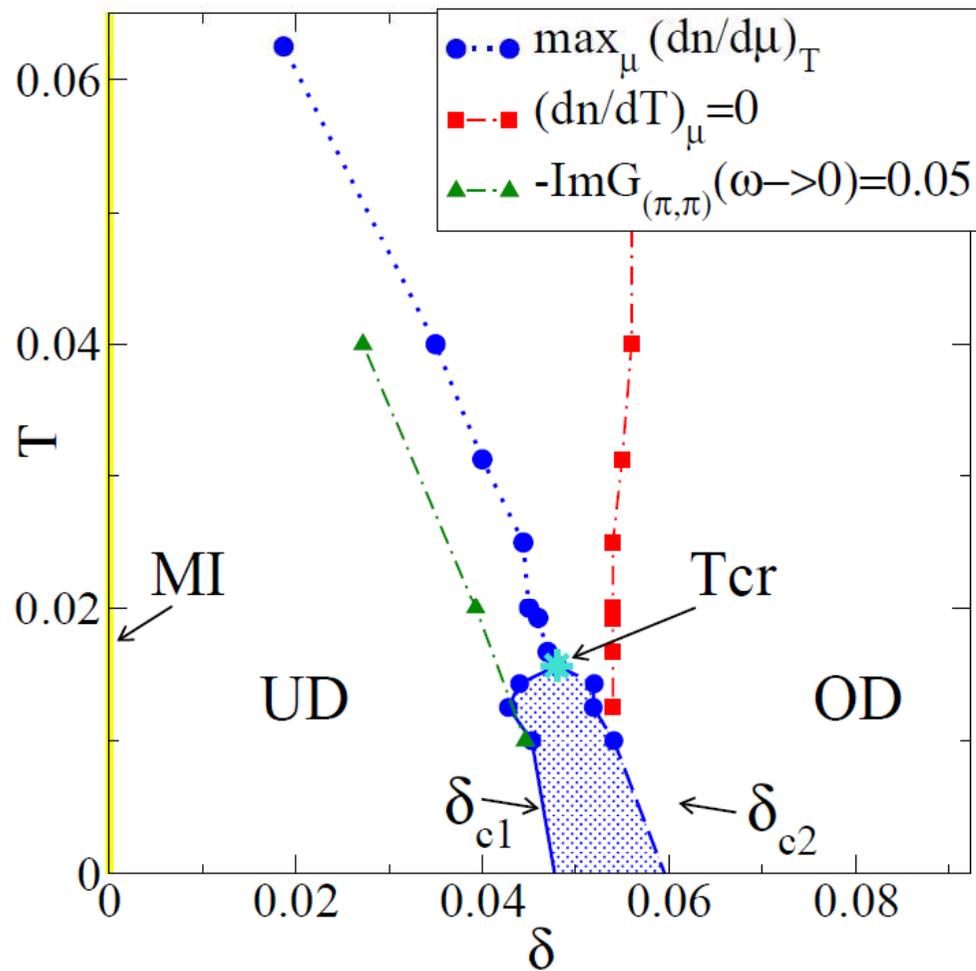
- ▶ UD and OD phases are stable ( $d^2\Omega < 0$ )
- ▶ UD and OD phases have a different filling
- ▶ UD has smaller entropy and smaller double occupation than the OD phase
- ▶ maximum of compressibility  $\max_{\mu}(dn/d\mu)_T$
- ▶ near the 1st order transition: crossing of isotherms  $dn/dT|_{\mu} = 0$   
 $\Rightarrow$  maximum of entropy ( $dS/d\mu = 0$ ) (cf Khatami PRB 2010)

# Phase characterisation: DOS at $E_F$



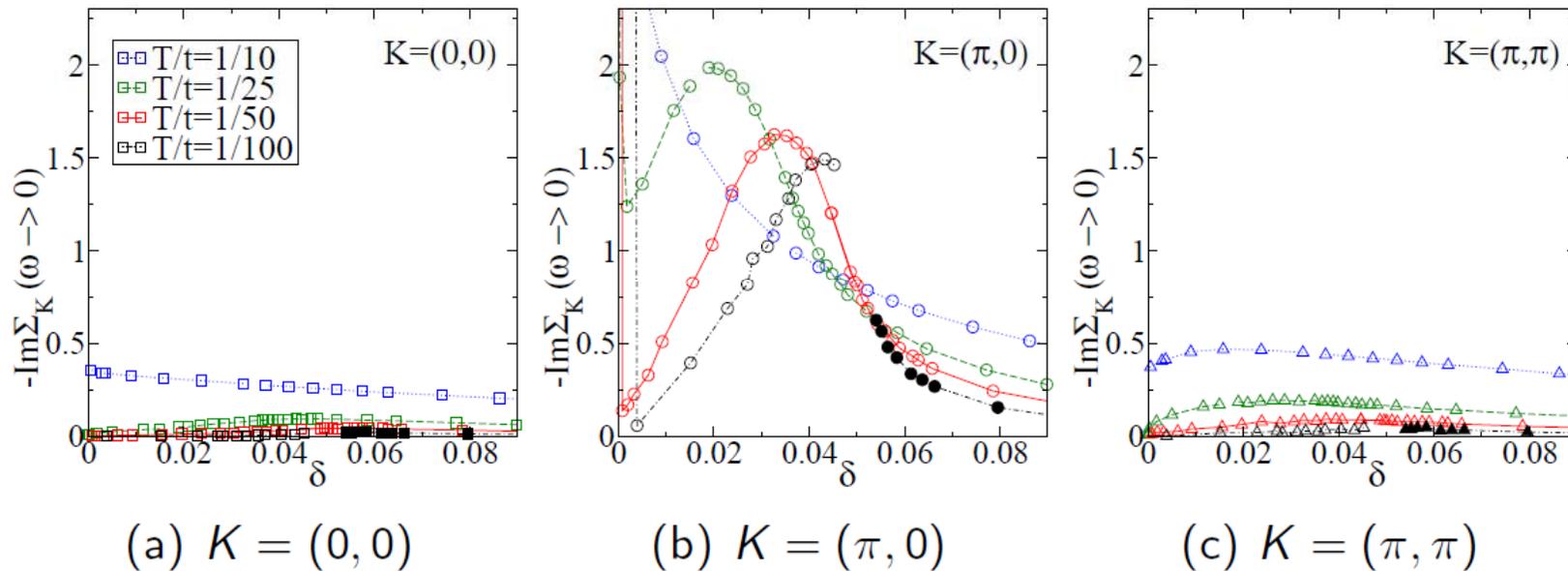
- ▶ MI: all cluster momenta are insulating
- ▶ UD: strong cluster momentum differentiation:  $K = (\pi, \pi)$  insulator,  $K = (\pi, 0), (0, 0)$  metallic
- ▶ OD: all cluster momenta are metallic

# Momentum differentiation at the transition



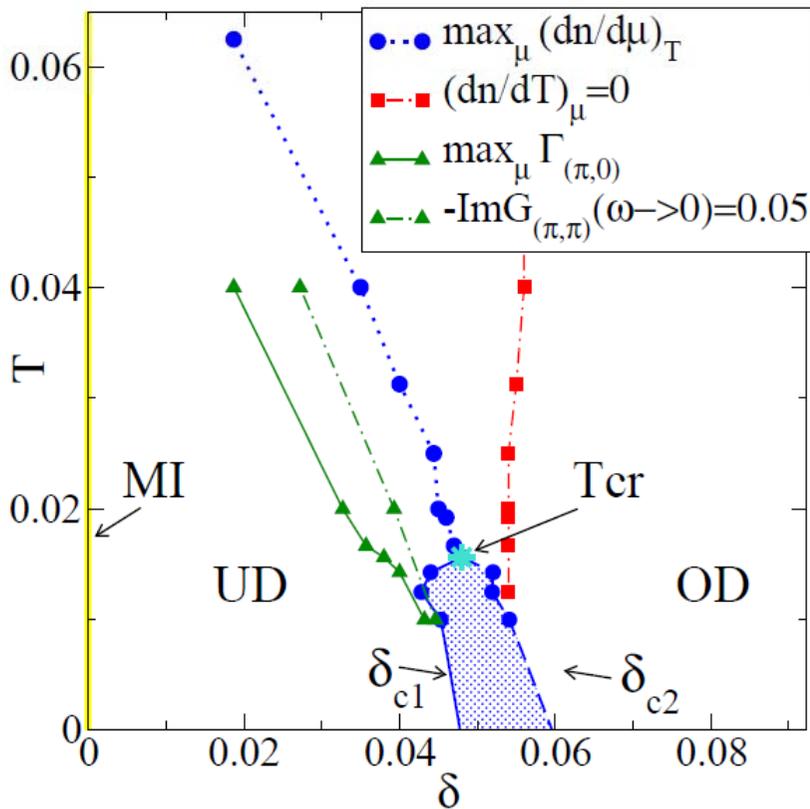
- ▶ UD phase: strong cluster momentum differentiation (cf Ferrero et al, EPL 2009, PRB 2009, Gull et al, PRB 2010)
- ▶ our contribution: phenomenon linked to first order transition at finite doping

# Phase characterisation: scattering rate

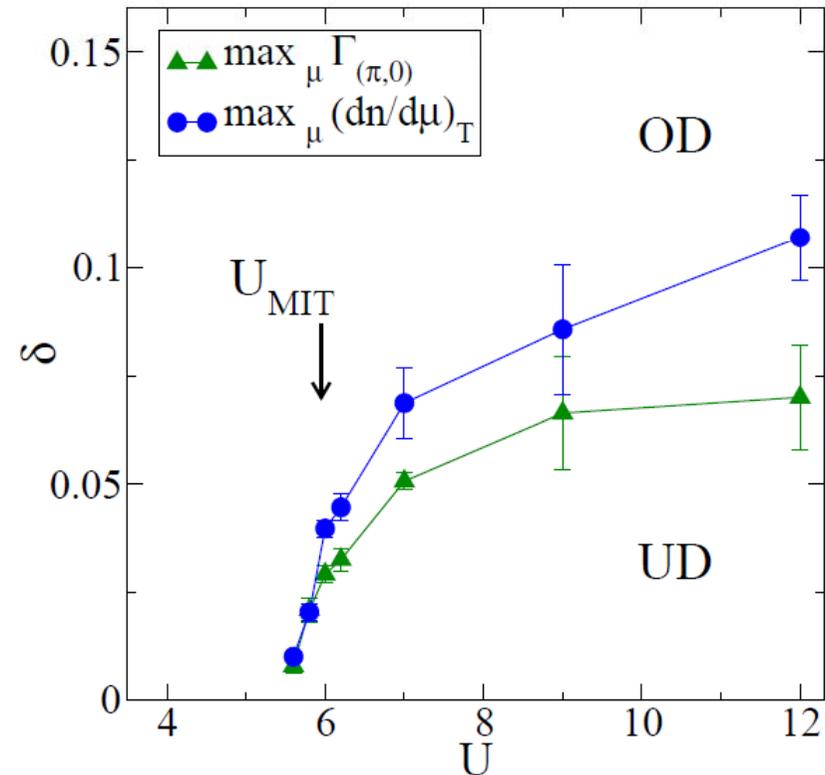


- ▶  $\text{Im}\Sigma_{K=(\pi,0)}(\omega \rightarrow 0)$  shows a maximum at UD/OD transition
- ▶ OD: coherent, FL
- ▶ UD: coherent, NFL: coherence does NOT result from QP propagations, but from another mechanism

# Maximum originates at the UD/OD transition



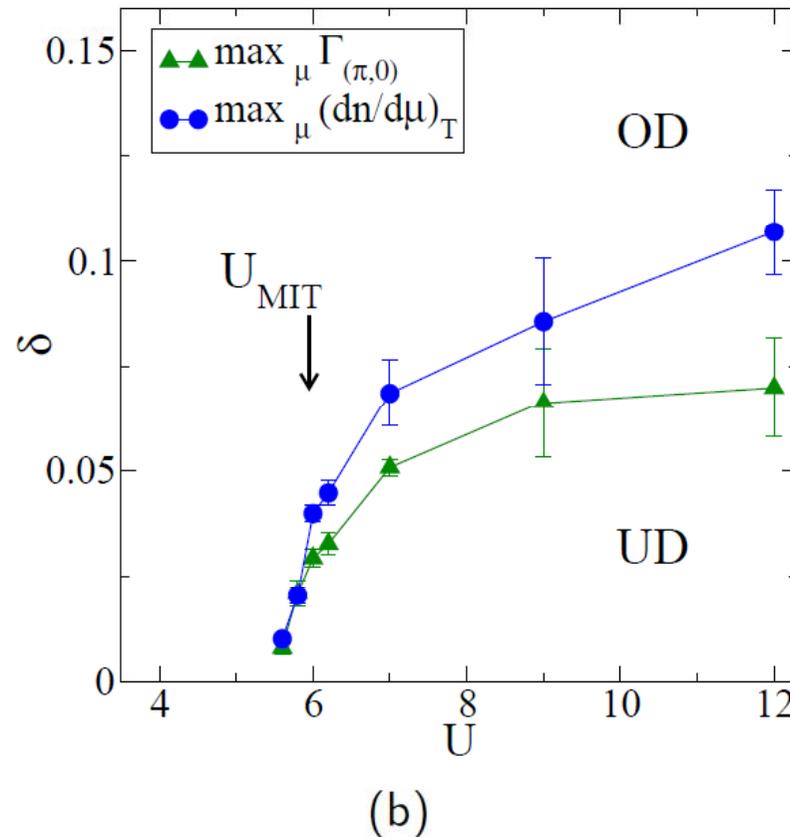
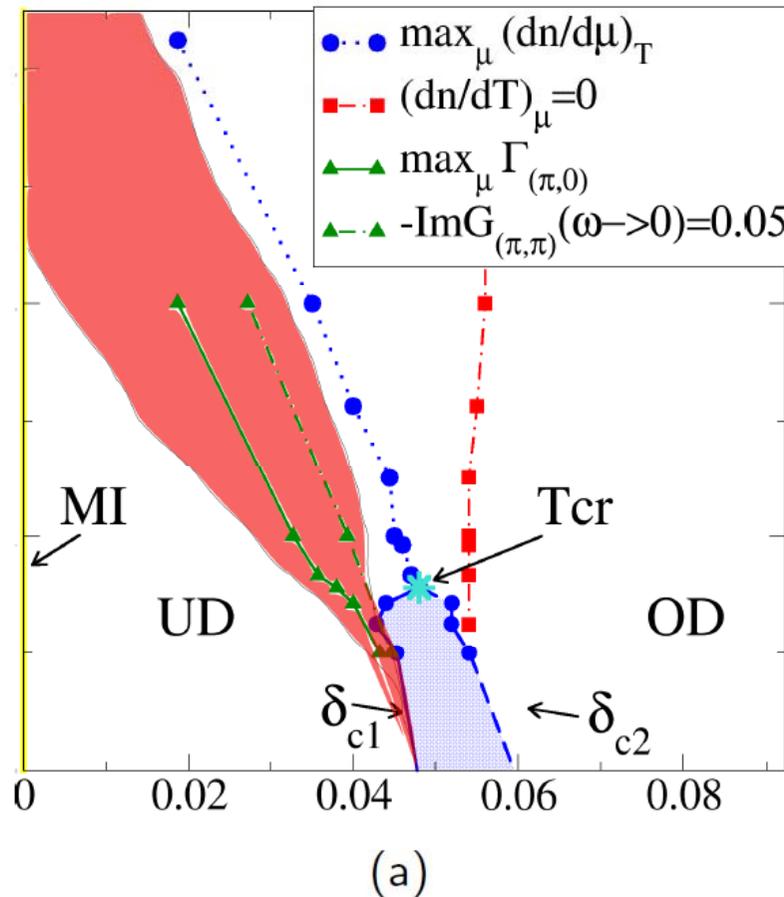
(a)



(b)

► Mott physics far away from half filling

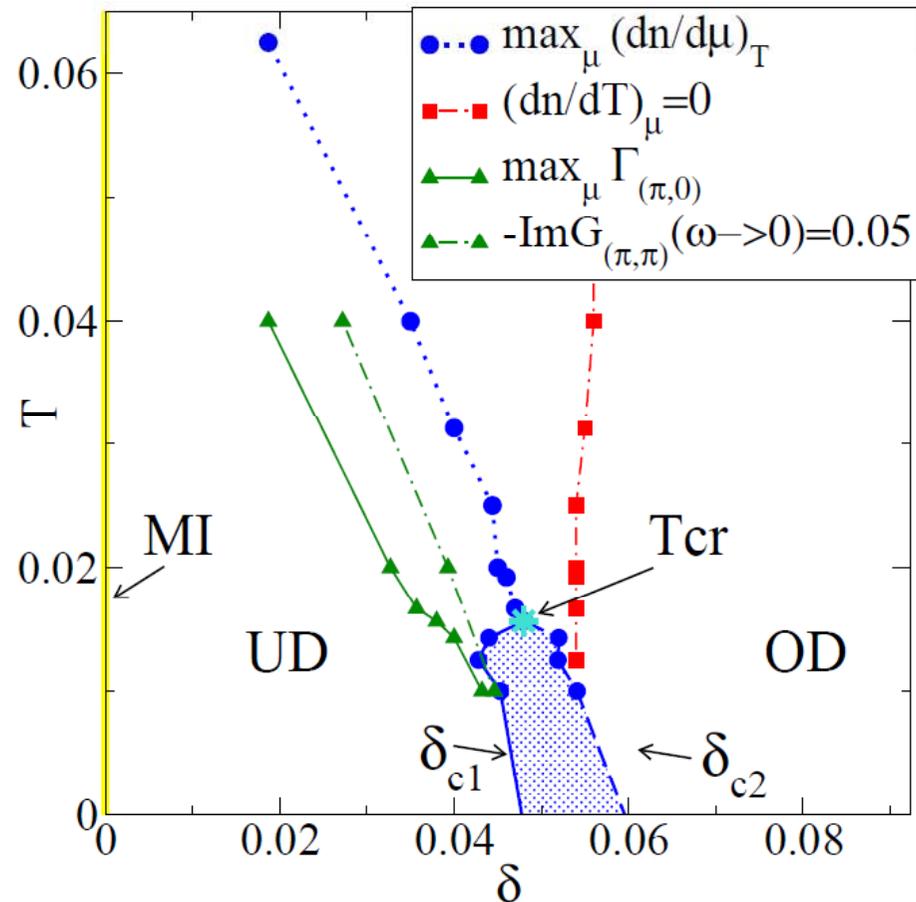
# Maximum originates at the UD/OD transition



- ▶ Mott physics far away from half filling
- ▶ “Strange metal” between the UD/OD transition

# Emerging physical picture

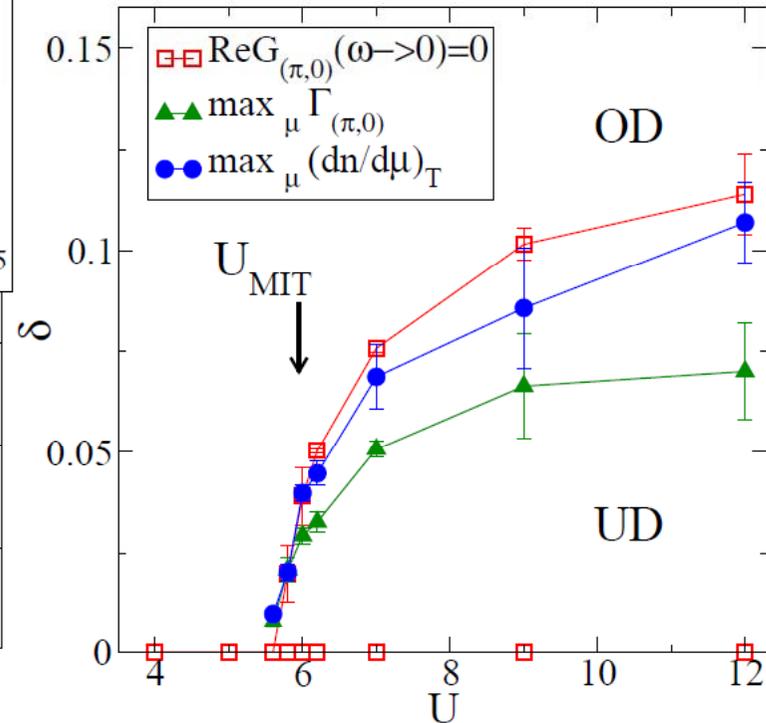
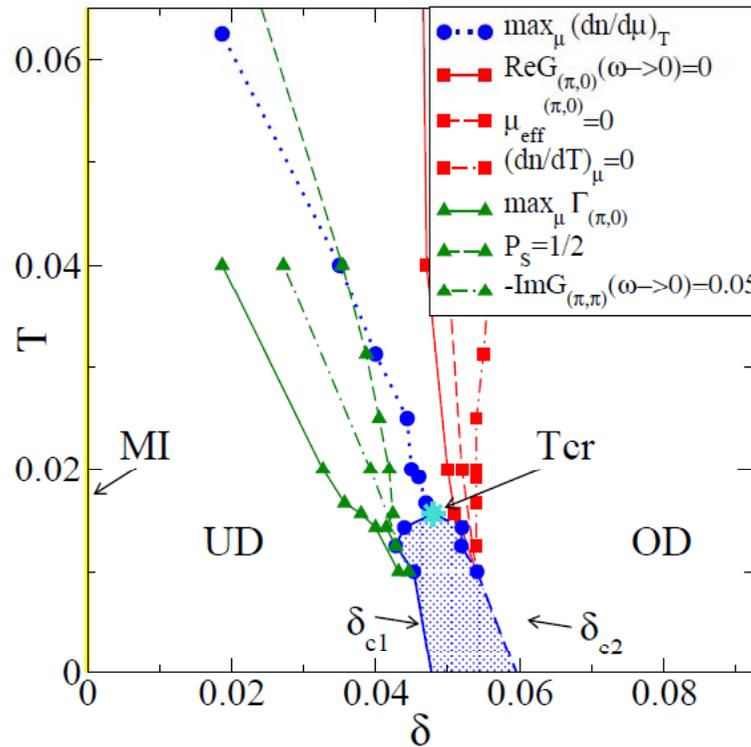
UD: coherent,  
NFL, holes  
propagating in a  
sea of singlets



OD: coherent, FL,  
Landau  
quasiparticles

Lev Tolstoj, AK: All Fermi liquids are all alike, each Non Fermi liquid is Non Fermi in its own way (© Shastry)

# Summary



- ▶ first order transition at finite doping between two metals
  - ▶ it is associated to Mott physics: all signatures of the first order transition can be traced back to Mott critical point
- $\Rightarrow$  signature of the Mott transition in the 2D Hubbard model **extends way beyond** half filling!

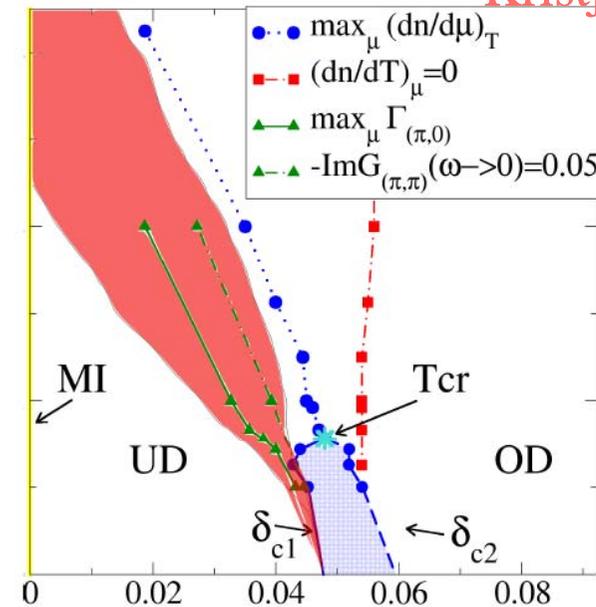
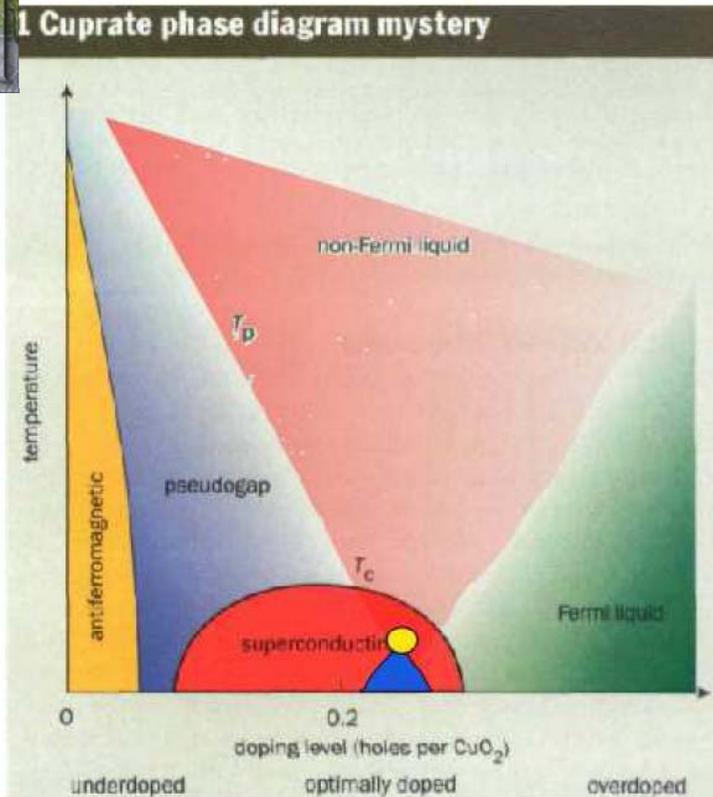
# Beneath the dome



Giovanni Sordi



Kristjan Haule



criticality originates from  
first order transition at finite doping  
coming from influence of Mott physics well beyond half filling!

G. Sordi, K. Haule, A.-M.S.T PRL, **104**, 226402 (2010)

and arXiv:1102.0463



Another property of the UD phase



# Underdoped metal very sensitive to anisotropy

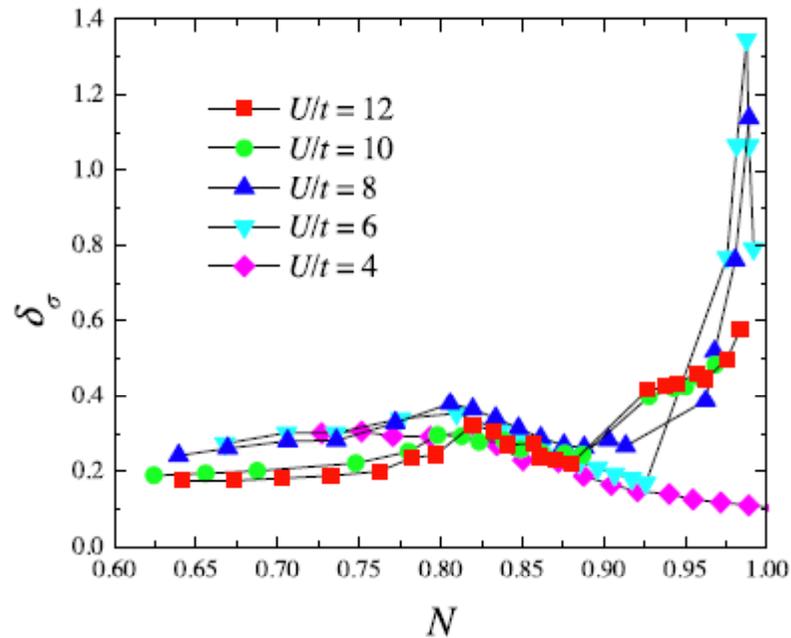
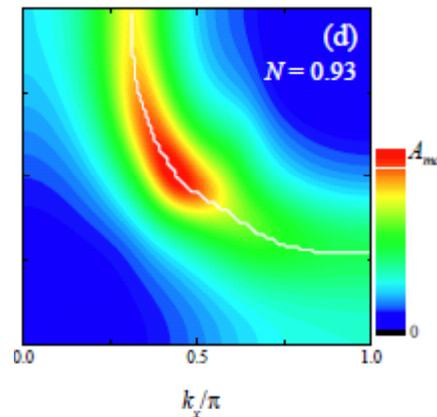
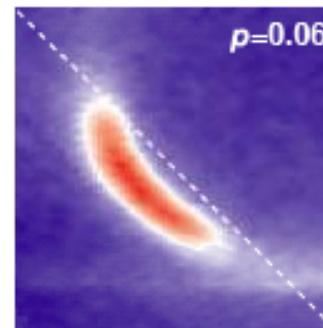


FIG. 3: (Color online) Anisotropy in the CDMFT conductivity  $\delta_\sigma = 2 [\sigma_x(0) - \sigma_y(0)] / [\sigma_x(0) + \sigma_y(0)]$  as a function of filling  $N$  for various values of  $U$  and  $\eta = 0.1$ ,  $\delta_0 = 0.04$ .

Okamoto, Sénéchal, Civelli, AMST  
Phys. Rev. B **82**, 180511R 2010



g



Satoshi Okamoto



David Sénéchal



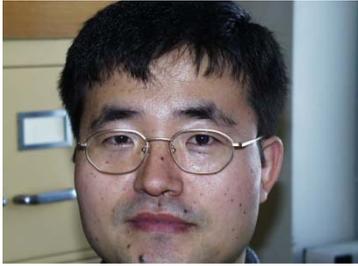
D. Fournier *et al.* Nature Physics ( Marcello Civelli )

# Superconductivity

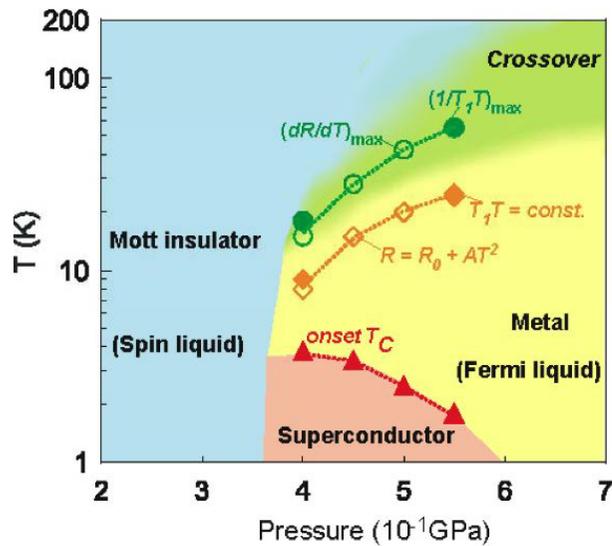
Phase diagram

Exact diagonalization as impurity  
solver ( $T=0$ ).



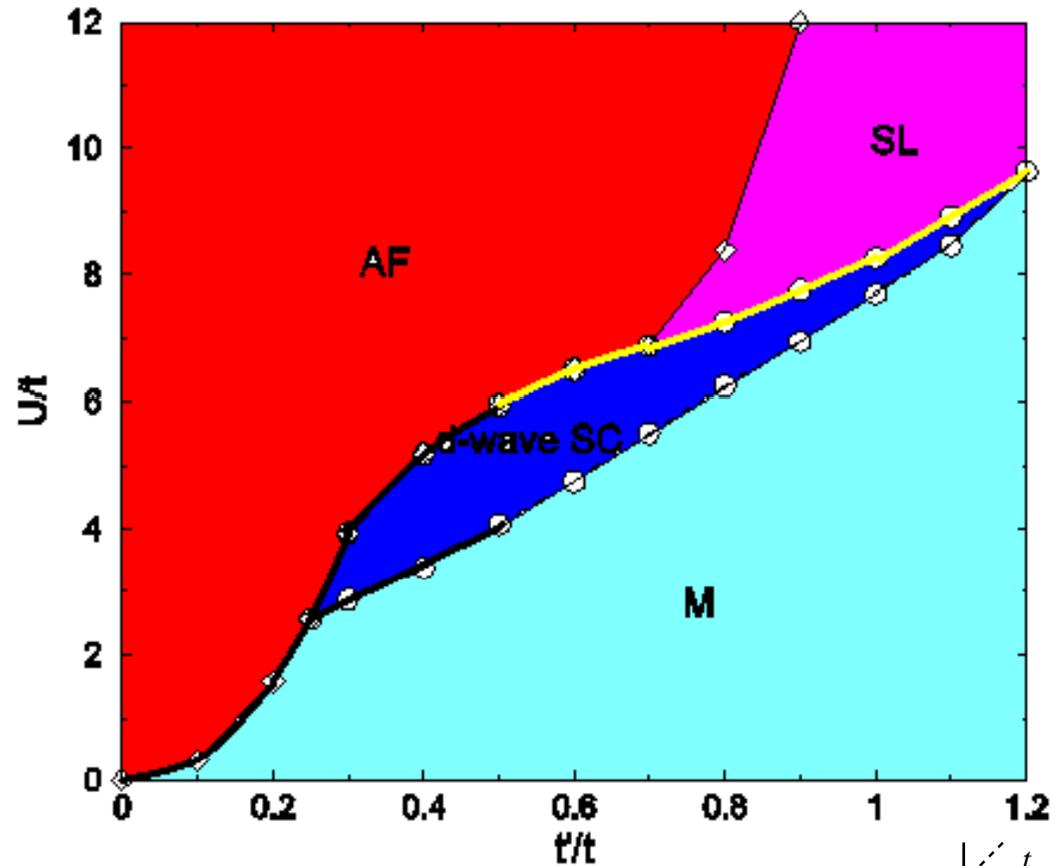


# Theoretical phase diagram BEDT

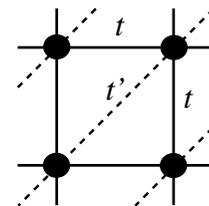


Y. Kurisaki, et al.

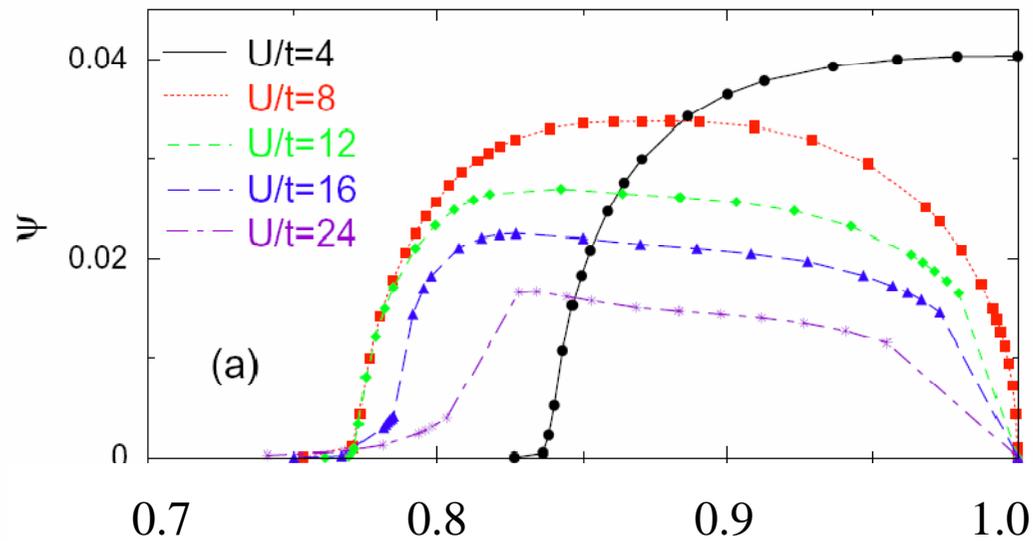
Phys. Rev. Lett. **95**, 177001(2005) Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)



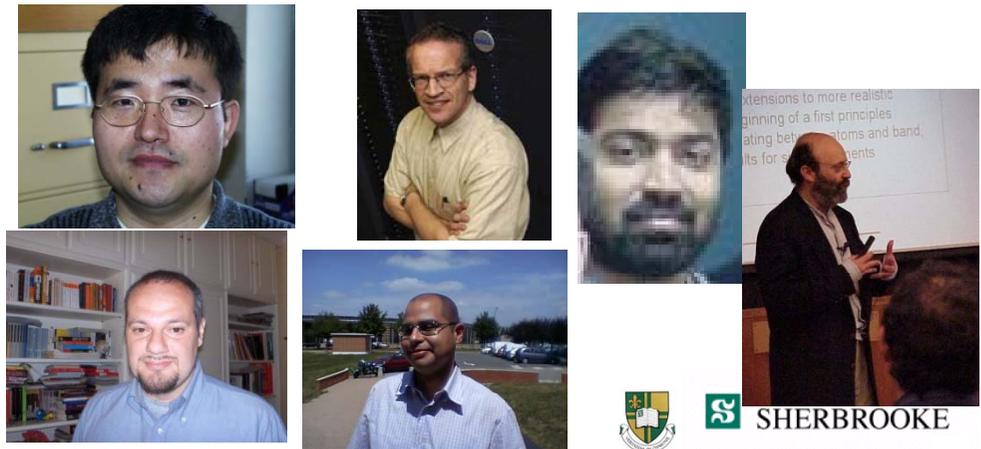
Kyung, A.-M.S.T. PRL 97, 046402 (2006)



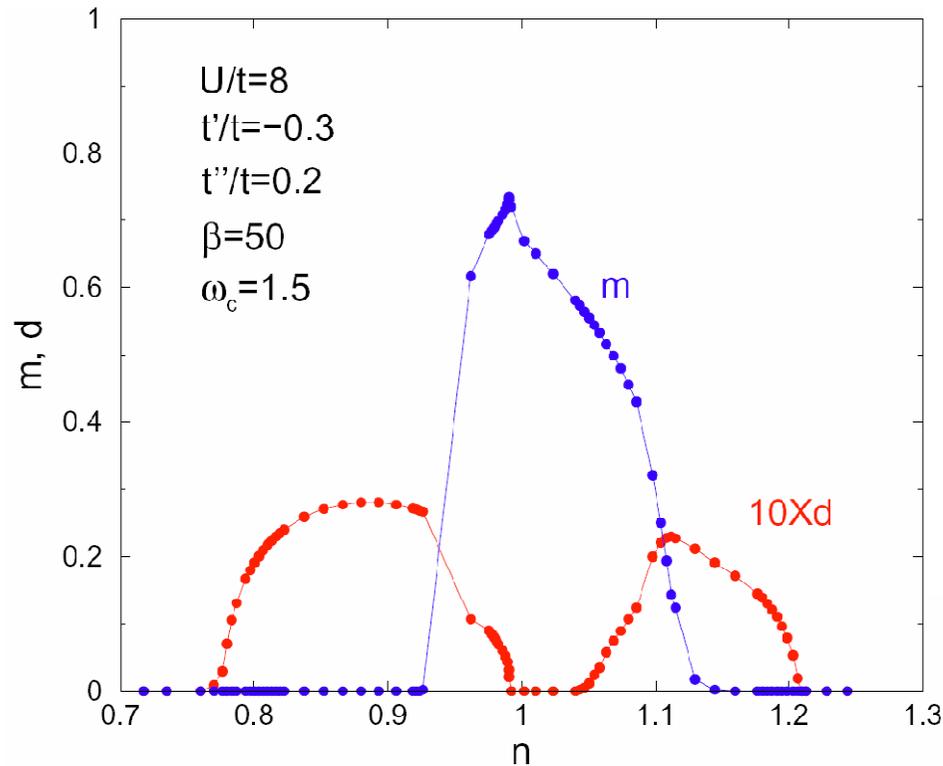
# Dome vs Mott (CDMFT)



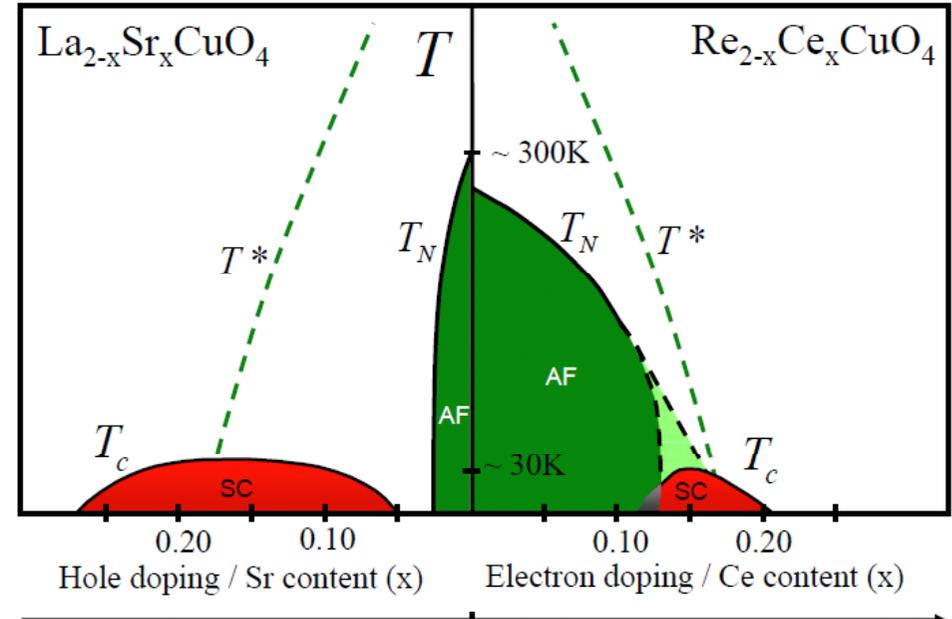
Kancharla, Kyung, Civelli,  
Sénéchal, Kotliar AMST  
Phys. Rev. B (2008)



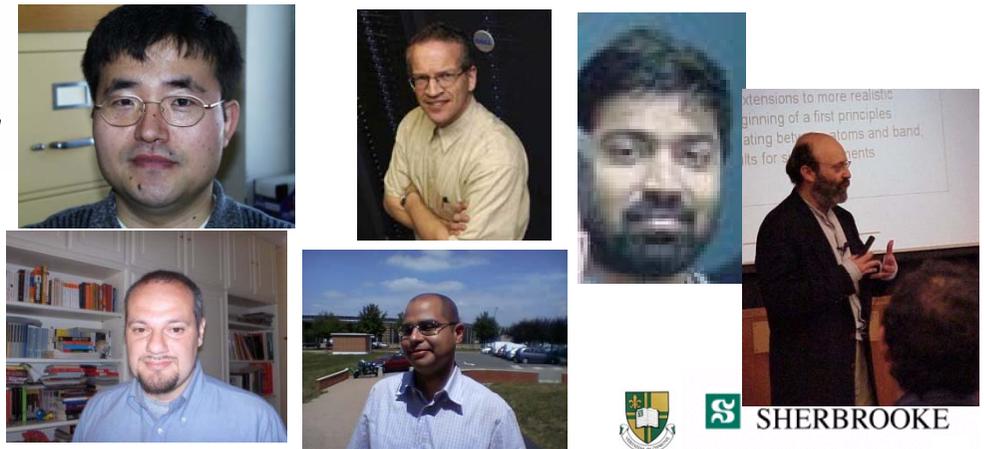
# CDMFT global phase diagram



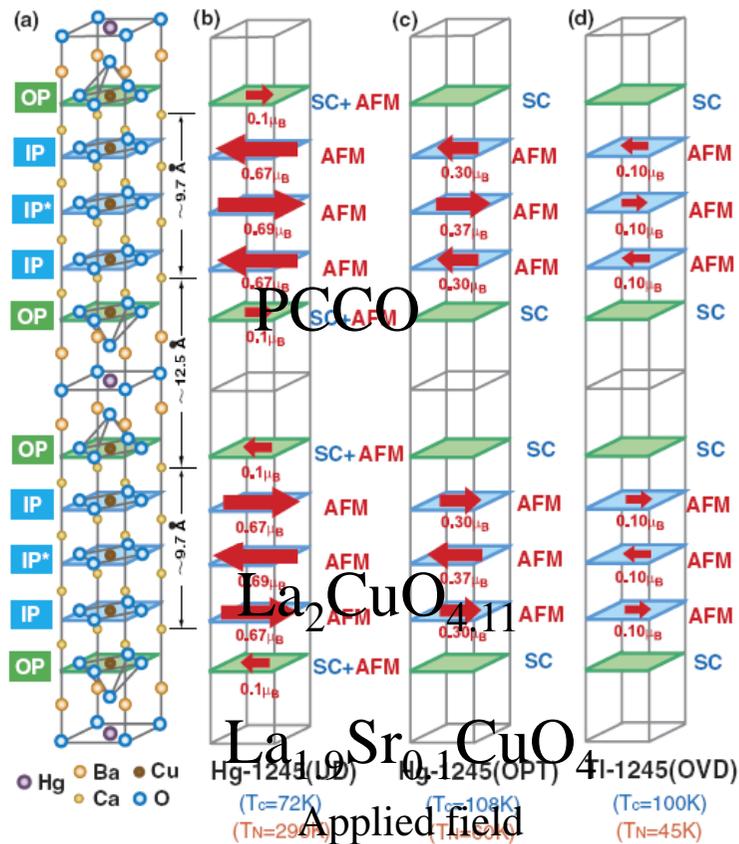
Kancharla, Kyung, Civelli,  
 Sénéchal, Kotliar AMST  
 Phys. Rev. B (2008)



Armitage, Fournier, Greene, RMP (2009)



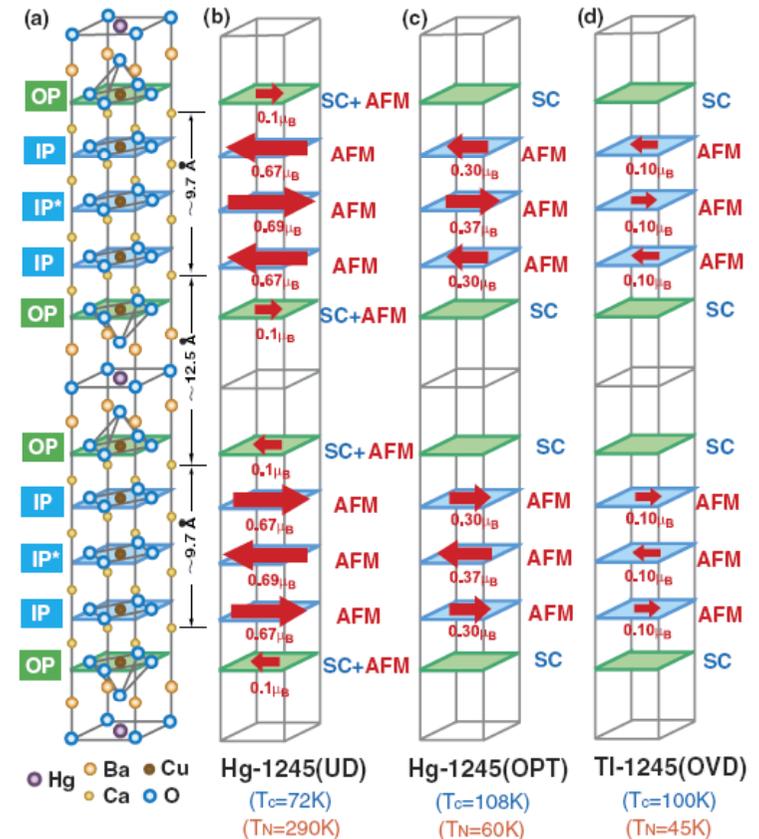
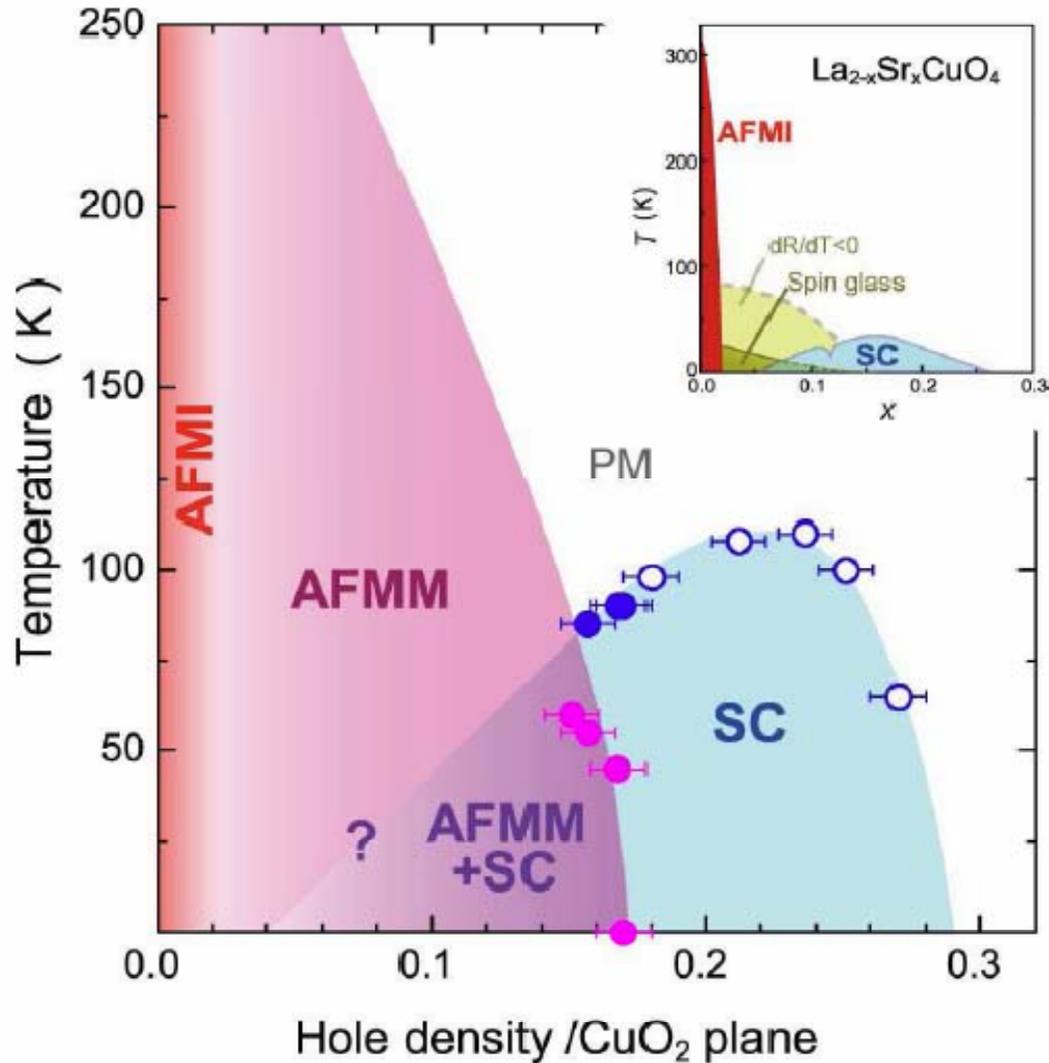
# Homogeneous coexistence (experimental)



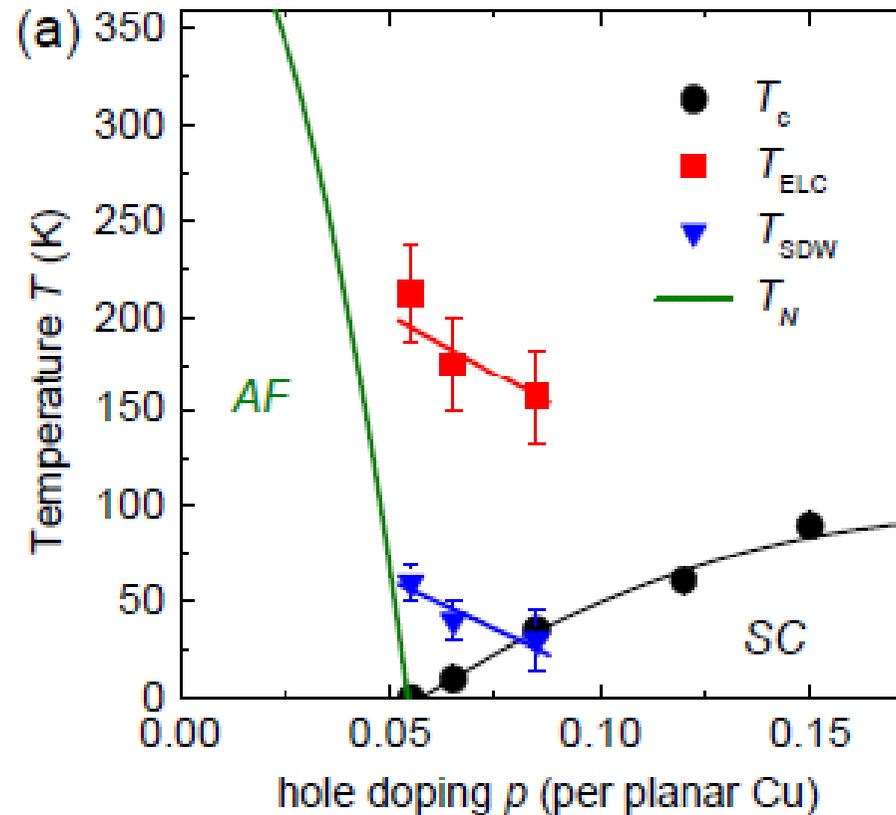
- H. Mukuda, M. Abe, Y. Araki, Y. Kitaoka, K. Tokiwa, T. Watanabe, A. Iyo, H. Kito, and Y. Tanaka, Phys. Rev. Lett. **96**, 087001 (2006).
- Pengcheng Dai, H. J. Kang, H. A. Mook, M. Matsuura, J. W. Lynn, Y. Kurita, Seiki Komiya, and Yoichi Ando, Phys. Rev. B **71**, 100502 R (2005).
- Robert J. Birgeneau, Chris Stock, John M. Tranquada and Kazuyoshi Yamada, J. Phys. Soc. Japan, **75**, 111003 (2006).
- Chang, ... Mesot PRB **78**, 104525 (2008).

# Consistent with following experiment

H. Mukuda, Y. Yamaguchi, S. Shimizu, ... A. Iyo JPSJ 77, 124706 (2008)



# Magnetic phase diagram of YBCO



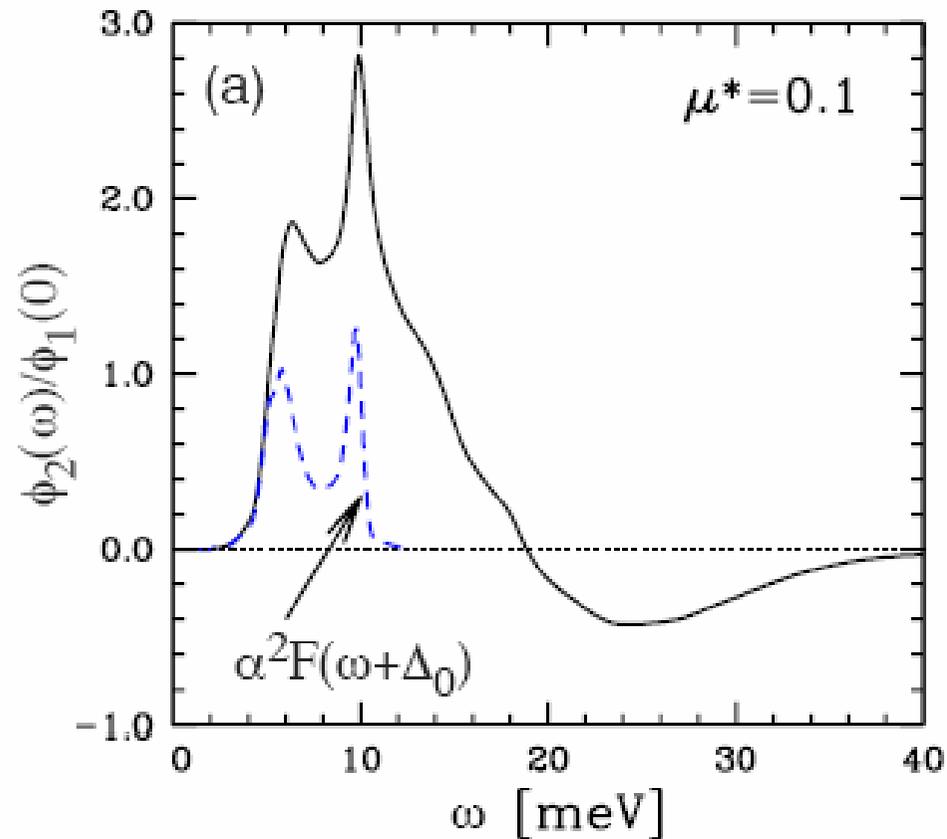
Haug, ... Keimer, arXiv:1008.4298



# The glue

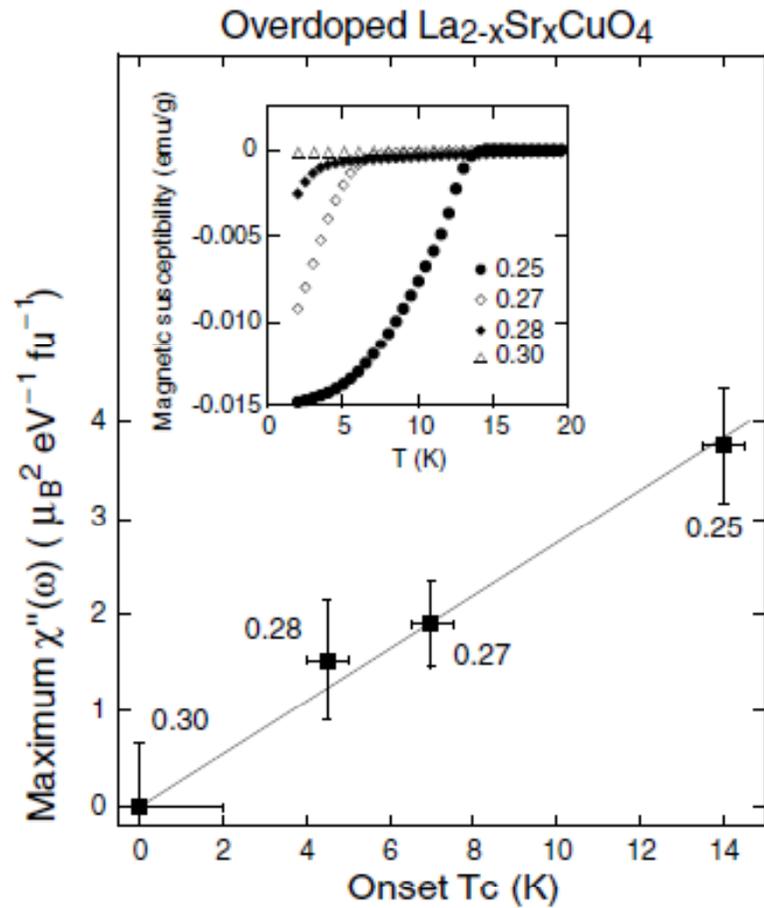
# $\text{Im } \Sigma_{an}$ and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

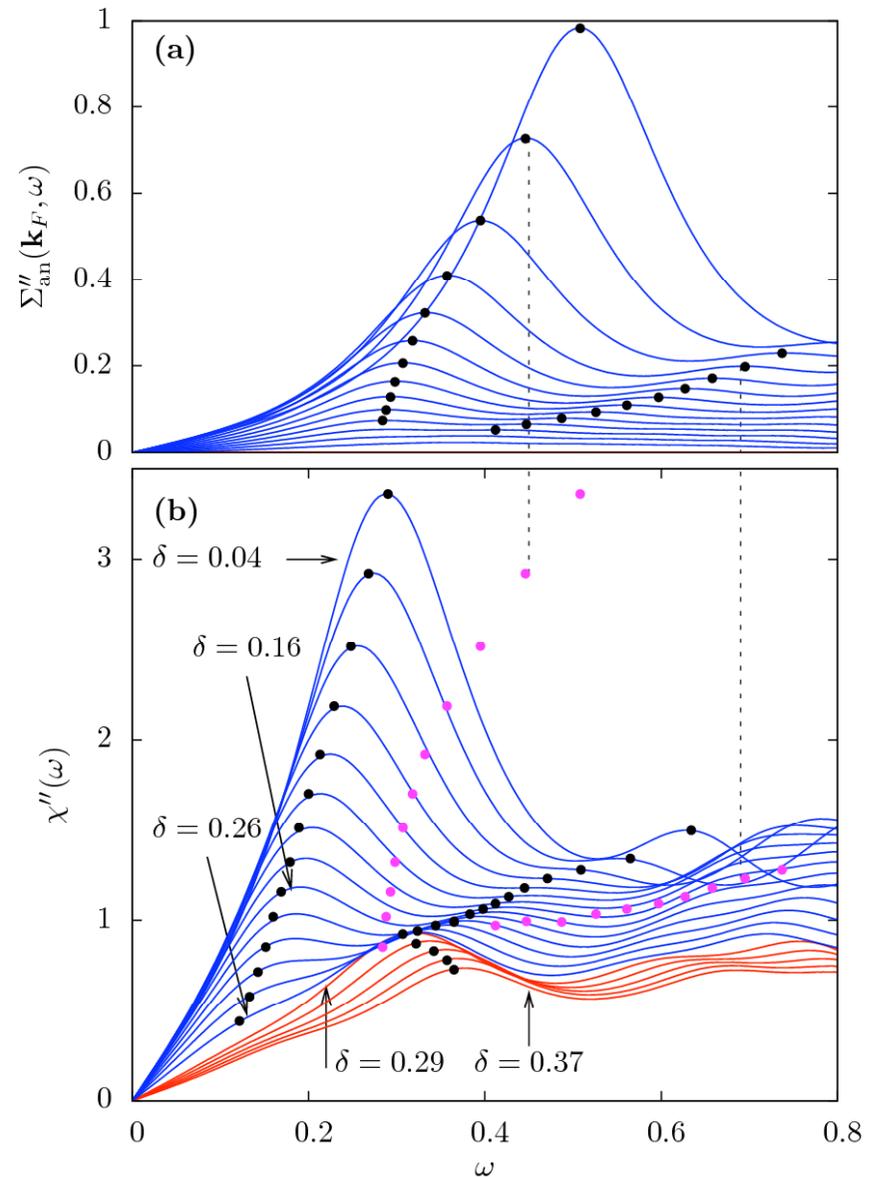


# The glue

Kyung, S en echal, Tremblay, Phys. Rev. B  
**80**, 205109 (2009)



Wakimoto ... Birgeneau  
 PRL (2004)



# The glue and neutrons

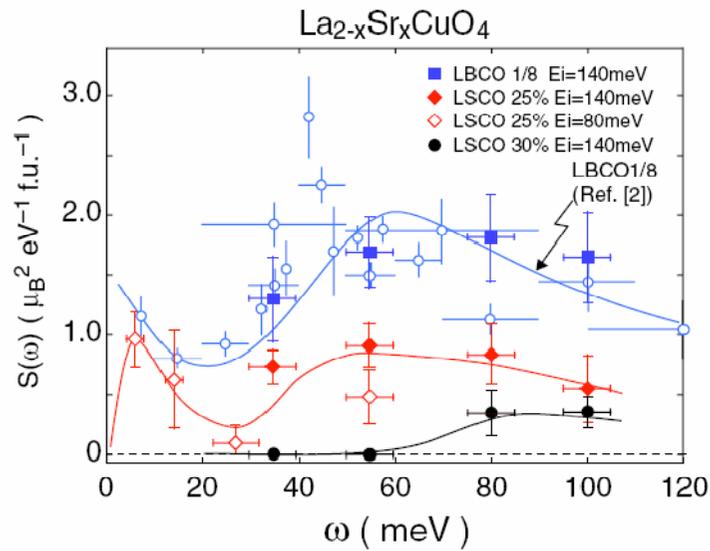
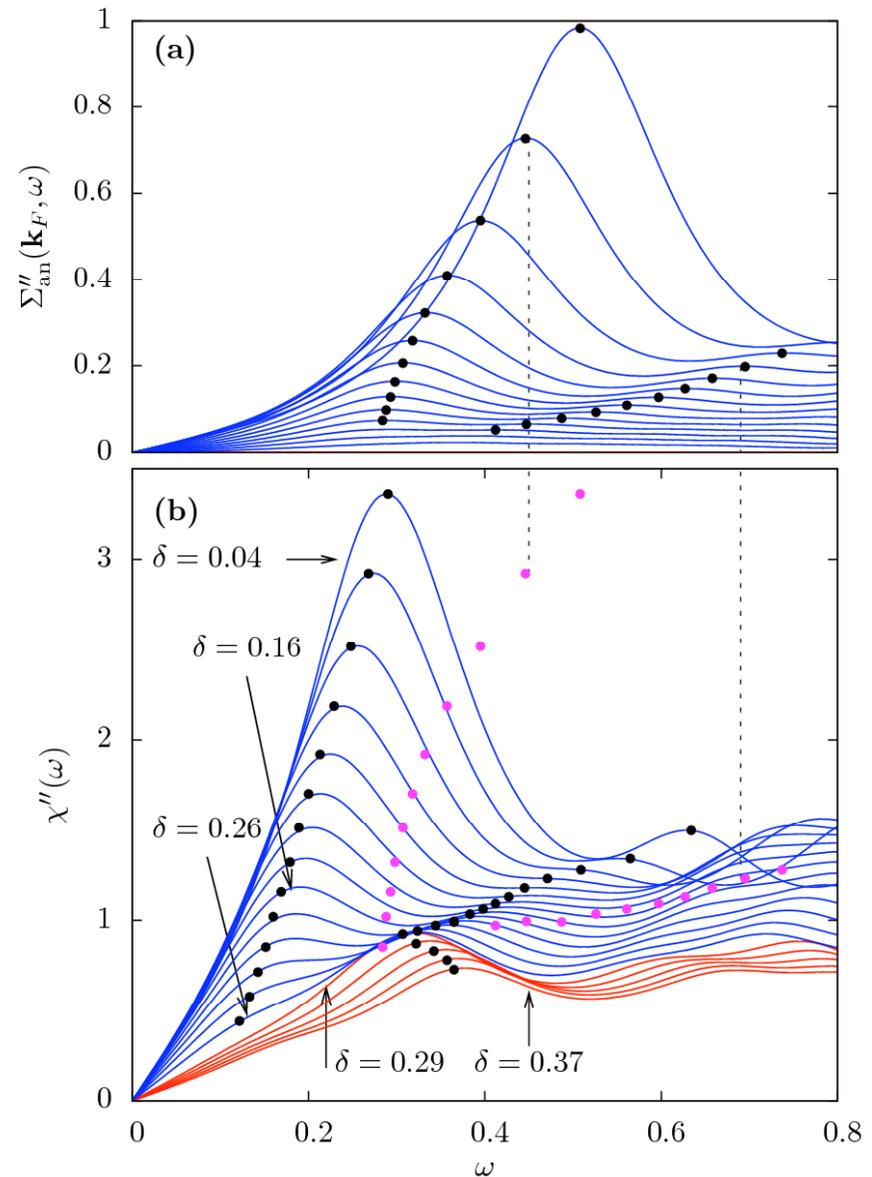


FIG. 3 (color online).  $\mathbf{Q}$ -integrated dynamic structure factor  $S(\omega)$  which is derived from the wide- $H$  integrated profiles for LBCO 1/8 (squares), LSCO  $x = 0.25$  (diamonds; filled for  $E_i = 140$  meV, open for  $E_i = 80$  meV), and  $x = 0.30$  (filled circles) plotted over  $S(\omega)$  for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO  $x = 0.25$  and  $0.30$  are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007);  
PRL (2004)



# Main collaborators



Giovanni Sordi



David Sénéchal



Kristjan Haule



Bumsoo Kyung



Marcello Civelli



Satoshi Okamoto

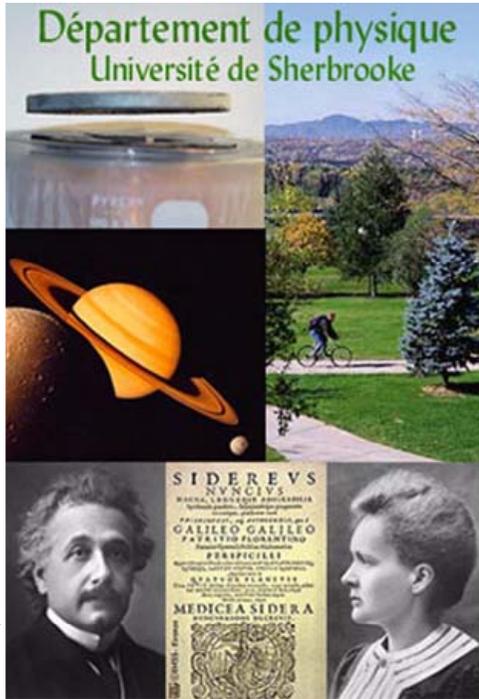


# Conclusions

- The influence of Mott Physics extends way beyond half-filling
- Conjecture that quantum-critical like behavior is constant  $U$  cut of our phase diagram, i.e. very low  $T$  critical point.
- Superconductivity follows naturally and retardation effects in pairing come from spin fluctuations.



# André-Marie Tremblay



Le regroupement québécois sur les matériaux de pointe



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Merci

Arigato

Thank you