

# T19. 1 : Importance of subleading corrections for the Mott critical point

8:00 AM–8:12 AM



A.-M. Tremblay

Patrick Sémon



APS-Baltimore, 21 March 2013



# « Conventional » Mott transition

Double occupancy: Ising universality class

C. Castellani et al., Phys. Rev. Lett. **43**, 1957 (1979).

G. Kotliar, et al. Phys. Rev. Lett. **84**, 5180 (2000).

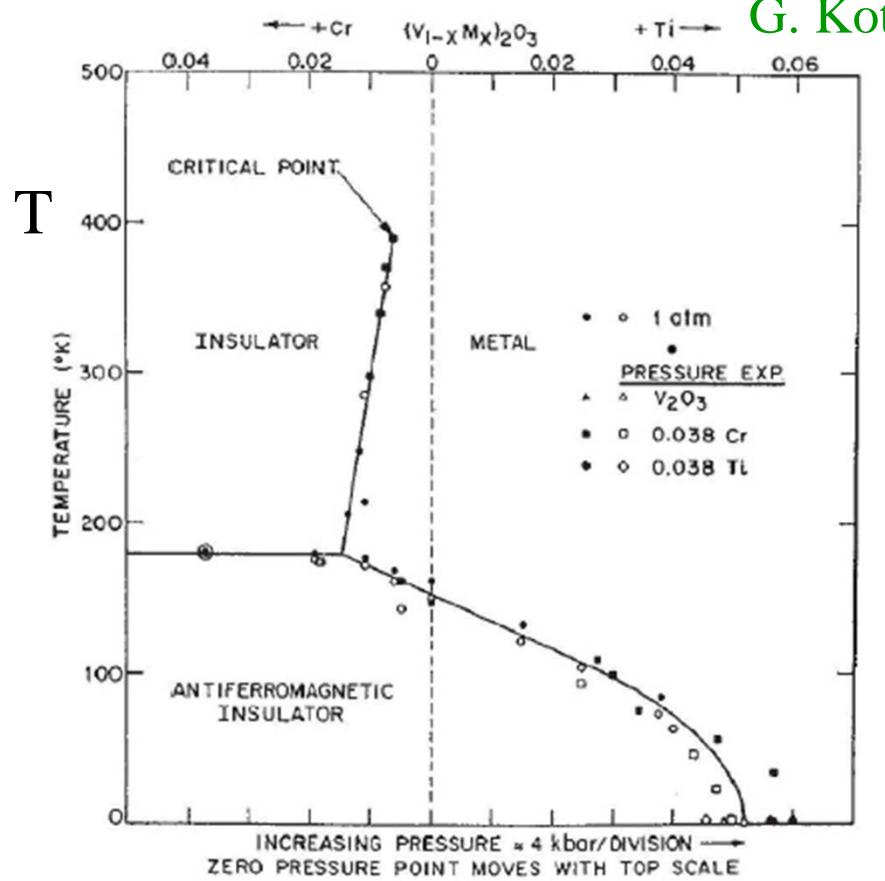


Figure: McWhan, PRB 1970; Limelette, Science 2003

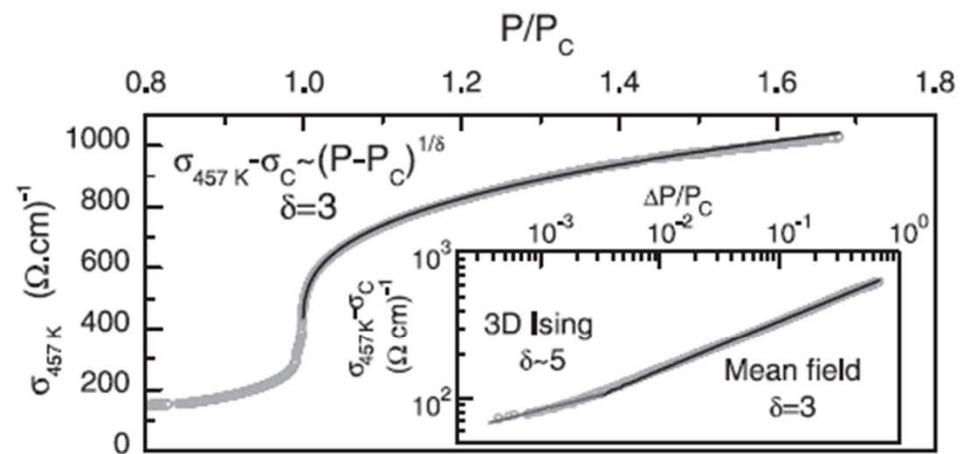
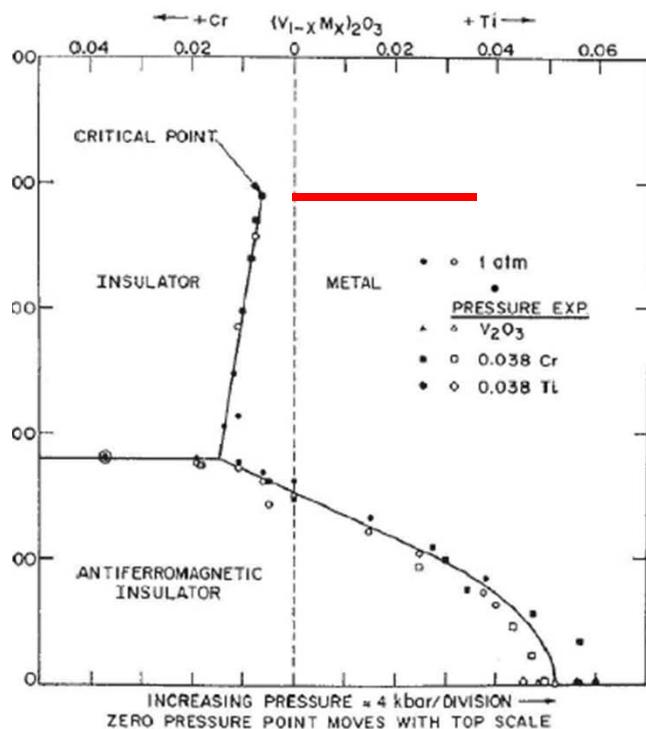


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# Critical behavior



Universality and Critical Behavior at the Mott Transition  
P. Limelette, et al.  
Science 302, 89 (2003);  
DOI: 10.1126/science.1088386

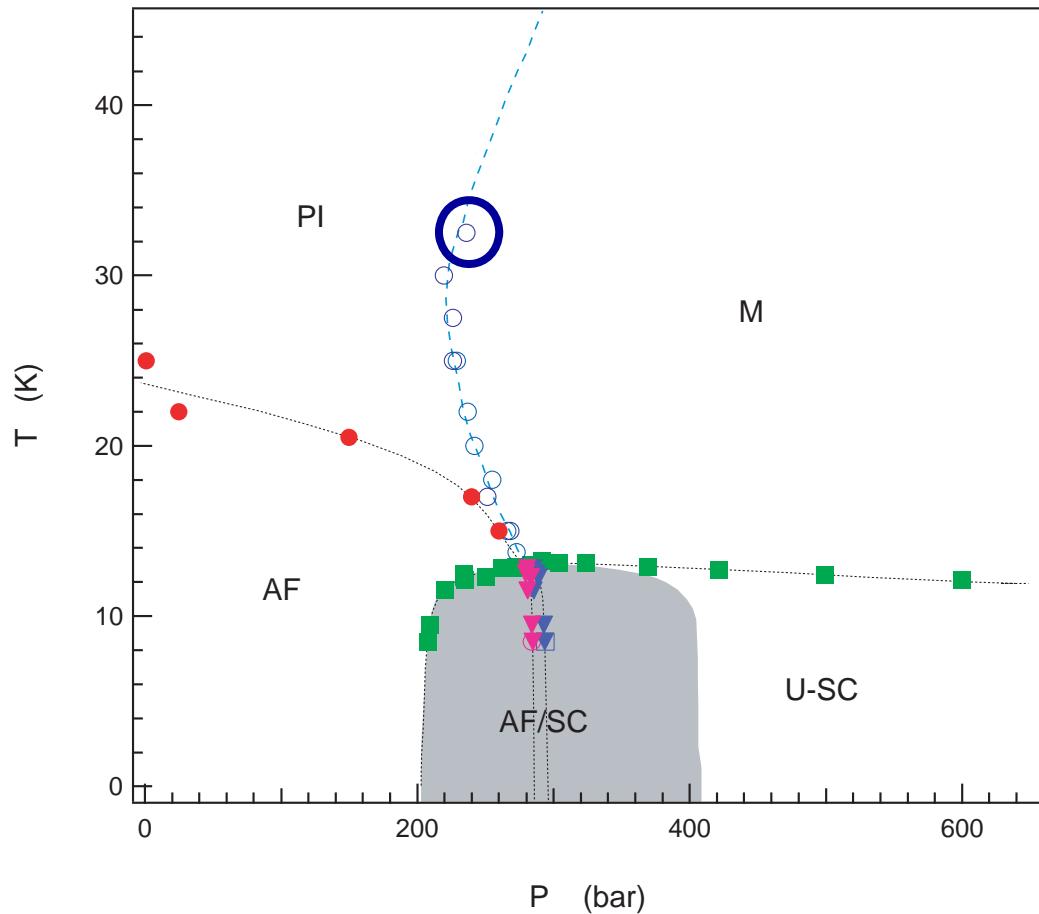


McWhan, PRB 1970; Limelette, Science 2003



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# Mott critical point in layered organics



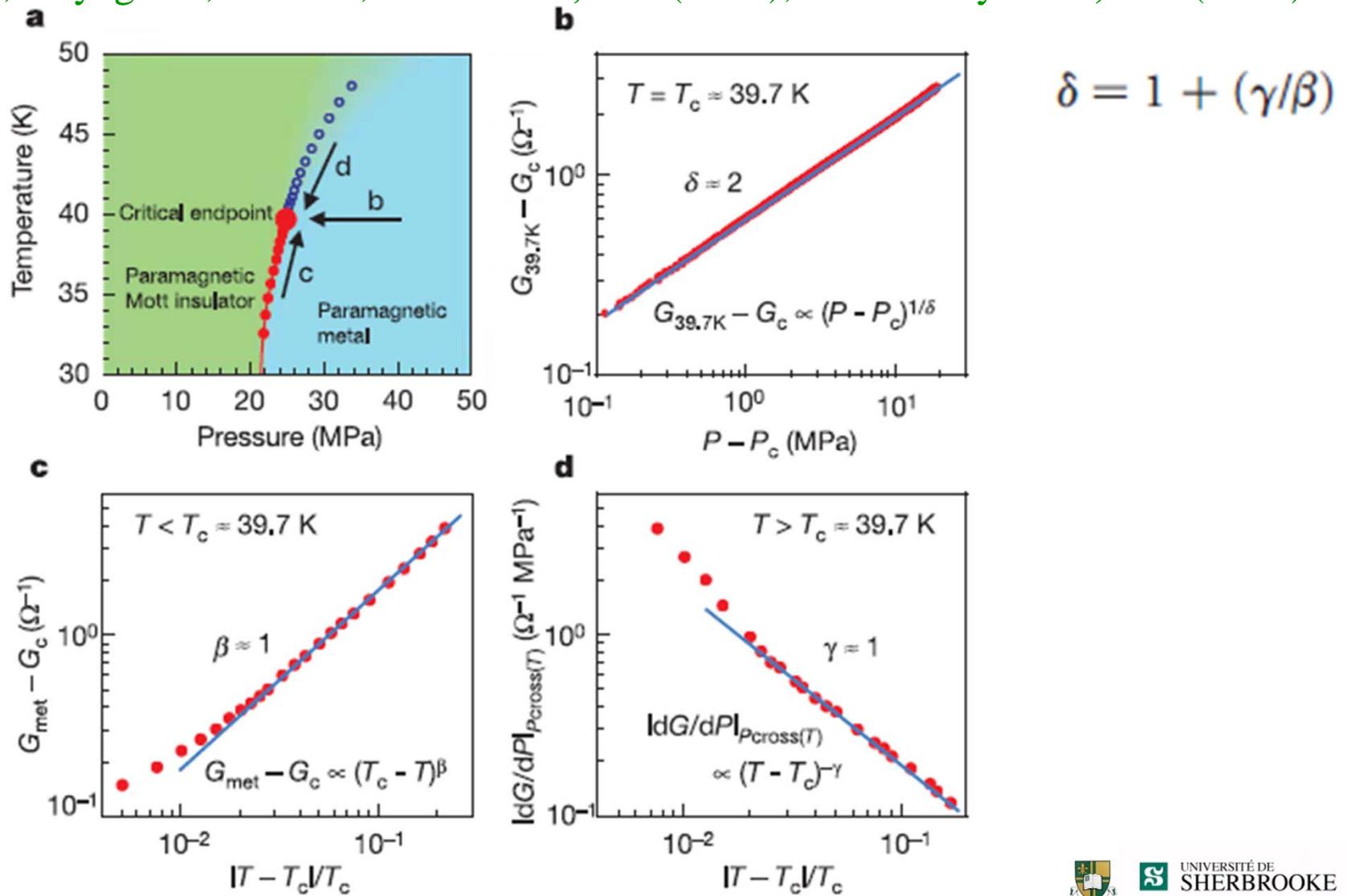
THIS TALK  
What is the critical behavior?

Phase diagram BEDT-X  
( $X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ )  
S. Lefebvre et al. PRL **85**, 5420 (2000),  
P. Limelette, et al. PRL **91** (2003)

F. Kagawa, K. Miyagawa, + K. Kanoda  
PRB **69** (2004) +Nature **436** (2005)

# Surprising critical behavior

Kagawa, Miyagawa, Kanoda, Nature 436, 534 (2005), Nature Physics 5, 880 (2009)



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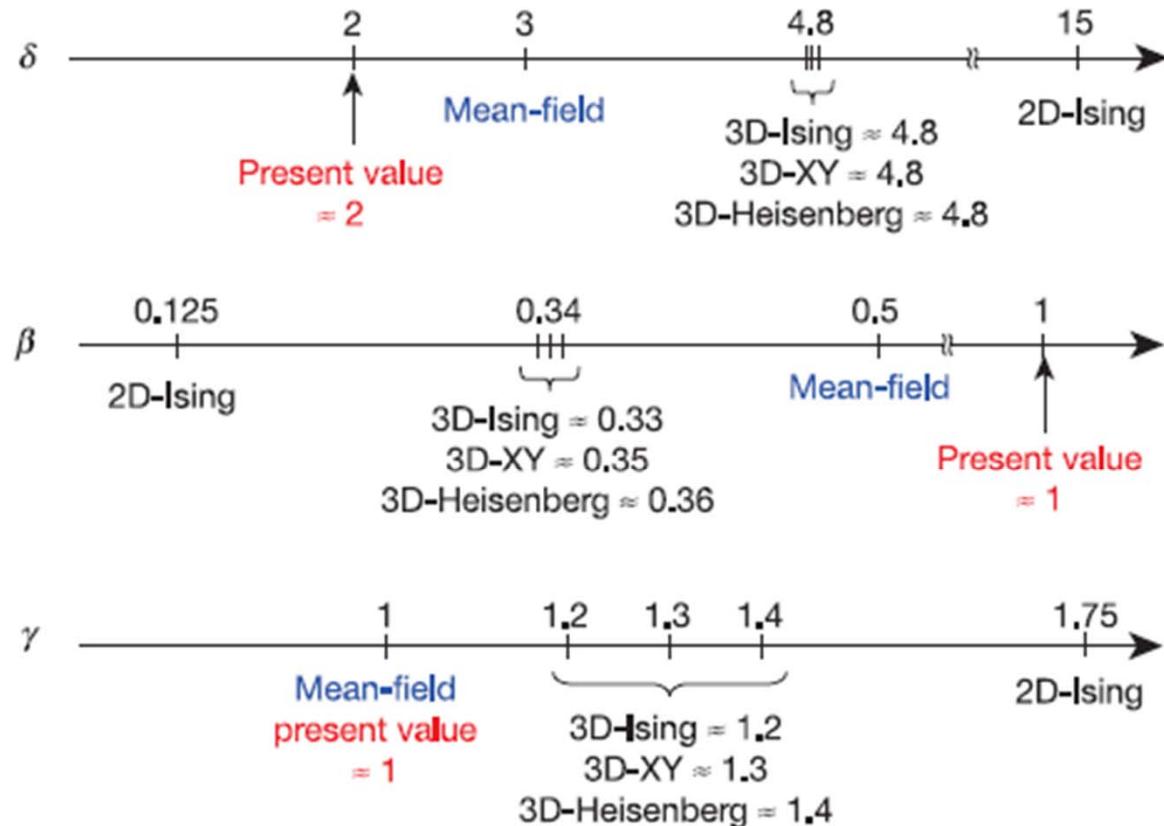
# Unconventional behavior

## Unconventional critical behaviour in a quasi-two-dimensional organic conductor

Nature

436, 534 (2005)

F. Kagawa<sup>1</sup>, K. Miyagawa<sup>1,2</sup> & K. Kanoda<sup>1,2</sup>

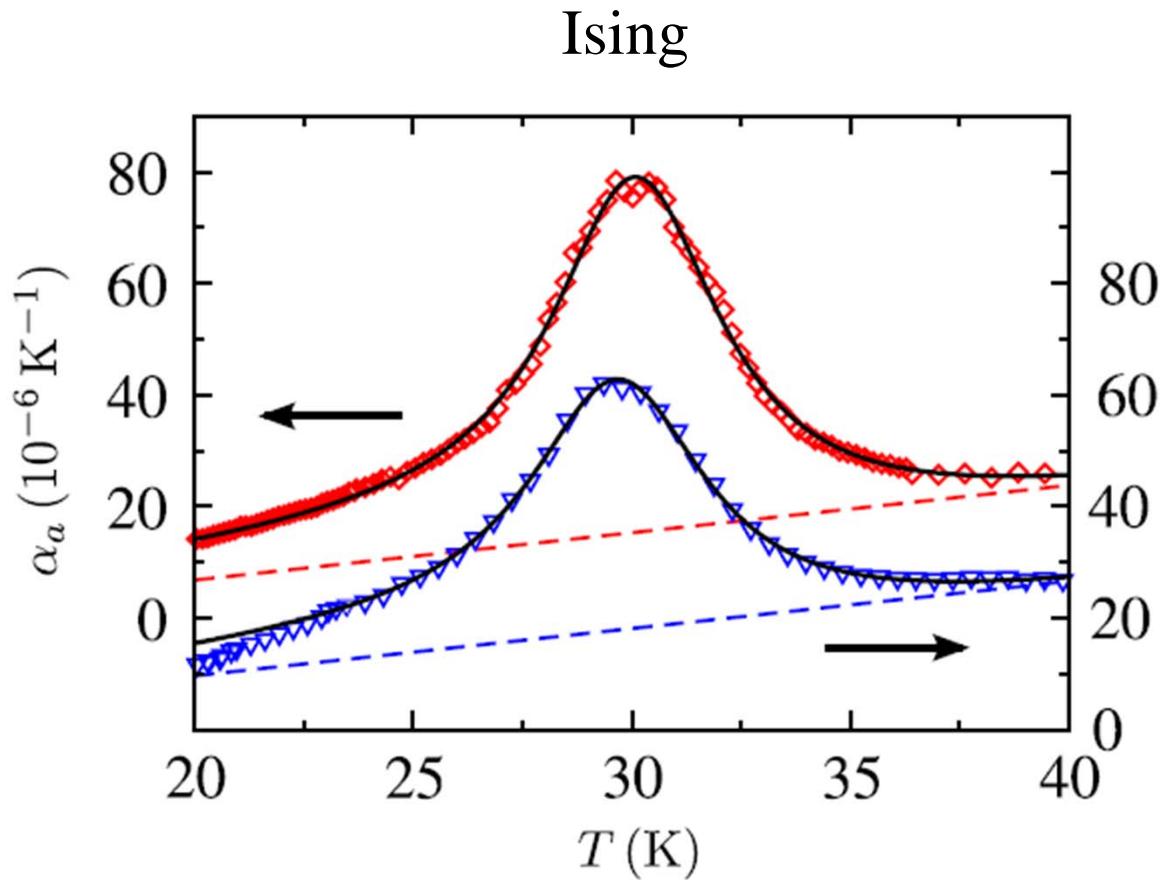


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# However, Ising for thermal expansion (Br ion)

M. de Souza et al., Phys. Rev. Lett. **99**, 037003 (2007)

L. Bartosch, M. de Souza, and M. Lang, Phys. Rev. Lett. **104**, 245701 (2010).



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# Possible explanations

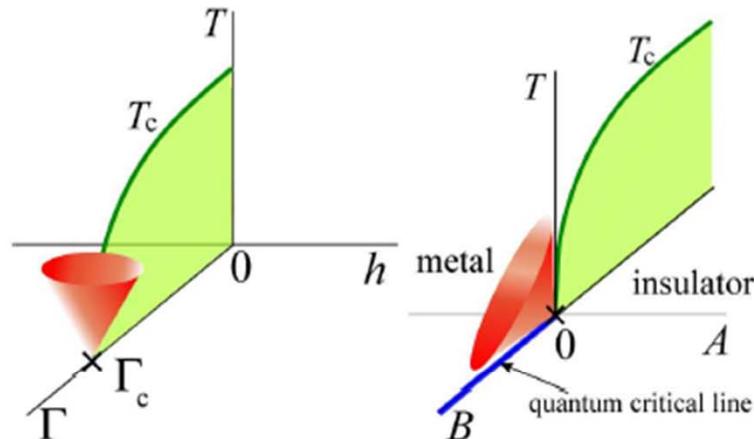


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# Control by a $T = 0$ marginal QCP

M. Imada, Phys. Rev. B **72**, 075113 (2005).

M. Imada, et al. J. Phys.: Condens.Matter **22**, 164206 (2010).



**Figure 1.** Phase diagram around conventional QCP (a)(left panel), and MQCP (b) (right panel for metal-insulator transition) in the parameter space of temperature  $T$ , fields to control transitions  $h$  or  $A$  and parameters to control quantum fluctuations  $\Gamma$  or  $B$ . The cone structures schematically illustrate the quantum critical regions of the QCP (a) and MQCP (b) depicted by the crosses. First-order transitions occur when one crosses shaded (green) walls. Quantum critical line (bold (blue) line) in (b) represents continuous topological transition at  $T = 0$ .

# Coupling to the energy density instead of OP

S. Papanikolaou, R. M. Fernandes, E. Fradkin, P. W. Phillips,  
J. Schmalian, and R. Sknepnek, Phys. Rev. Lett. **100**, 026408 (2008).

$$\sigma \propto \eta^\theta$$

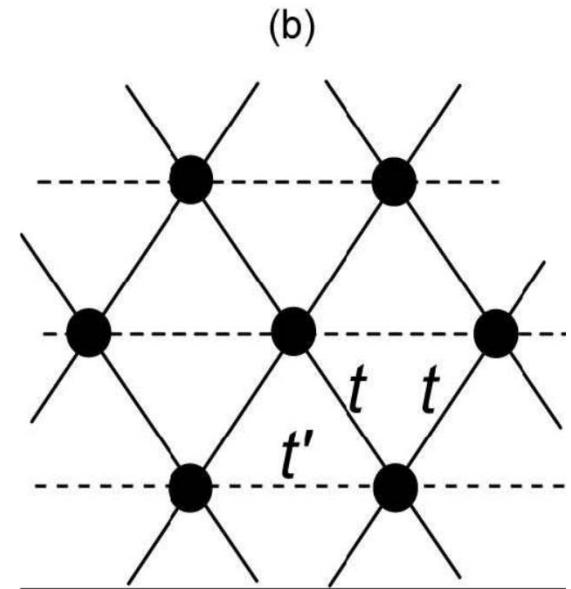
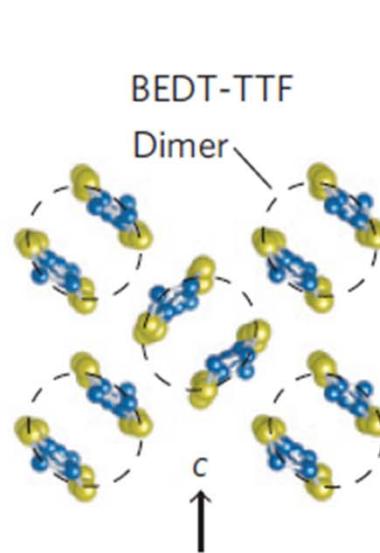
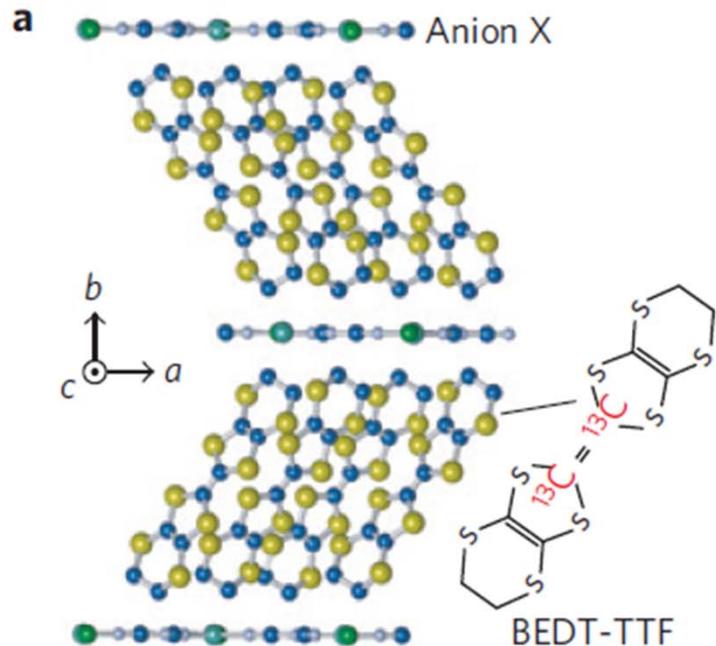
$$\theta = (1 - \alpha)/\beta$$

$$(\beta_\sigma, \gamma_\sigma, \delta_\sigma) = (1, \frac{7}{8}, \frac{15}{8})$$

# The model

# Hubbard on anisotropic triangular lattice

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),  
R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



Kagawa *et al.*  
Nature Physics  
**5**, 880 (2009)

$$t \approx 50 \text{ meV}$$

$$\Rightarrow U \approx 400 \text{ meV}$$
$$t'/t \sim 0.6 - 1.1$$

$$H = \sum_{ij\sigma} (t_{ij} - \delta_{ij}\mu) c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$





## Importance of subleading corrections for the Mott critical point

Patrick Sémond<sup>1</sup> and A.-M. S. Tremblay<sup>1,2</sup>

### The method

Cellular dynamical mean-field theory  
Continuous-time quantum Monte Carlo  
Hybridization expansion

P. Werner, et al., Phys. Rev. Lett. **97**, 076405 (2006).

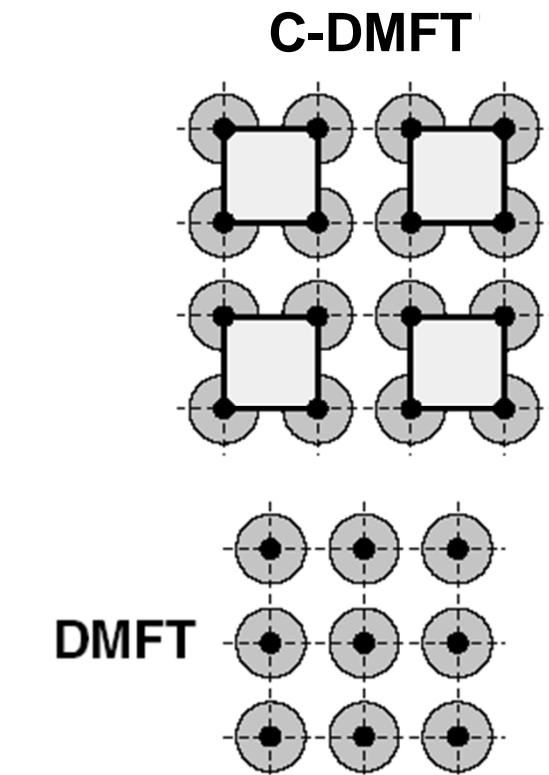
P. Werner and A. J. Millis, Phys. Rev. B **74**, 155107 (2006).

E. Gull, et al., Rev. Mod. Phys. **83**, 349 (2011).

K. Haule, Phys. Rev. B **75**, 155113 (2007).

# 2d Hubbard: Quantum cluster method physics

- Observed behavior is a transient from a QCP?
- Quantum fluctuations
- Cluster necessary in  $d = 2$  for
- Short-range spatial fluctuations
- Disentangle effects of  $J$
- No low  $q$  spatial fluctuations



## REVIEWS

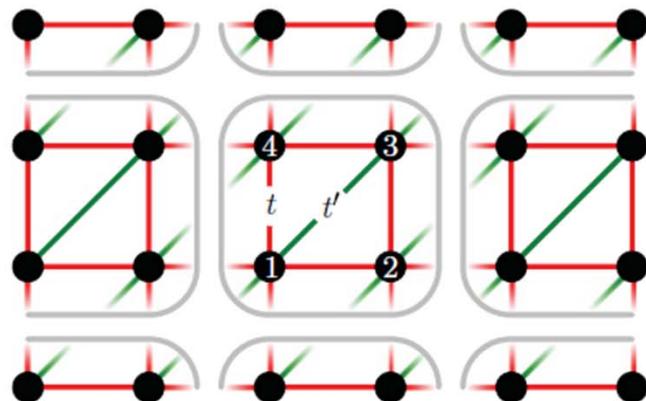
Maier, Jarrell et al., RMP. (2005)  
Kotliar et al. RMP (2006)  
AMST et al. LTP (2006)



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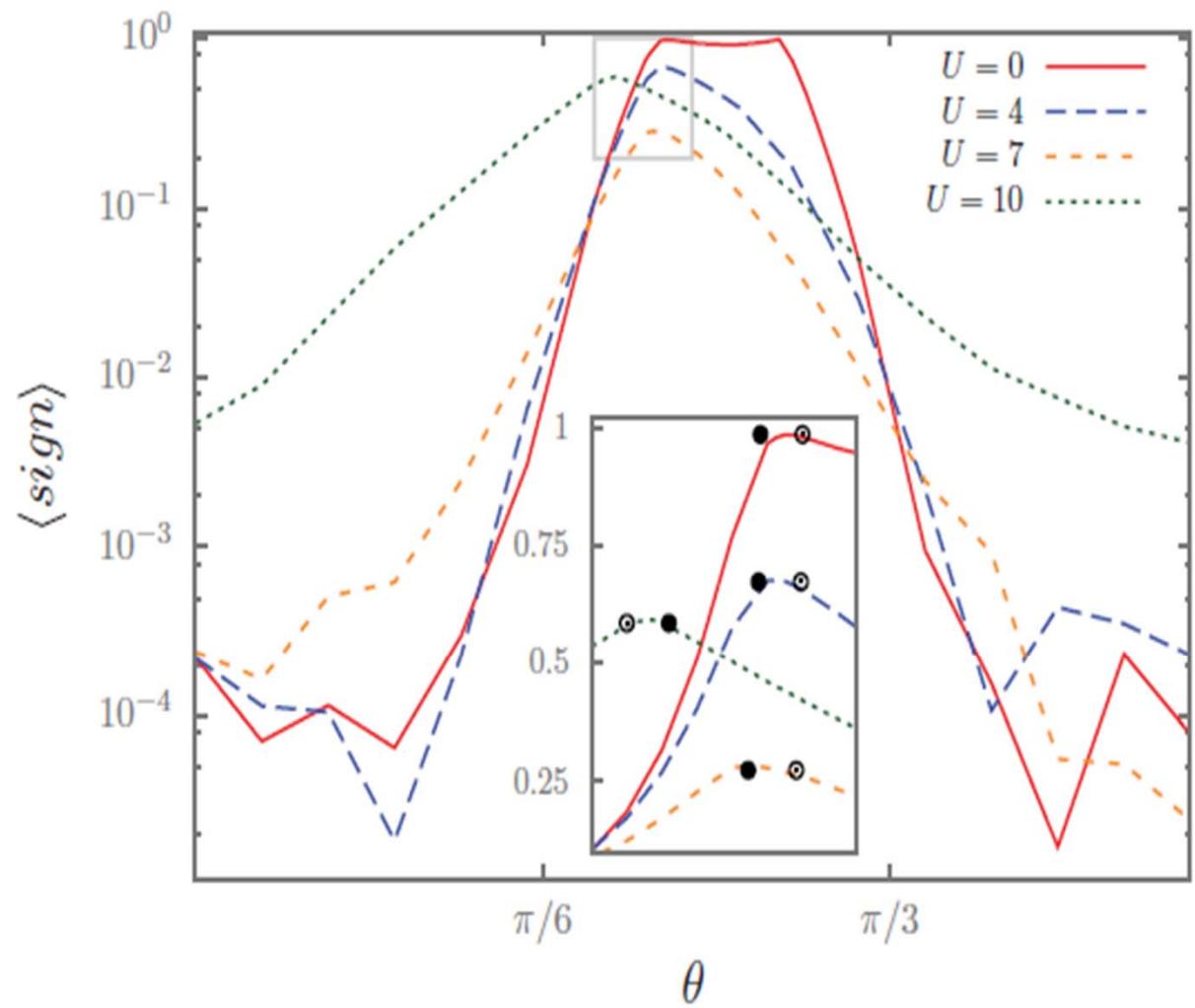
# Reducing the sign problem

$$\cos \theta c'_{A_1\sigma} - \sin \theta c_{A_1\sigma}, \quad \sin \theta c'_{A_1\sigma} + \cos \theta c_{A_1\sigma}$$



$$t'/t = 0.8$$

$C_{2v}$   
2A<sub>1</sub>, B<sub>1</sub>, B<sub>2</sub>



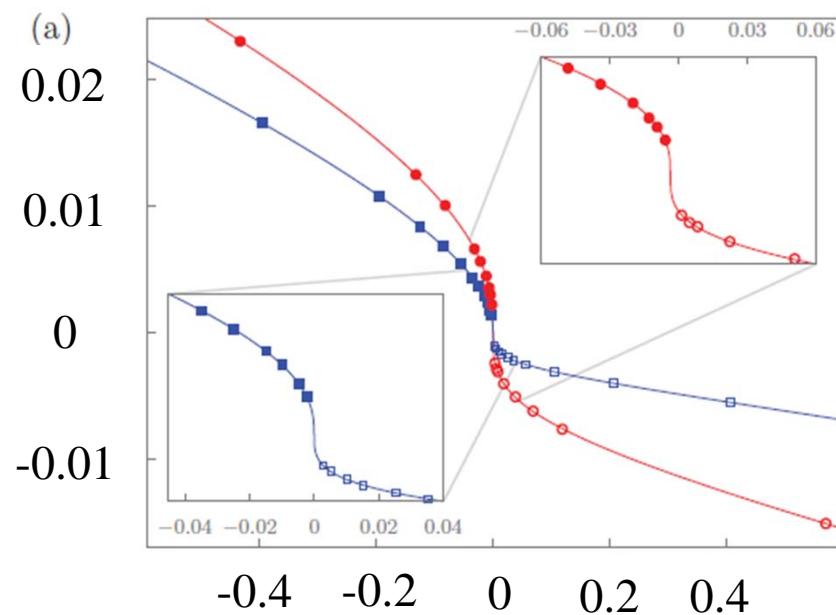
# Numerical results



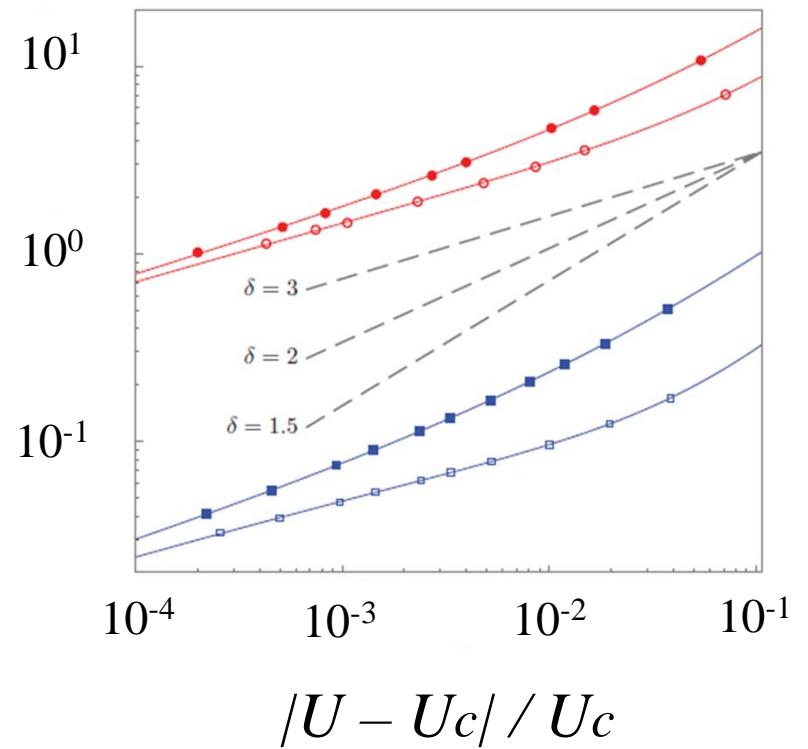
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# Double occupancy ( $\delta$ )

$D - D_c$



$|D - D_c| / D_c$



Red circles: CDMFT

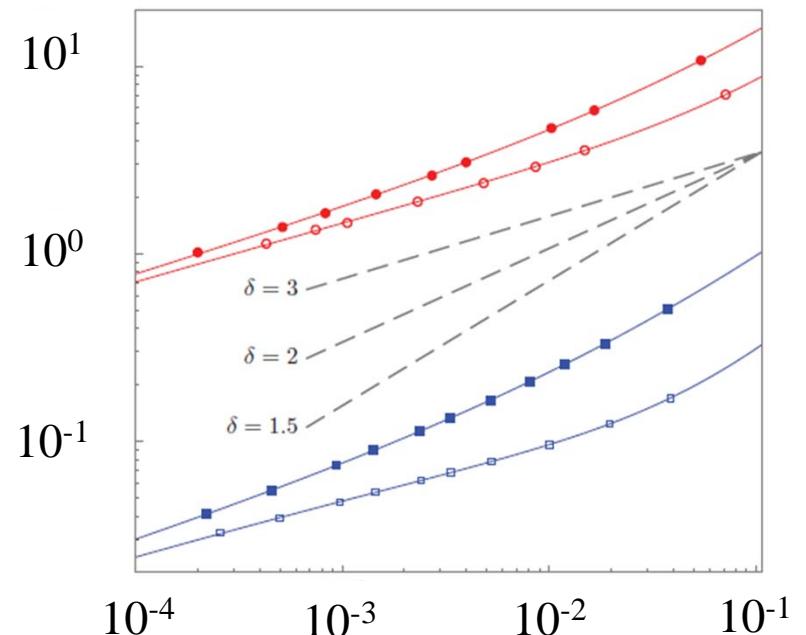
Blue squares: single-site DMFT



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# Double occupancy ( $\delta$ )

$$|D - D_c| / D_c$$



M. Sentef, P. Werner, E. Gull, and A. P. Kampf,  
Phys. Rev. B **84**, 165133 (2011).

$|U - U_c| / U_c$   
Fit with single exponent:  $\delta = 2$

Red circles: CDMFT

Blue squares: single-site DMFT : Should give  $\delta = 3$



PHYSICAL REVIEW B 85, 201101(R) (2012)

## Importance of subleading corrections for the Mott critical point

Patrick Sémond<sup>1</sup> and A.-M. S. Tremblay<sup>1,2</sup>

# Importance of subleading corrections



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# Subleading corrections to mean-field

VOLUME 84, NUMBER 22

PHYSICAL REVIEW LETTERS

29 MAY 2000

## Landau Theory of the Finite Temperature Mott Transition

G. Kotliar,<sup>1</sup> E. Lange,<sup>1</sup> and M. J. Rozenberg<sup>2</sup>

$$p\eta + c\eta^3 = h \quad \delta = 3$$

$$p \equiv p_1(U - U_c) + p_2(T - T_c)$$

$$p_1\delta U\eta + c\eta^3 = h_1\delta U$$

$$\eta = \sum_{i=1}^{\infty} \delta U^{i/3} \eta_i$$

$$D = D_c + a_1\eta + a_2\eta^2$$

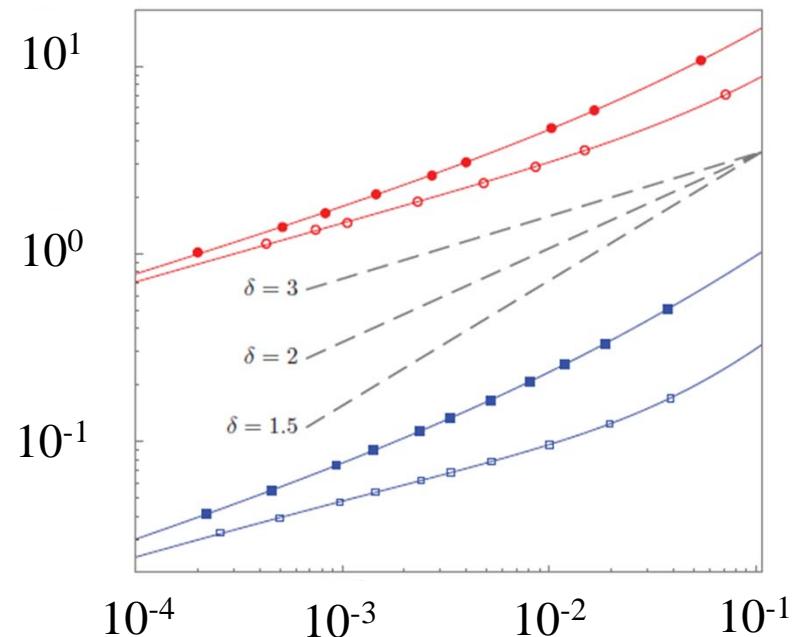
$$D - D_c = c_1 \operatorname{sgn}(\delta U) |\delta U|^{1/\delta} + c_2 |\delta U|^{2/\delta} + c_3 \delta U$$



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# Double occupancy ( $\delta$ )

$$|D - D_c| / D_c$$



$$D - D_c = c_1 \operatorname{sgn}(\delta U) |\delta U|^{1/\delta} + c_2 |\delta U|^{2/\delta} + c_3 \delta U$$

$$|U - U_c| / U_c$$

Red circles: CDMFT

$$\delta = 3.04 \pm 0.25$$

Blue squares: single-site DMFT

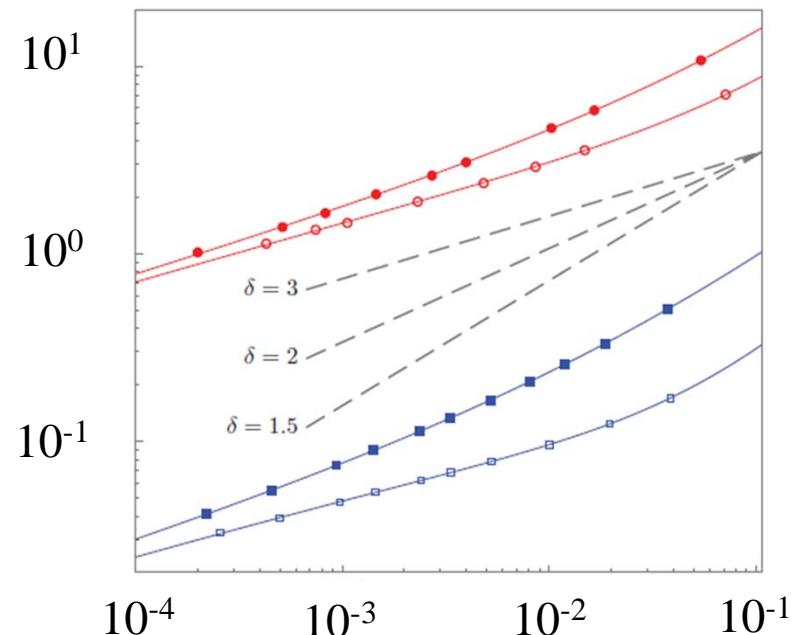
$$\delta = 2.93 \pm 0.15$$



# Conclusion

- Drastic reduction of sign problem allows:
- CDMFT and DMFT over almost three decades to give  $\delta = 2$ 
  - If fit over whole range with one exponent.
  - Consistent with experiment.
- Incorrect. Indeed, know that  $\delta = 3$  (mean field) in DMFT
  - Must take into account subleading corrections
  - Then,  $\delta = 3$  better fits for both DMFT and CDMFT (with same number of parameters as  $\delta = 2$  )
  - Should reanalyze experiments

$$|D - D_c| / D_c$$



$$|U - U_c| / U_c$$

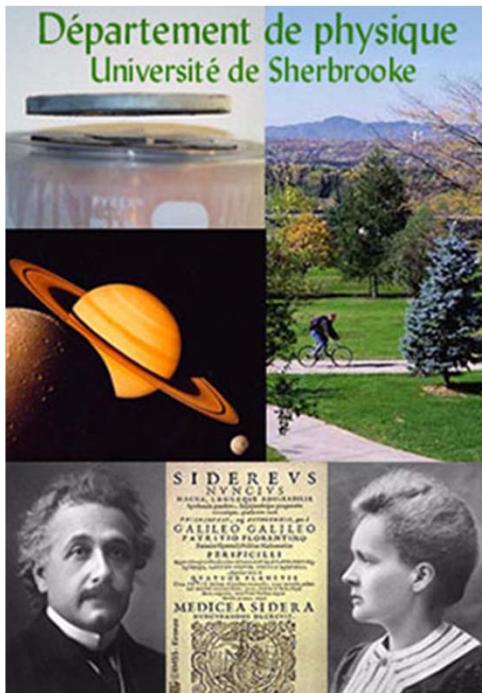
Red circles: CDMFT

$$\delta = 3.04 \pm 0.25$$

Blue squares: single-site DMFT

$$\delta = 2.93 \pm 0.15$$

# André-Marie Tremblay



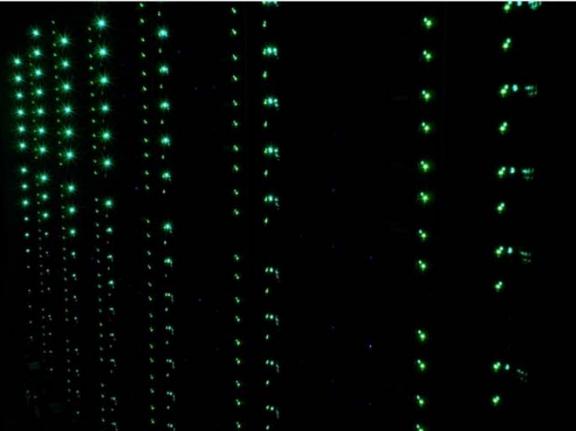
Le regroupement québécois sur les matériaux de pointe



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merci

thank you

# Comparisons to experiment

Charles-David Hébert



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FIG. 3. (Color online) Double occupancy as a function of  $U$  near the Mott critical point for the Hubbard model on an anisotropic triangular lattice with  $t'/t = 0.8$  ( $t \equiv 1$ ) at half filling and fixed critical inverse temperature  $\beta = 11.15$  (squares) for DMFT and  $\beta = 9.9$  (circles, shifted by  $\times 10^{1.5}$ ) for CDMFT on a  $2 \times 2$  plaquette. The solid lines show a fit to  $f(U) = c_1 \text{sgn}(\delta U) |\delta U|^{1/\delta} + c_2 |\delta U|^{2/\delta} + c_3 \delta U + D_c$  ( $\delta U \equiv U - U_c$ ) with the same parameters  $c_1, c_2, c_3, D_c, U_c$ , and  $\delta$  for the metallic (filled symbols) and the insulating region (open symbols). The best fitting values  $(U_c, D_c, \delta)$  are  $(10.445, 0.0325, 2.93)$  for DMFT and  $(7.932, 0.0679, 3.04)$  for CDMFT. (a) Linear plot centered at  $(U_c, D_c)$ . The insets zoom on the regions close to the critical point. (b) Logarithmic plot in reduced units relative to the critical point with CDMFT data shifted by a factor of  $10^{1.5}$  along the  $y$  axis. The dashed lines show the function  $\propto |U - U_c|^{1/\delta}$  with  $\delta$  as indicated. In the critical regime, up to 500 iterations are necessary for convergence in the iterative solution of the (C)DMFT equation. Once convergence is reached, we take the average over hundreds of iterations. Monte Carlo sweeps per iteration:  $6 \times 10^9$  for DMFT and  $10^9$  for CDMFT.