

Phase diagram of the cuprates: Where is the mystery?

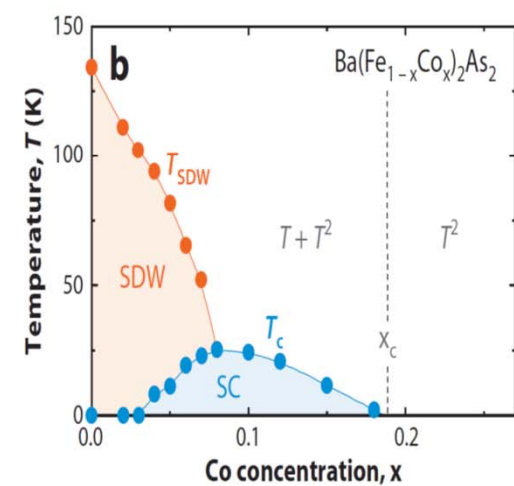
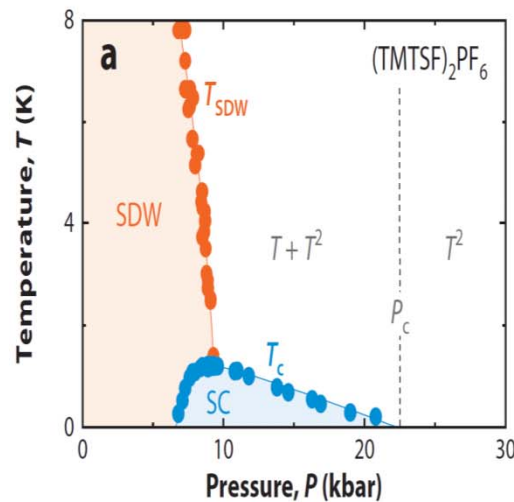
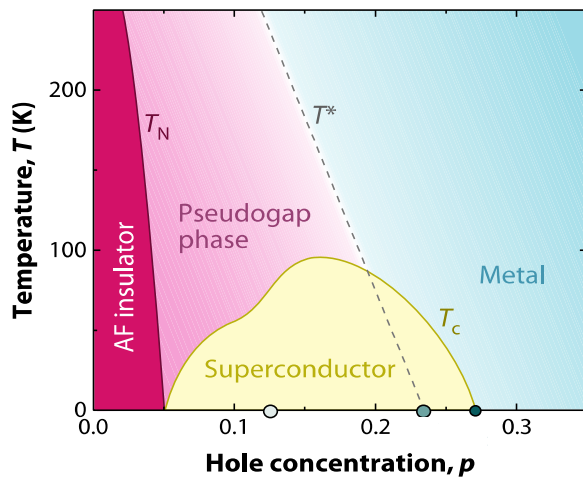
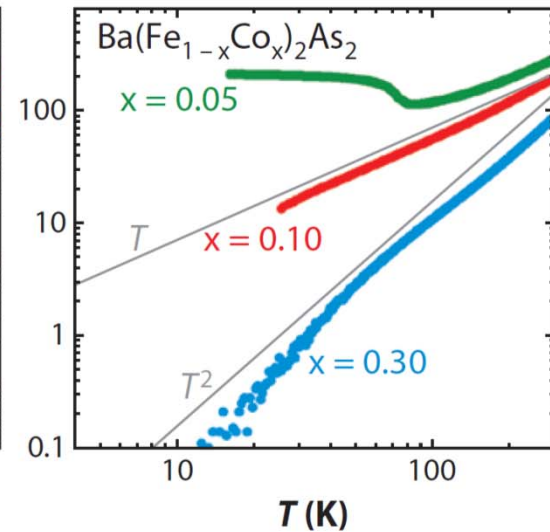
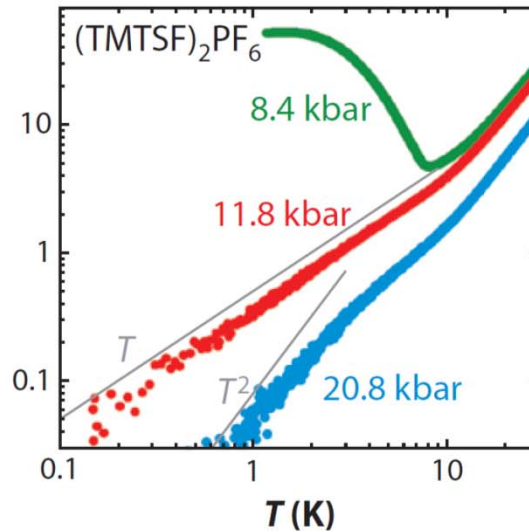
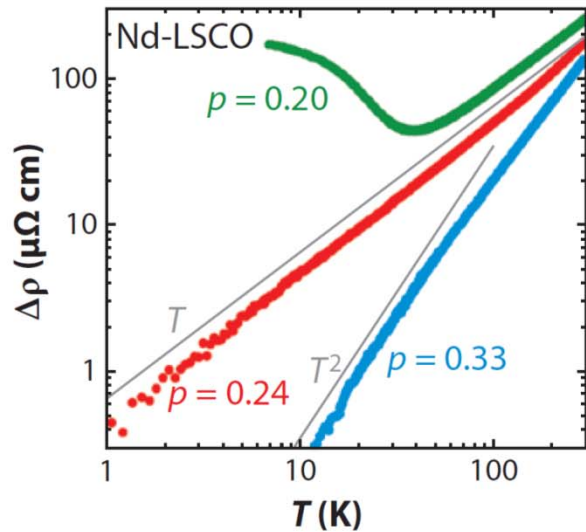
A.-M. Tremblay



I- Similarities between phase diagram and quantum critical points



Quantum Criticality in 3 Families of Superconductors

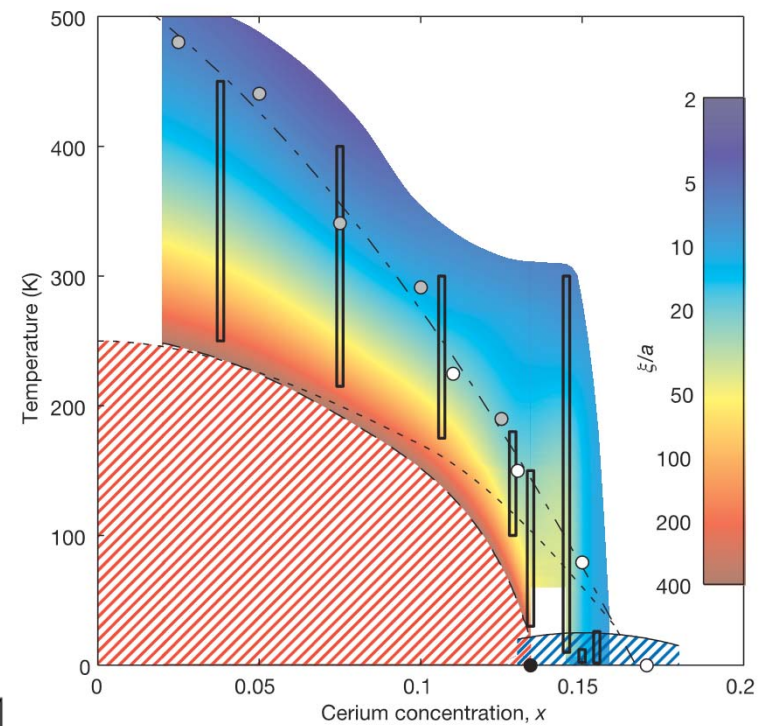
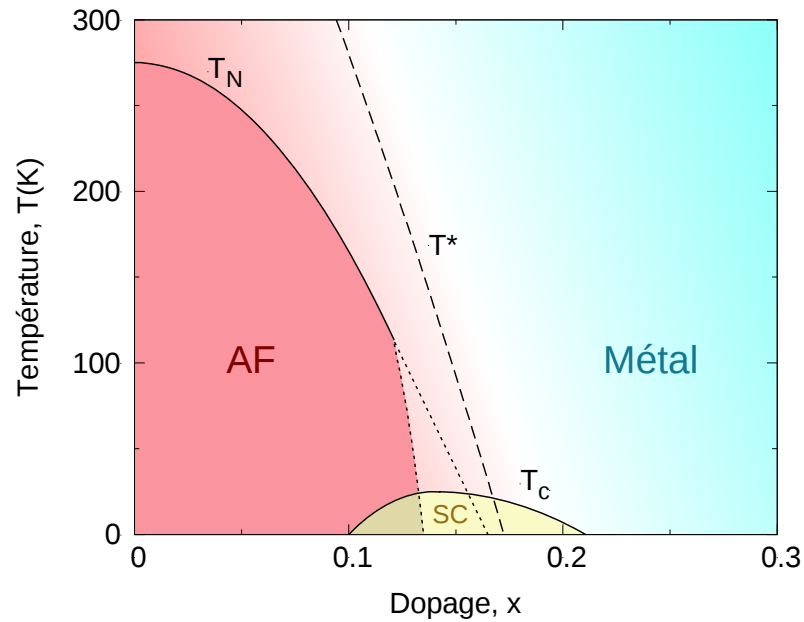


L. Taillefer, Annual Reviews of CMP 2010

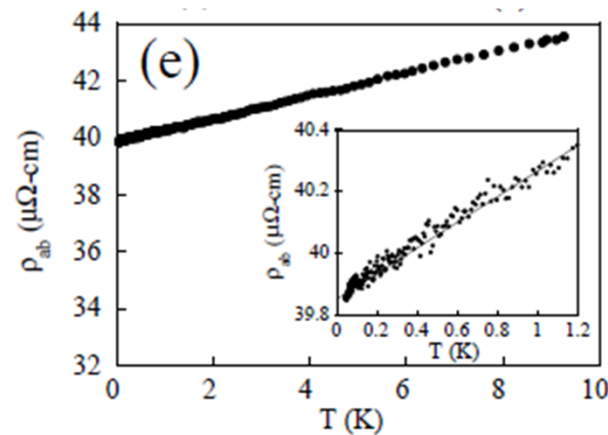


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Electron-doped cuprates



PCCO



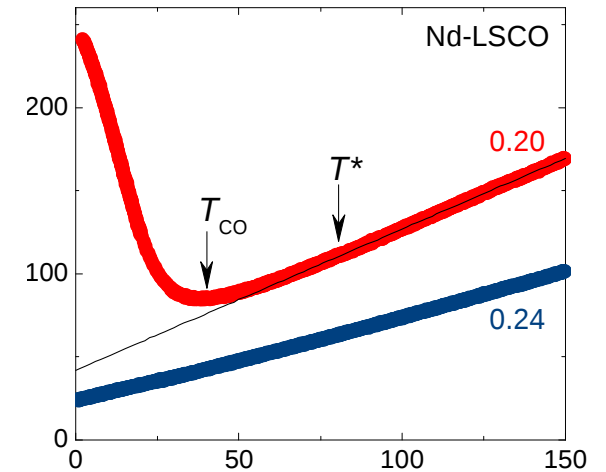
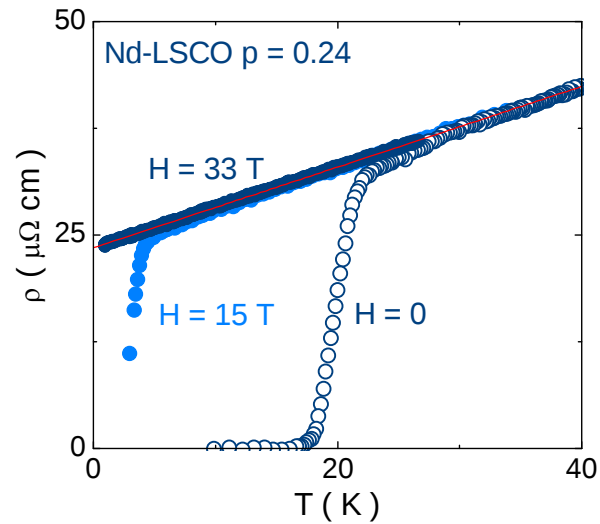
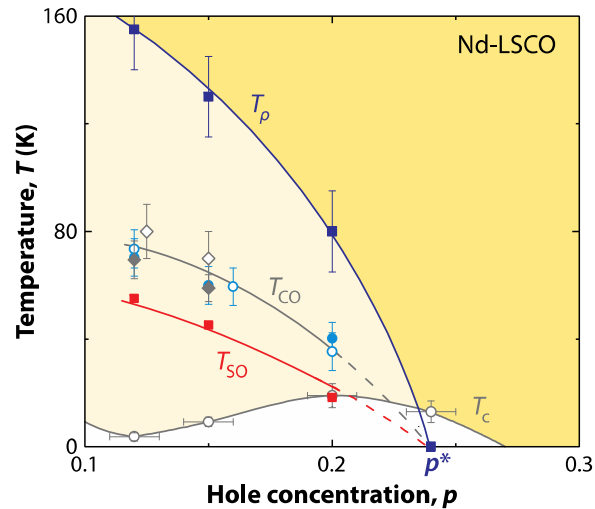
Fournier et al. PRL **81**, 4720 (1998)



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Quantum critical point

Nd-LSCO



Neutrons: Ichikawa *et al.*, PRL 2000

NMR: Hunt *et al.*, PRB 2001

X-rays: Niemoller *et al.*, EPJB 1999

$$T^* \sim 2 T_{CO}$$

Daou *et al.*, Nature Physics 2009



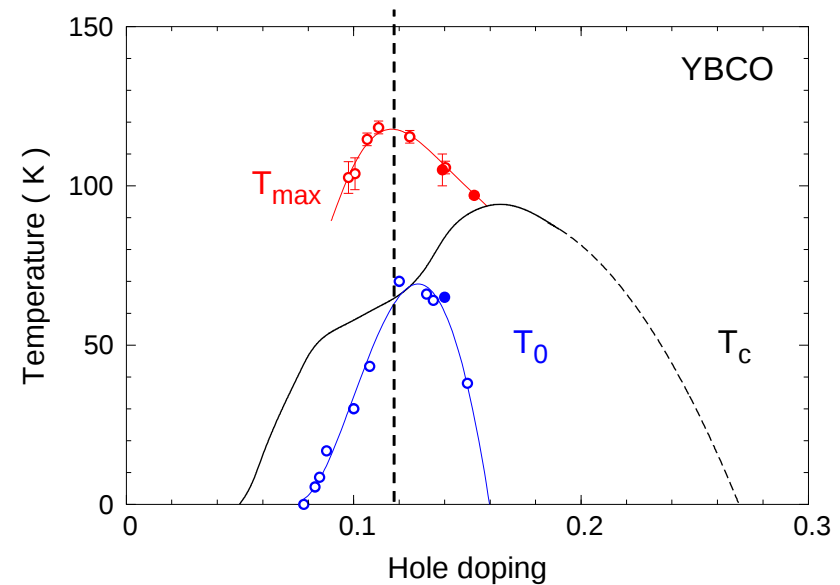
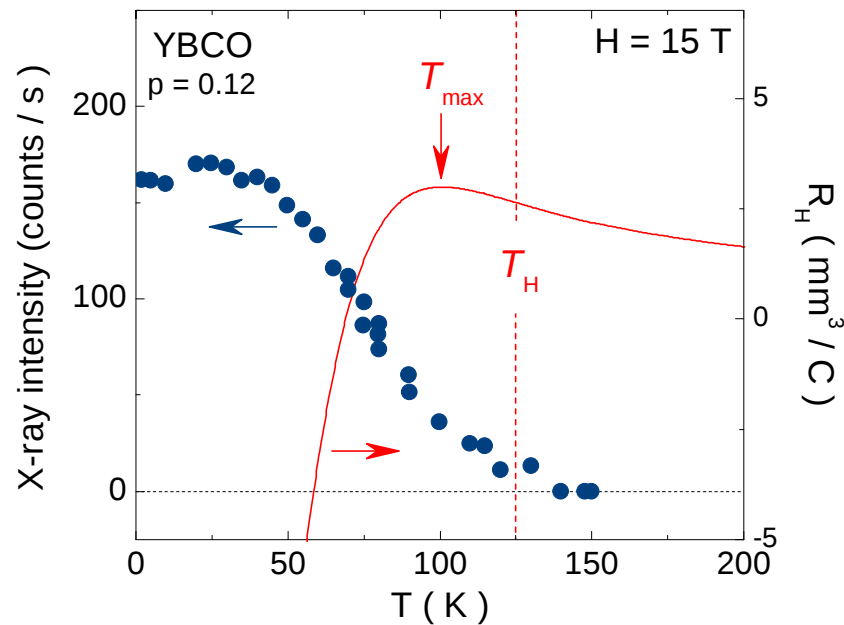
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II- Charge order is the competing phase
in the hole-doped cuprates



Charge order & FS reconstruction

YBCO



Charge order causes Fermi-surface reconstruction

X-rays: Chang *et al.*, Nature Physics 2012

Hall: LeBoeuf *et al.*, Nature 2007

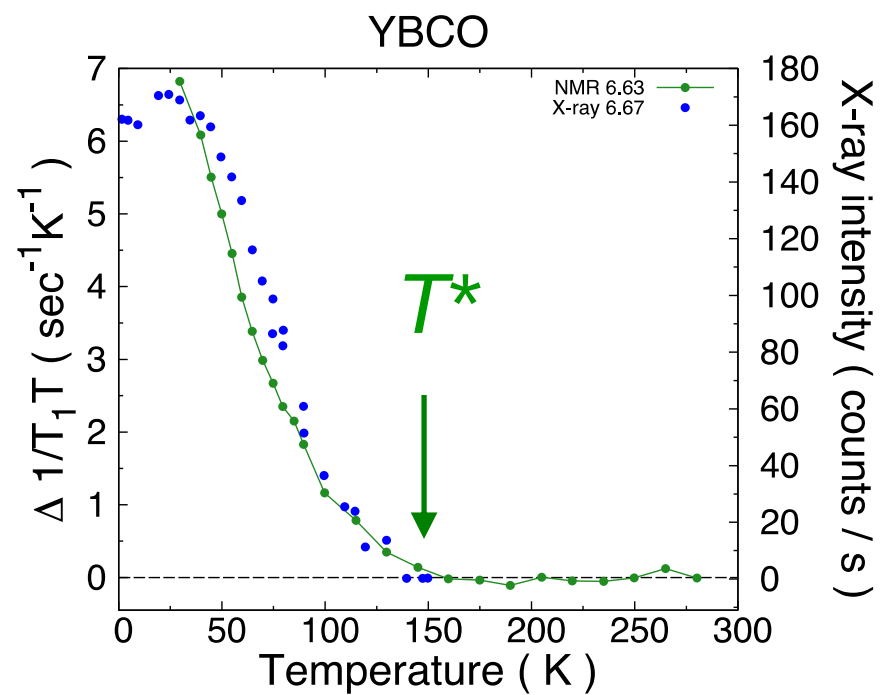
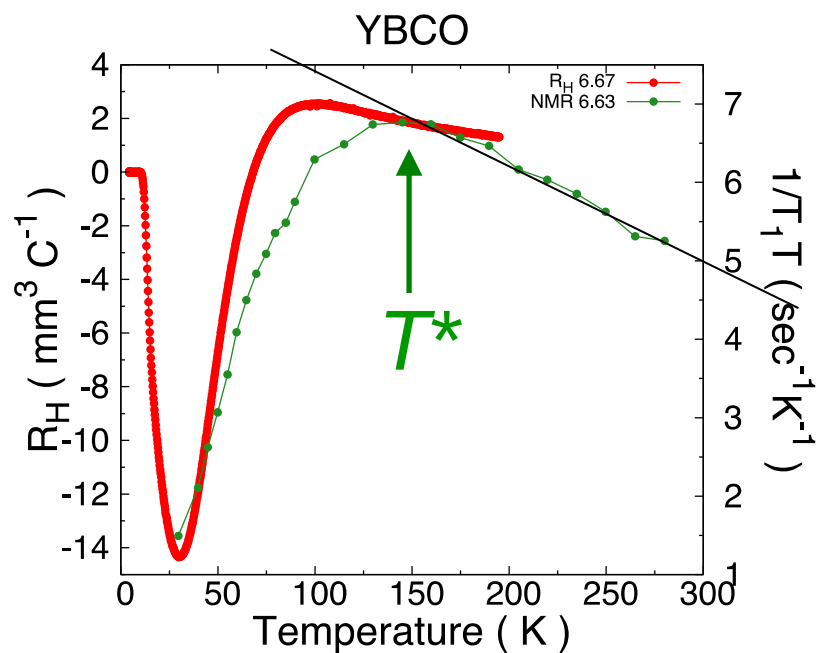
LeBoeuf *et al.*, PRB 2011



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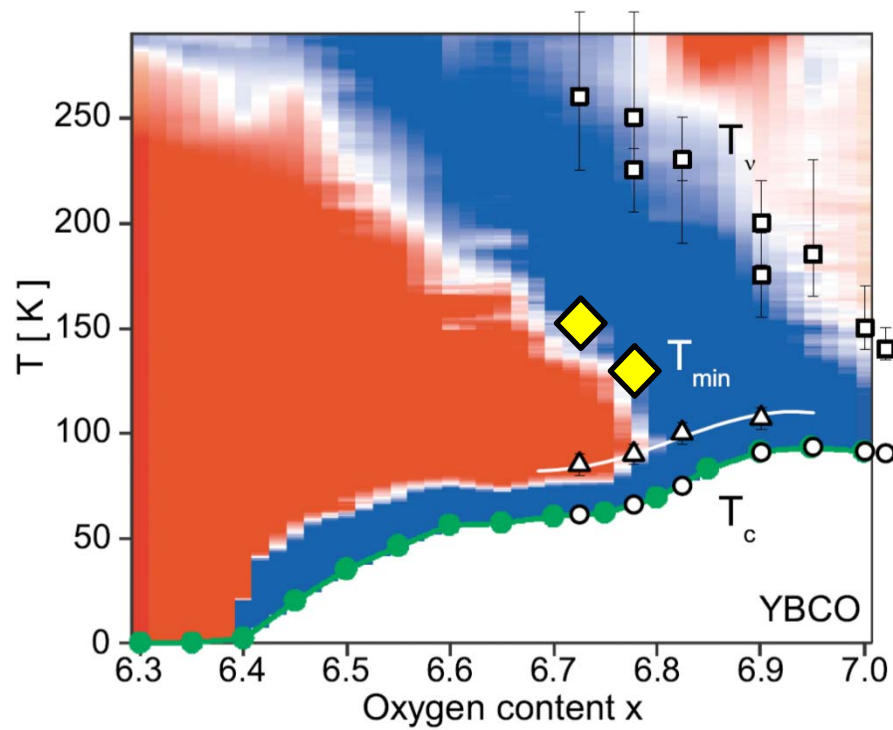
Pseudogap phase YBCO

NMR

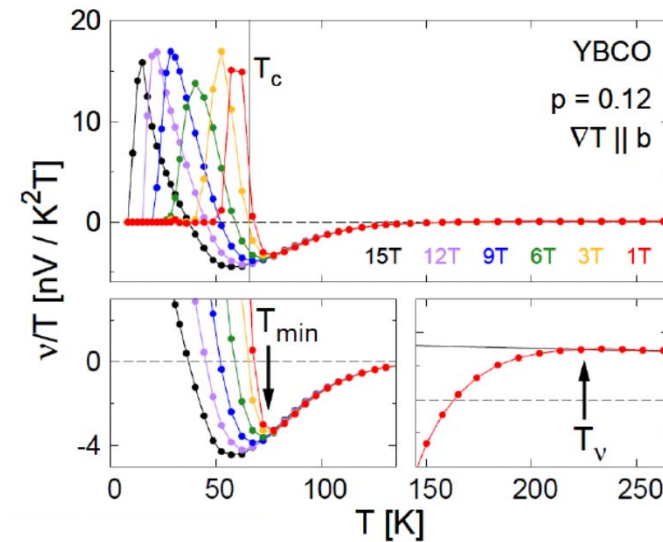
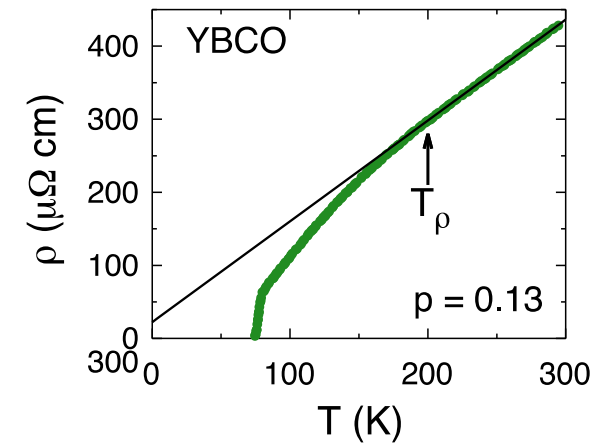


Pseudogap regime

YBCO



[Color map : Ando et al., PRL 2004](#)



Daou et al., *Nature* **463**, 519 (2010)



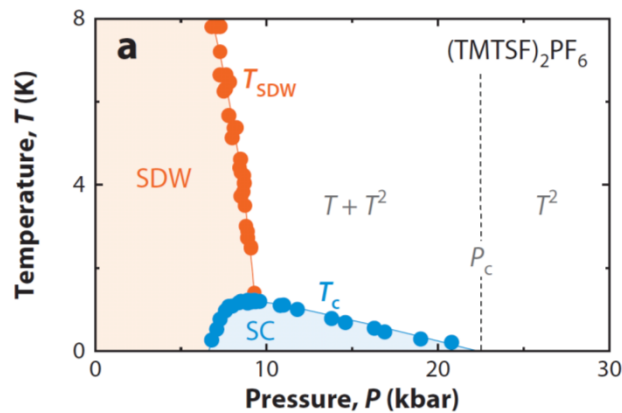
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III- Coefficient of linear term vs T_c

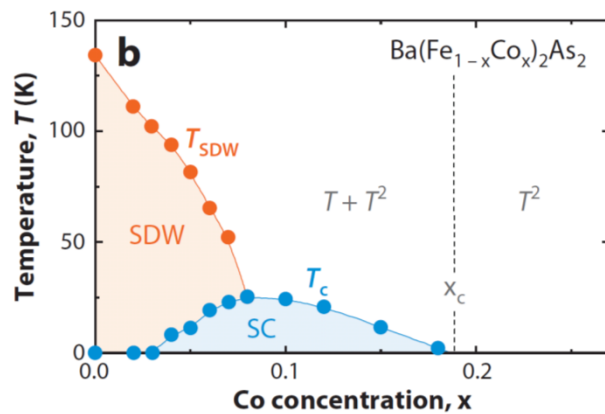


Organics & Pnictides

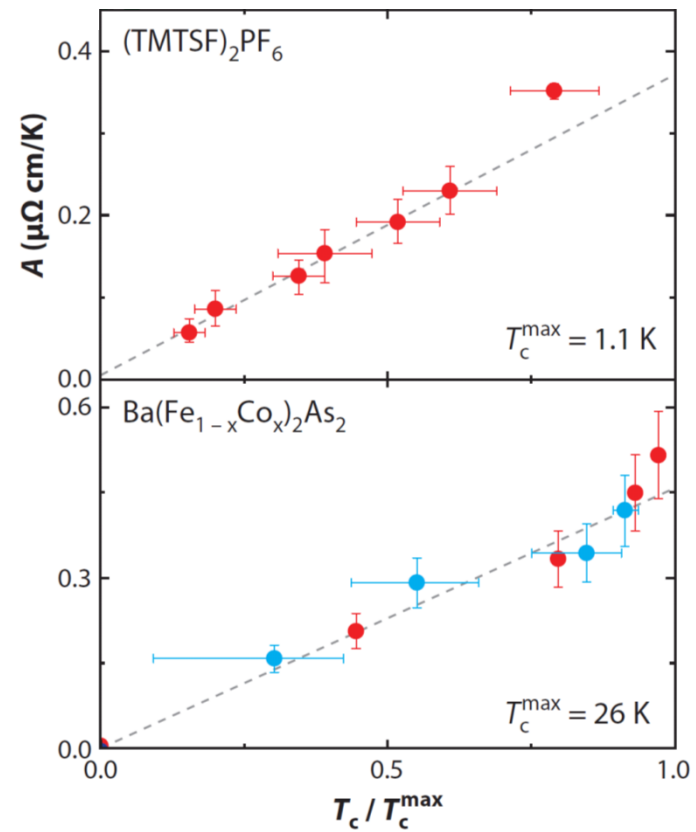
Organic



Pnictide



Bourbonnais, Sedeki, 2012



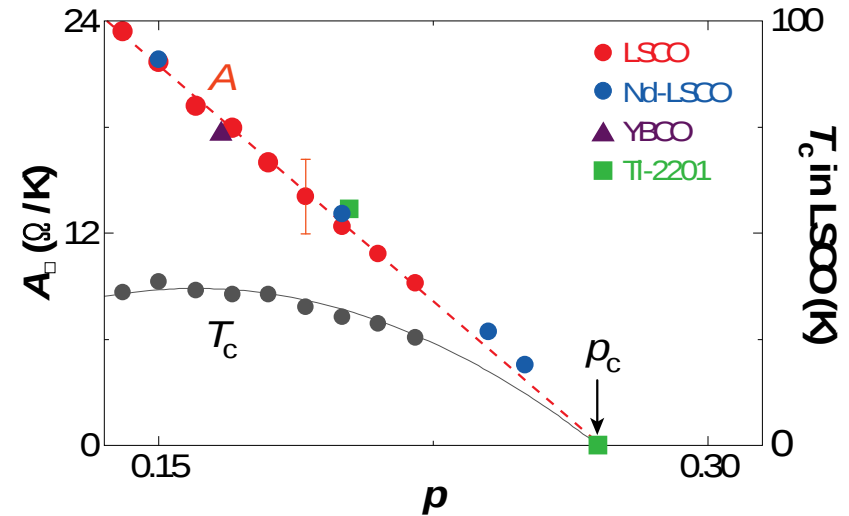
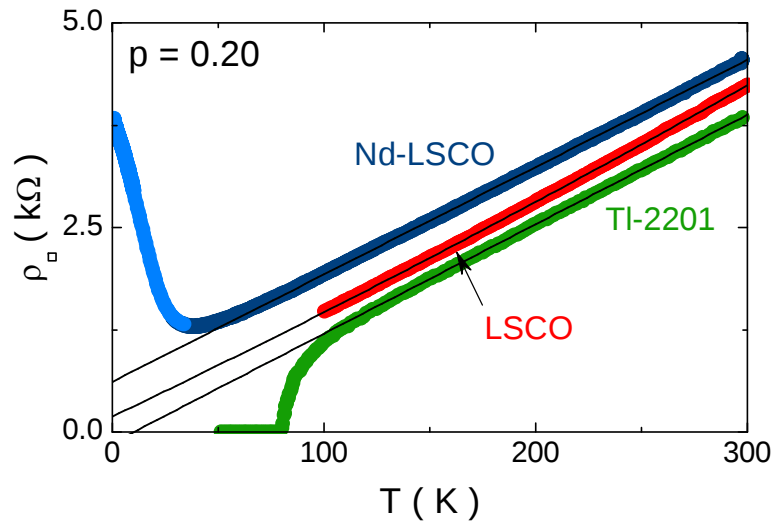
Doiron-Leyraud et al., PRB **80**, 214531 (2009)



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Hole-doped cuprates

Linear- T resistivity



Linear- T resistivity is universal in hole-doped cuprates

Correlation between linear- T resistivity and T_c

Doiron-Leyraud *et al.*, arXiv:0905.0964

Taillefer, Annual Review of CMP **1**, 51 (2010)



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Outline

- Phases of matter: Strong vs weak coupling
- Standard model: pillars
- Breakdown of band theory + BCS
- Model
- QCP in e-doped near optimal doping
- Hole –doped cuprates as doped Mott insulators.



What is a phase of matter?



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« Phase » and emergent properties

- Emergent properties
 - e.g. Fermi surface
 - Shiny
 - Quantum oscillations (in B field)
- Many microscopic models will do the same
 - Electrons in box or atoms in solid, Fermi surface
 - Often hard to « derive » from first principles (fractionalization - gauge theories)



Antiferromagnetic phase: emergent properties

- Some broken symmetries
 - Time reversal symmetry
 - Translation by one lattice spacing
 - Unbroken Time-reversal times translation by lattice vector $\mathbf{a}$
- Spin waves
- Single-particle gap



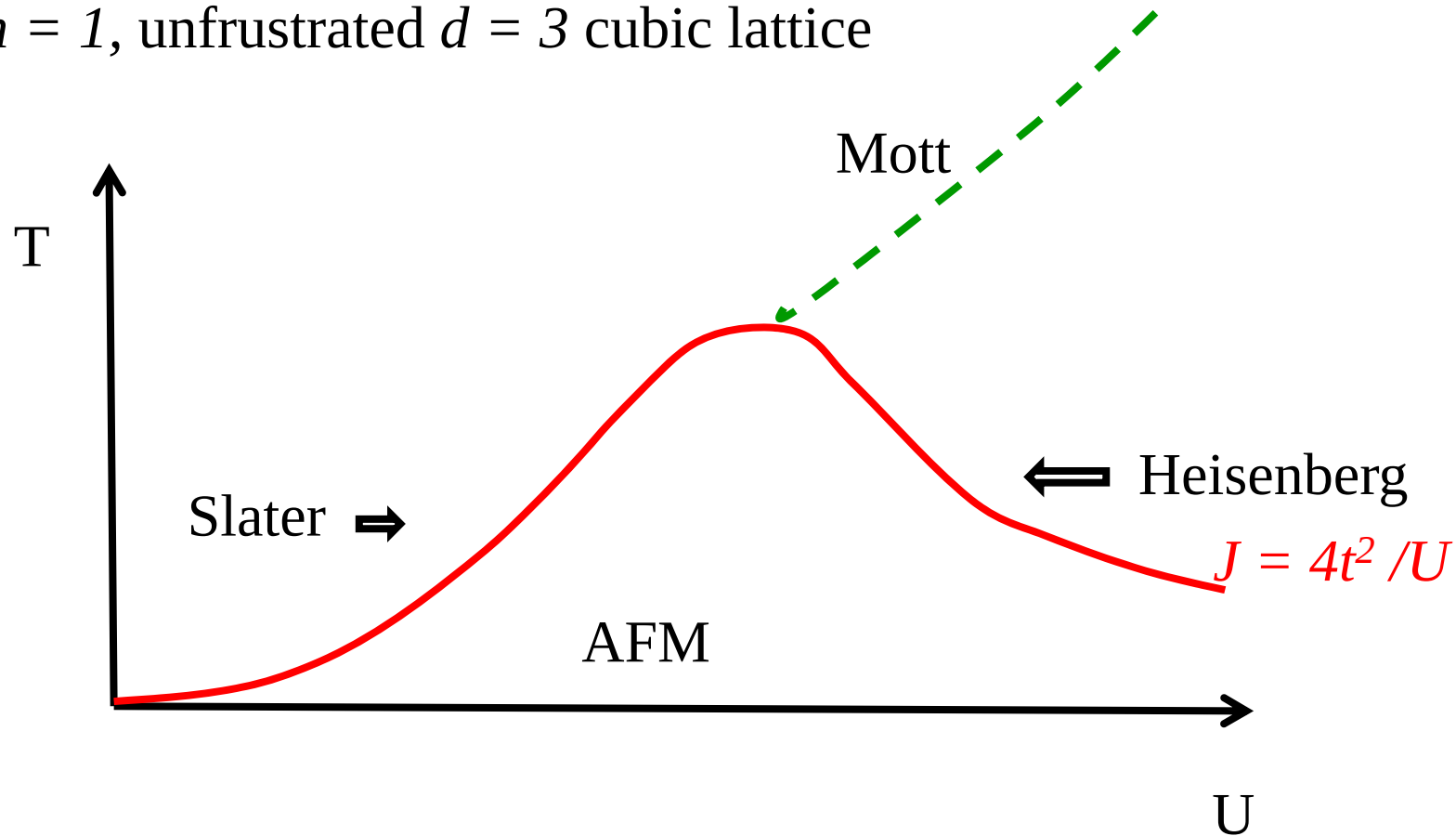
Differences between weakly and strongly correlated

- Different in ordered phase (finite frequency)
 - Ordered moment
 - Landau damping
 - Spin waves all the way or not to J
- Different, even more, in the normal state:
 - metallic in $d = 3$ if weakly correlated
 - Insulating if strongly correlated



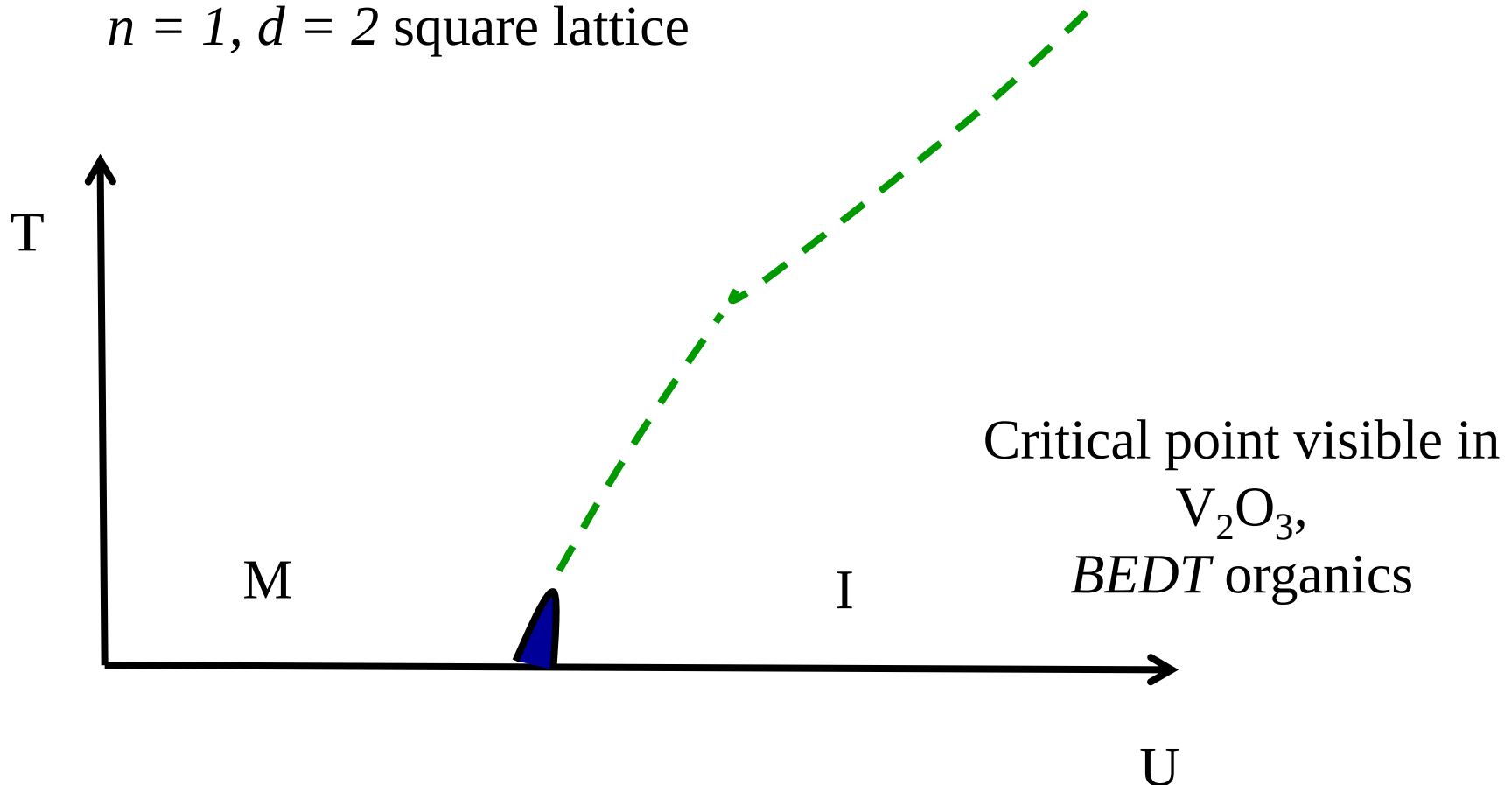
Local moment and Mott transition

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition

$n = 1, d = 2$ square lattice



Understanding finite temperature phase from a *mean-field theory* down to $T = 0$



Superconducting phase: identical properties

- Emergent:
 - Same broken symmetry $U(1)$ for s-wave,
 - $U(1)$ and C_{4v} for d-wave
 - Single-Particle gap, point or line node.
 - T dependence of C_p and κ at low T
 - Goldstone modes (Higgs)



Superconductivity not universal even with phonons: weak or strong coupling

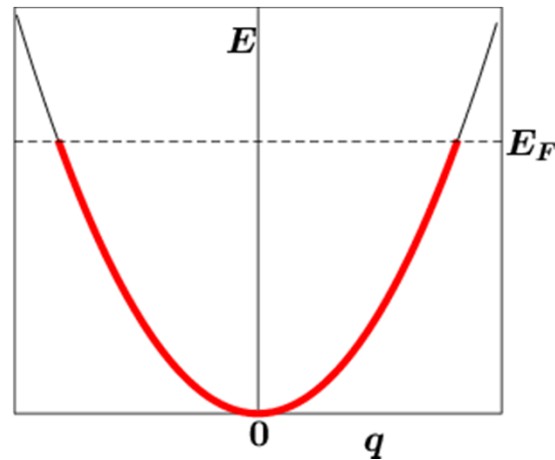
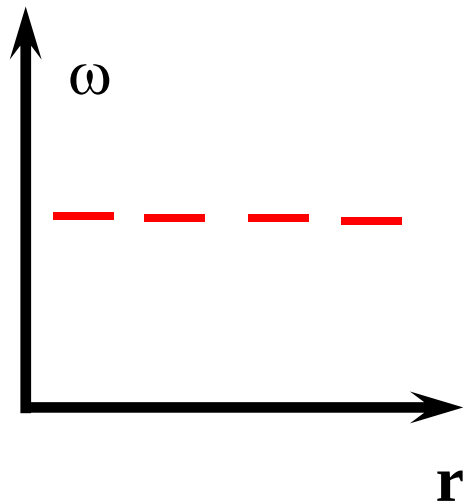
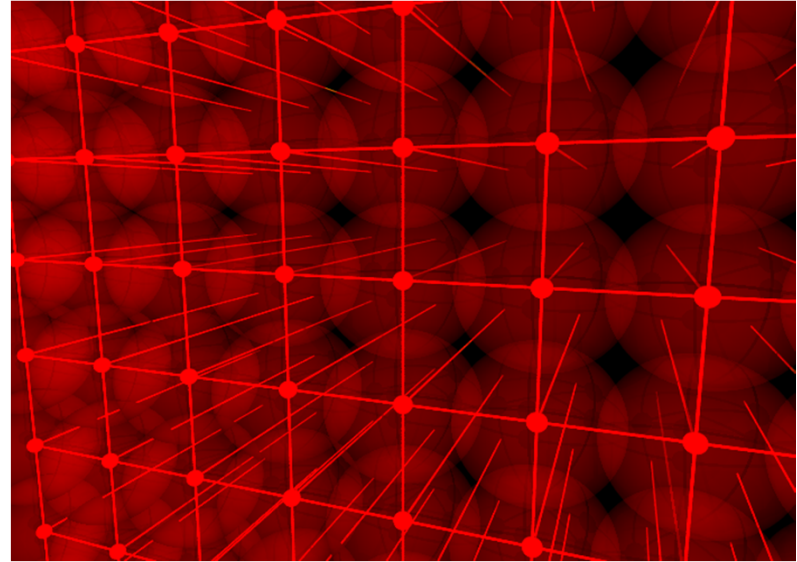
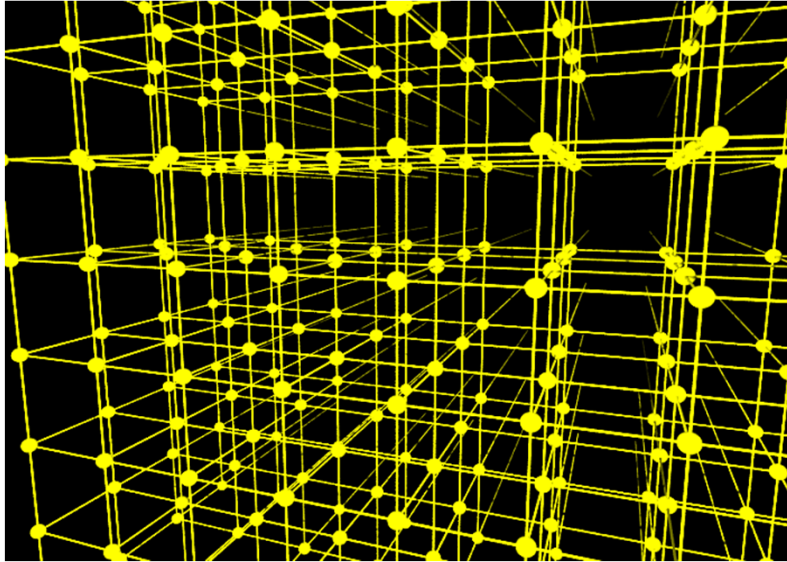
- In BCS universal ratios: e.g. $\Delta/k_B T_c$
 - Would never know the mechanism for sure if only BCS!



Two pillars of Solid State Physics



How to make a metal



Courtesy, S. Julian

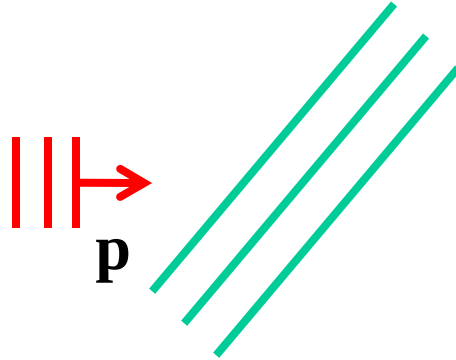


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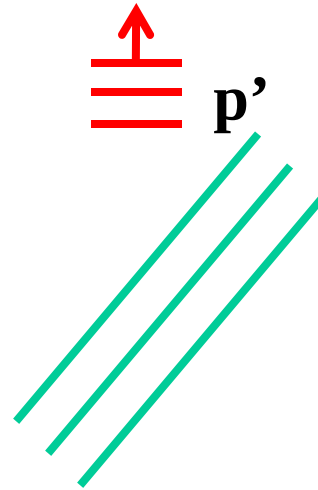
Superconductivity



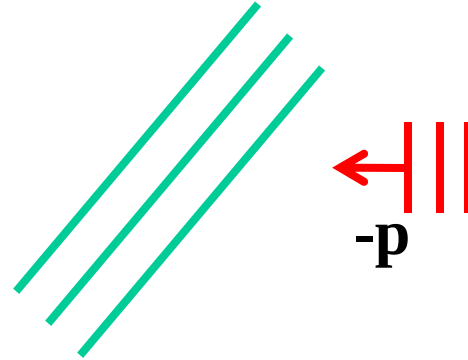
Attraction mechanism in the metallic state



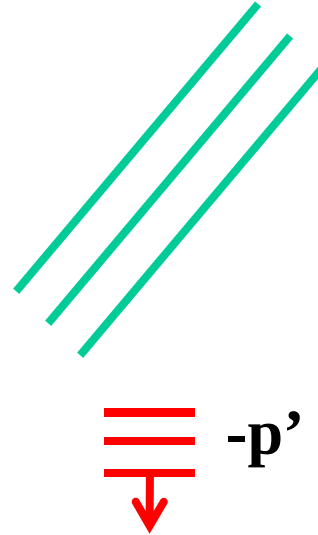
Attraction mechanism in the metallic state



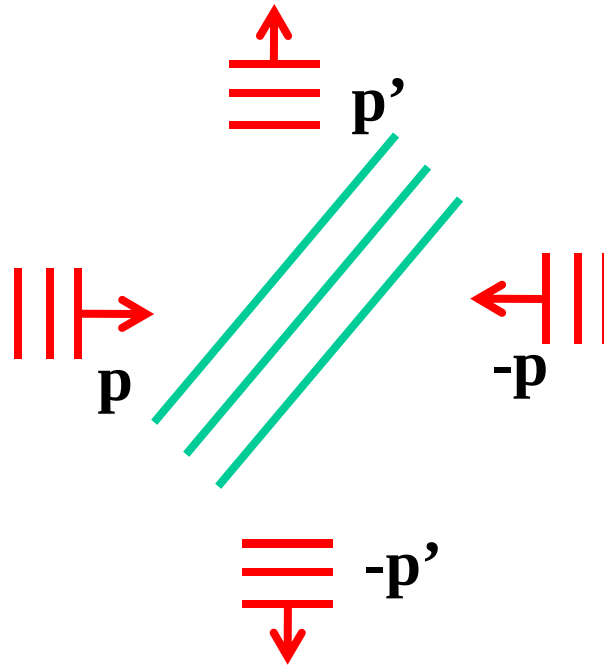
Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



Attraction mechanism in the metallic state



#1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^*$$

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left(\langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* + \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \right)$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$



Breakdown of band theory

Half-filled band is metallic?



Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937



Mott, 1949



« Conventional » Mott transition

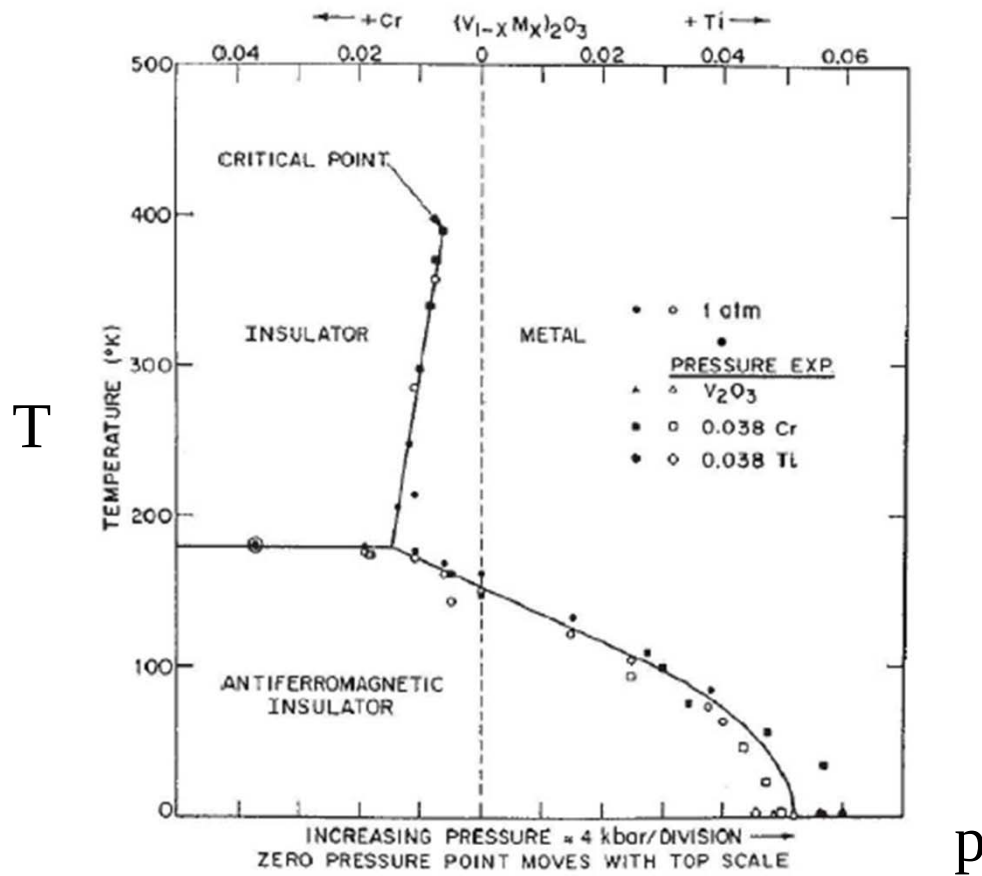
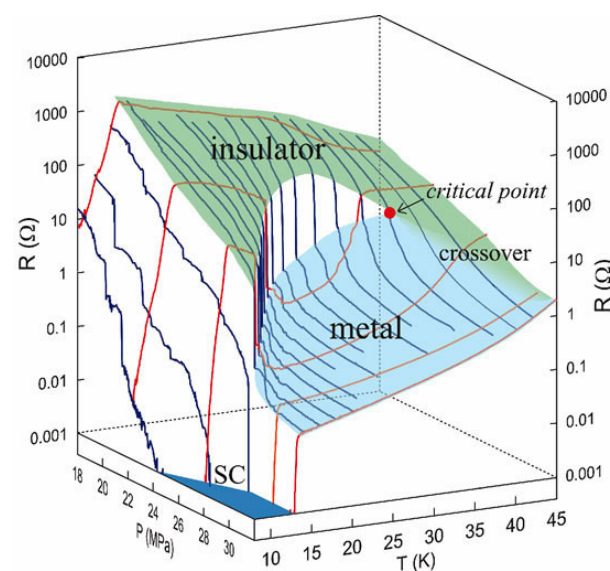
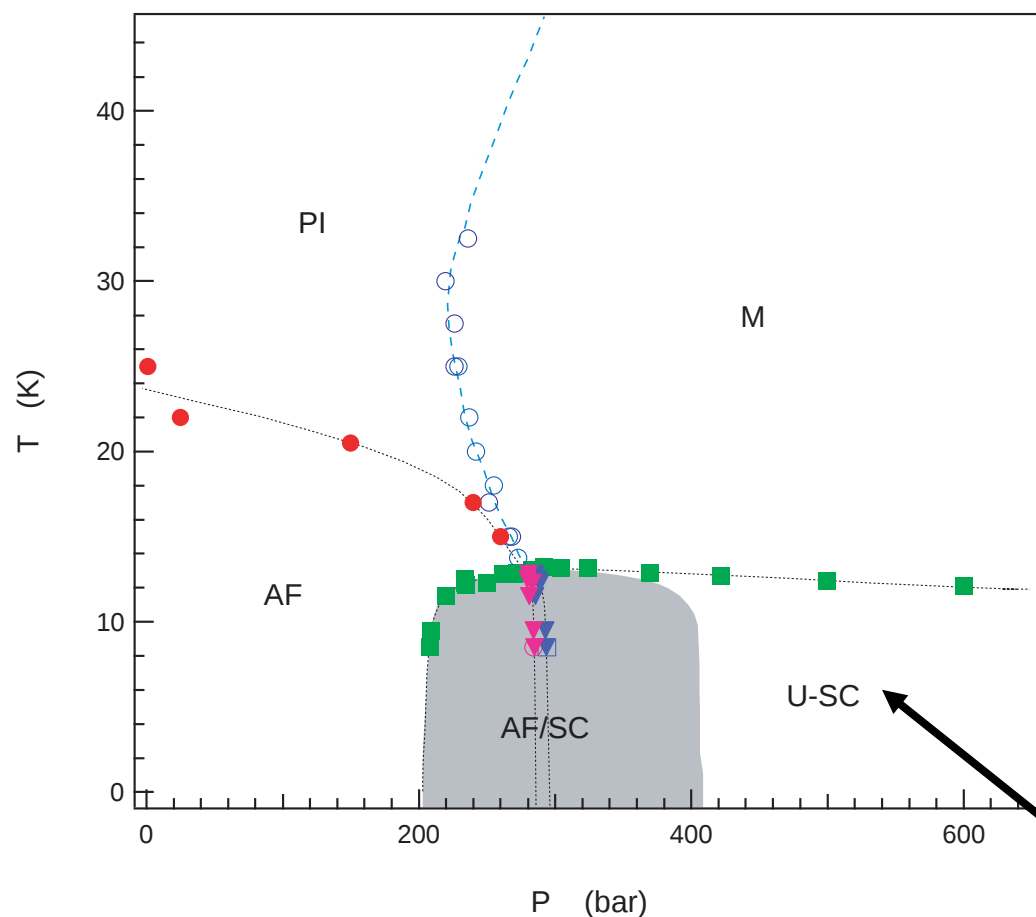


Figure: McWhan, PRB 1970; Limelette, Science 2003

Experimental phase diagram for Cl



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

B_g for C_{2h} and B_{2g} for D_{2h}

Powell, McKenzie cond-mat/0607078

Phase diagram ($X = \text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)



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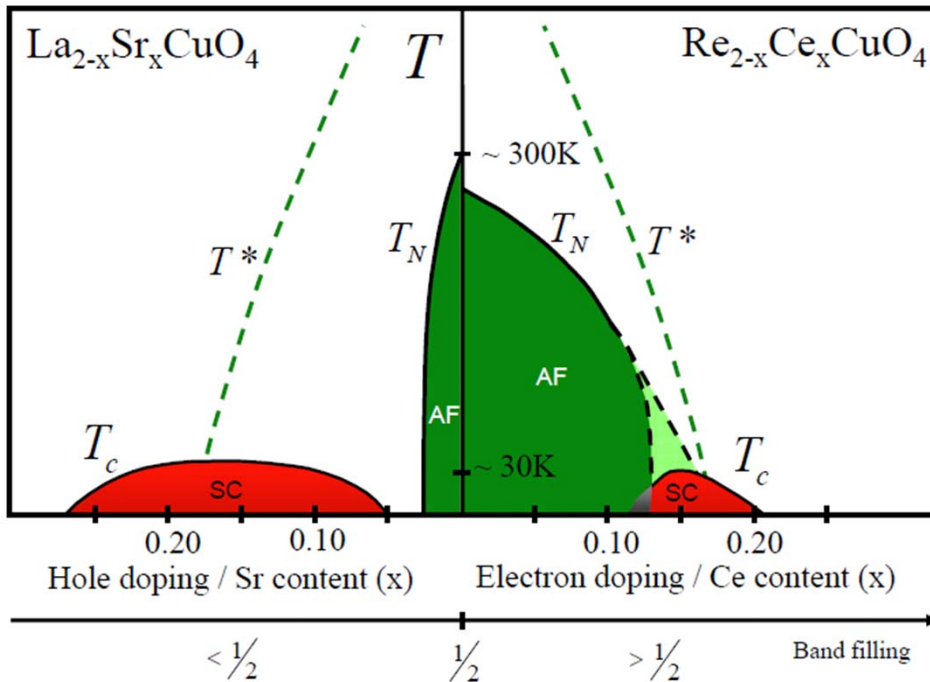
Where is the mystery?



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High-temperature superconductors

Armitage, Fournier, Greene, RMP (2009)



- Competing order

- Current loops: Varma, PRB **81**, 064515 (2010)
- Stripes or nematic: Kivelson et al. RMP 75 1201(2003); J.C.Davis
- d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
- SDW: Sachdev PRB **80**, 155129 (2009) ...

- Or Mott Physics?

- RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

What is under the dome?
Mott Physics away from $n = 1$



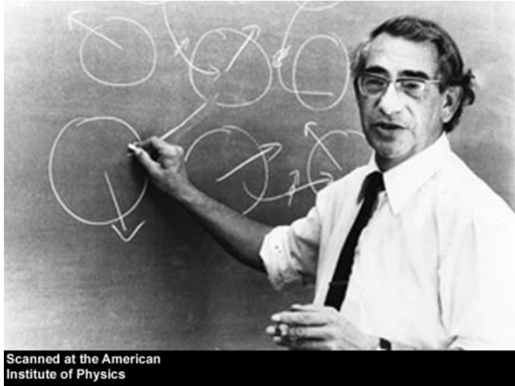
Anomalous properties

- Strong particle-hole asymmetry in DOS even in SC state.
- Mott Ioffe-Regel limit
- Spectral weight transfer in XAS
- No sharp phase transition at pseudogap
- Carriers are hole measured with respect to half-filling (Hall, thermopower etc...)
- Superfluid density not BCS

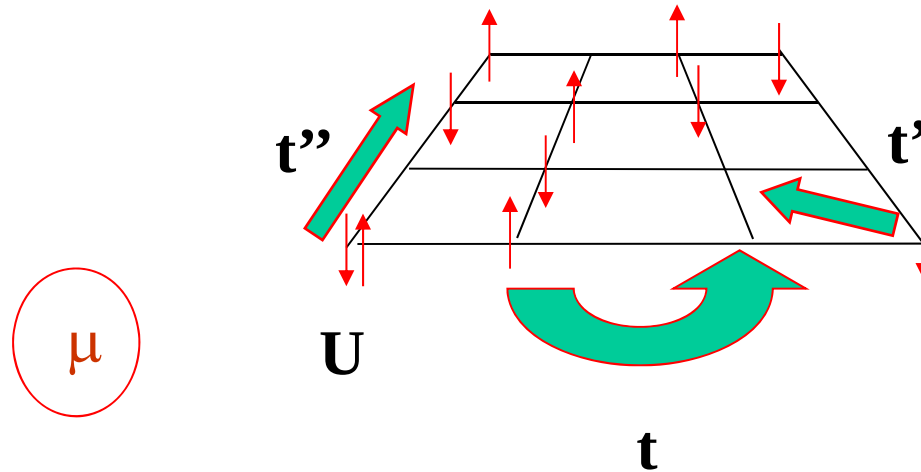


Model

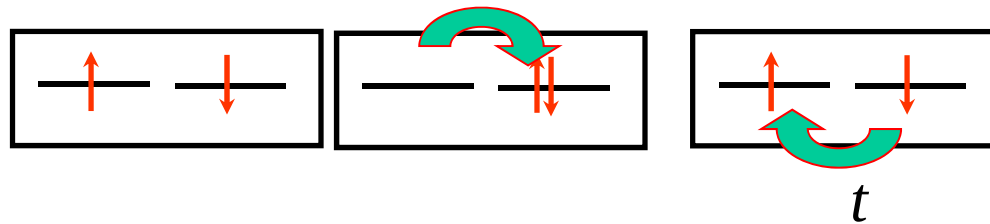
Hubbard model



1931-1980



$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$t = 1$

Effective model, Heisenberg: $J = 4t^2 / U$



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The normal state is not always normal
A first case: MIR



Mott-Ioffe-Regel limit

$$\sigma = \frac{ne^2\tau}{m}$$

$$n = \frac{1}{2\pi d} k_F^2$$

$$\sigma = \left(\frac{1}{2\pi d} k_F^2 \right) \frac{e^2\tau}{m}$$

$$\ell = \left(\frac{\hbar k_F}{m} \right) \tau$$

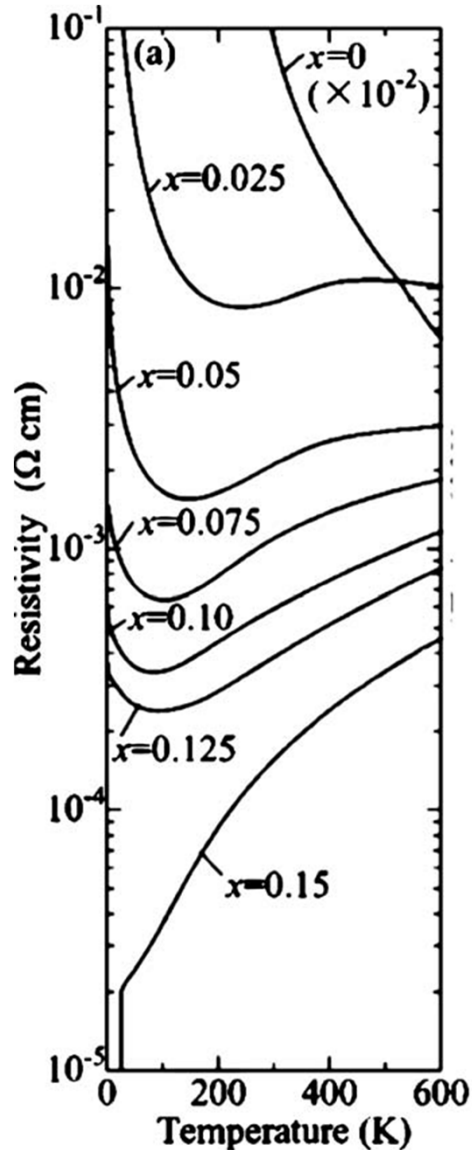
$$\sigma = \frac{1}{2\pi d} k_F e^2 \left(\frac{\ell}{\hbar} \right)$$

$$k_F \ell = \frac{2\pi}{\lambda_F} \ell \sim 2\pi$$

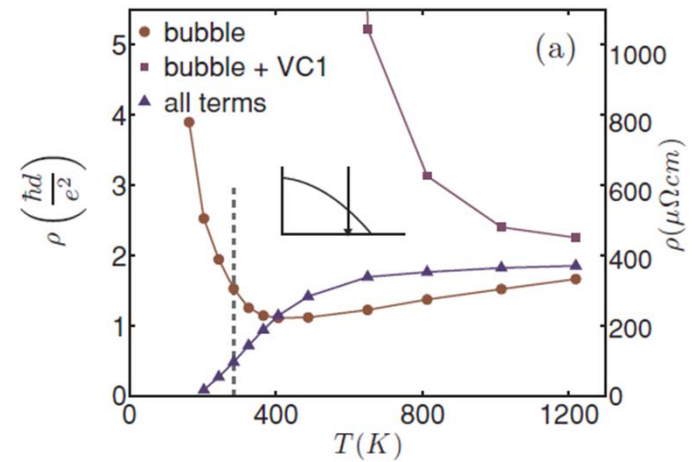
$$\sigma_{MIR} = \frac{e^2}{\hbar d}$$



Electron-doped and MIR limit



NCCO



Dominic Bergeron et al. TPSC
PRB **84**, 085128 (2011)

Onose et al. 2004

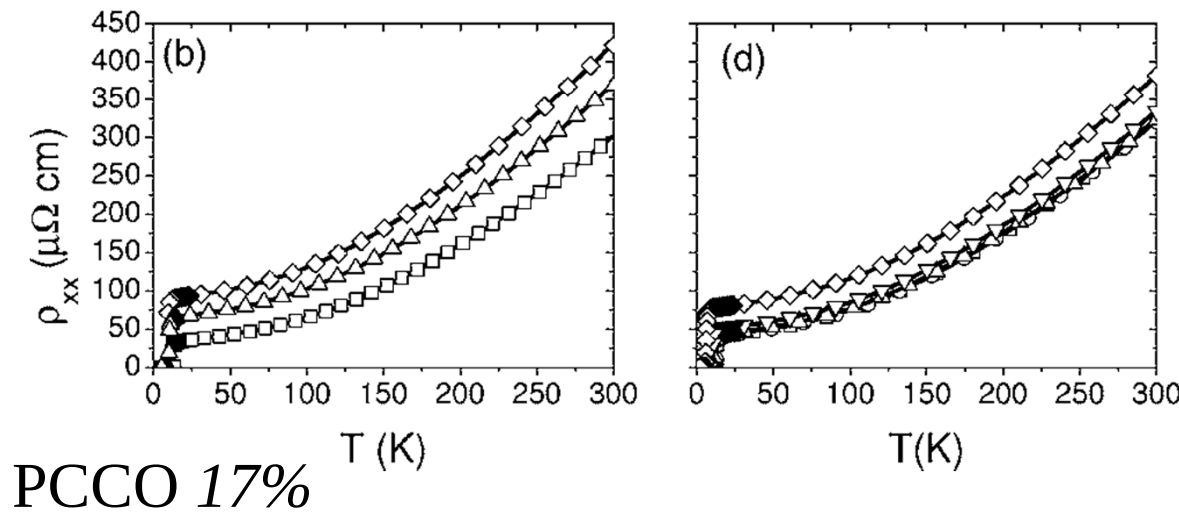


Quantum criticality vs AFM in e-doped

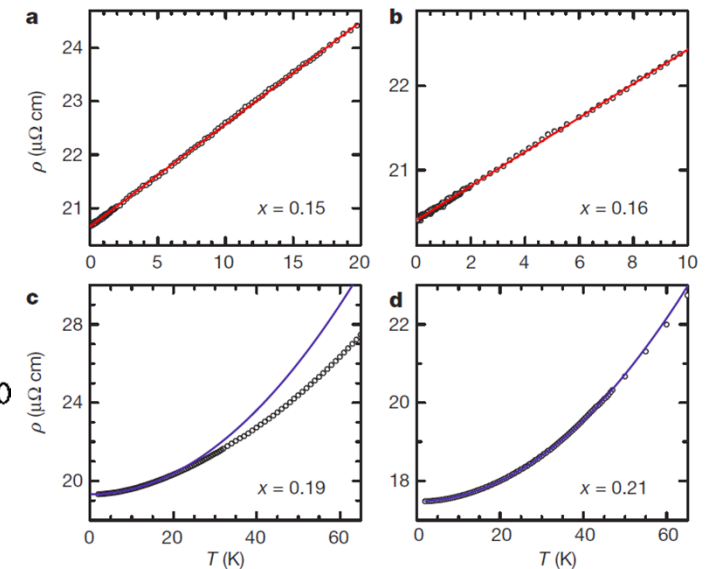


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Electron-doped and MIR limit

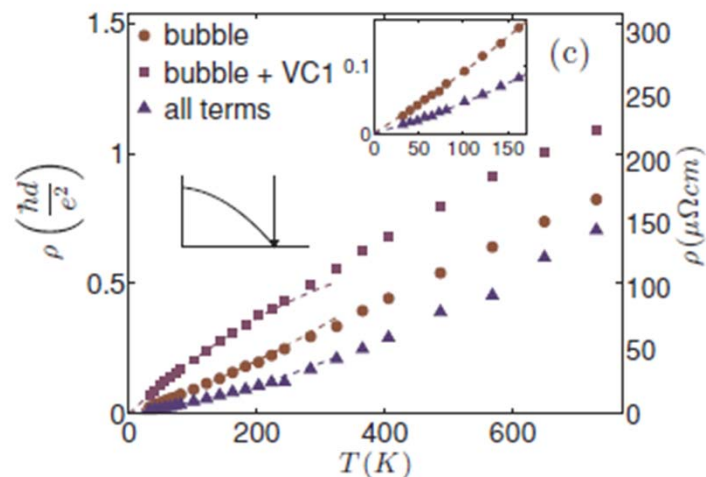


Higgins et al. PRB 73, 104510 (2006)



LCCO

Jin et al., Nature 2011

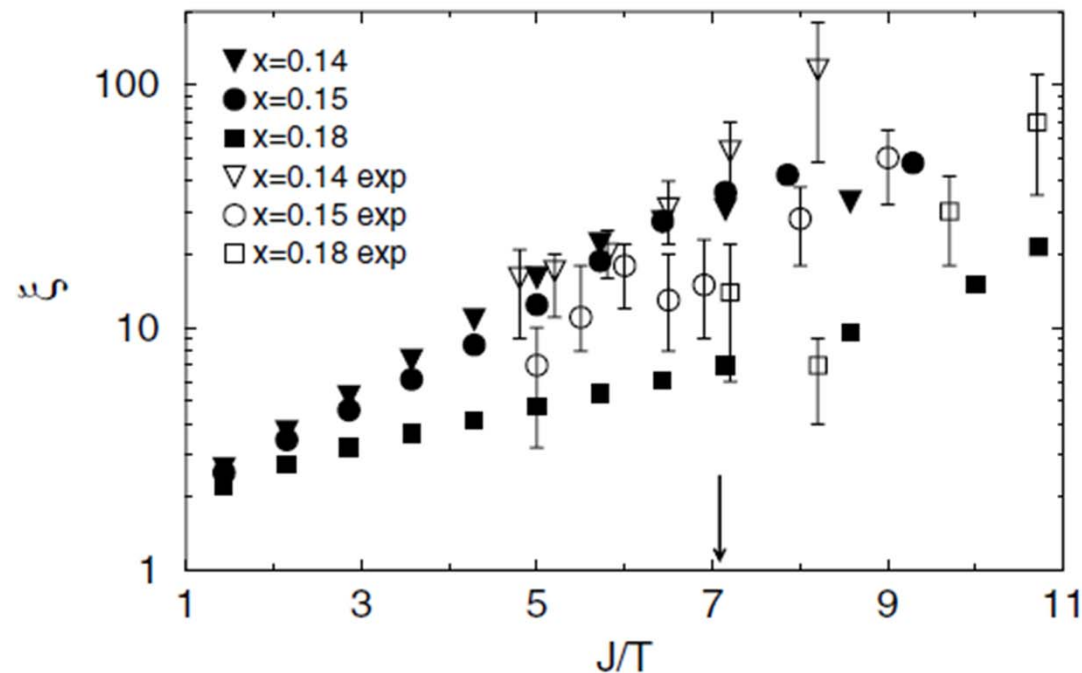


Dominic Bergeron et al. TPSC
PRB 84, 085128 (2011)



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TPSC vs experiment for ξ



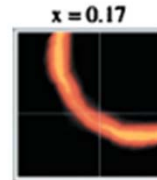
Kyung et al. PRL **93**, 147004 (2004)

P. K. Mang et al., Phys. Rev. Lett. **93**, 027002 (2004).

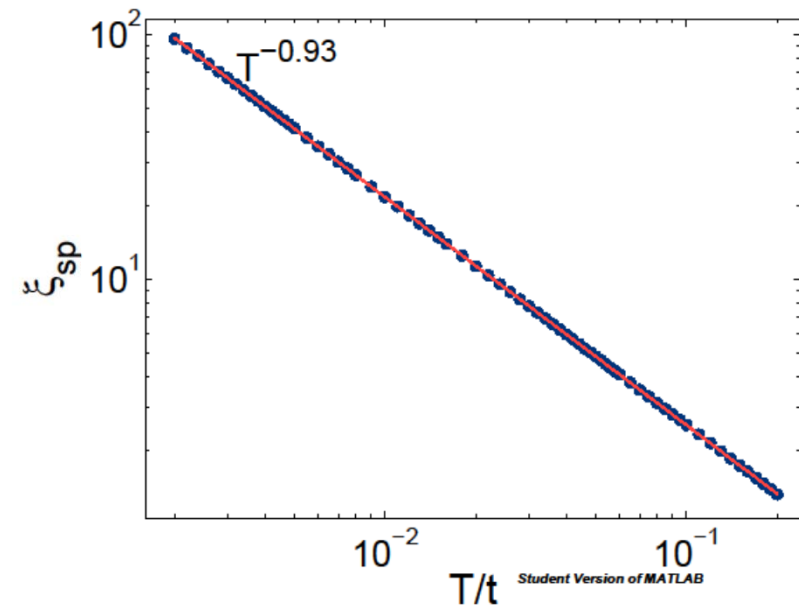
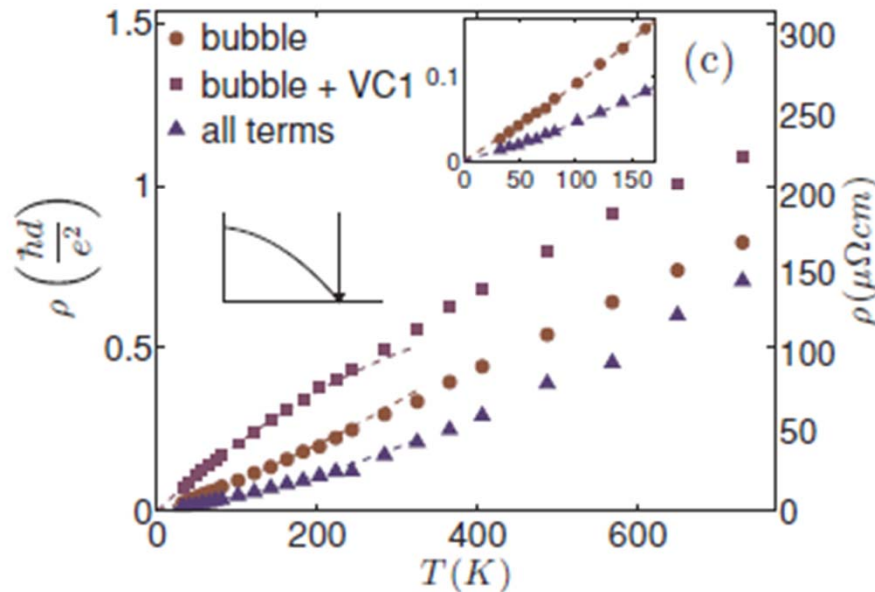
M. Matsuda et al., Phys. Rev. B **45**, 12 548 (1992).



$\xi(T)$ at the QCP



NCCO
Matsui et al. PRB 2007



$z = 1$ Motoyama, Nature 2007

$$U=6, t'=-0.175, t''=0.05, n=1.2007$$

Dominic Bergeron TPSC



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AFM QCP means there is also a pseudogap driven by AFM fluctuations



Three broad classes of mechanisms for pseudogap

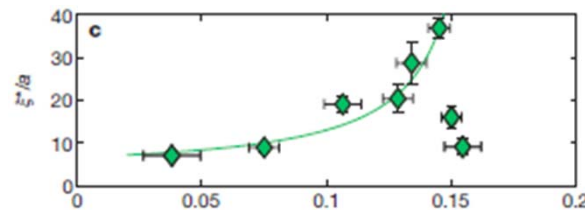
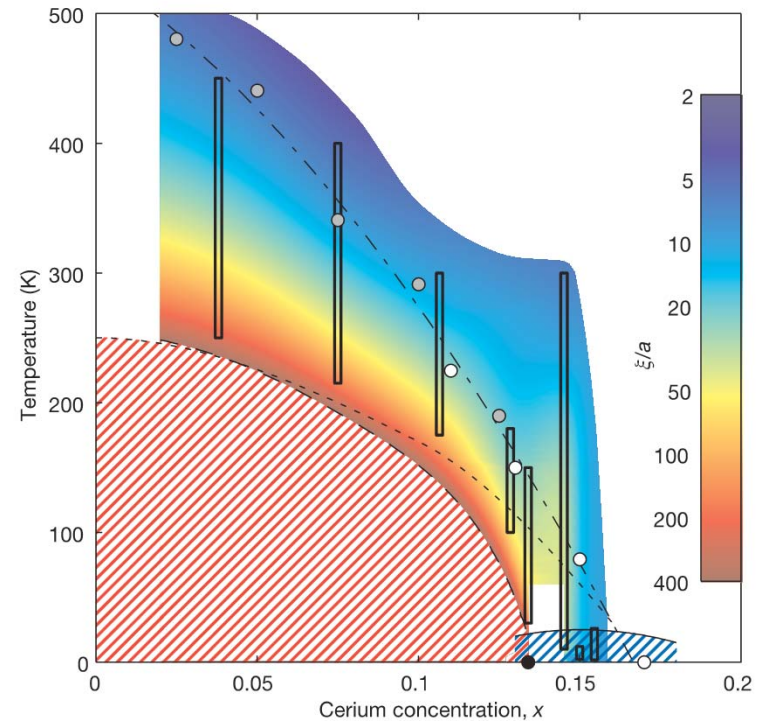
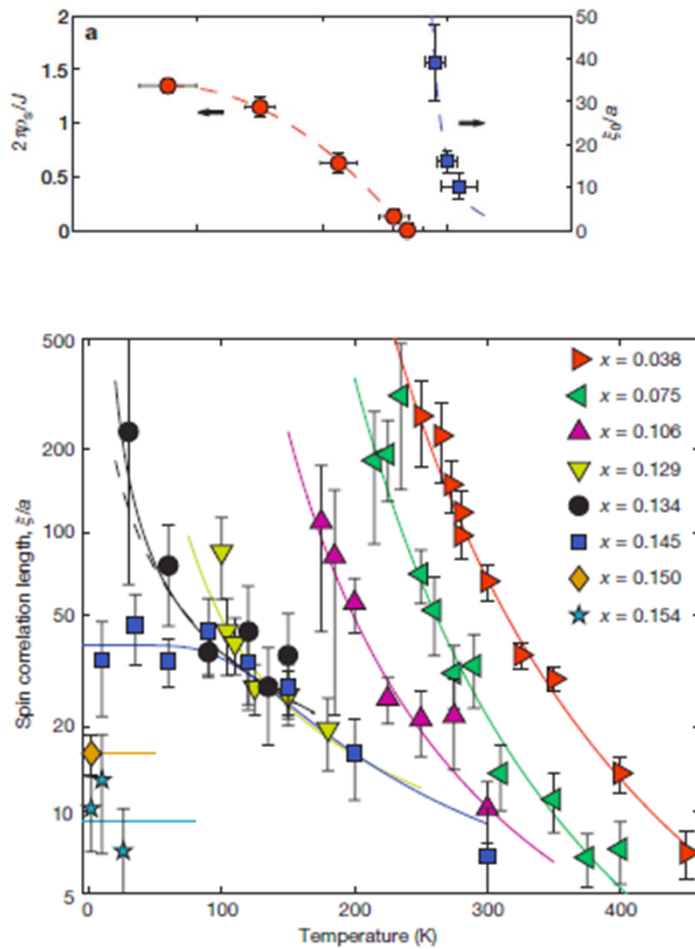
- Rounded first order transition
- $d = 2$ precursor to a lower temperature broken symmetry phase
- Mott physics
- Competing order
 - Current loops: Varma, PRB **81**, 064515 (2010)
 - Stripes or nematic: Kivelson et al. RMP 75 1201(2003); J.C.Davis
 - d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
 - SDW: Sachdev PRB **80**, 155129 (2009) ...
- Or Mott Physics?
 - RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)



E-doped quantum critical

NCCCO

Motoyama, E. M. et al.. Nature 445, 186–189 (2007).



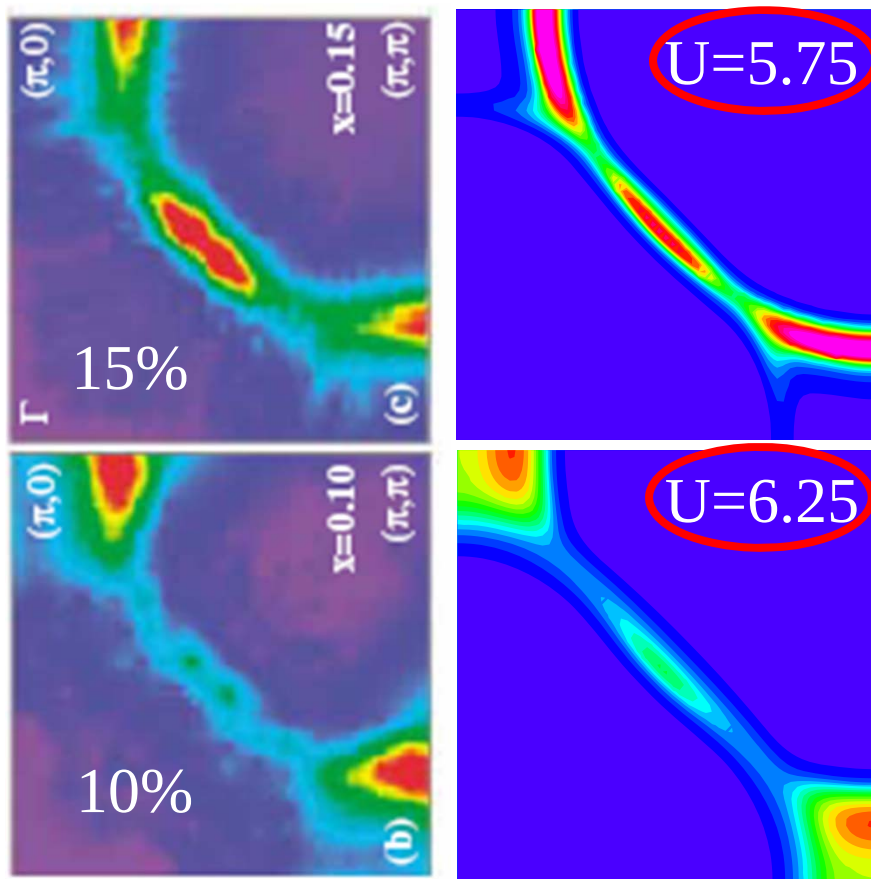
$$\xi^* = 2.6(2)\xi_{th}$$



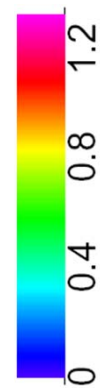
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Fermi surface plots

Hubbard repulsion U has to...



be not too large



increase for
smaller doping

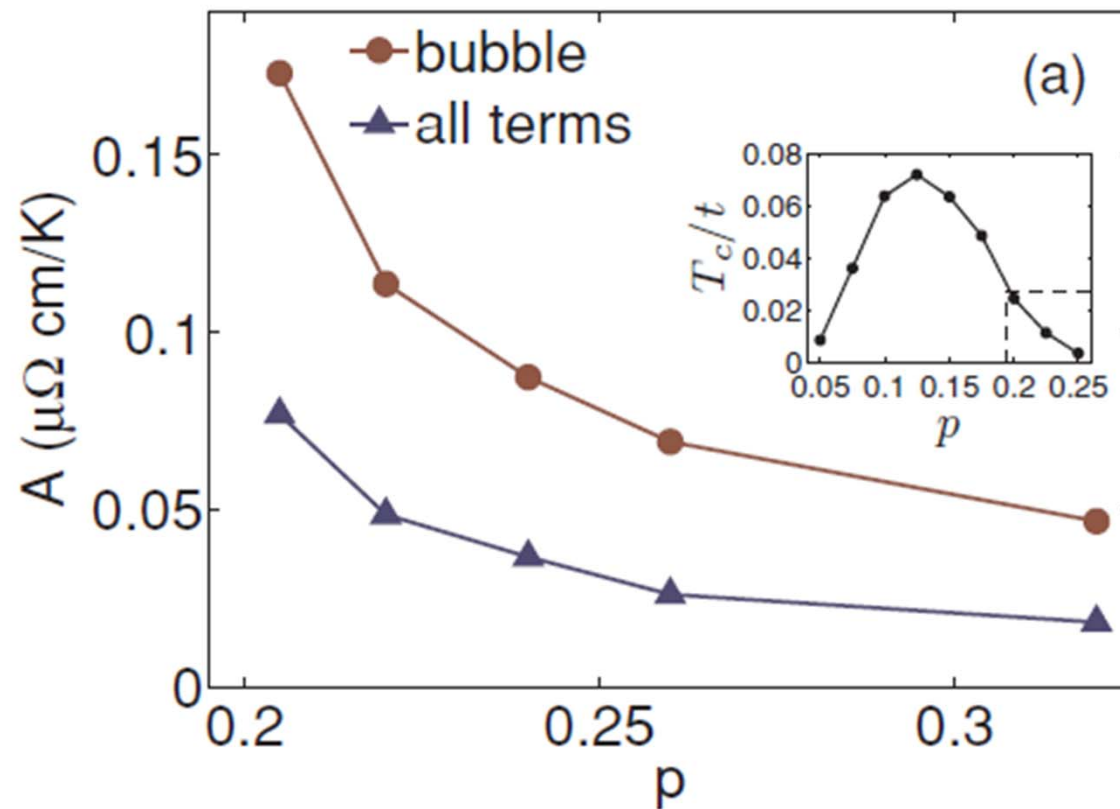
Hankevych, Kyung, A.-M.S.T., PRL, sept. 2004

B.Kyung *et al.*, PRB **68**, 174502 (2003)



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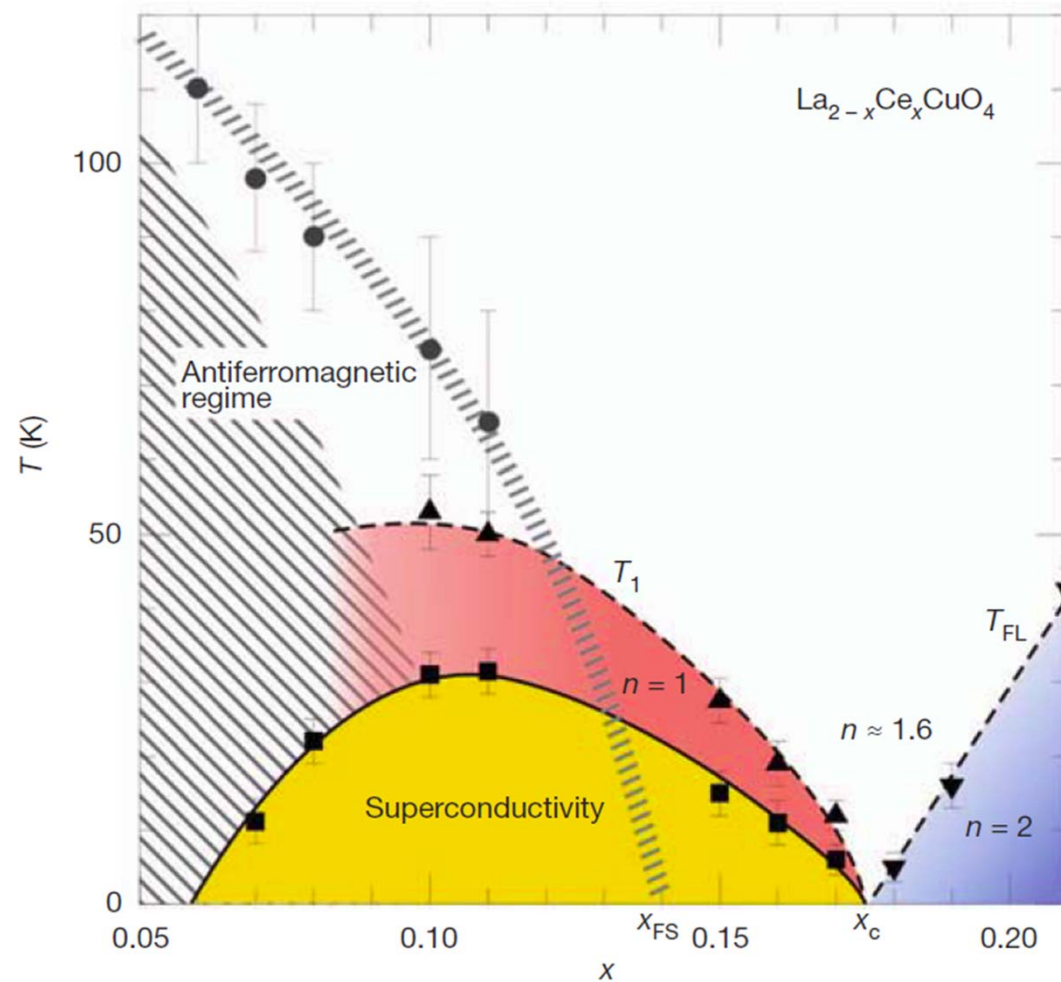
Correlation resistivity vs T_c



Dominic Bergeron et al. TPSC
PRB **84**, 085128 (2011)



Extended quantum criticality



Jin et al. Nature 2011



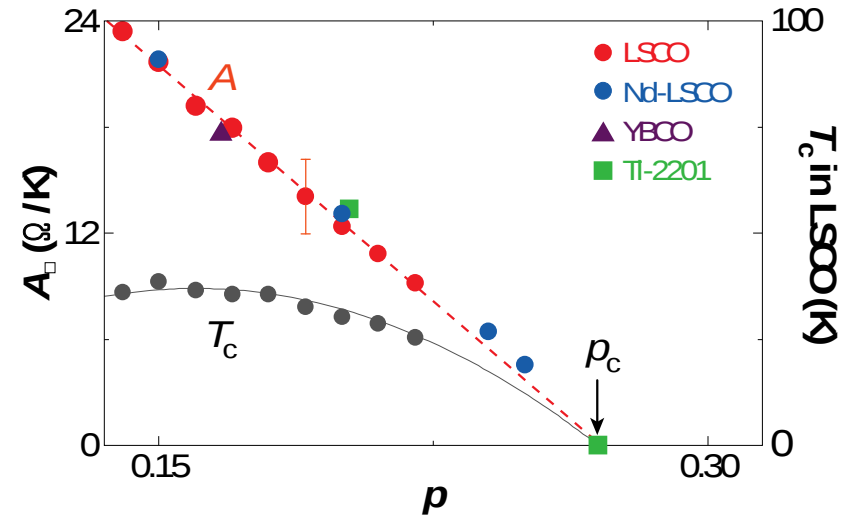
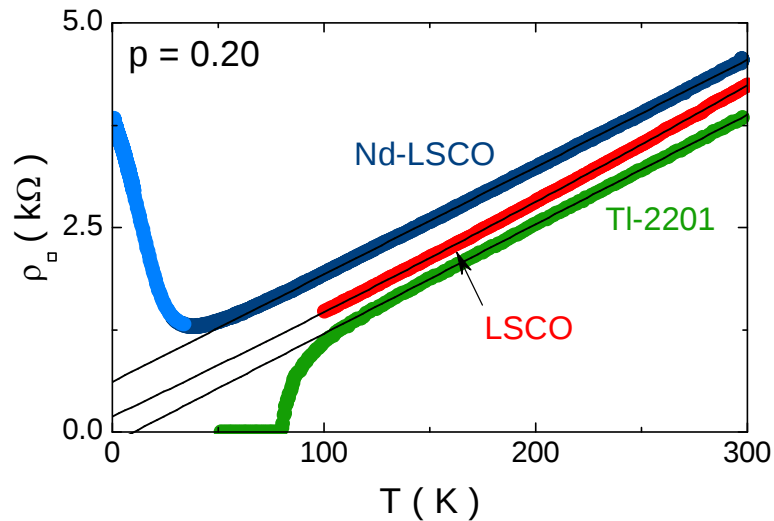
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Hole-doped cuprates as Mott insulators



Hole-doped cuprates

Linear- T resistivity



Linear- T resistivity is universal in hole-doped cuprates

Correlation between linear- T resistivity and T_c

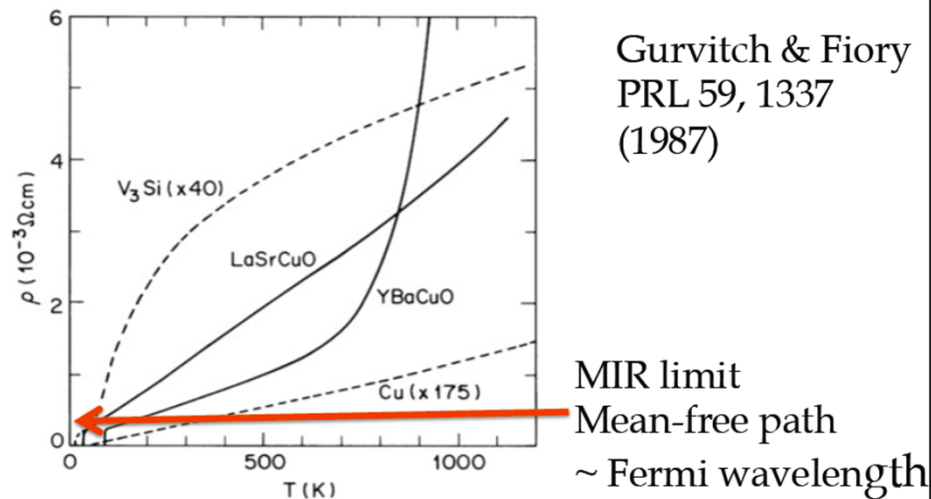
Doiron-Leyraud *et al.*, arXiv:0905.0964

Taillefer, Annual Review of CMP **1**, 51 (2010)

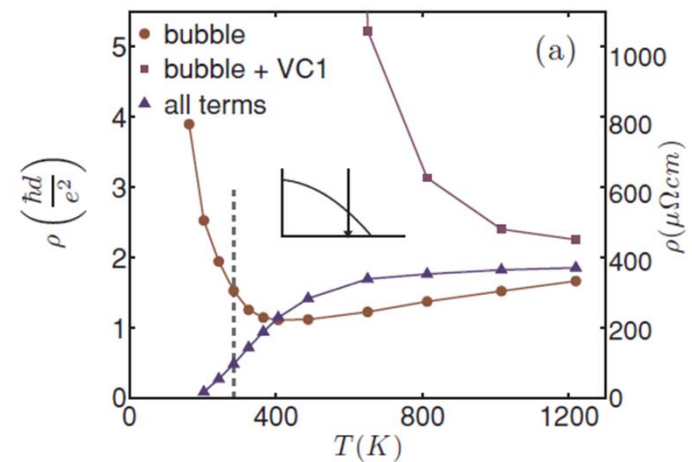


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Hole-doped cuprates and MIR limit



LSCO 17%, YBCO optimal



Dominic Bergeron TPSC

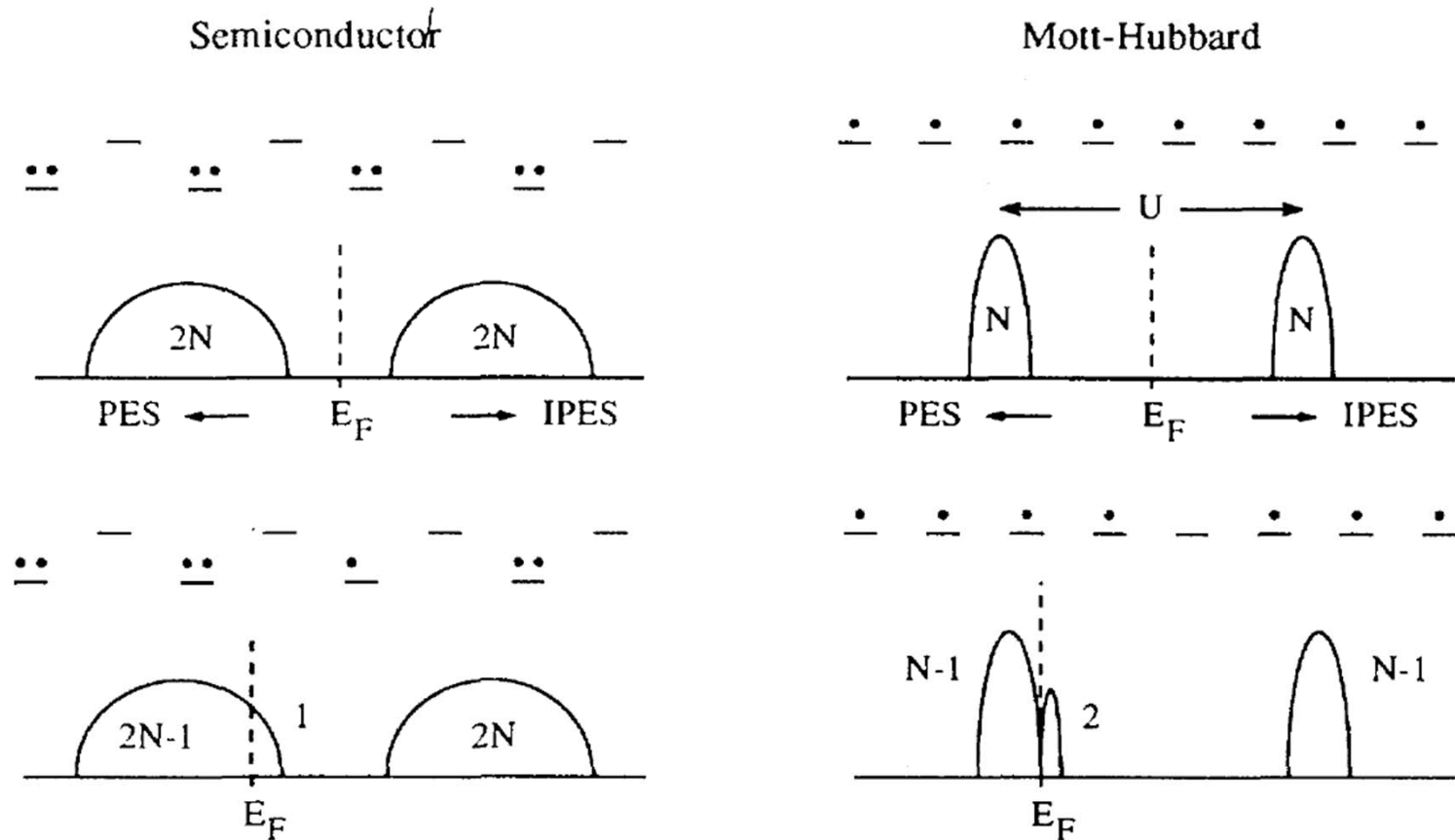
PHYSICAL REVIEW B **84**, 085128 (2011)

Optical and dc conductivity of the two-dimensional Hubbard model in the pseudogap regime and across the antiferromagnetic quantum critical point including vertex corrections



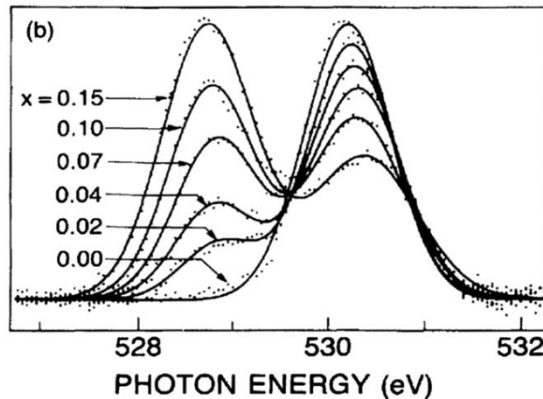
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Spectral weight transfer

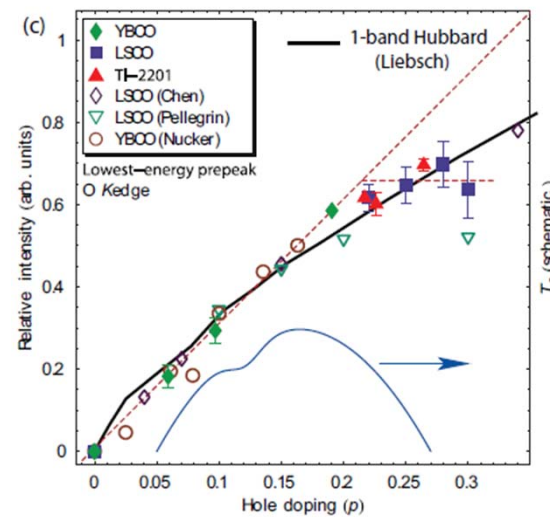


Meinders *et al.* PRB **48**, 3916 (1993)

Experiment: X-Ray absorption



Chen et al. PRL **66**, 104 (1991)



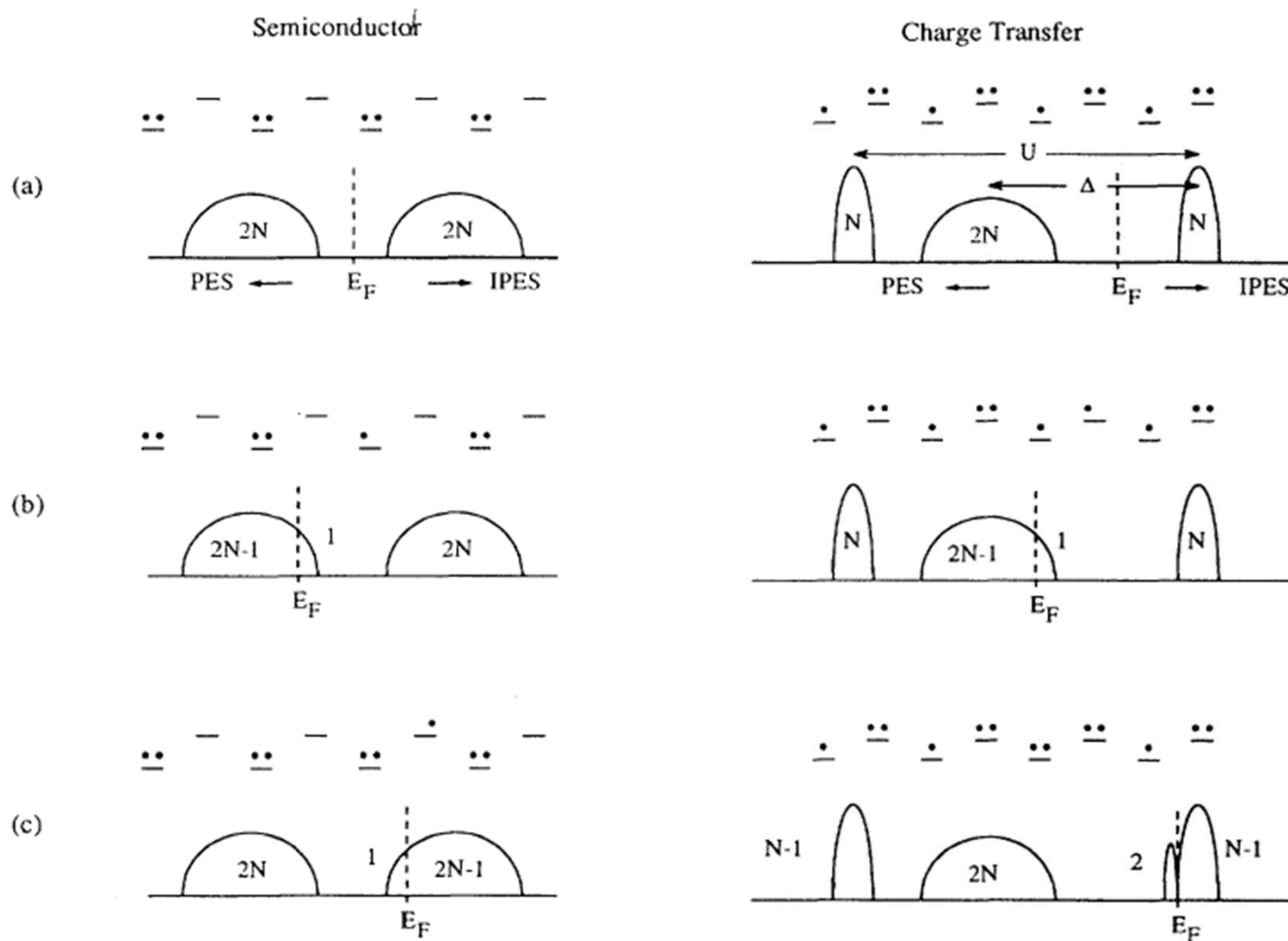
Peets et al. PRL **103**, (2009),

Phillips, Jarrell PRL , vol. **105**, 199701 (2010)

Number of low energy states above $\omega = 0$ scales as $2x +$
Not as $1+x$ as in Fermi liquid

Meinders *et al.* PRB **48**, 3916 (1993)

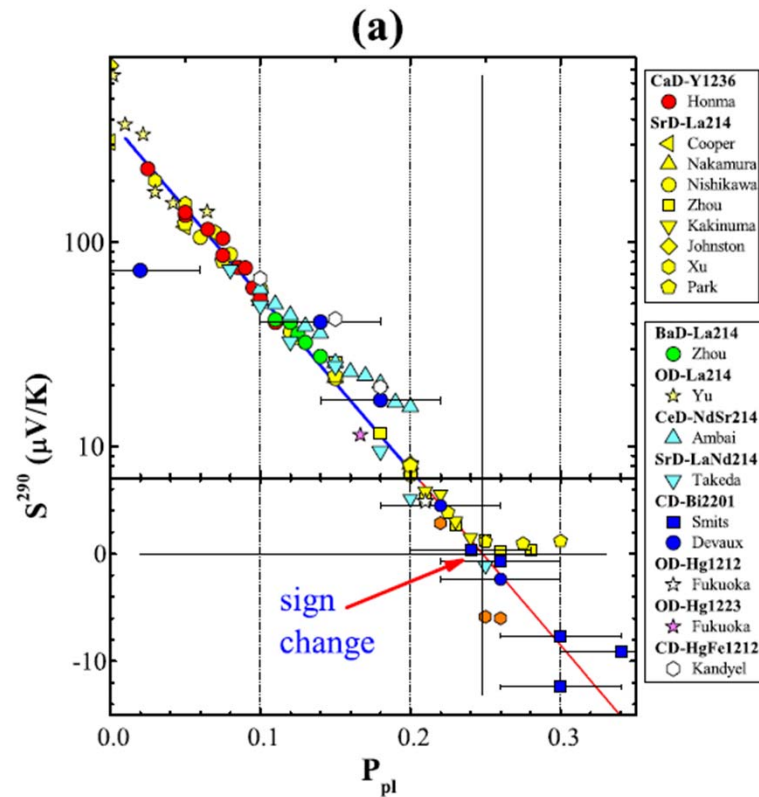
Charge-transfer insulator



Meinders *et al.* PRB **48**, 3916 (1993)

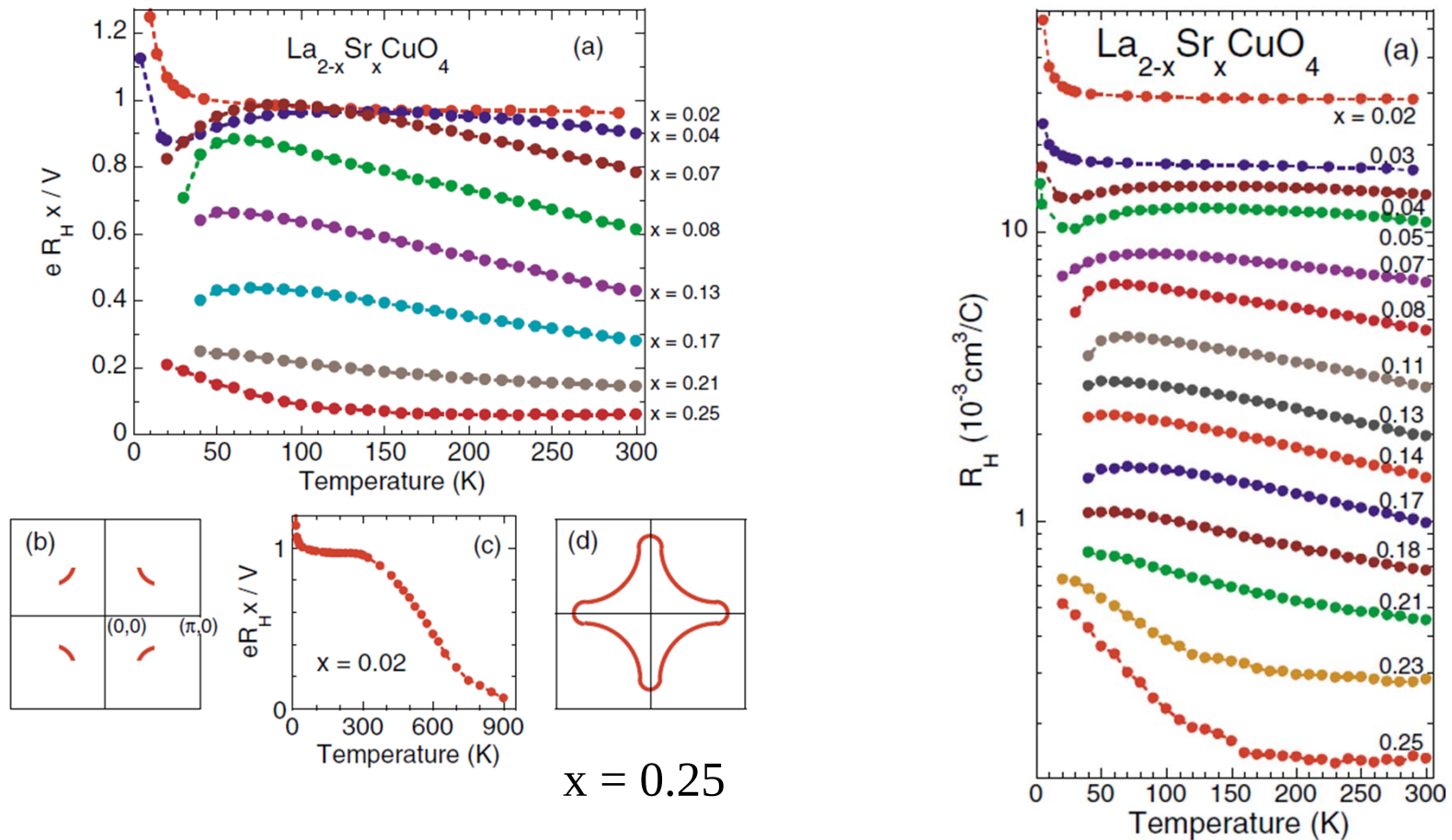


Thermopower



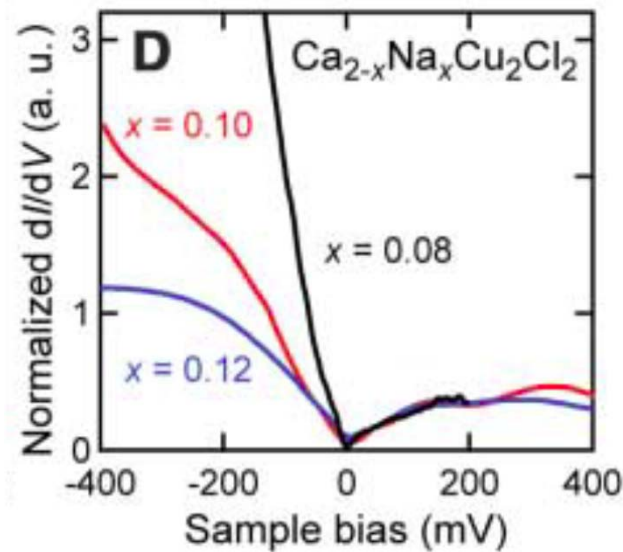
T. Honma and P. H. Hor, Phys. Rev. B 77, 184520 (2008).

Hall coefficient



Ando et al. PRL **92**, 197001 (2004)

Density of states (STM)



Khosaka et al. *Science* **315**, 1380 (2007);

