

Insulators, metals, pseudogaps and high-temperature superconductors

A.-M. Tremblay

G. Sordi, K. Haule, D. Sénéchal,
P. Sémon, B. Kyung, G. Kotliar

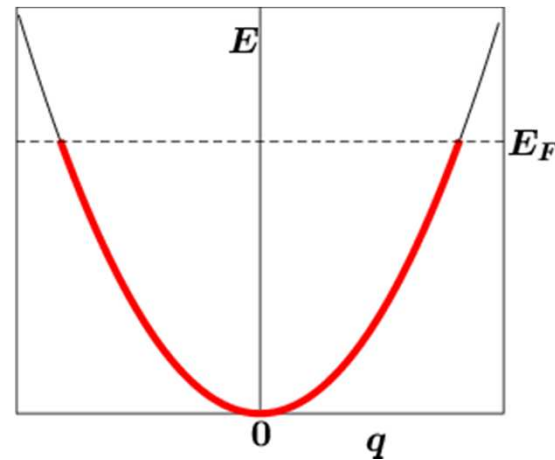
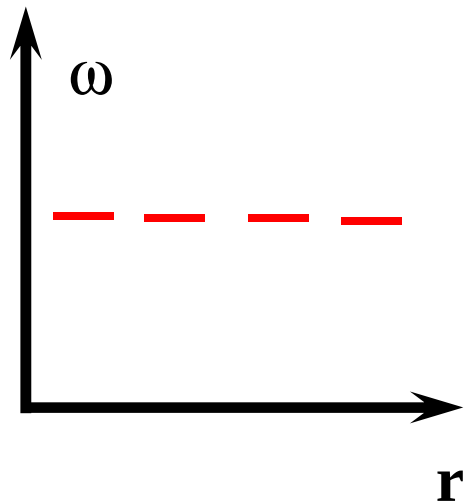
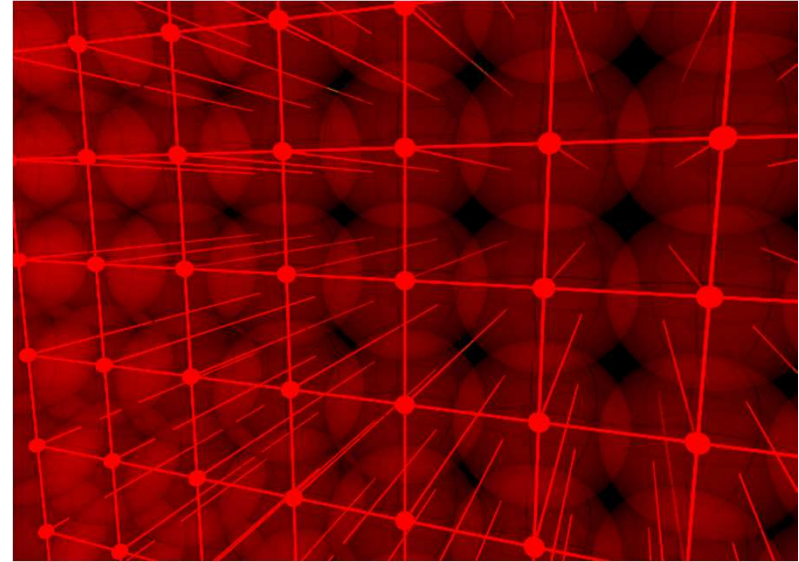
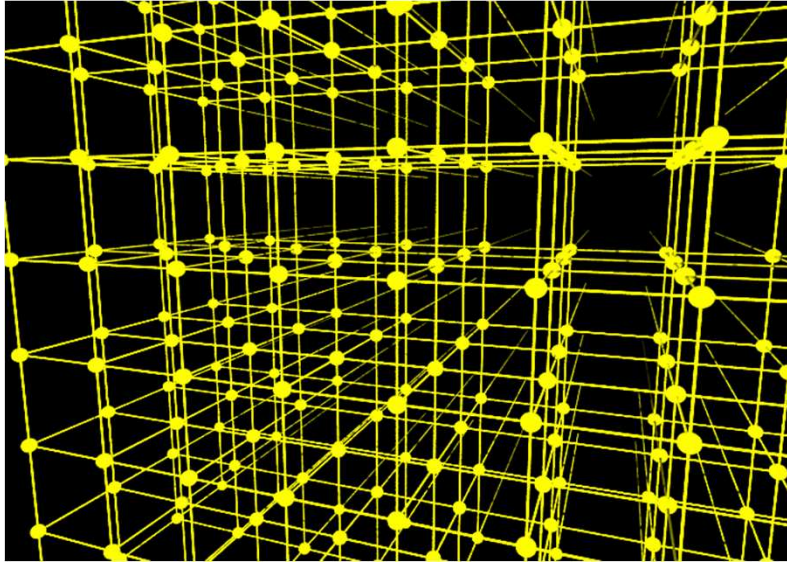


Rice, 16 April, 2012



UNIVERSITÉ DE
SHERBROOKE

How to make a metal



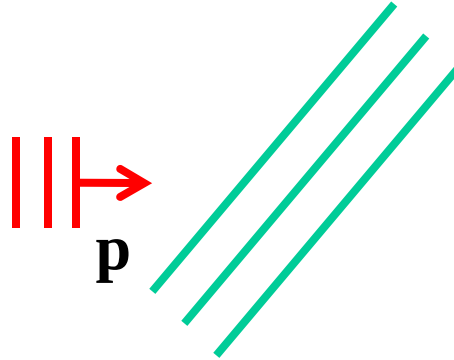
Courtesy, S. Julian



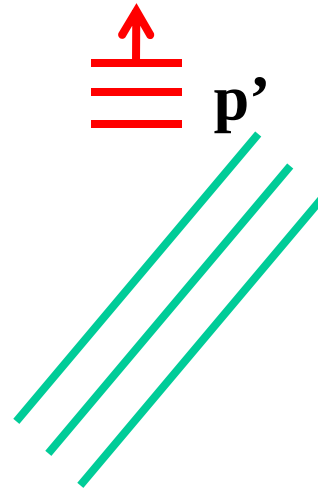
Superconductivity



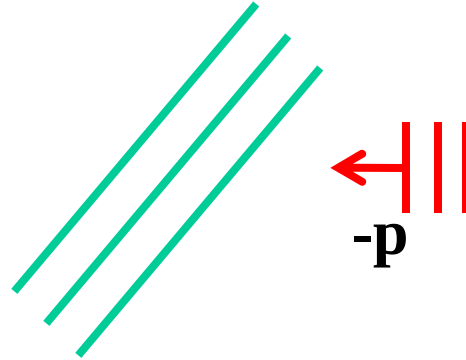
Attraction mechanism in the metallic state



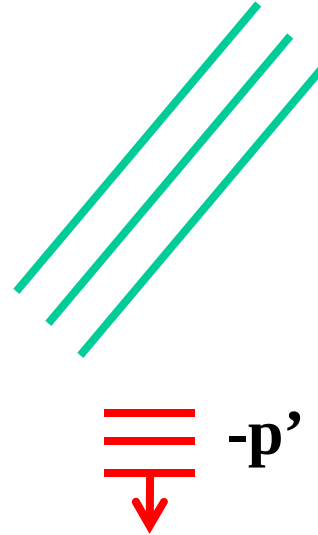
Attraction mechanism in the metallic state



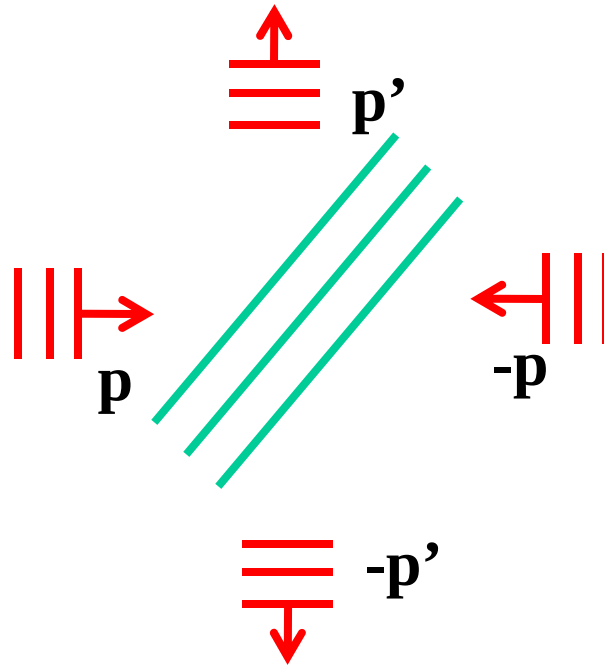
Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



#1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^*$$

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left(\langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* + \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \right)$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$



Half-filled band is metallic?

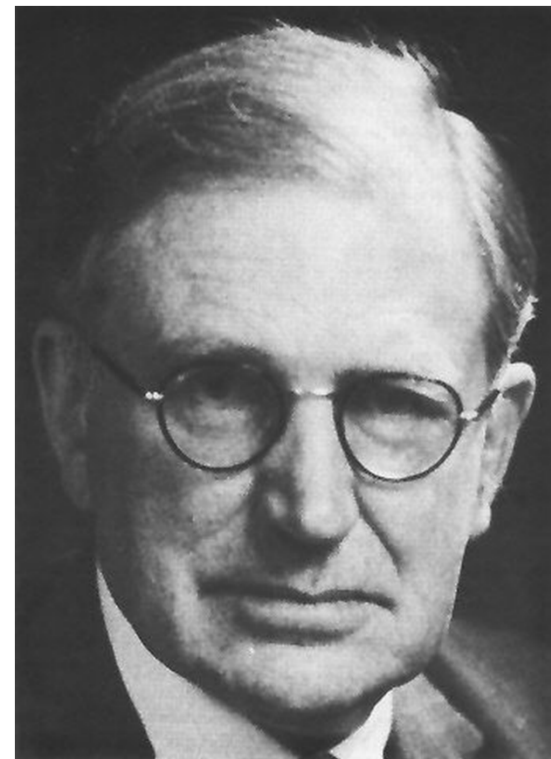


Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937



Mott, 1949



UNIVERSITÉ DE
SHERBROOKE

« Conventional » Mott transition

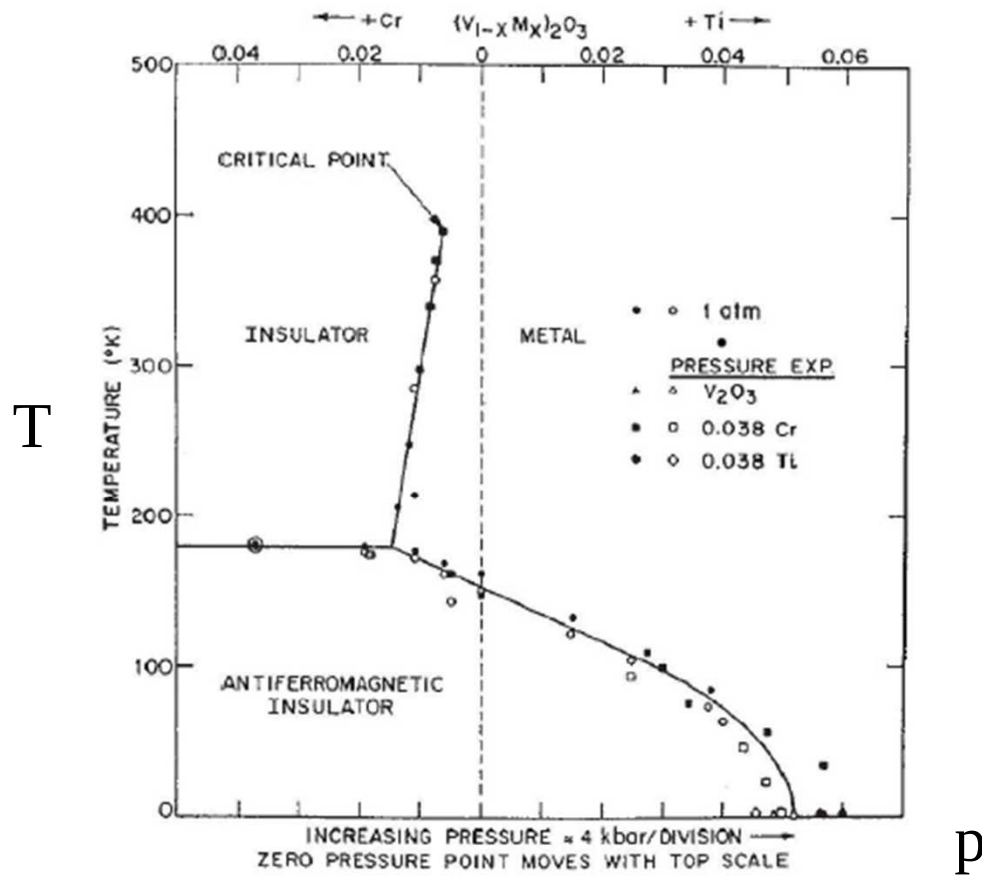
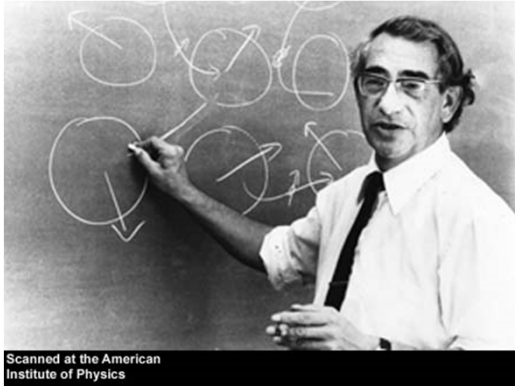
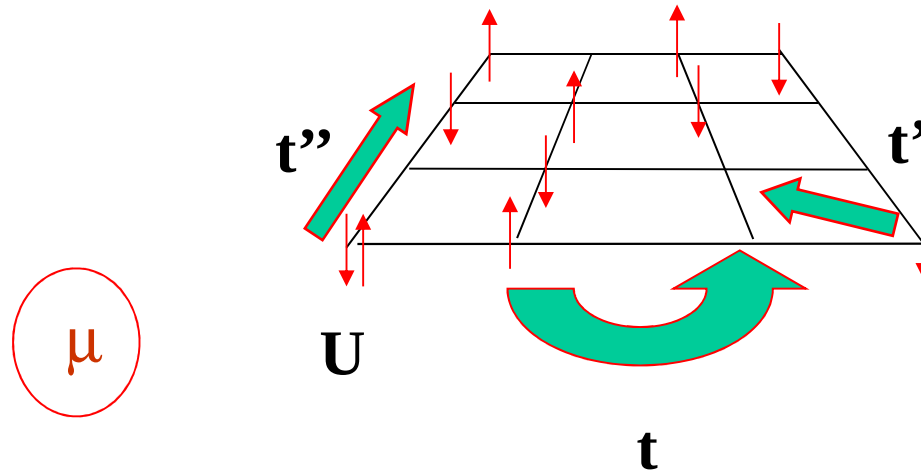


Figure: McWhan, PRB 1970; Limelette, Science 2003

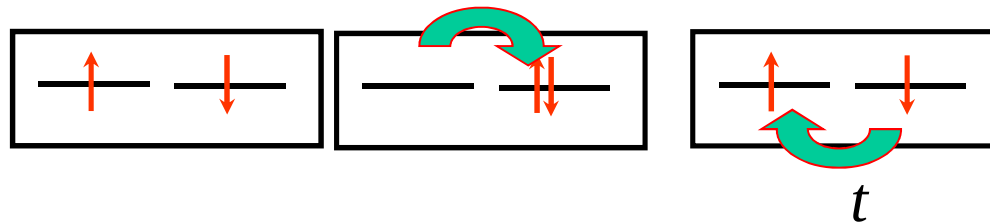
Hubbard model



1931-1980



$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} \left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



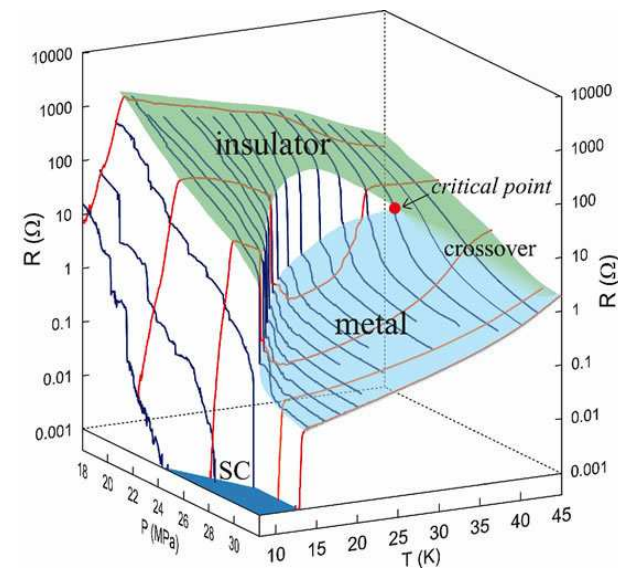
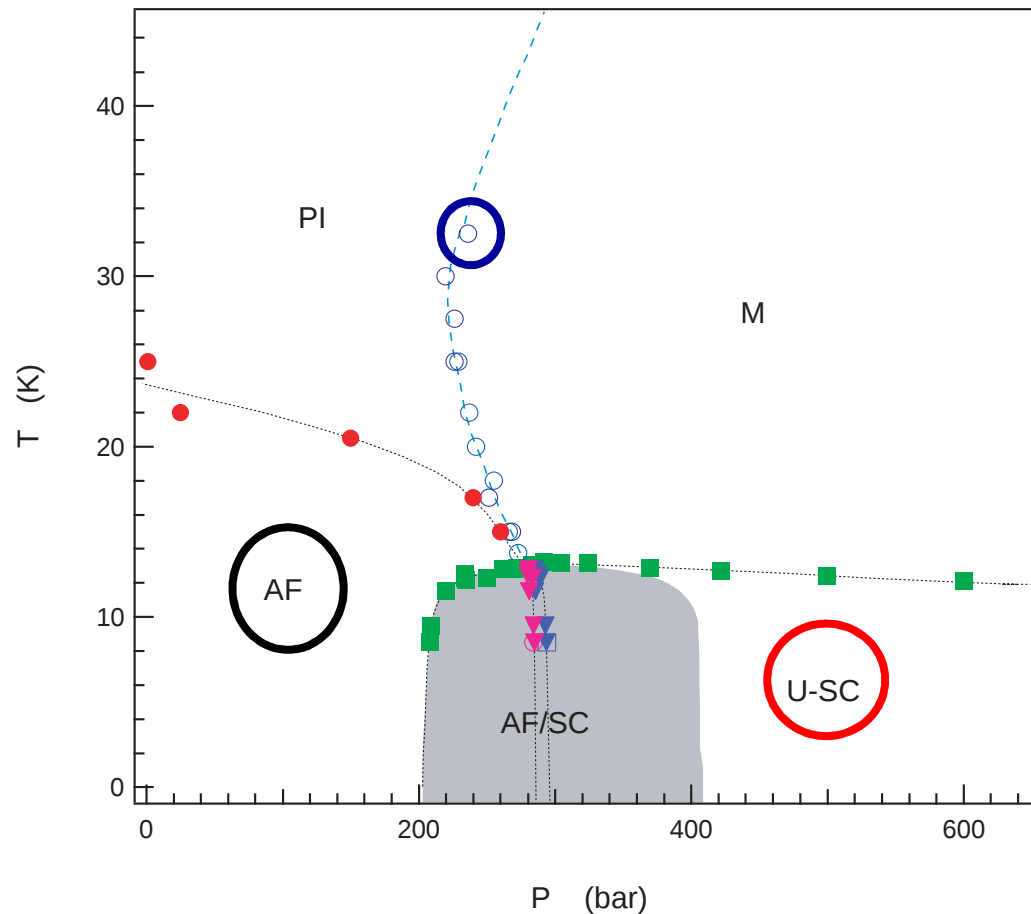
Effective model, Heisenberg: $J = 4t^2 / U$



Superconductivity and attraction?



Bare Mott critical point in organics



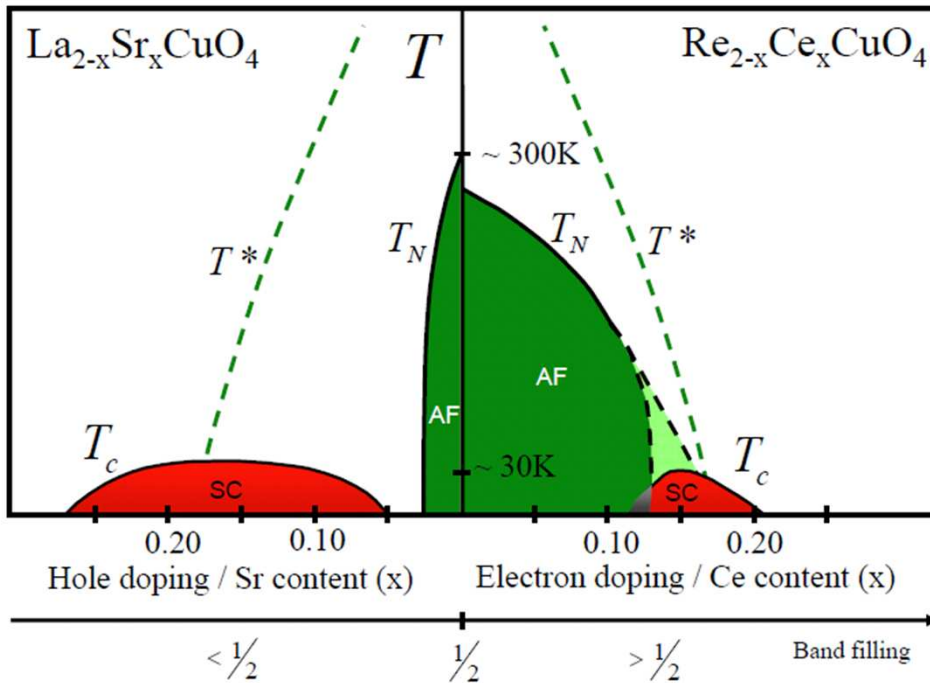
F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

Phase diagram ($X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

High-temperature superconductors

Armitage, Fournier, Greene, RMP (2009)



- Competing order

- Current loops: Varma, PRB **81**, 064515 (2010)
- Stripes or nematic: Kivelson et al. RMP 75 1201(2003); J.C.Davis
- d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
- SDW: Sachdev PRB **80**, 155129 (2009) ...

- Or Mott Physics?

- RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

What is under the dome?
Mott Physics away from $n = 1$

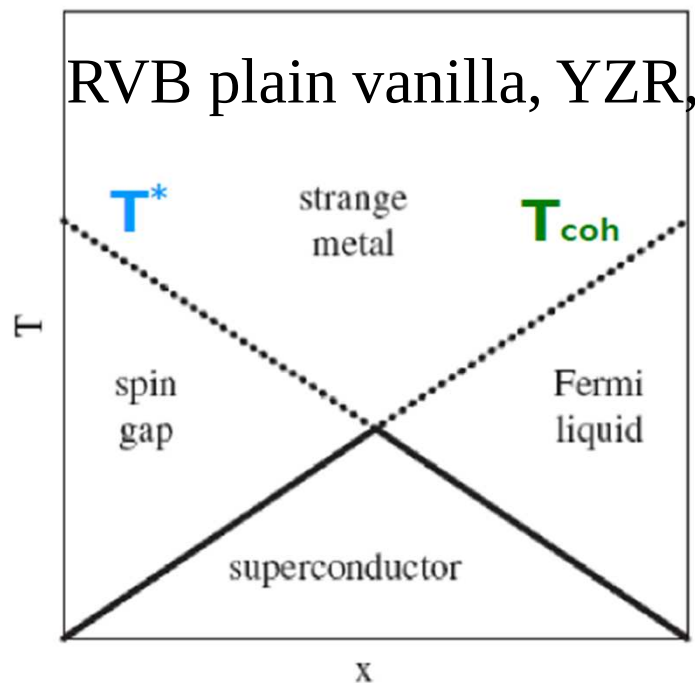


Two views (caricature)

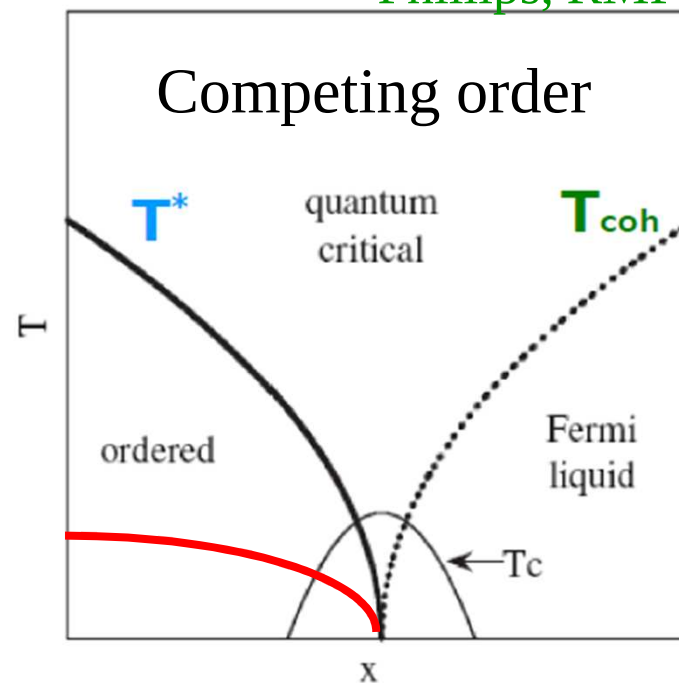
Norman, Adv. Phys. (2005)

Broun, Nat. Phys. (2006)

Phillips, RMP (2010)



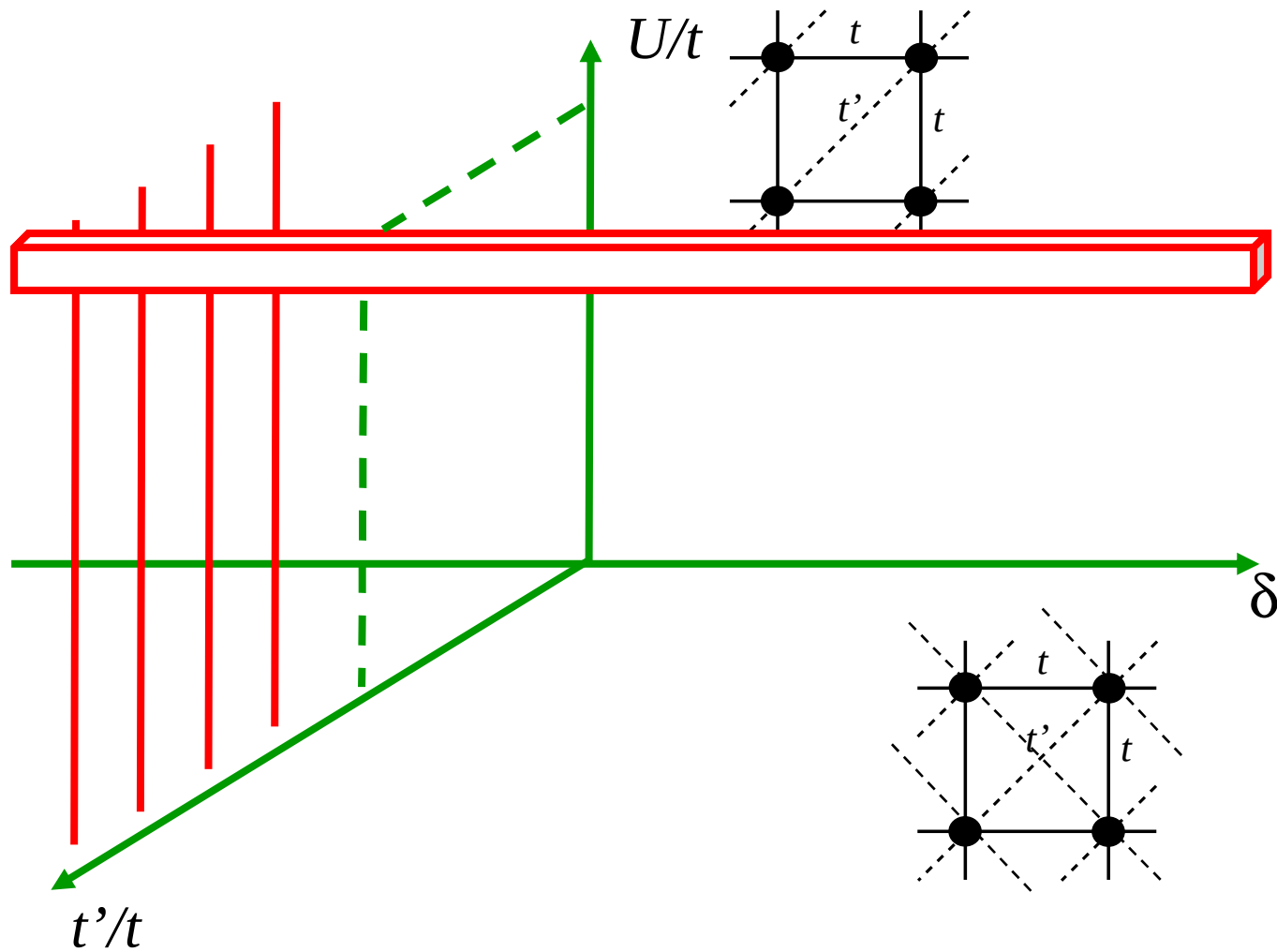
Why T_c decreases?
What is the origin of T^* ?
What is the strange metal?



Broken symmetry or not.
What lies beneath the dome.
Mott Physics away from $n = 1$



Perspective



Outline

- Method
- $T=0$ phase diagram
 - The « glue »
- Finite T phase diagram
 - Normal state
 - First order transition
 - Widom line and pseudogap
 - Superconductivity

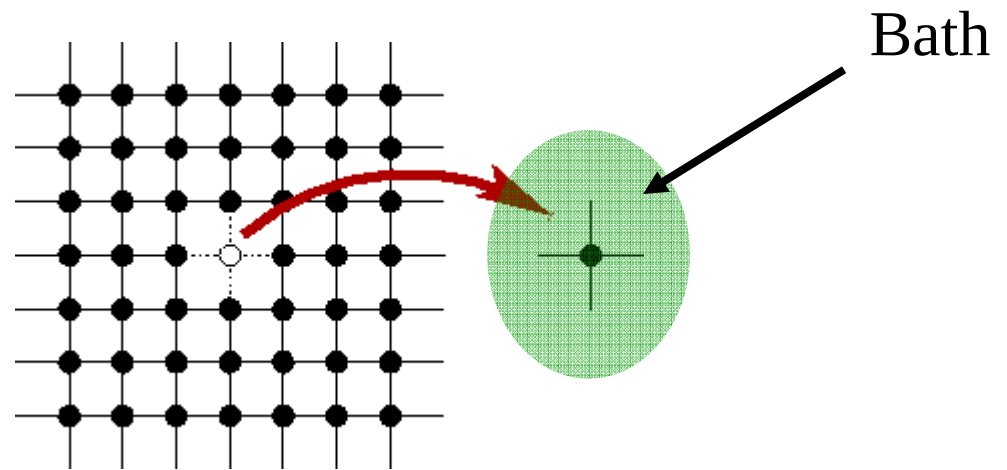


Method

Mott transition and Dynamical Mean-Field Theory.

The beginnings in $d = \text{infinity}$

- Compute scattering rate (self-energy) of impurity problem.
- Use that self-energy (ω dependent) for lattice.
- Project lattice on single-site and adjust bath so that single-site DOS obtained both ways be equal.



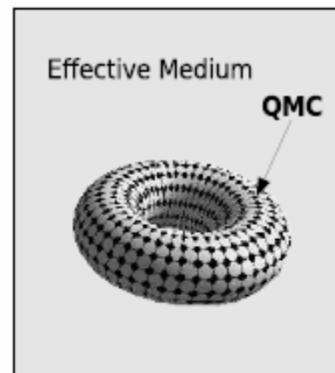
W. Metzner and D. Vollhardt, PRL (1989)

A. Georges and G. Kotliar, PRB (1992)

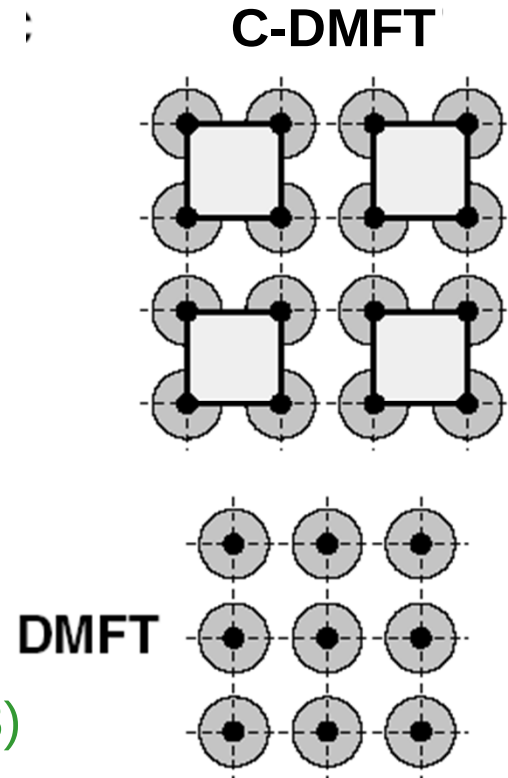
M. Jarrell PRB (1992)

DMFT, ($d = 3$)

2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)



UNIVERSITÉ DE
SHERBROOKE

Not perfect!

- Long range order:
 - Allow symmetry breaking in the bath (mean-field)
- Missing:
 - Long wavelength fluctuations
- Included:
 - Short-range dynamical and spatial correlations

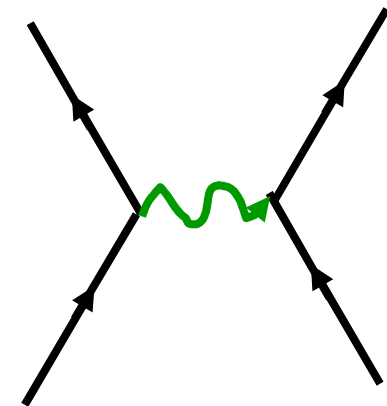
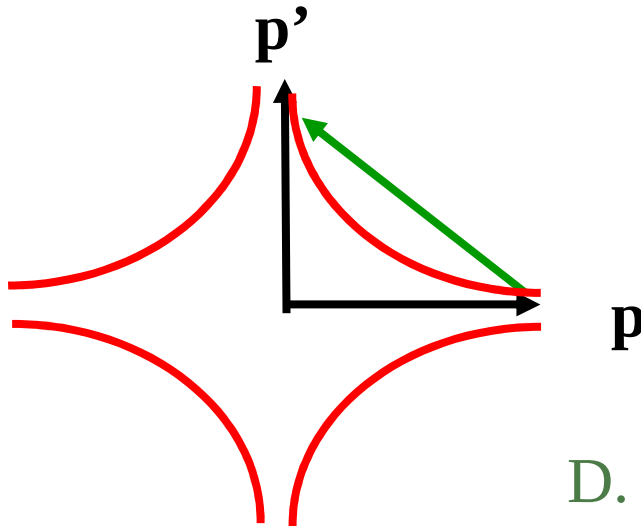


A bit of physics: superconductivity and repulsion



Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Exchange of spin waves?

Kohn-Luttinger

T_c with pressure

P.W. Anderson Science 317, 1705 (2007)

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).

Béal-Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Kohn, Luttinger, P.R.L. **15**, 524 (1965).



UNIVERSITÉ DE
SHERBROOKE

A cartoon strong coupling picture

P.W. Anderson Science 317, 1705 (2007)

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

Kotliar and Liu, P.R. B **38**, 5142 (1988)

Miyake, Schmitt-Rink, and Varma

P.R. B **34**, 6554-6556 (1986)



d-wave superconductivity

- Weak coupling

- C. J. Halboth and W. Metzner, Phys. Rev. Lett. 85, 5162 (2000).
- B. Kyung, J.-S. Landry, and A. M. S. Tremblay, Phys. Rev. B 68, 174502 (2003).
- C. Bourbonnais and A. Sedeki, Physical Review B 80, 085105 (2009).
- D. J. Scalapino, Physica C: Superconductivity 470, Supplement 1, S1 (2010), ISSN 0921-4534, proceedings of the 9th International Conference on Materials and Mechanisms of Superconductivity.

- Renormalized Mean-Field Theory

- P. W. Anderson, P. A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, Journal of Physics: Condensed Matter 16, R755 (2004).
- K.-Y. Yang, T. M. Rice, and F.-C. Zhang, Phys. Rev. B 73, 174501 (2006).

- Slave particles

- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. 78, 17 (2006).
- M. Imada, Y. Yamaji, S. Sakai, and Y. Motome, Annalen der Physik 523, 629 (2011)

- Variational approaches

- T. Giamarchi and C. Lhuillier, Phys. Rev. B 43, 12943 (1991).
- A. Paramekanti, M. Randeria, and N. Trivedi, Phys. Rev. B 70, 054504 (2004).



d-wave superconductivity

- Quantum cluster methods

- T. Maier, M. Jarrell, T. Pruschke, and J. Keller, Phys. Rev. Lett. 85, 1524 (2000).
- David Sénéchal, P.-L. Lavertu, M.-A. Marois and A.-M. S. Tremblay, PRL **94**, 156404 (2005).
- T. A. Maier, M. Jarrell, T. C. Schulthess, P. R. C. Kent, and J. B. White, Phys. Rev. Lett. 95, 237001 (2005).
- K. Haule and G. Kotliar, Phys. Rev. B 76, 104509 (2007).
- Kancharla, Kyung, Civelli, Sénéchal, Kotliar AMST , Phys. Rev. B (2008)
- Many more...



But...

QMC constrained path

S. Zhang, Carlson, Gubernatis Phys. Rev. Lett. 78, 4486 (1997)

Refined variational approach: no

Aimi and Imada, J. Phys. Soc. Jpn (2007)



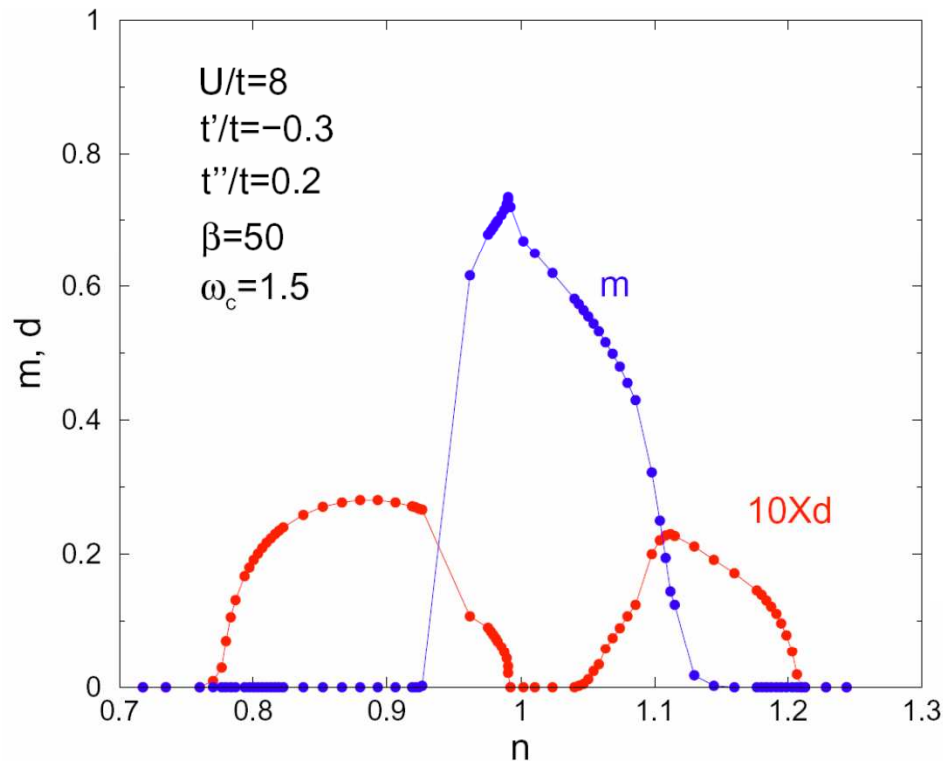
$T = 0$ phase diagram: cuprates

Phase diagram

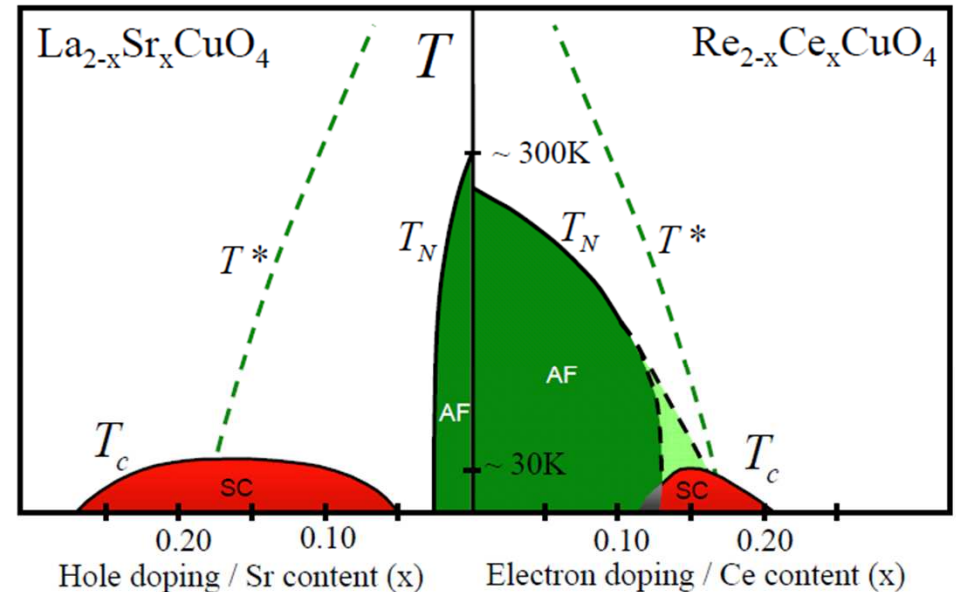
Exact diagonalization as impurity
solver ($T=0$).



CDMFT global phase diagram



Kancharla, Kyung, Civelli,
 Sénéchal, Kotliar AMST
 Phys. Rev. B (2008)
 AND Capone, Kotliar PRL (2006)



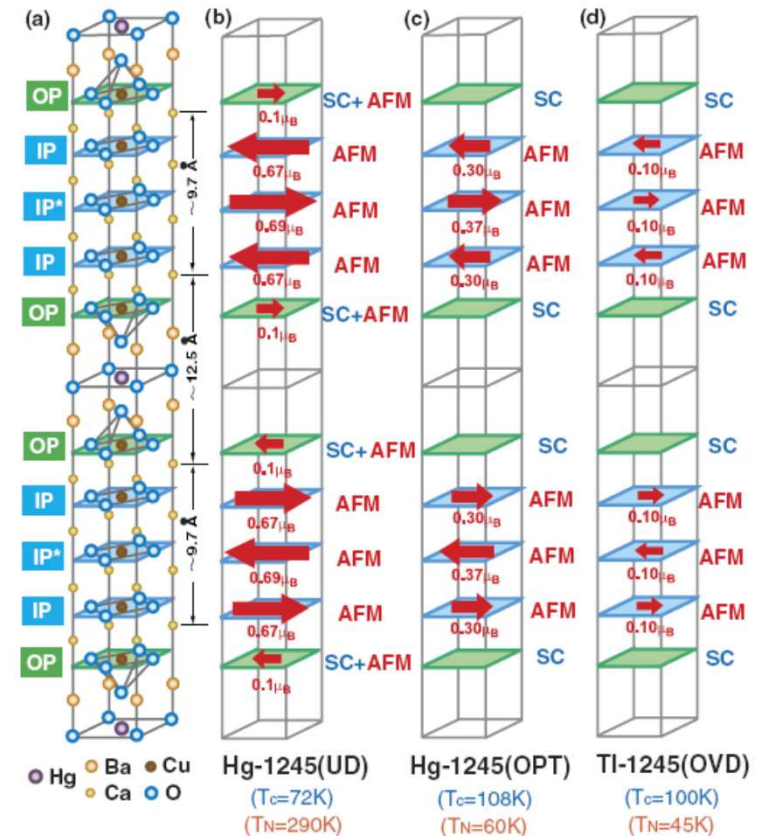
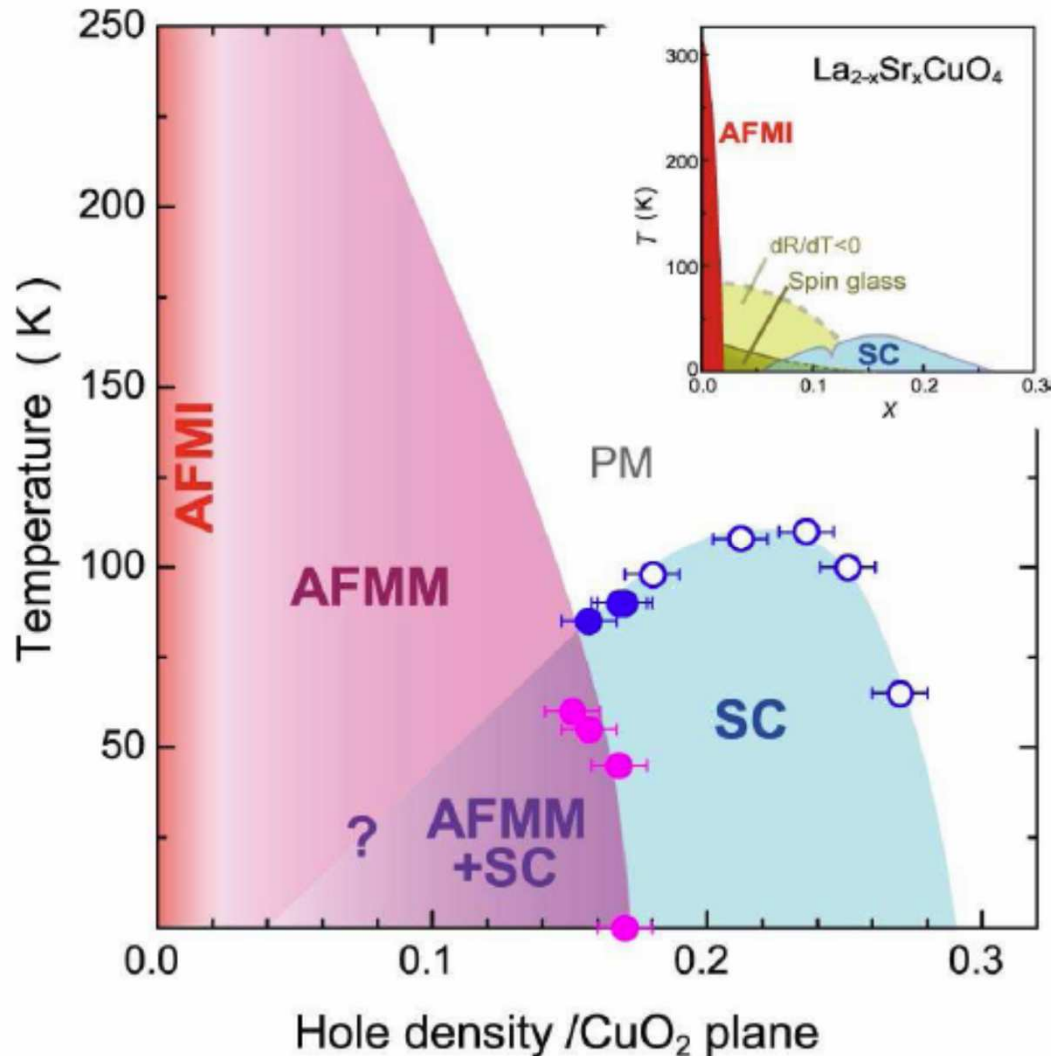
Armitage, Fournier, Greene, RMP (2009)



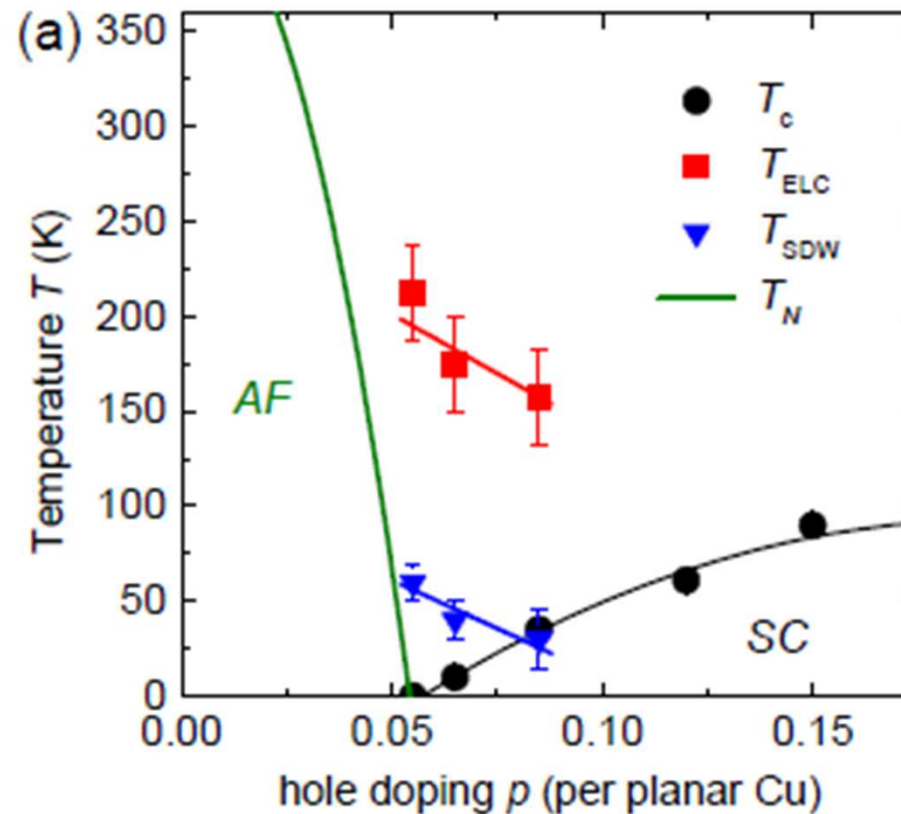
UNIVERSITÉ DE
SHERBROOKE

Consistent with following experiments

H. Mukuda, Y. Yamaguchi, S. Shimizu, ... A. Iyo JPSJ 77, 124706 (2008)



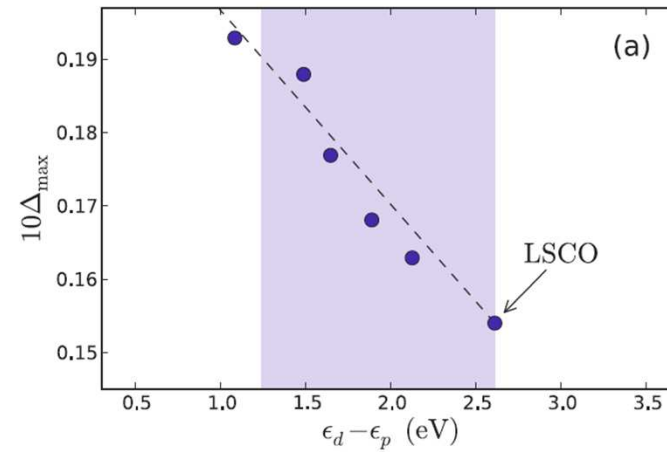
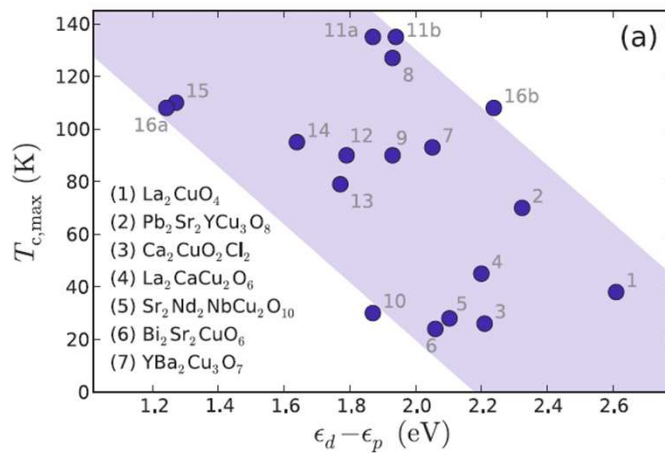
Magnetic phase diagram of YBCO



Haug, ... Keimer, New J. Phys. 12, 105006 (2010)



Materials dependent properties



C. Weber, C.-H. Yee, K. Haule, and G. Kotliar, ArXiv e-prints (2011), 1108.3028.

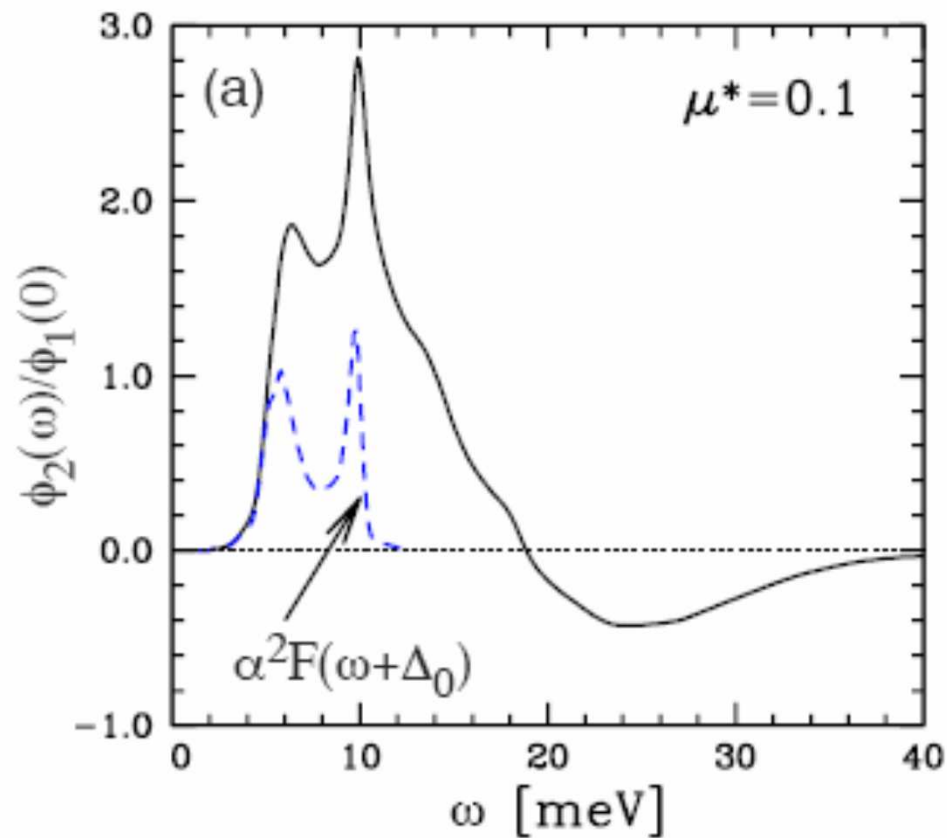
$T = 0$ phase diagram

The glue



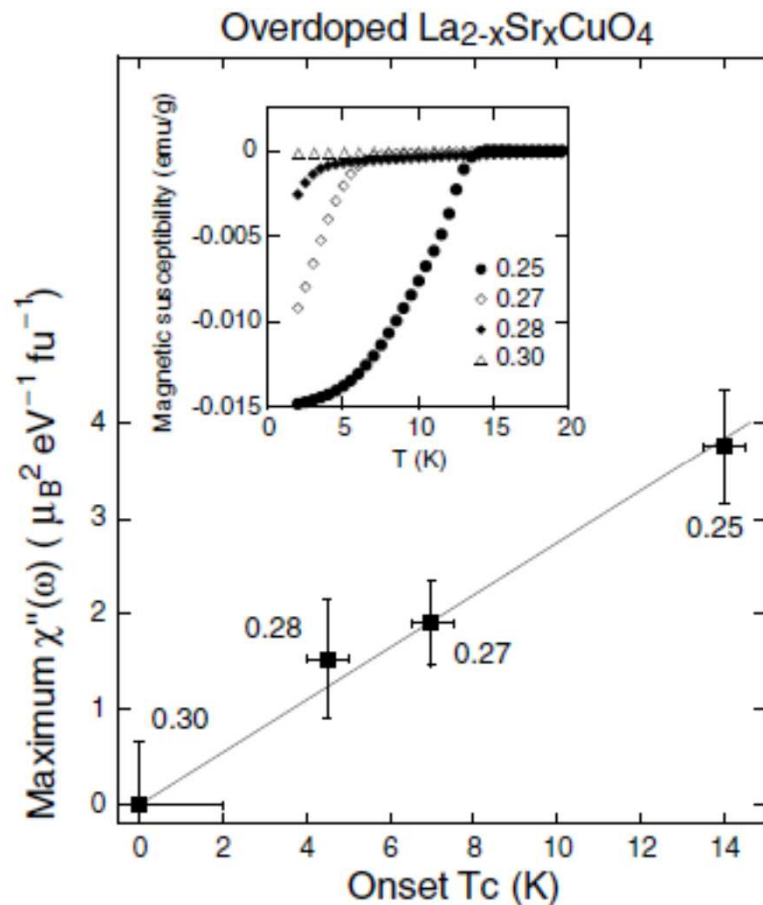
$\text{Im } \Sigma_{\text{an}}$ and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

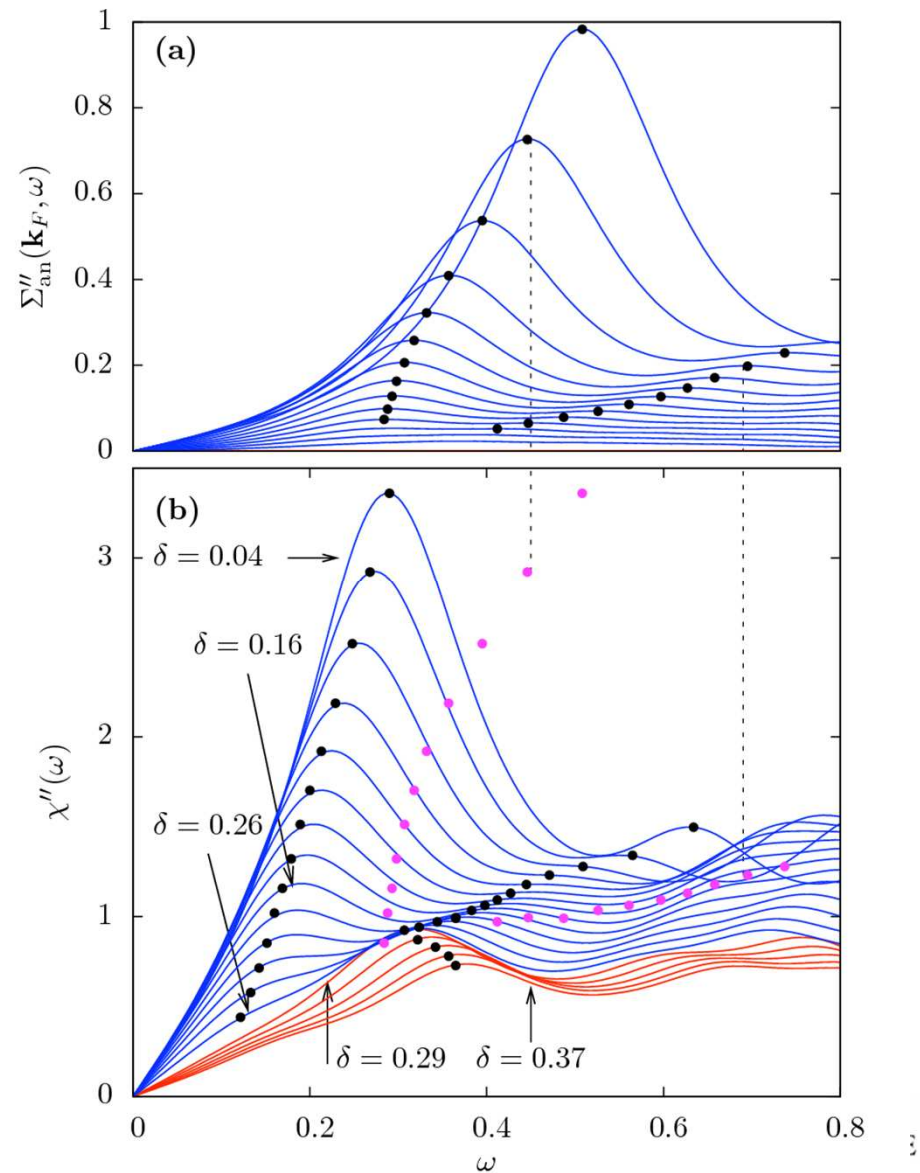


The glue

Kyung, Sénéchal, Tremblay, Phys. Rev. B
80, 205109 (2009)



Wakimoto ... Birgeneau
 PRL (2004)



The glue and neutrons

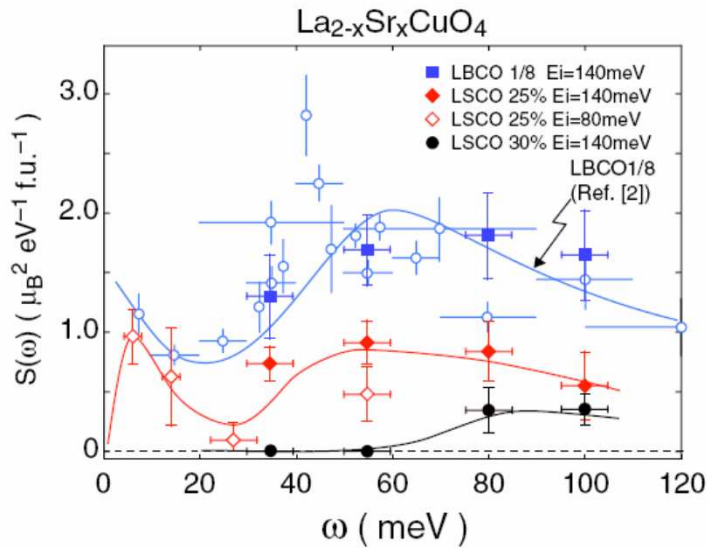
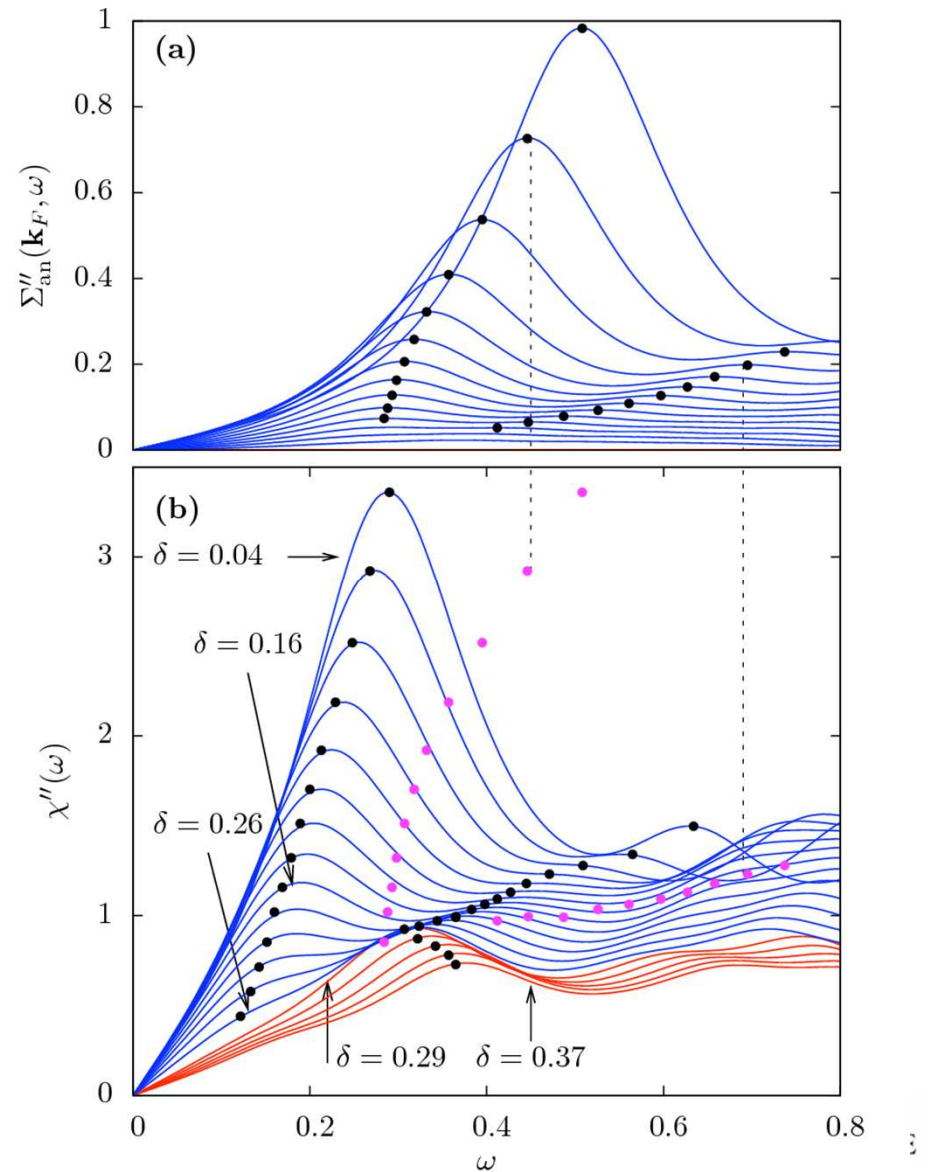


FIG. 3 (color online). $\mathbf{Q}$ -integrated dynamic structure factor $S(\omega)$ which is derived from the wide- H integrated profiles for LBCO 1/8 (squares), LSCO $x = 0.25$ (diamonds; filled for $E_i = 140$ meV, open for $E_i = 80$ meV), and $x = 0.30$ (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO $x = 0.25$ and 0.30 are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007);
PRL (2004)



Outline

- Method
- $T=0$ phase diagram
 - The « glue »
- Finite T phase diagram
 - Normal state
 - First order transition
 - Widom line and pseudogap
 - Superconductivity



Finite T phase diagram

Normal state of the cuprates



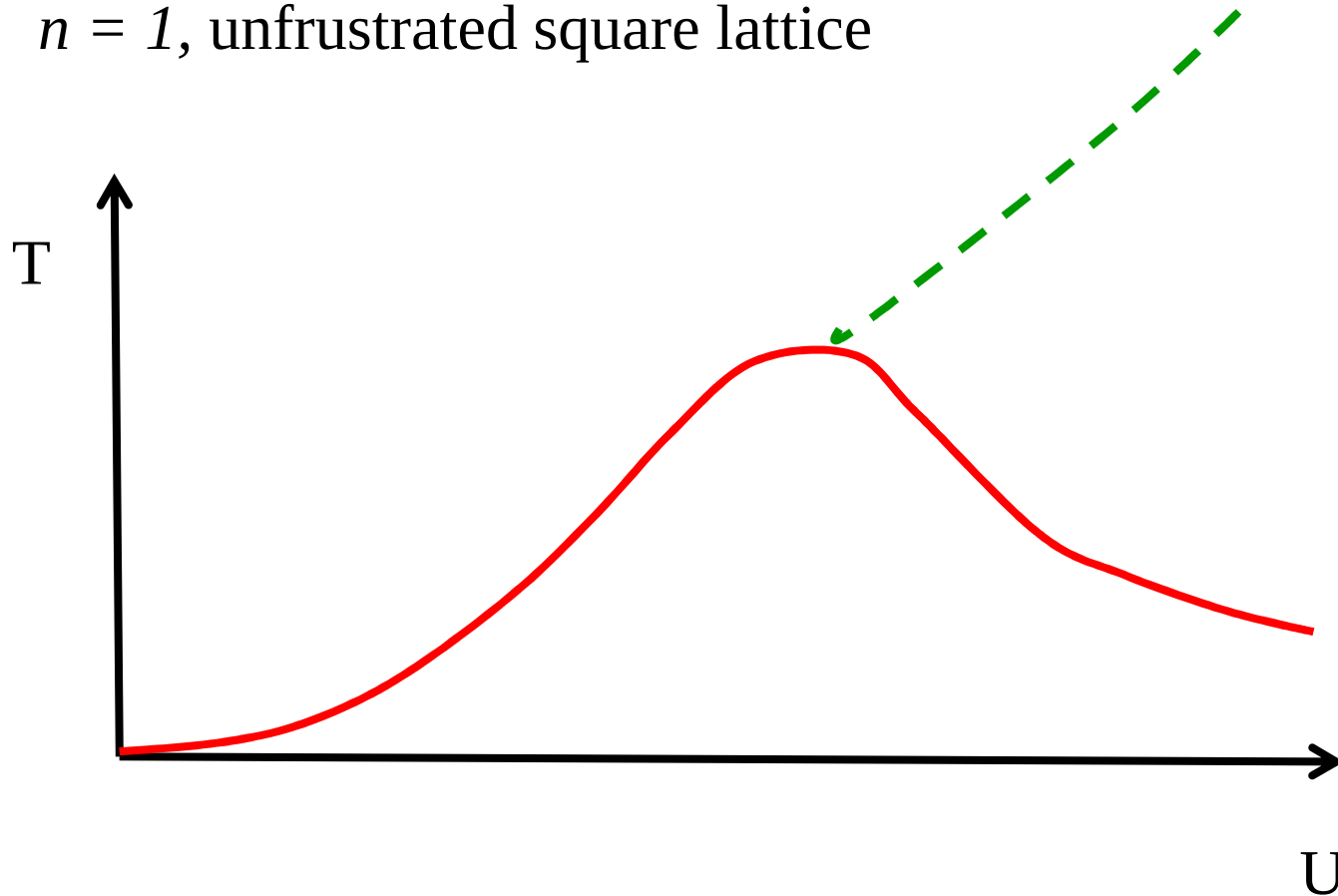
Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

- Fermi liquid
 - Start from Fermi sea
 - Self-energy analytical
 - One to one correspondence of elementary excitations
 - Landau parameters
 - Long-wavelength collective modes can become unstable
- Mott insulator
 - Hubbard model
 - Atomic limit
 - Self-energy singular
 - DMFT
 - How many sites in the cluster determines how low in temperature your description of the normal state is valid.
 - Long-wavelength collective modes can become unstable



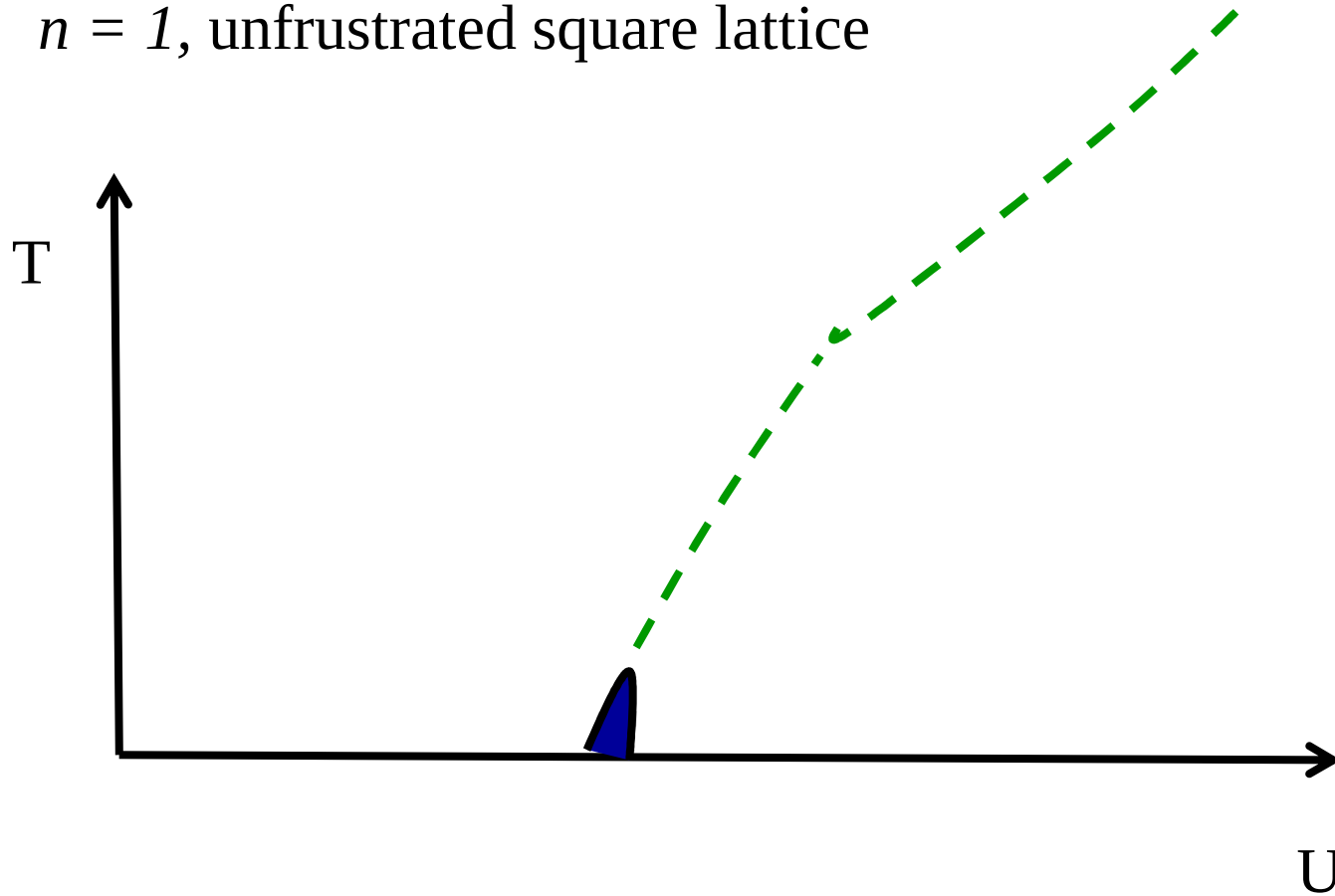
Local moment and Mott transition

$n = 1$, unfrustrated square lattice

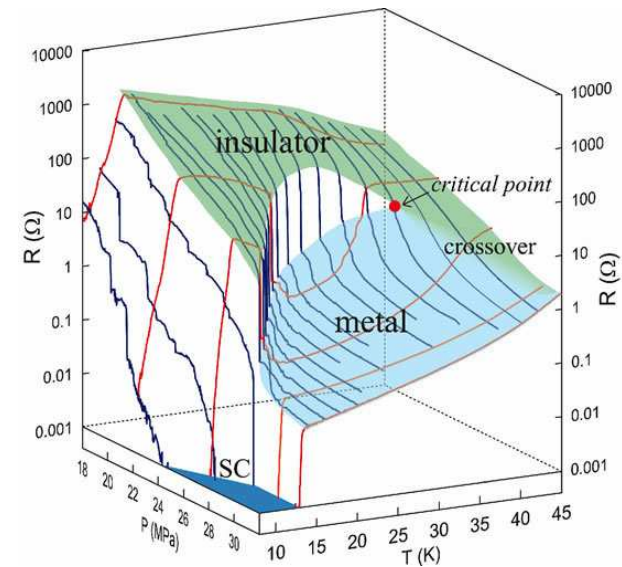
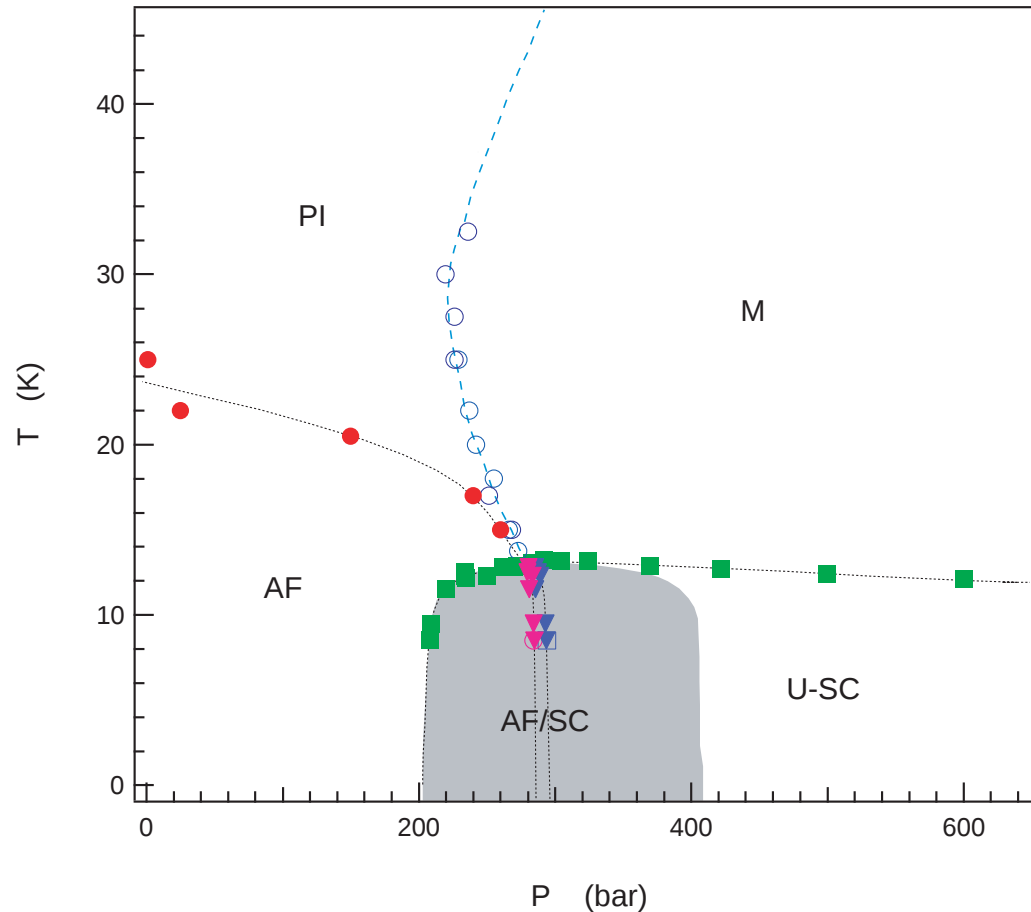


Local moment and Mott transition

$n = 1$, unfrustrated square lattice



Bare Mott critical point in organics



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

Phase diagram ($X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

Finite T phase diagram

Normal state of the cuprates





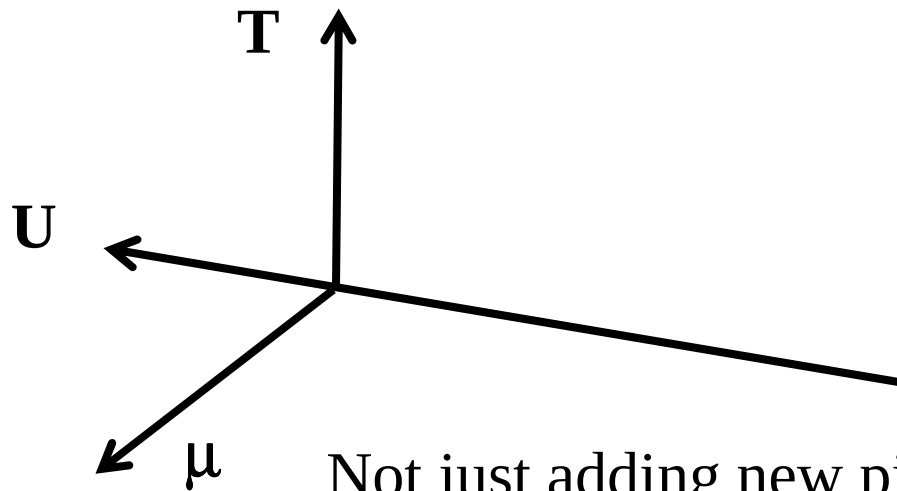
Giovanni Sordi

G. Sordi, K. Haule, A.-M.S.T
PRL, **104**, 226402 (2010)

and

Phys. Rev. B. **84**, 075161 (2011)

Doping-induced Mott transition ($t'=0$)



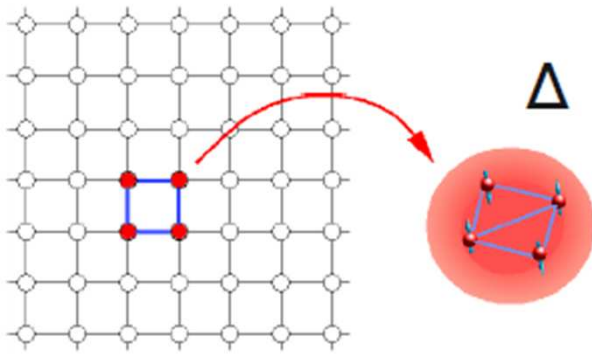
Kristjan Haule

Not just adding new piece:
Lesson from DMFT, first order transition + critical
point governs phase diagram



UNIVERSITÉ DE
SHERBROOKE

C-DMFT



Mean-field is not a trivial problem! Many impurity solvers.

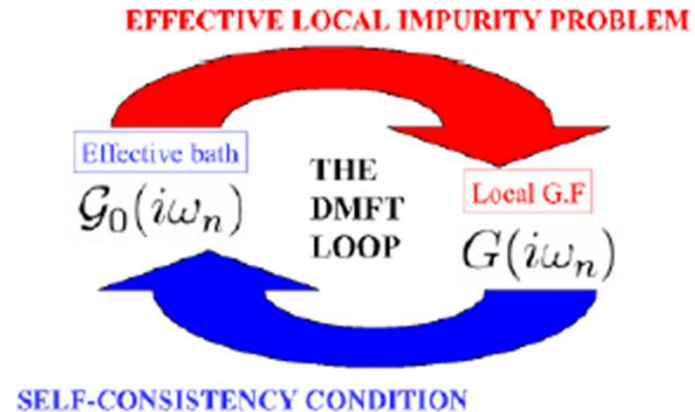
Here: continuous time QMC

P. Werner, PRL 2006

P. Werner, PRB 2007

K. Haule, PRB 2007

$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta_{\mathbf{k}}(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$- \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

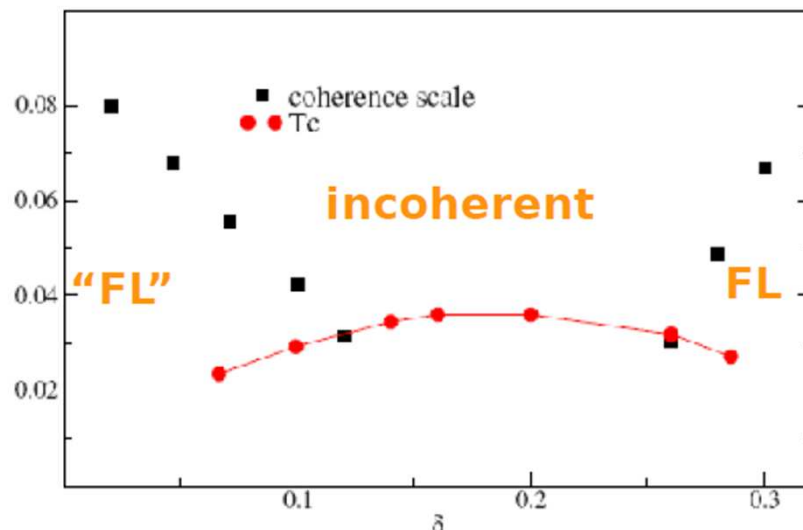
Solving cluster in a bath problem

- Continuous-time Quantum Monte Carlo calculations to sum all diagrams generated from expansion in powers of hybridization.
 - P. Werner, A. Comanac, L. de' Medici, M. Troyer, and A. J. Millis, Phys. Rev. Lett. **97**, 076405 (2006).
 - K. Haule, Phys. Rev. B **75**, 155113 (2007).

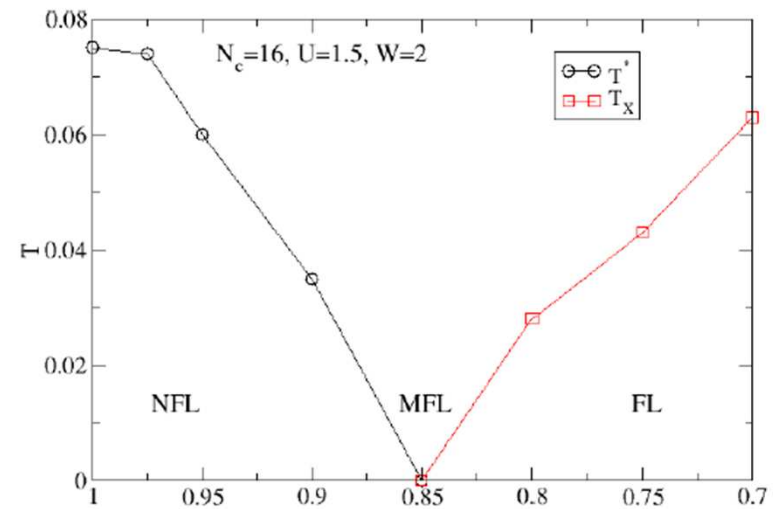


Doping driven Mott transition, $t' = 0$

Method	t'	Orbital selective	U	Critical point	Ref.
D+C+H 8			7		Werner et al. cond-mat (2009)
D+C+H 4					Gull et al. EPL (2008)
	-0.3		10,6		Liebsch, Merino... (2008)
					Ferrero et al. PRB (2009)
D+C+H 8			7		Gull, et al. PRB (2009)

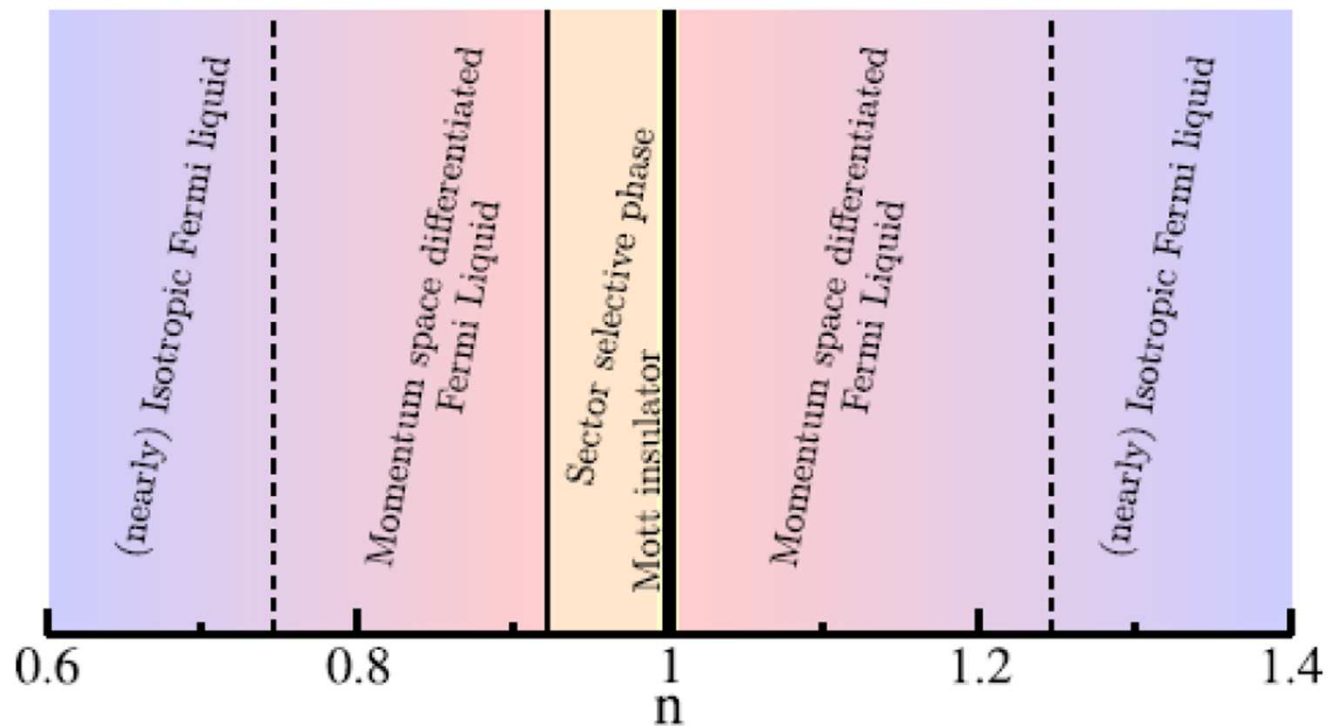


K. Haule, G. Kotliar, PRB (2008)



Vildhyadhiraja, PRL (2009)

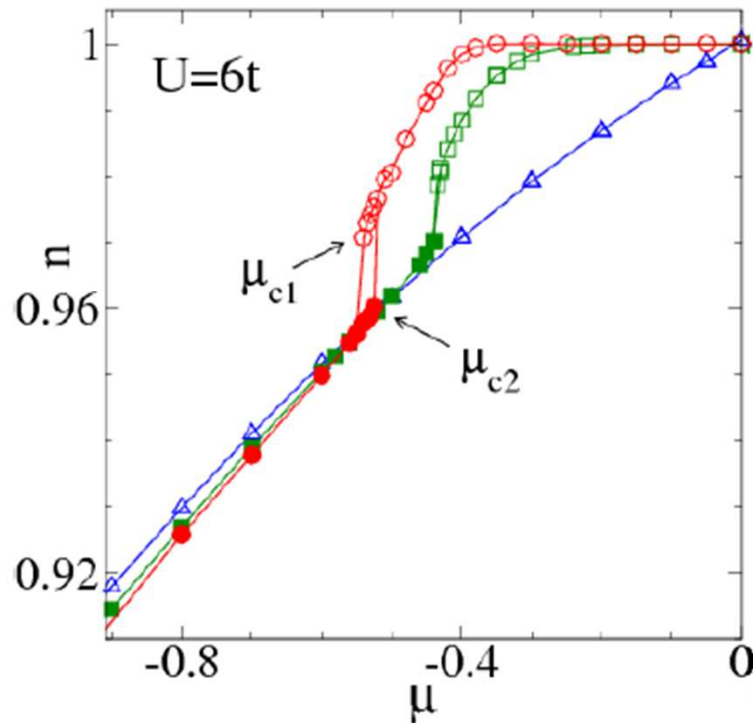
Doping driven Mott transition



Gull, Werner, Millis, (2009)

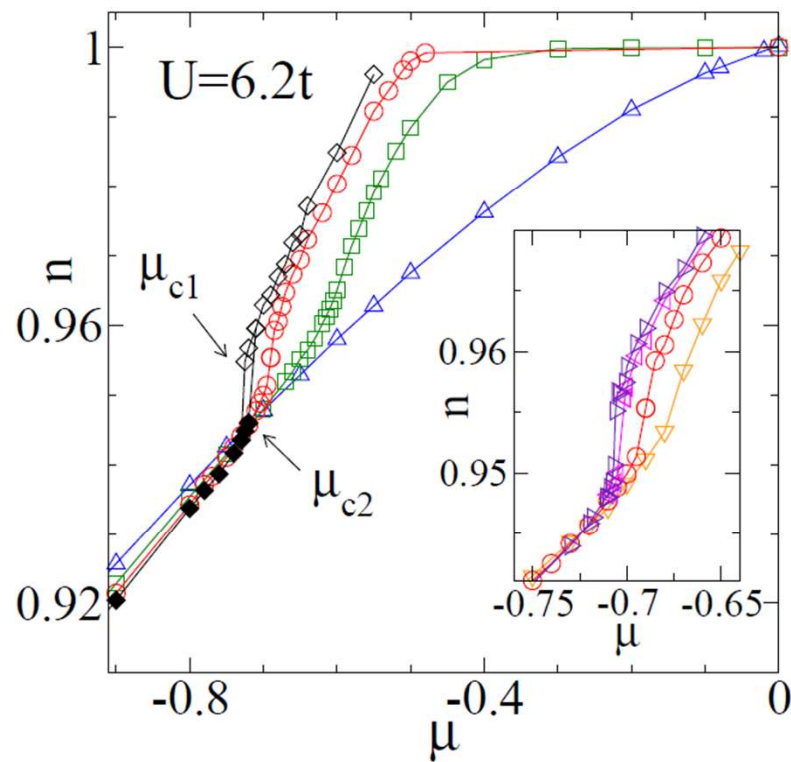


First order transition at finite doping

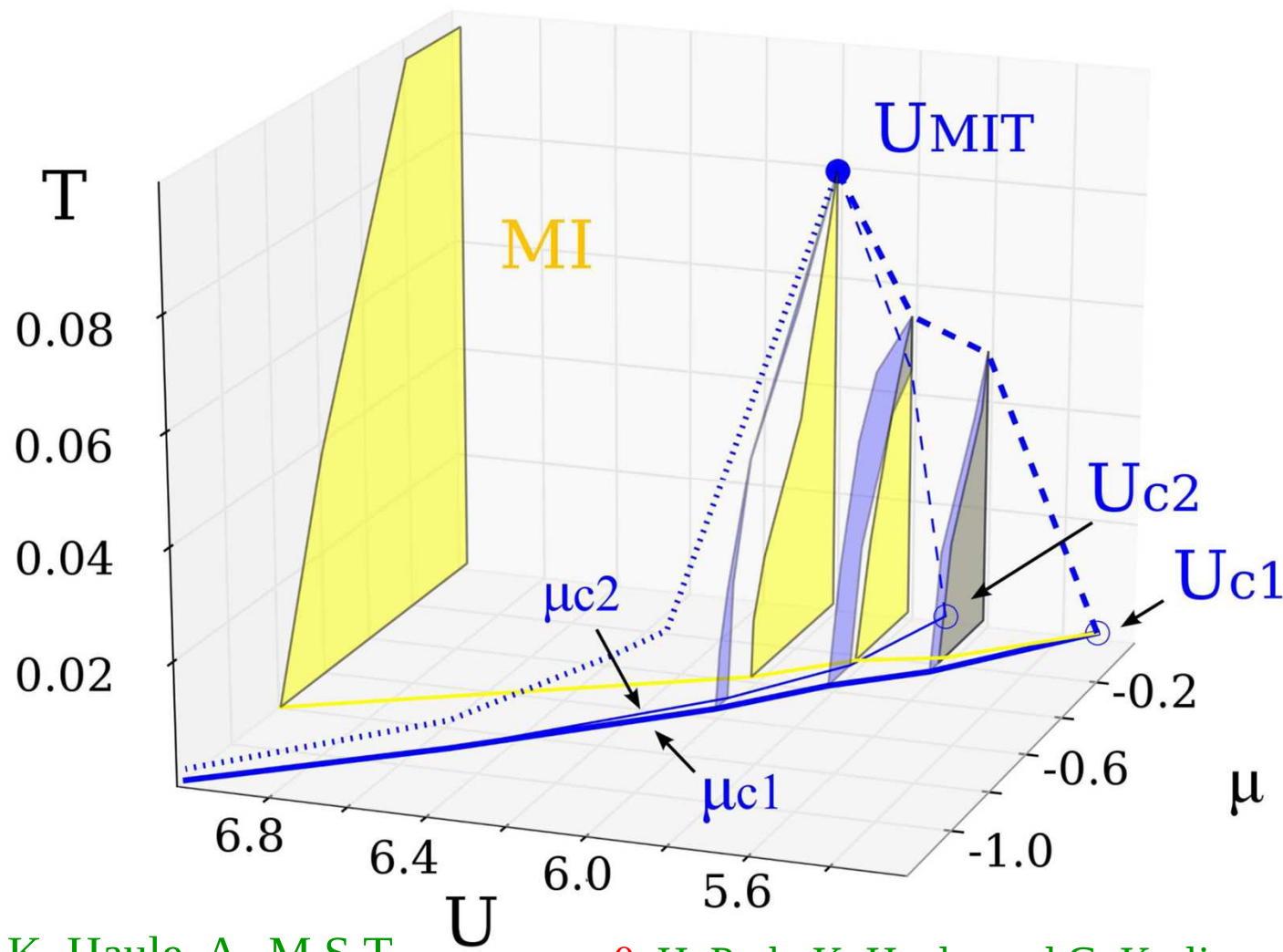


$n(\mu)$ for several temperatures:
 $T/t = 1/10$, $1/25$, $1/50$

The critical point



Normal state phase diagram



G. Sordi, K. Haule, A.-M.S.T
PRL, **104**, 226402 (2010)

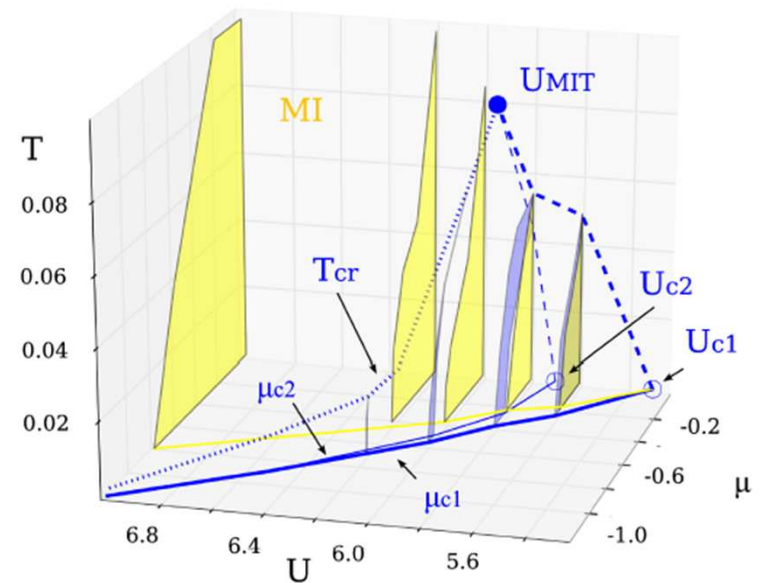
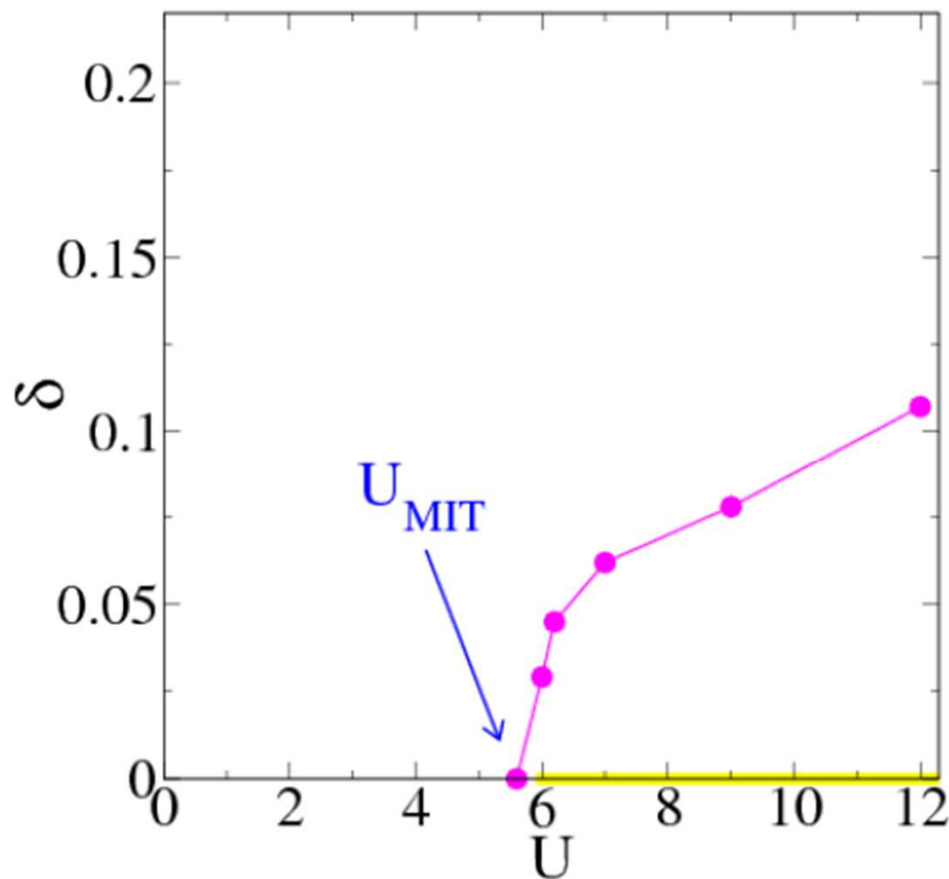
$\mu = 0$, H. Park, K. Haule, and G. Kotliar,
Phys. Rev. Lett. 101, 186403 (2008).



UNIVERSITÉ DE
SHERBROOKE

Link to Mott transition up to optimal doping

Doping dependence of critical point as a function of U



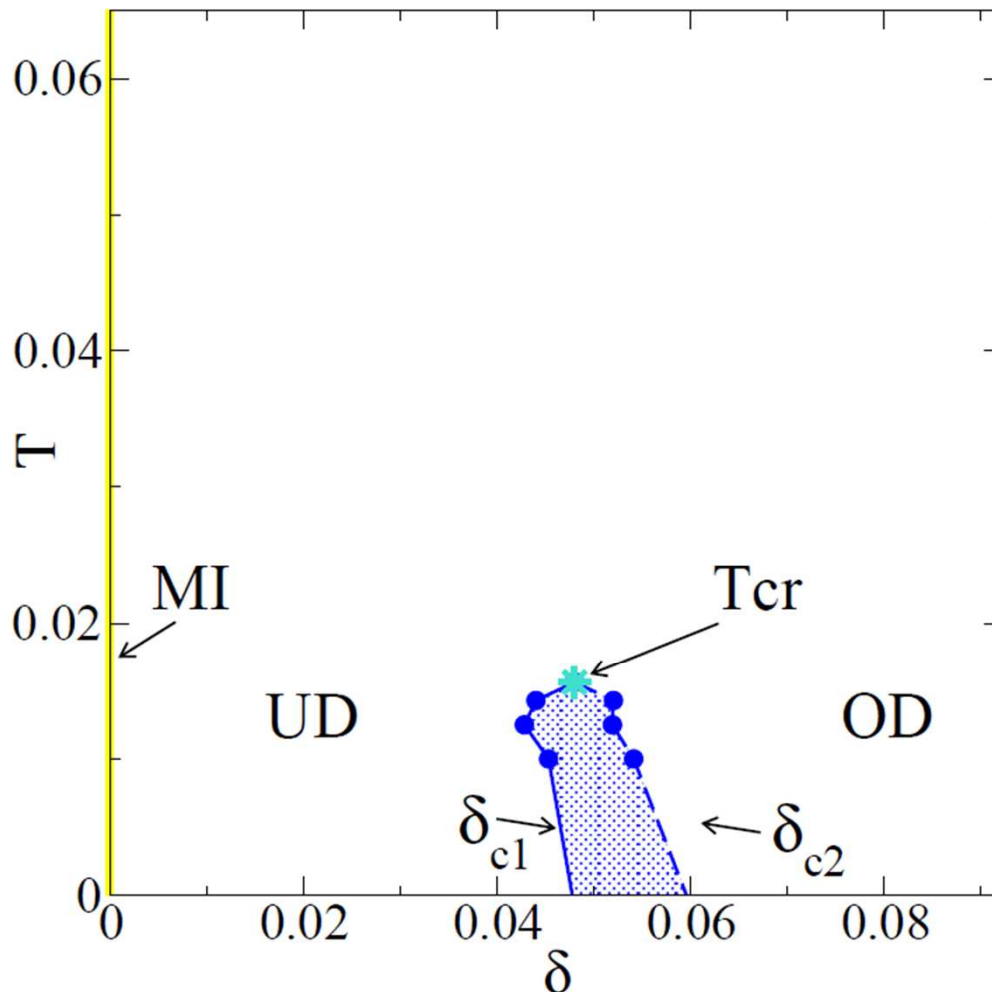
Characterisation of the phases ($U=6.2t$)

$U > U_{\text{MIT}}$:

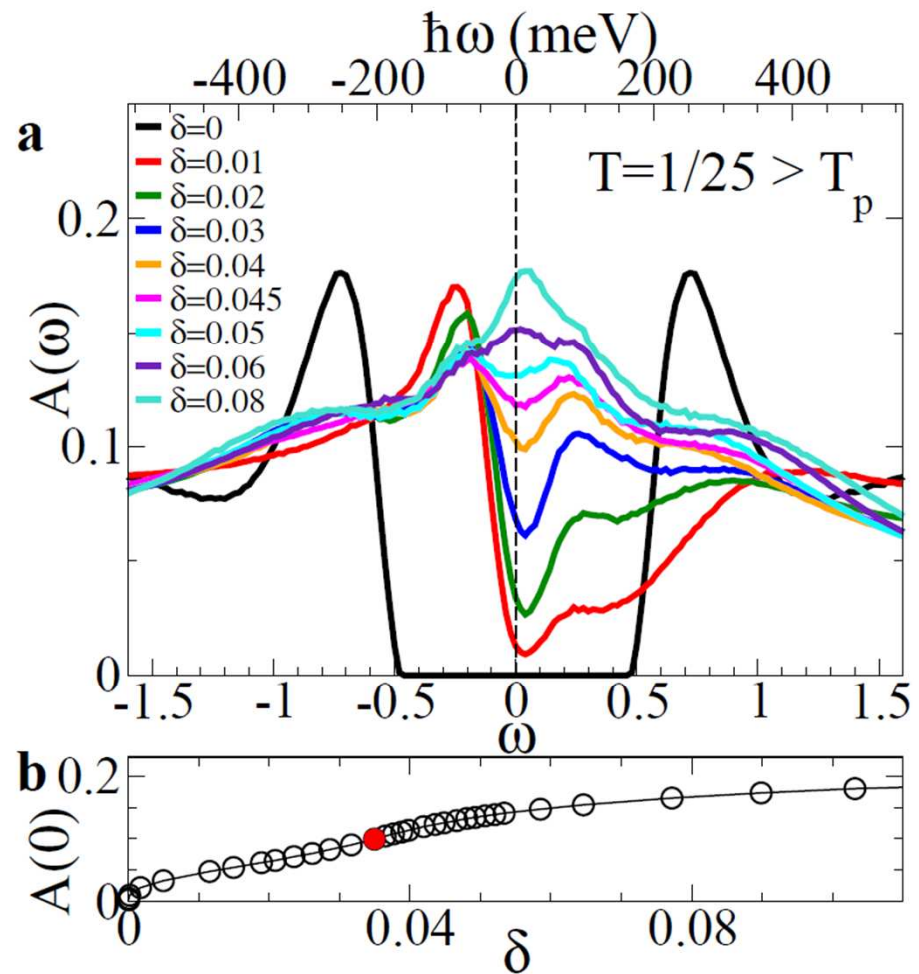
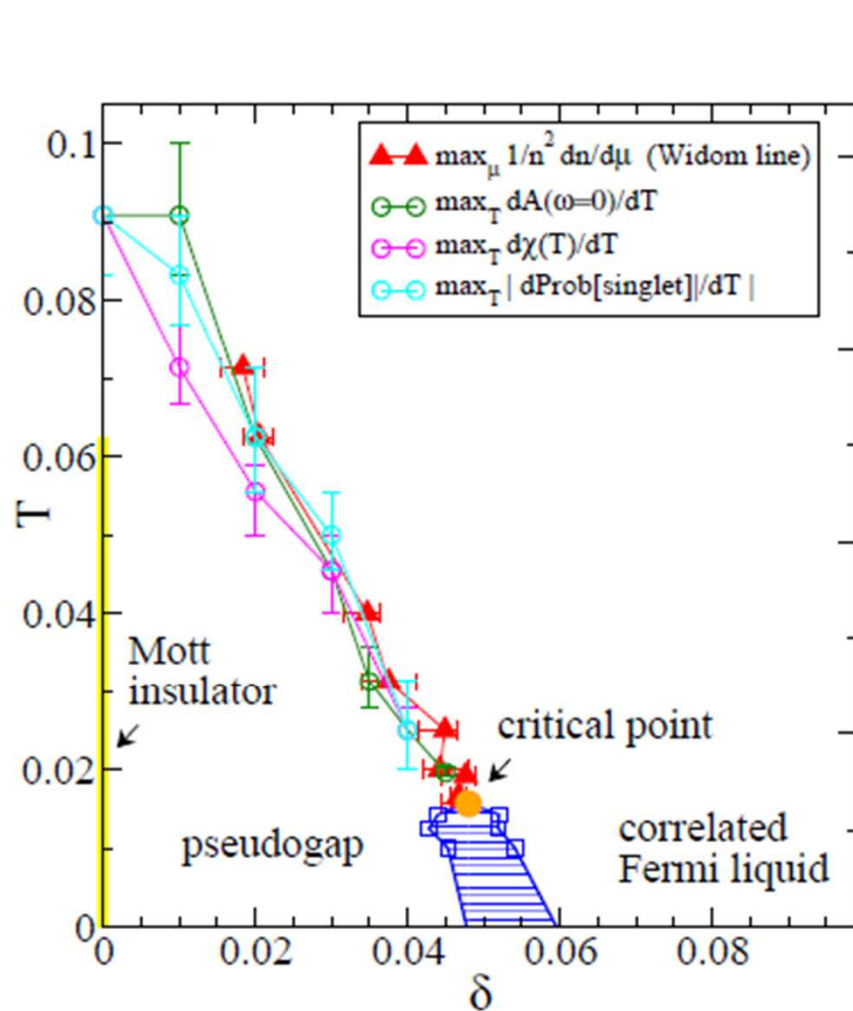
1. Mott insulator (MI)
2. Underdoped phase (UD):
 $\delta < \delta_c$
3. Overdoped phase (OD):
 $\delta > \delta_c$
4. Coexistence/forbidden region

Here “optimal doping” δ_c =
doping at which the 1st order
transition occurs

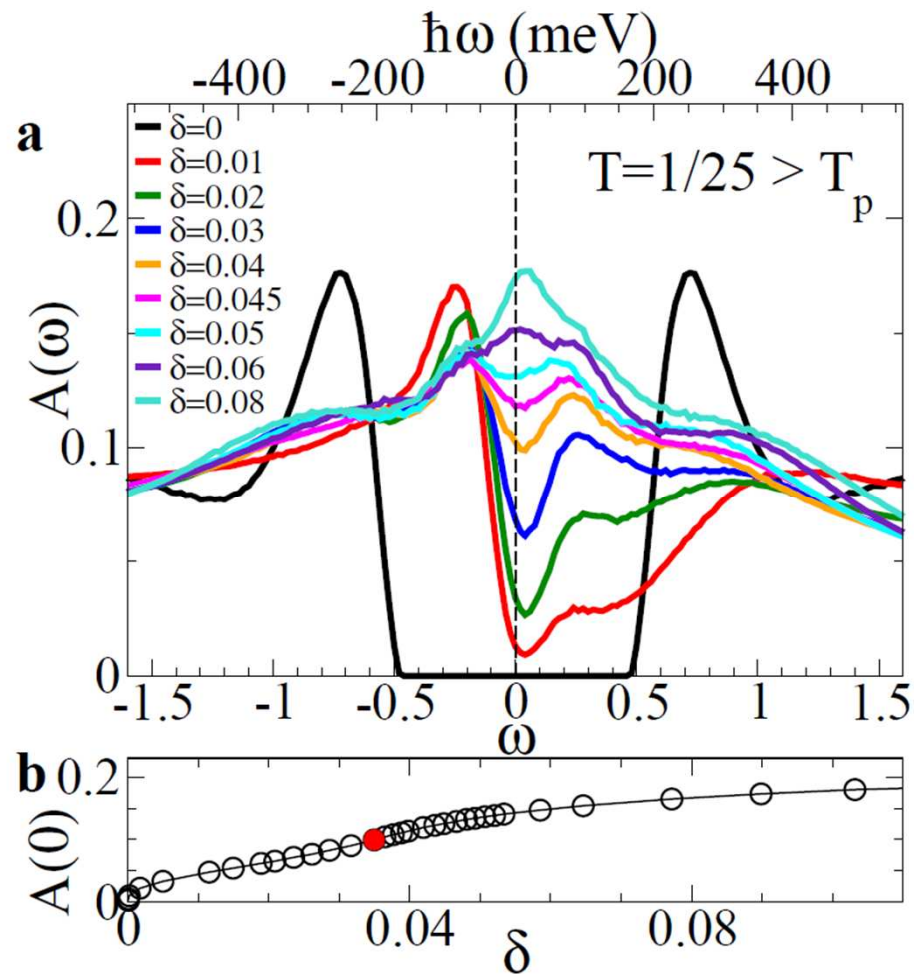
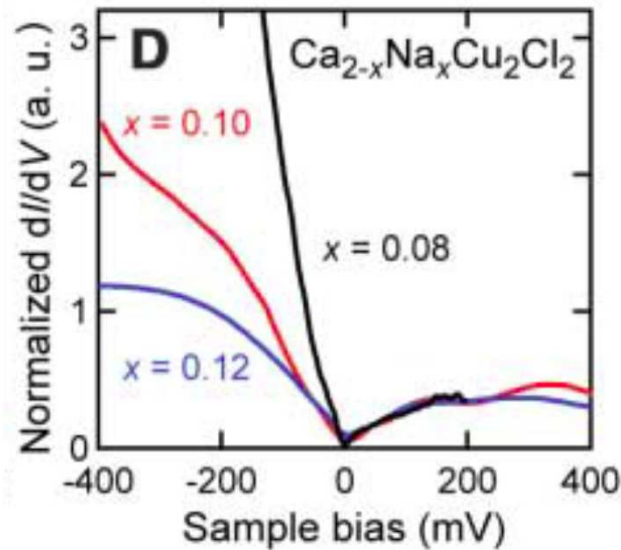
How does the UD phase differ
from the OD phase?



Density of states

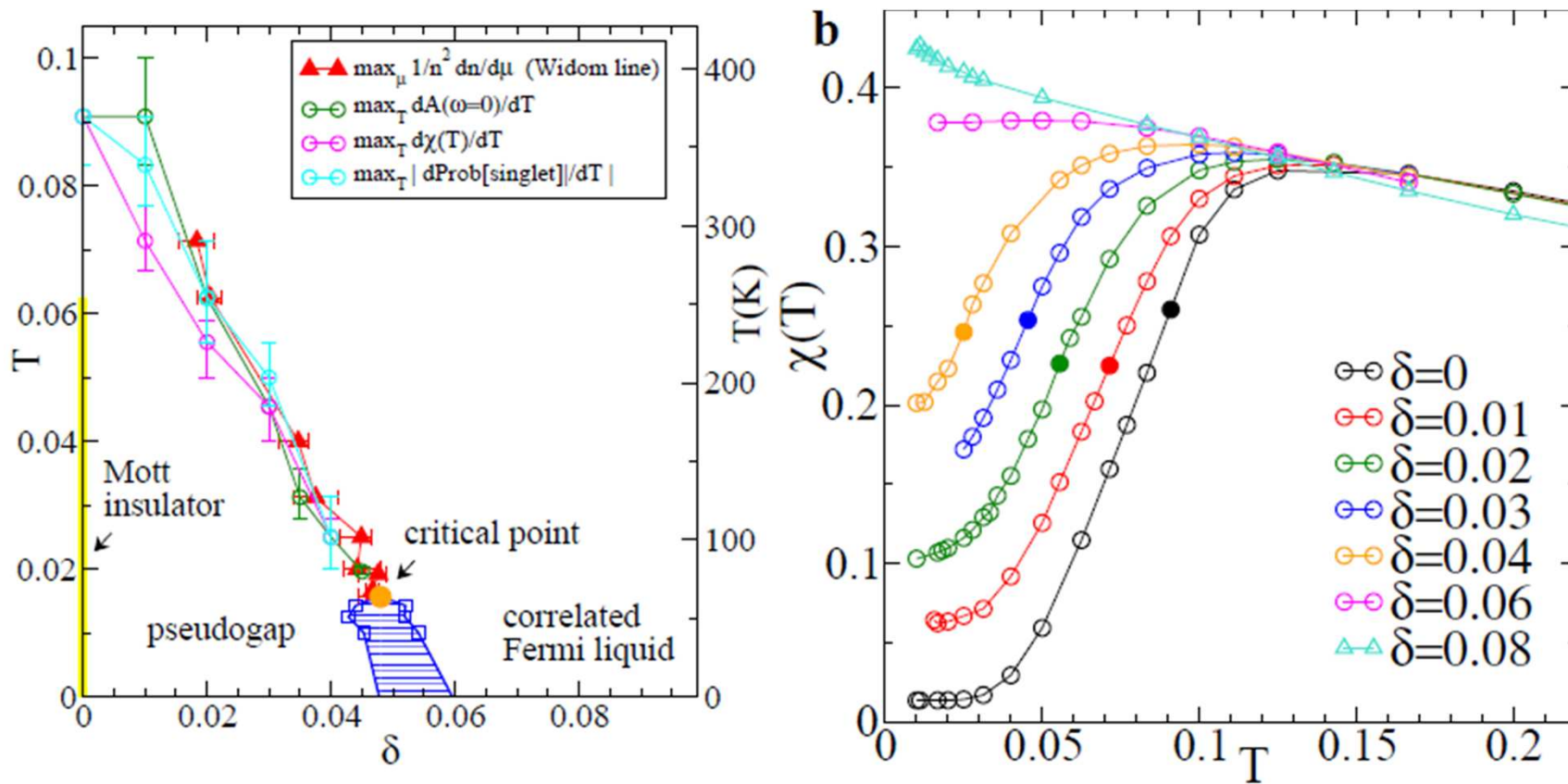


Density of states

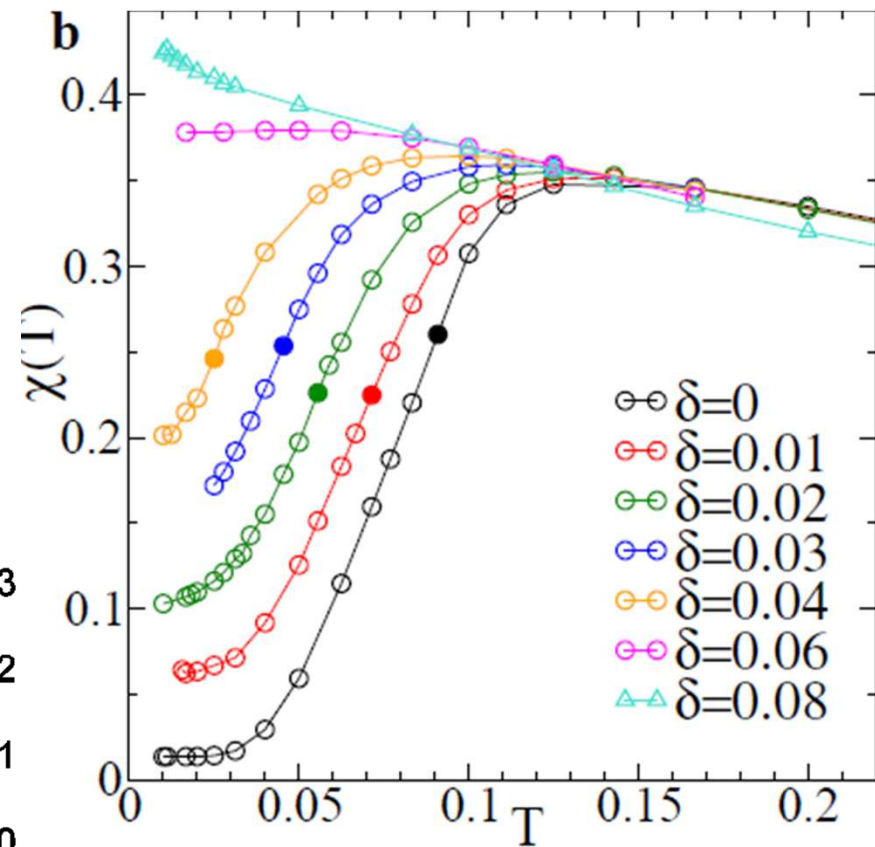
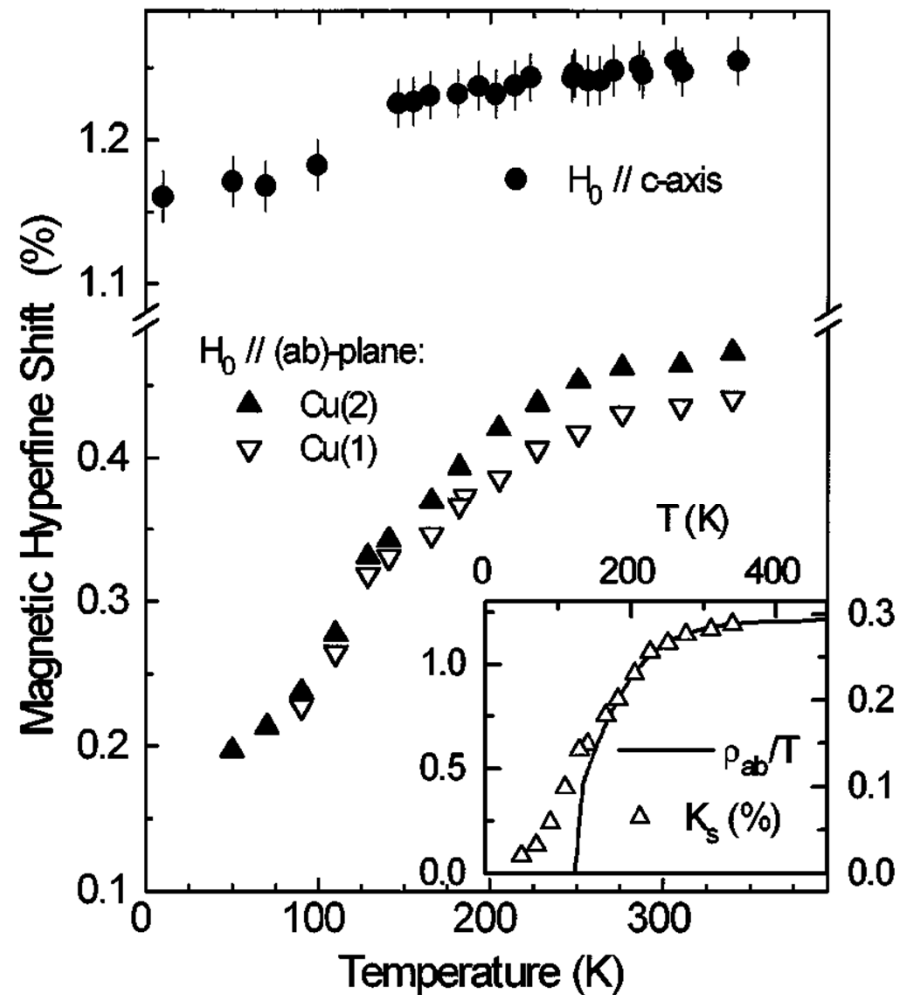


Khosaka et al. *Science* **315**, 1380 (2007);

Spin susceptibility



Spin susceptibility



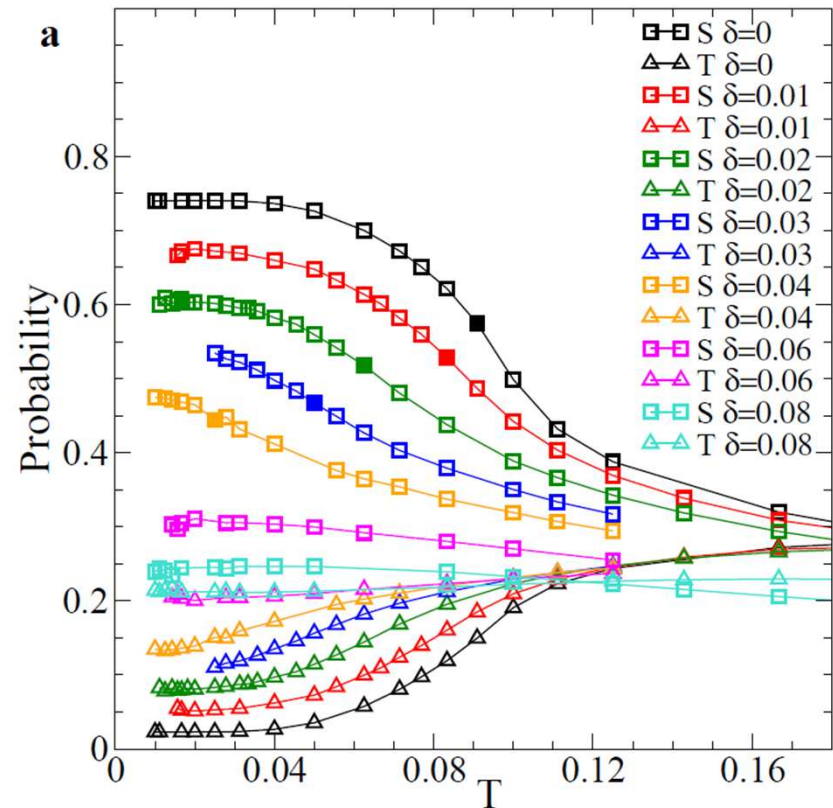
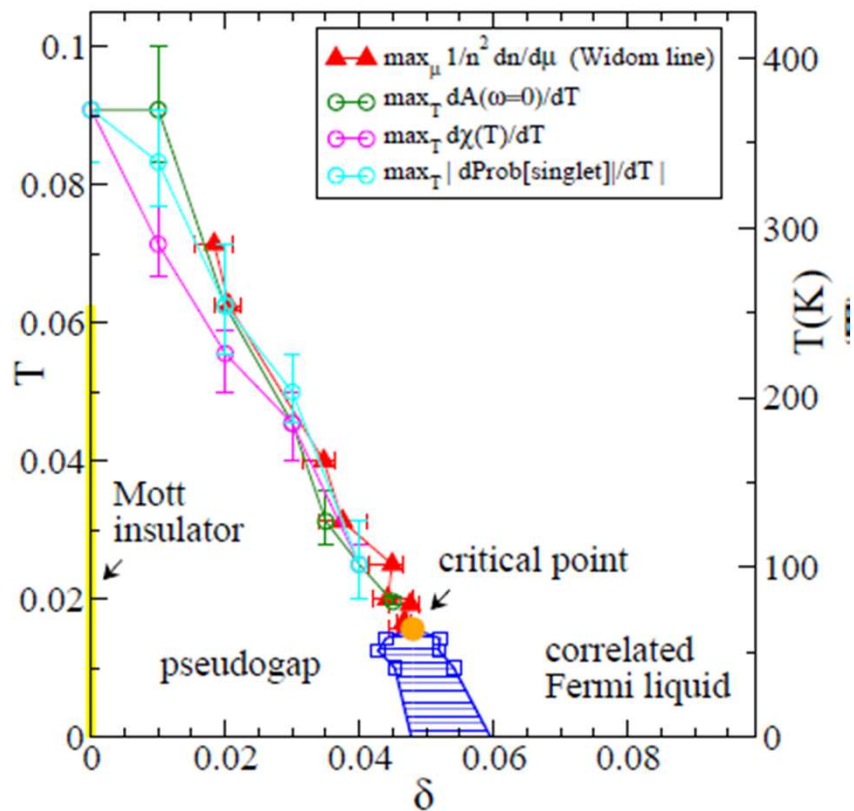
Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)

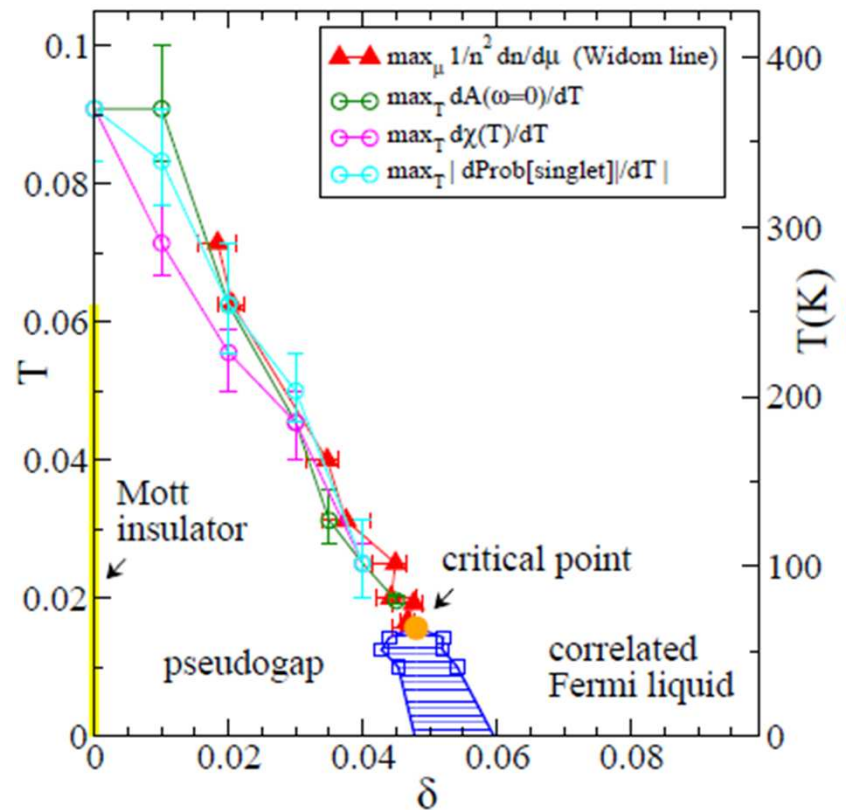


UNIVERSITÉ DE
SHERBROOKE

Plaquette eigenstates



Pseudogap T^* along the Widom line





Giovanni Sordi



Patrick Sémon



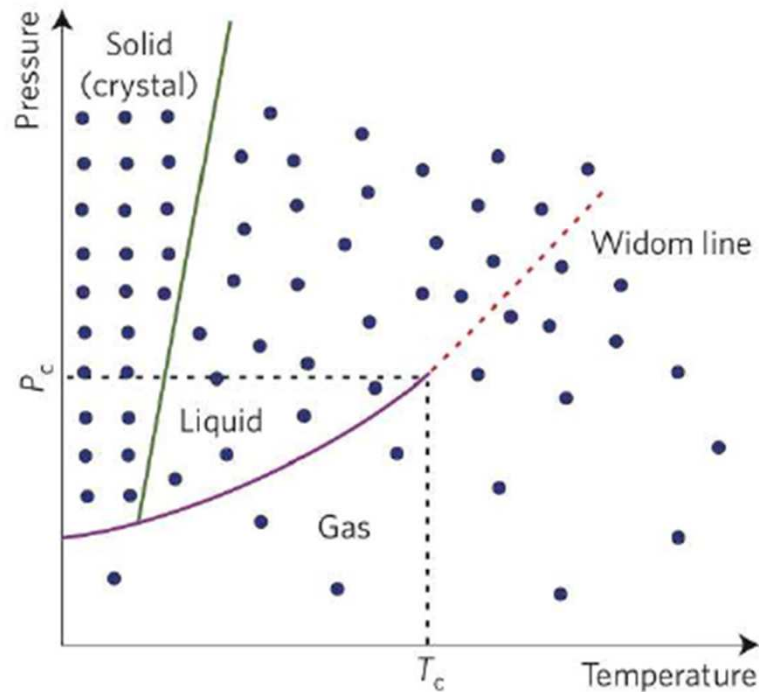
Kristjan Haul

Finite T phase diagram

Pseudogap in the normal state and the
Widom line



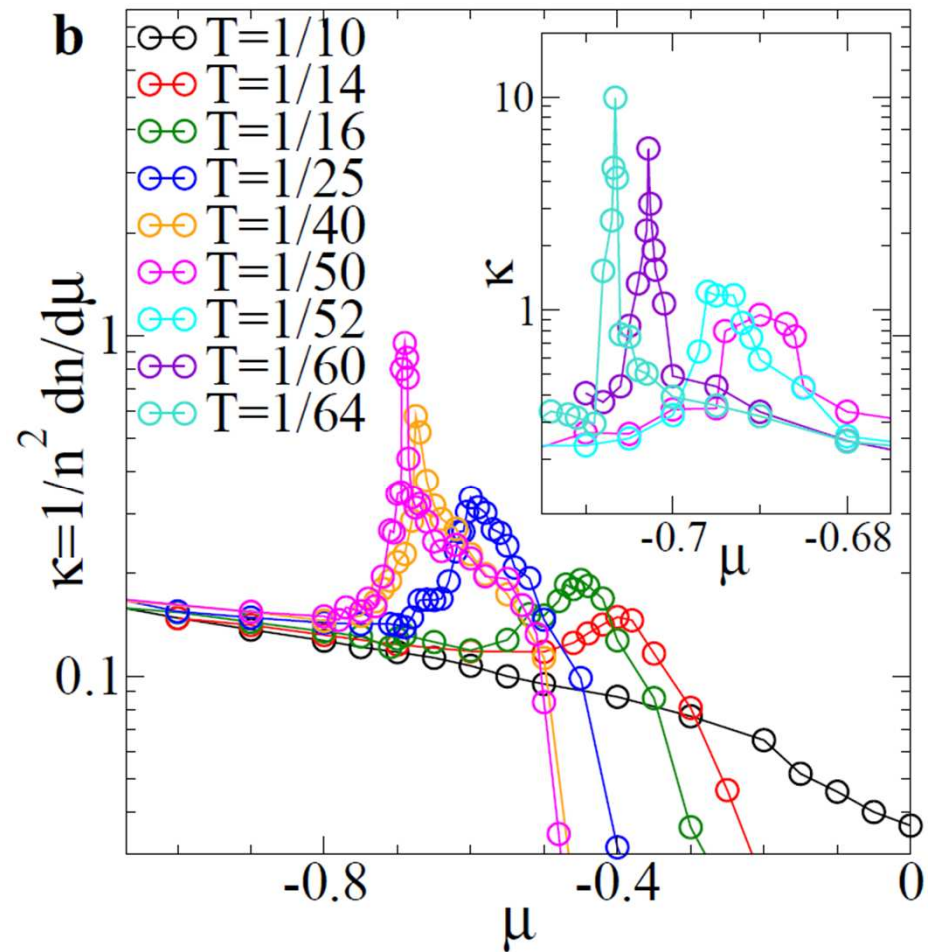
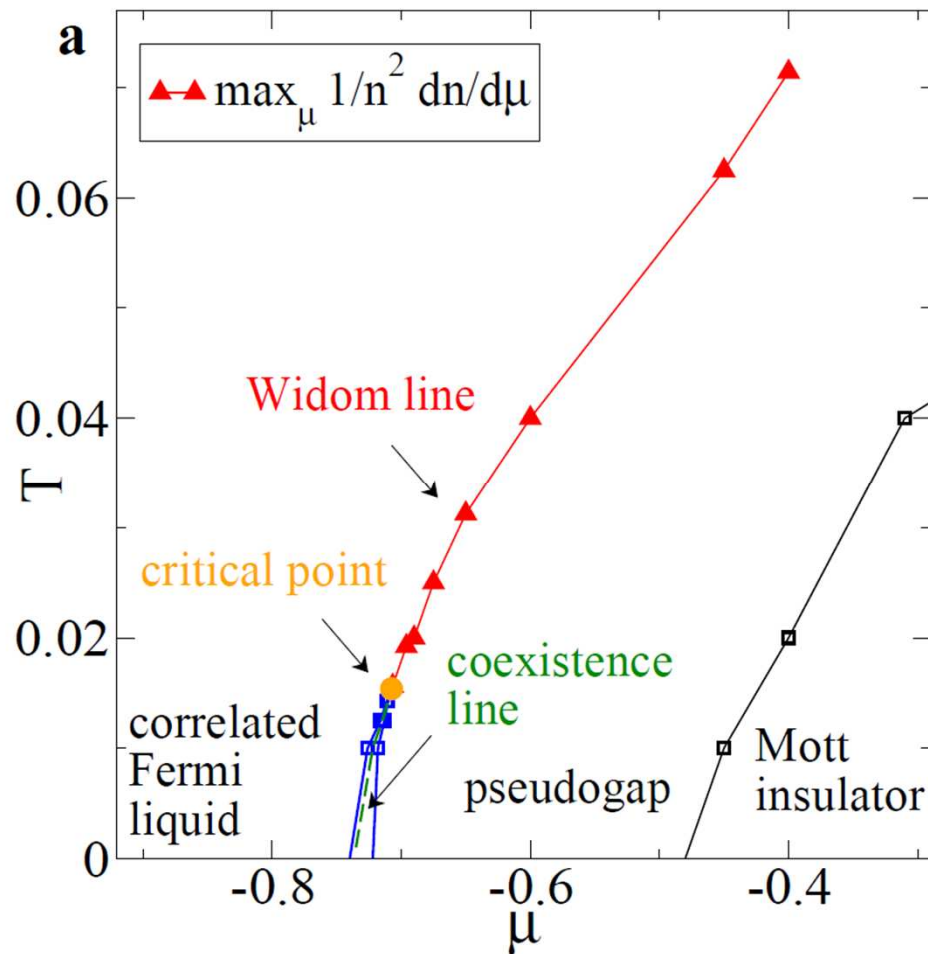
What is the Widom line?



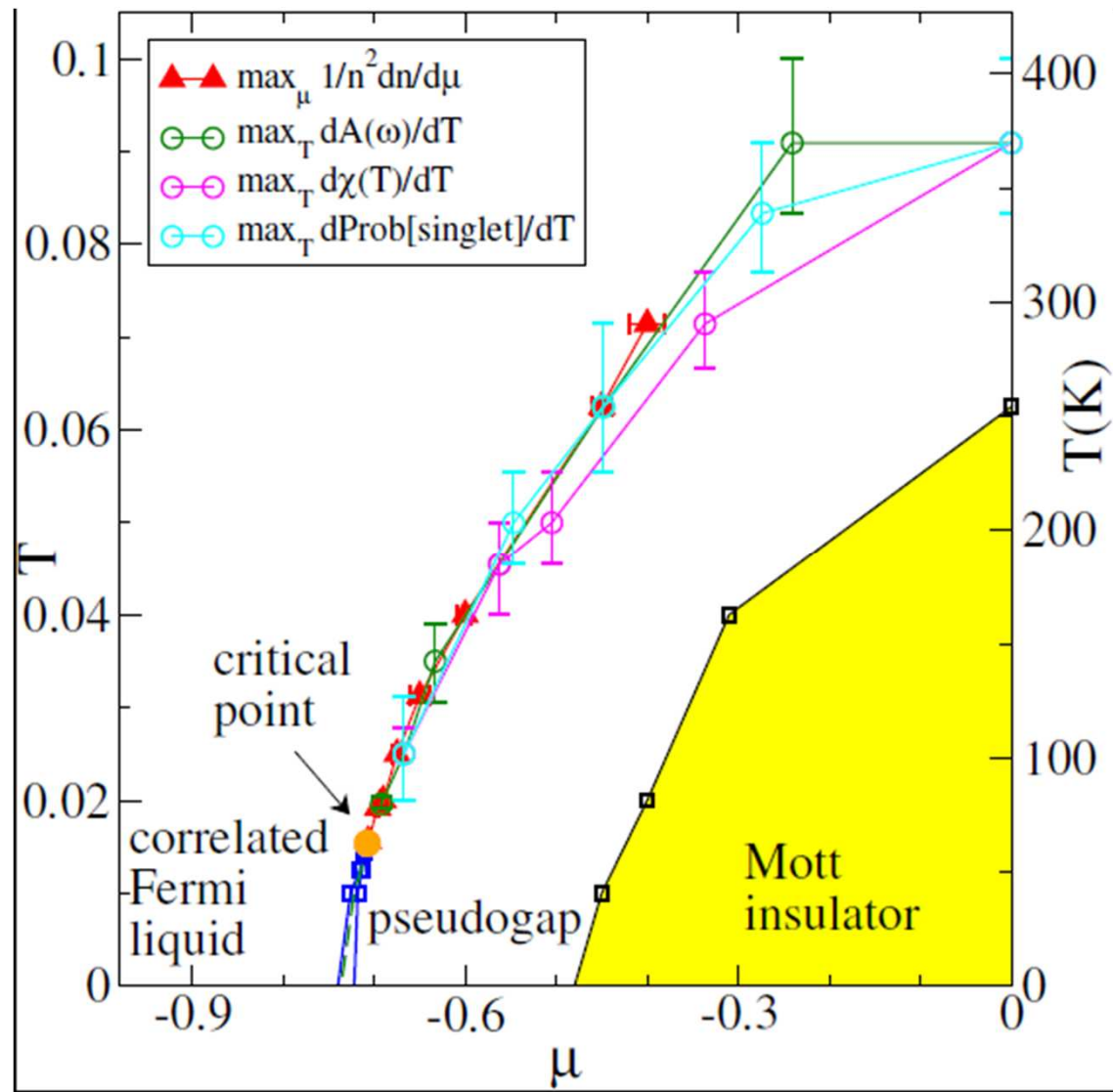
McMillan and Stanley, Nat Phys 2010

- ▶ it is the continuation of the coexistence line in the supercritical region
- ▶ line where the **maxima of different response functions** touch each other asymptotically as $T \rightarrow T_p$
- ▶ liquid-gas transition in water: max in isobaric heat capacity C_p , isothermal compressibility, isobaric heat expansion, etc
- ▶ **DYNAMIC crossover arises from crossing the Widom line!**
water: Xu et al, PNAS 2005, Simeoni et al Nat Phys 2010

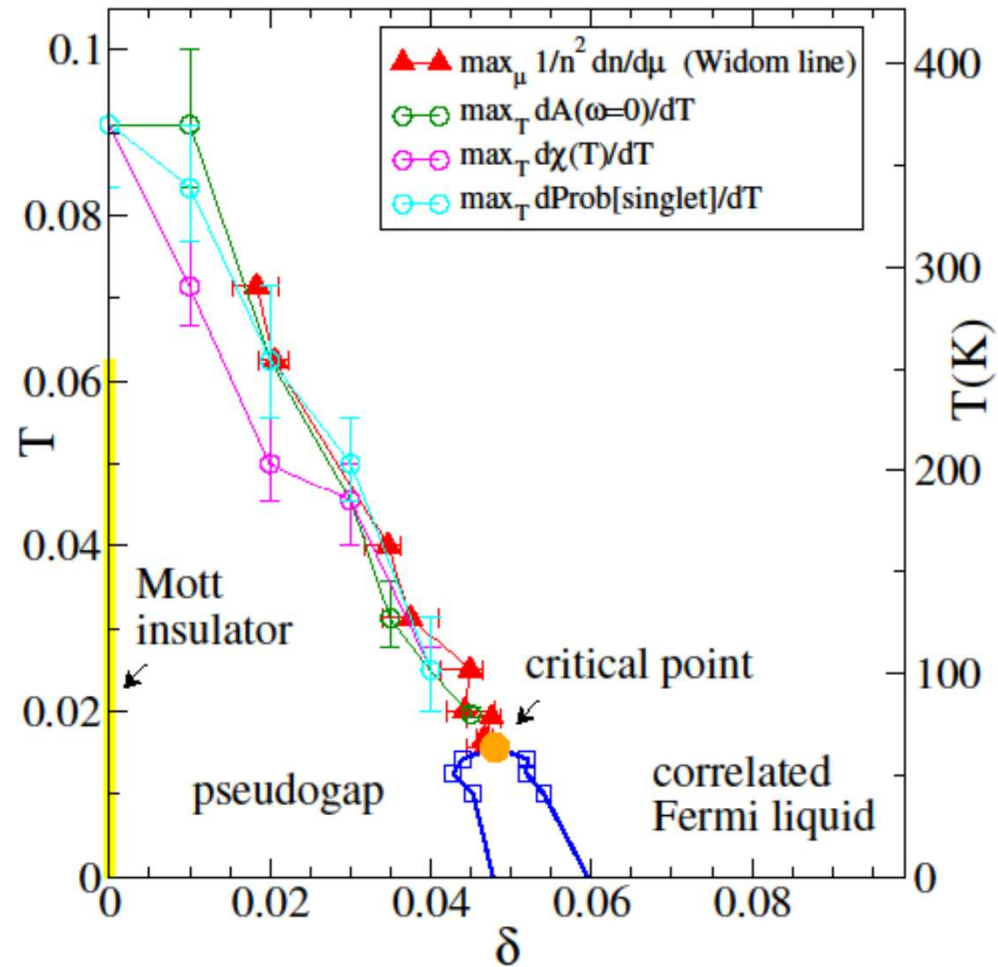
The Widom line



Rapid change also in dynamical quantities



Phase diagram





Giovanni Sordi



Patrick Sémon



Kristjan Haul

Finite T phase diagram

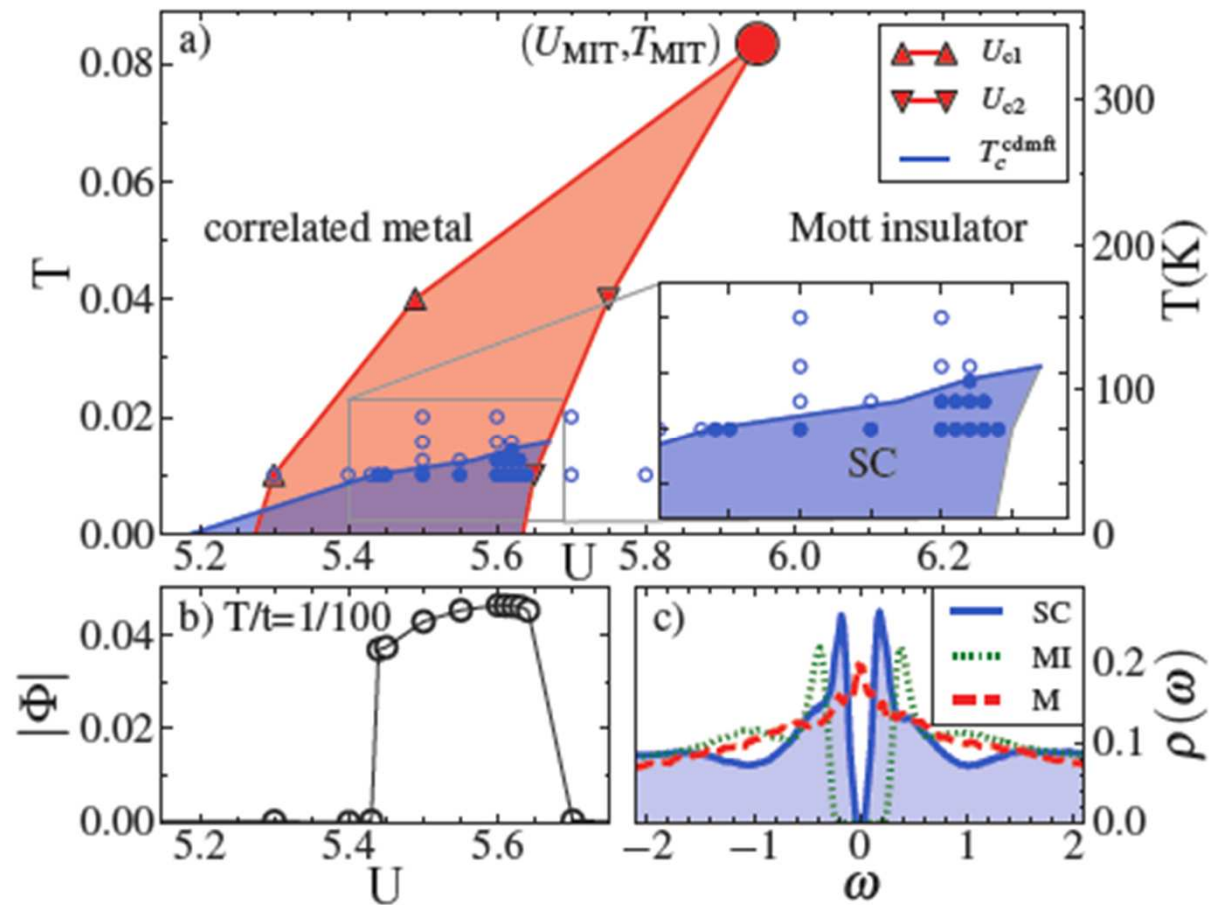
Superconductivity

arXiv:1201.1283v1

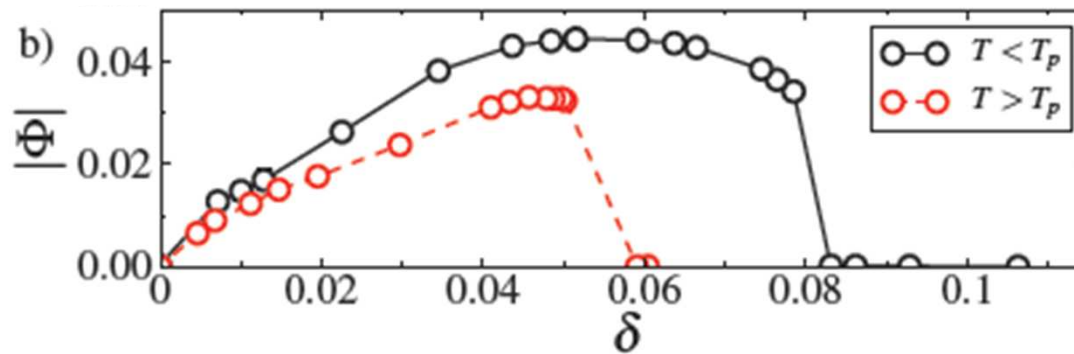


UNIVERSITÉ DE
SHERBROOKE

$n = 1$, Almost layered organics BEDT

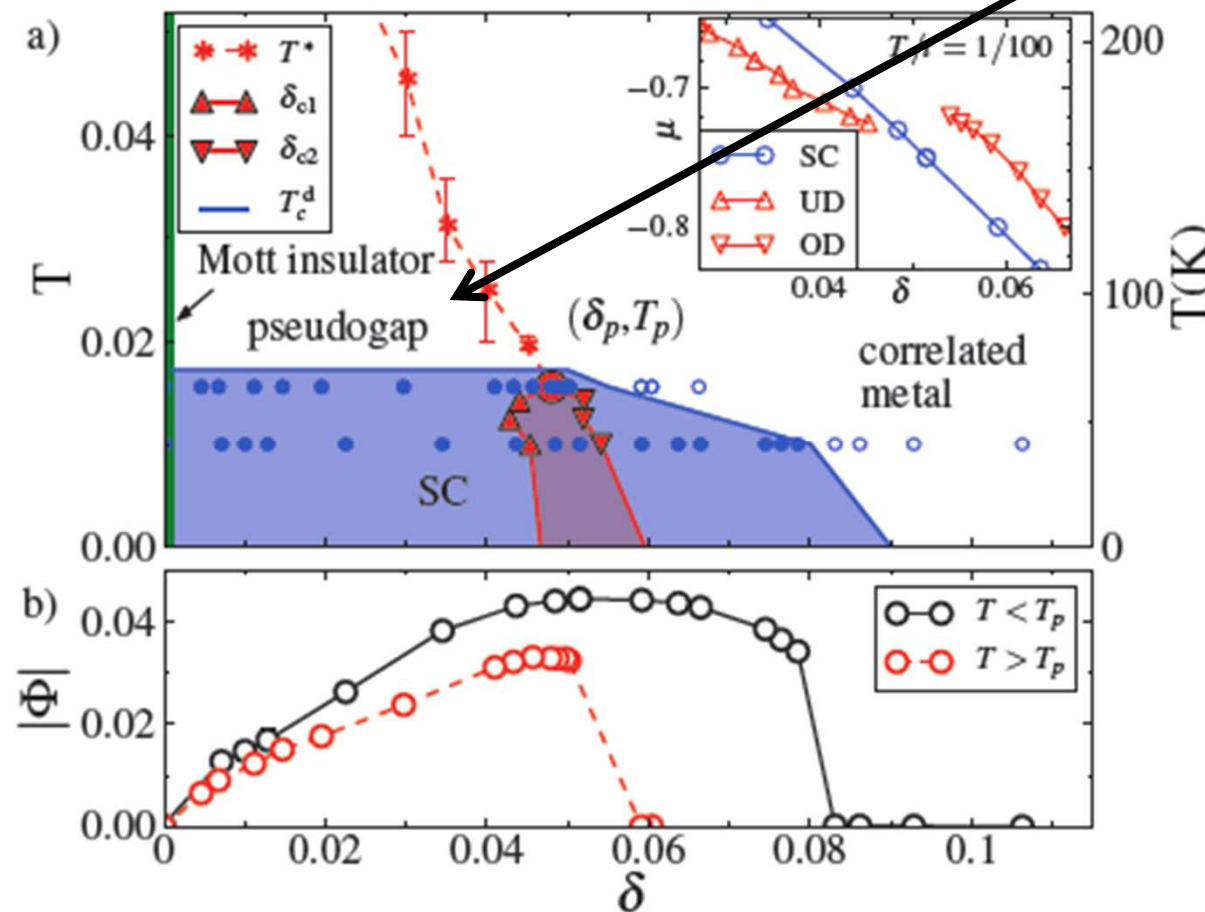


Cuprates (doping driven transition)



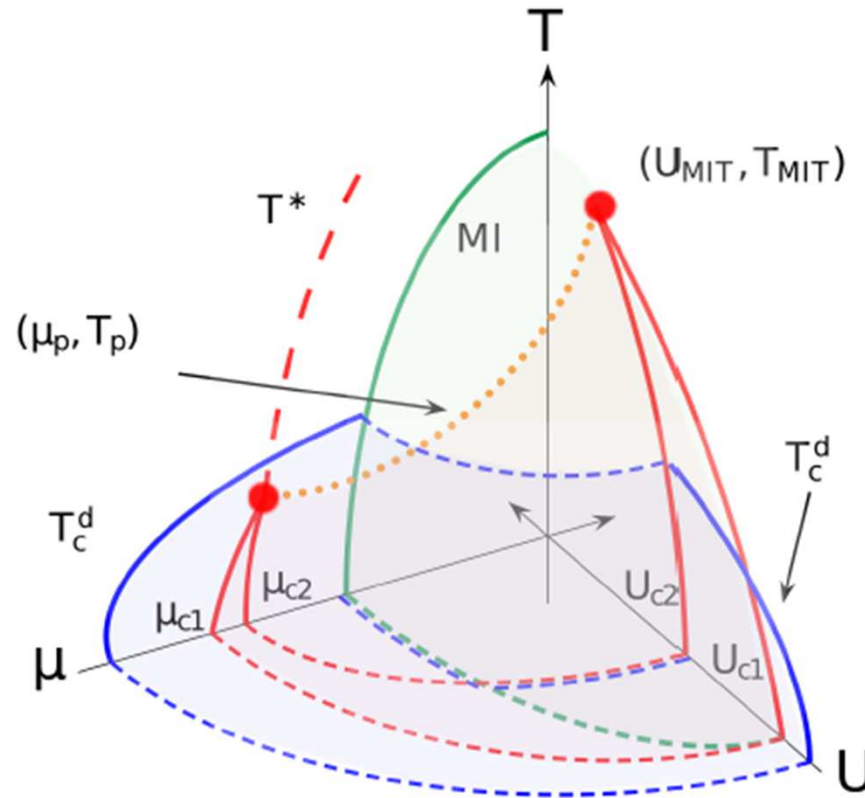
Cuprates (doping driven transition)

Pseudogap vs pair



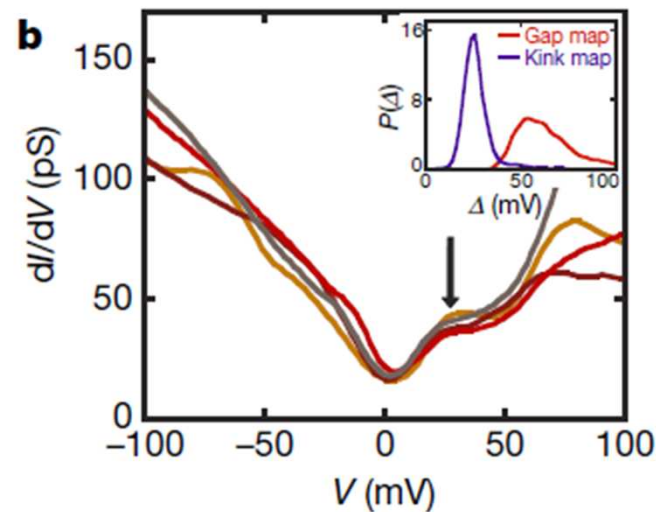
F. Rullier-Albenque, H. Alloul, and G. Rikken, Phys. Rev. B **84**, 014522 (2011).

Unified phase diagram



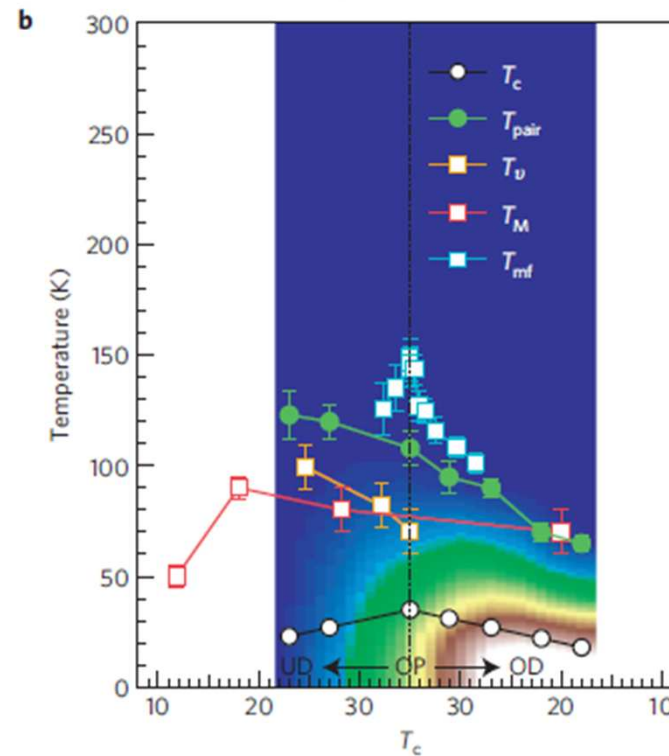
Meaning of T_c^d

- Local pair formation



K. K. Gomes, A. N. Pasupathy, A. Pushp,
S. Ono, Y. Ando, and A. Yazdani,
Nature **447**, 569 (2007)

T_{pair}



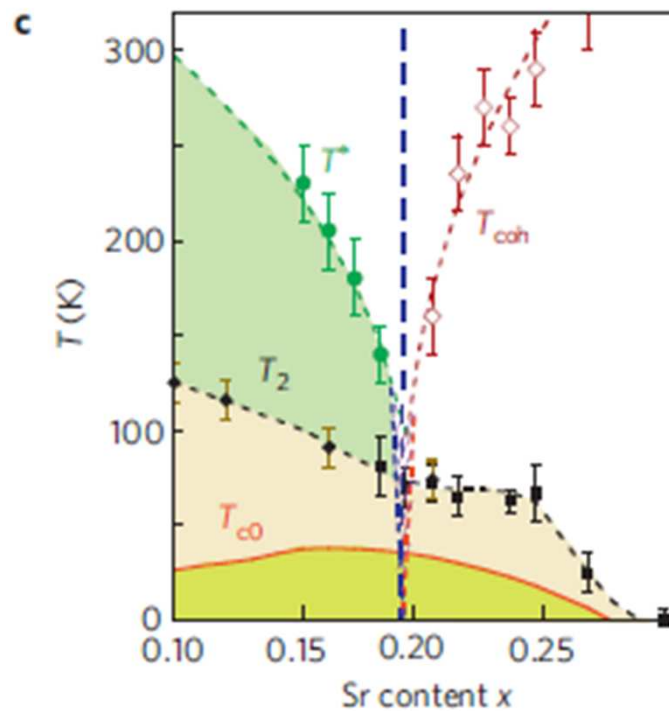
ARPES
Bi2212

Kondo, Takeshi, et al. Kaminski Nature
Physics **2011**, 7, 21-25



UNIVERSITÉ DE
SHERBROOKE

$$T_2$$



Magnetoresistance, LSCO
Fluctuating vortices

Patrick M. Rourke, et al. *Nature Physics* 7, 455–458 (2011)



Actual T_c in underdoped

- Quantum and classical phase fluctuations
 - V. J. Emery and S. A. Kivelson, Phys. Rev. Lett. **74**, 3253 (1995).
 - V. J. Emery and S. A. Kivelson, Nature **374**, 474 (1995).
 - D. Podolsky, S. Raghu, and A. Vishwanath, Phys. Rev. Lett. **99**, 117004 (2007).
 - Z. Tesanovic, Nat Phys **4**, 408 (2008).
- Magnitude fluctuations
 - I. Ussishkin, S. L. Sondhi, and D. A. Huse, Phys. Rev. Lett. **89**, 287001 (2002).
- Competing order
 - E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, Annual Review of Condensed Matter Physics **1**, 153 (2010).
- Disorder
 - F. Rullier-Albenque, H. Alloul, F. Balakirev, and C. Proust, EPL (Europhysics Letters) **81**, 37008 (2008).
 - H. Alloul, J. Bobro, M. Gabay, and P. J. Hirschfeld, Rev. Mod. Phys. **81**, 45 (2009).



Phase fluctuations and disorder?

Monolayer LSCO, field doped

A. T. Bollinger et al. & I. Božović, *Nature* **472**, 458–460

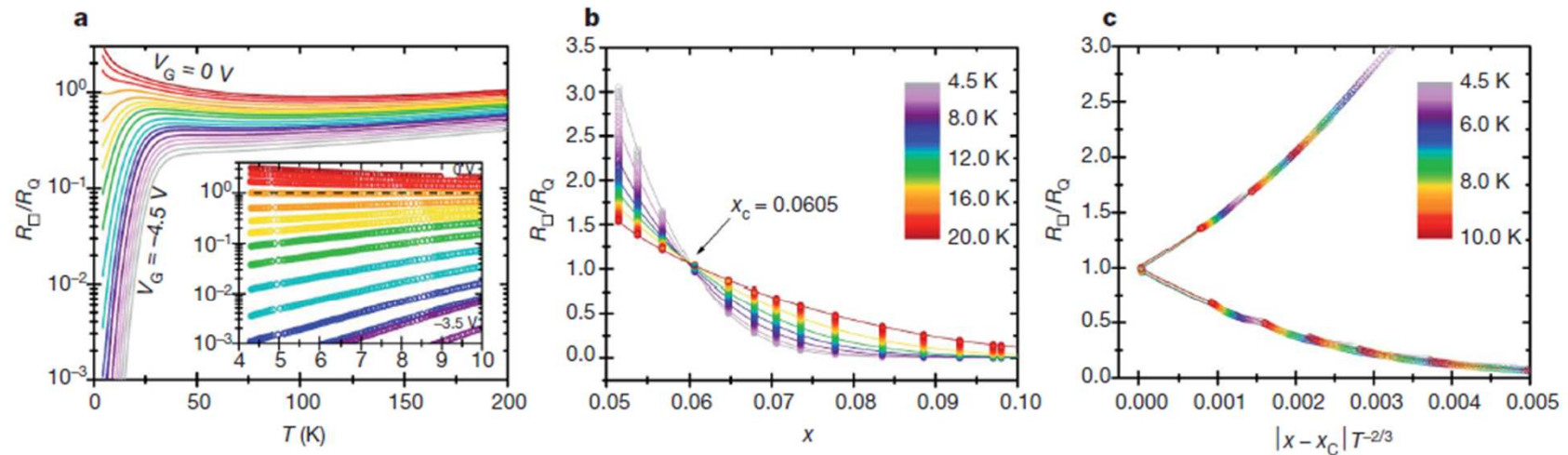
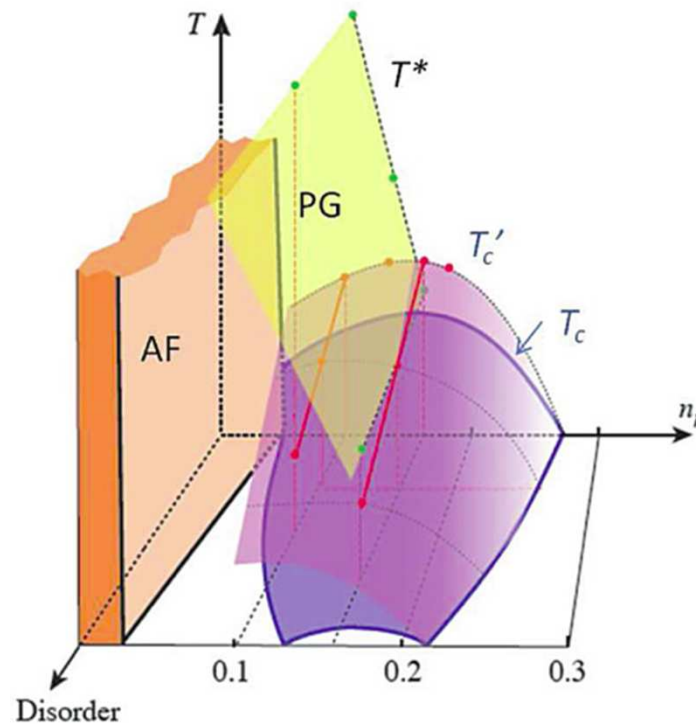


Figure 2 | Superconductor–insulator transition driven by electric field. a, Temperature dependence of normalized resistance $r = R_{\square}(x, T)/R_Q$ of an initially heavily underdoped and insulating film (see Supplementary Fig. 12 for linear scale). The device (Supplementary section B) employs a coplanar Au gate and DEME-TFSI ionic liquid. The carrier density, fixed for each curve, is tuned by varying the gate voltage from 0 V to -4.5 V in 0.25 V steps; an insulating film becomes superconducting via a QPT. The inset highlights a separatrix independent of temperature below 10 K. The open circles are the actual raw data points; the black dashed line is $R_{\square}(x_c, T) = R_Q = 6.45$ k Ω . b, The inverse representation of the same data, that is, the $r_T(x)$ dependence at fixed temperatures below 20 K. Each vertical array of (about 100) data points corresponds to one fixed carrier density, that is, to one $r_x(T)$ curve in Fig. 2a.

The colours refer to the temperature, and the continuous lines are interpolated for selected temperatures (4.5, 6.0, 8.0, 10.0, 12.0, 15.0 and 20.0 K). The crossing point defines the critical carrier concentration $x_c = 0.06 \pm 0.01$, and the critical resistance $R_c = 6.45 \pm 0.10$ k Ω . c, Scaling of the same data with respect to a single variable $u = |x - x_c| T^{-1/z\nu}$, with $z\nu = 1.5$. This figure is derived by folding panel b at x_c and scaling the abscissa of each $r_T(|x - x_c|)$ curve by $T^{-2/3}$. For 4.3 K $< T < 10$ K, the discrete groups of points of Fig. 2b collapse accurately onto a two-valued function, with one branch corresponding to x larger and the other to x smaller than x_c . The critical exponents are identical on both sides of the superconductor–insulator transition. The raw data points cover the interpolation lines almost completely, except close to the origin.

Effect of disorder

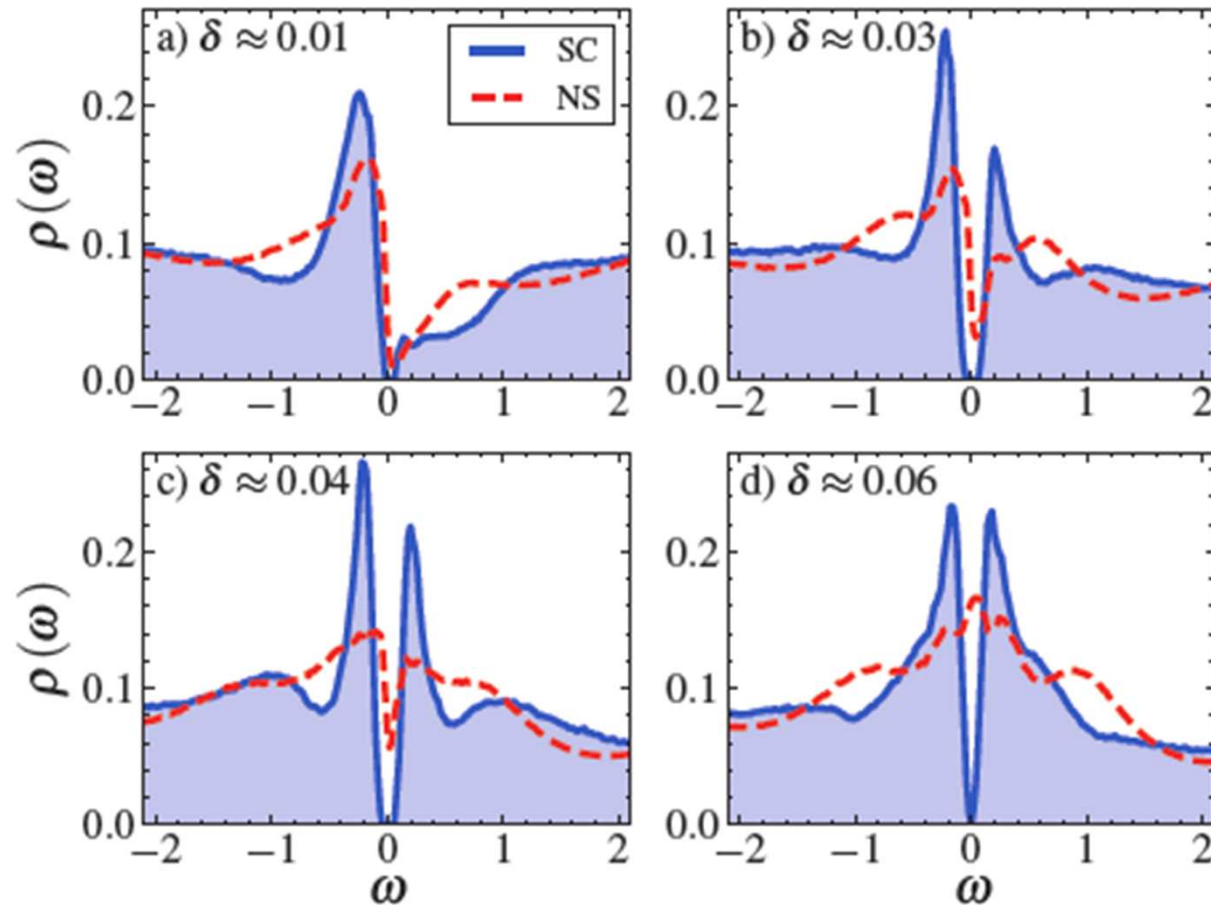


F. Rullier-Albenque, H. Alloul, and G. Rikken,
Phys. Rev. B **84**, 014522 (2011).

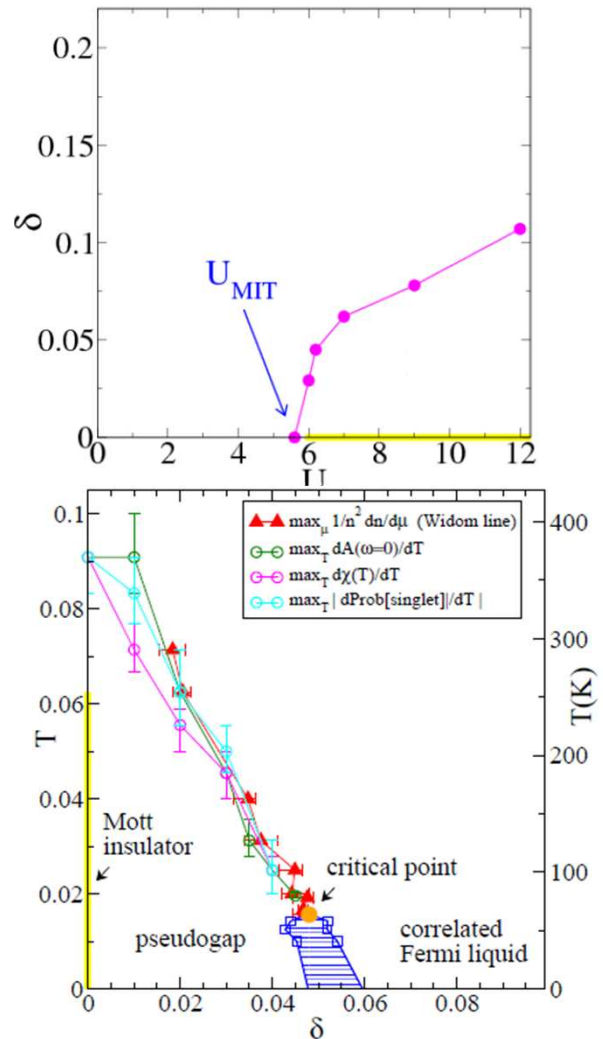
Superconductivity in underdoped vs BCS



First-order transition leaves its mark

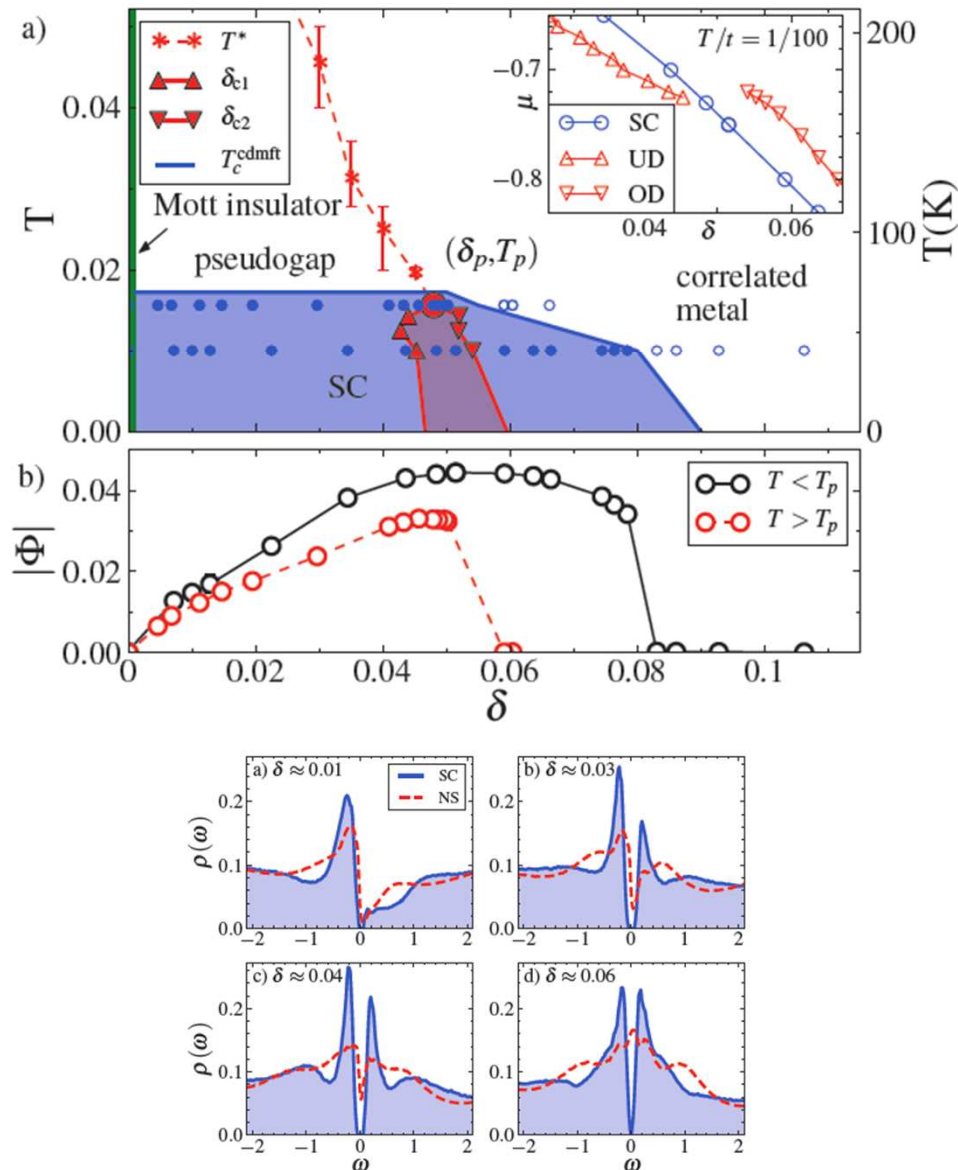


Summary



- Mott physics extends way beyond half-filling
- Pseudogap is a phase
- Pseudogap T^* is a Widom line
- High compressibility (stripes?)

Summary



- Below the dome finite T critical point (not QCP) controls normal state
- First-order transition destroyed but traces in the dynamics
- T^* different from T_c^d
- Actual T_c in underdoped
 - Competing order
 - Long wavelength fluctuations (see O.P.)
 - Disorder

Merci

Thank you