

# Superconductivity, antiferromagnetism and Mott critical point in the BEDT family



A.-M. Tremblay

P. Sémon, G. Sordi, K. Haule,  
B. Kyung, D. Sénéchal



ISCOM 2013, 14 – 19 July 2013



# Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937

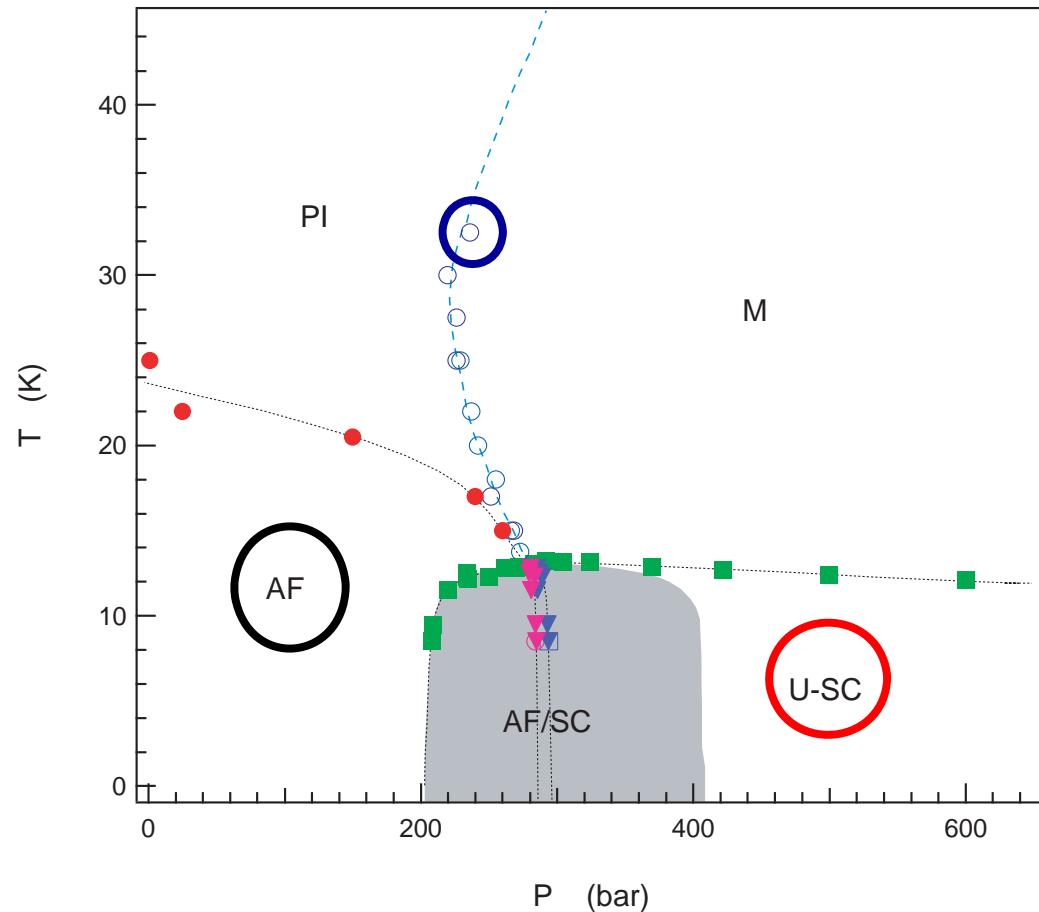


Mott, 1949



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# Mott, antiferromagnetism, superconductivity

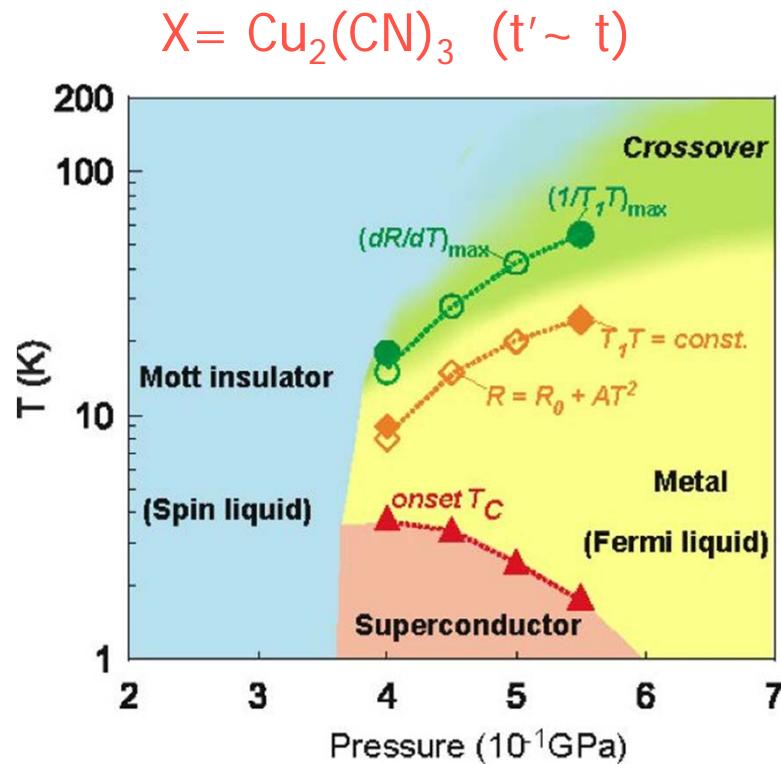


F. Kagawa, K. Miyagawa, + K. Kanoda  
PRB **69** (2004) +Nature **436** (2005)

Phase diagram ( $X = \text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ )

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

# « Spin liquid »



Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)

Y. Kurisaki, et al.  
Phys. Rev. Lett. **95**, 177001(2005)

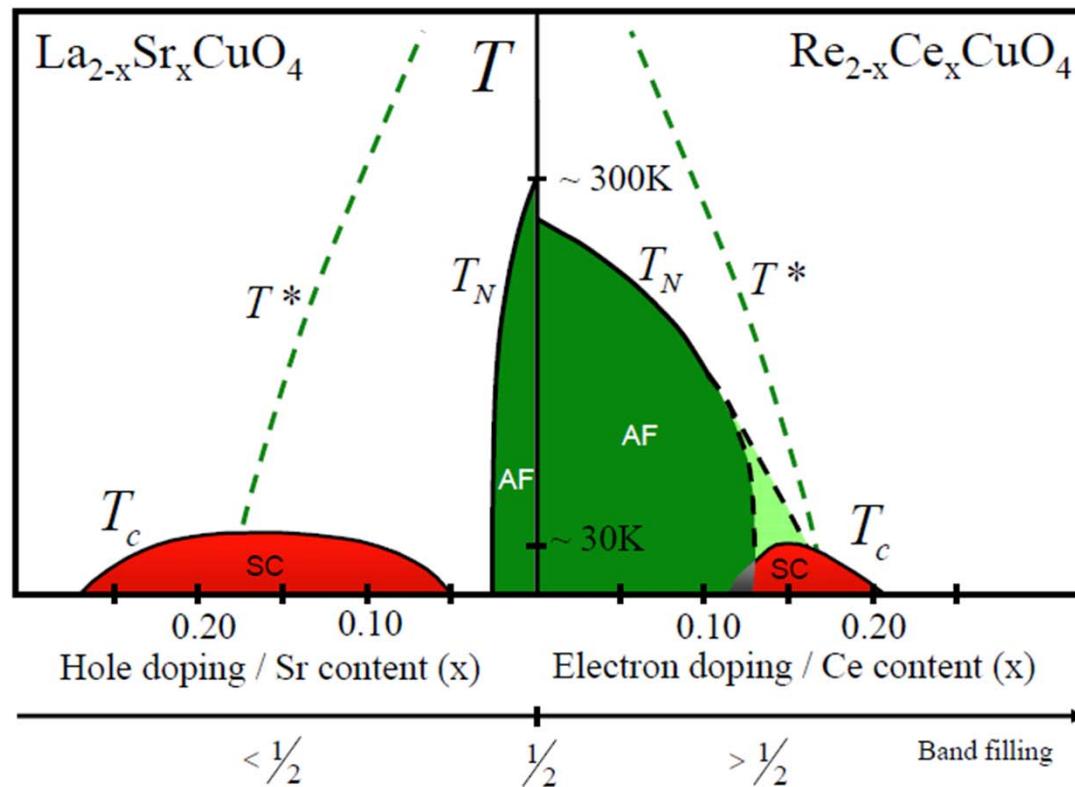
Reviews:

B J Powell and Ross H McKenzie, Rep. Prog. Phys. **74** (2011) 056501

K. Kanoda and R. Kato, Annu. Rev. Condens.Matter Phys. **2011**. 2:167–88

# Competing order or Mott Physics ?

Armitage, Fournier, Greene, RMP (2009)



# Outline

- Model and method
  - $T=0$
  - Finite  $T$
- Phase diagram: Organics and cuprates
  - $T = 0$
  - Finite  $T$
- The Mott critical point and critical exponents  
PII-36

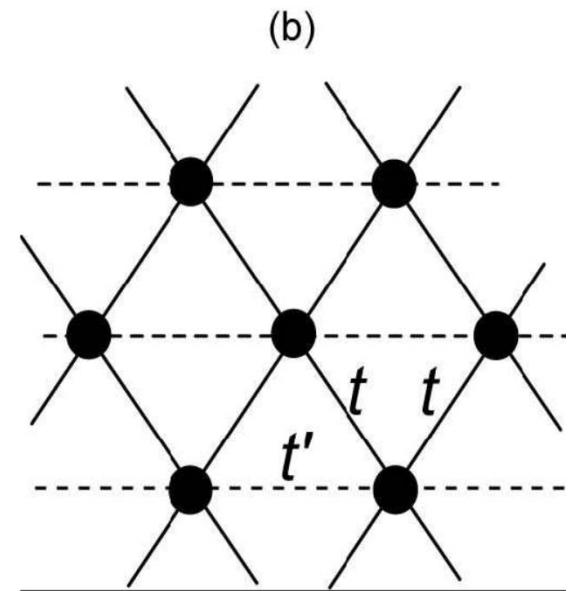
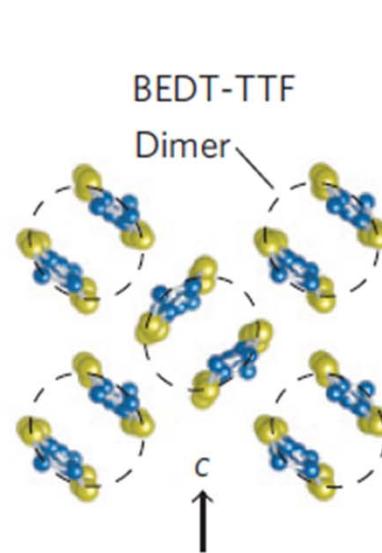
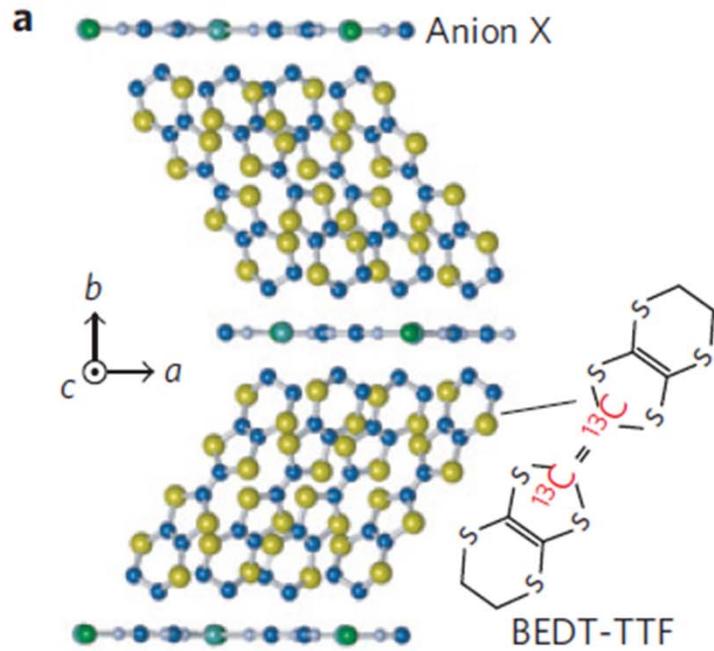


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# The model

# Hubbard on anisotropic triangular lattice

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),  
R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



Kagawa *et al.*  
Nature Physics  
**5**, 880 (2009)

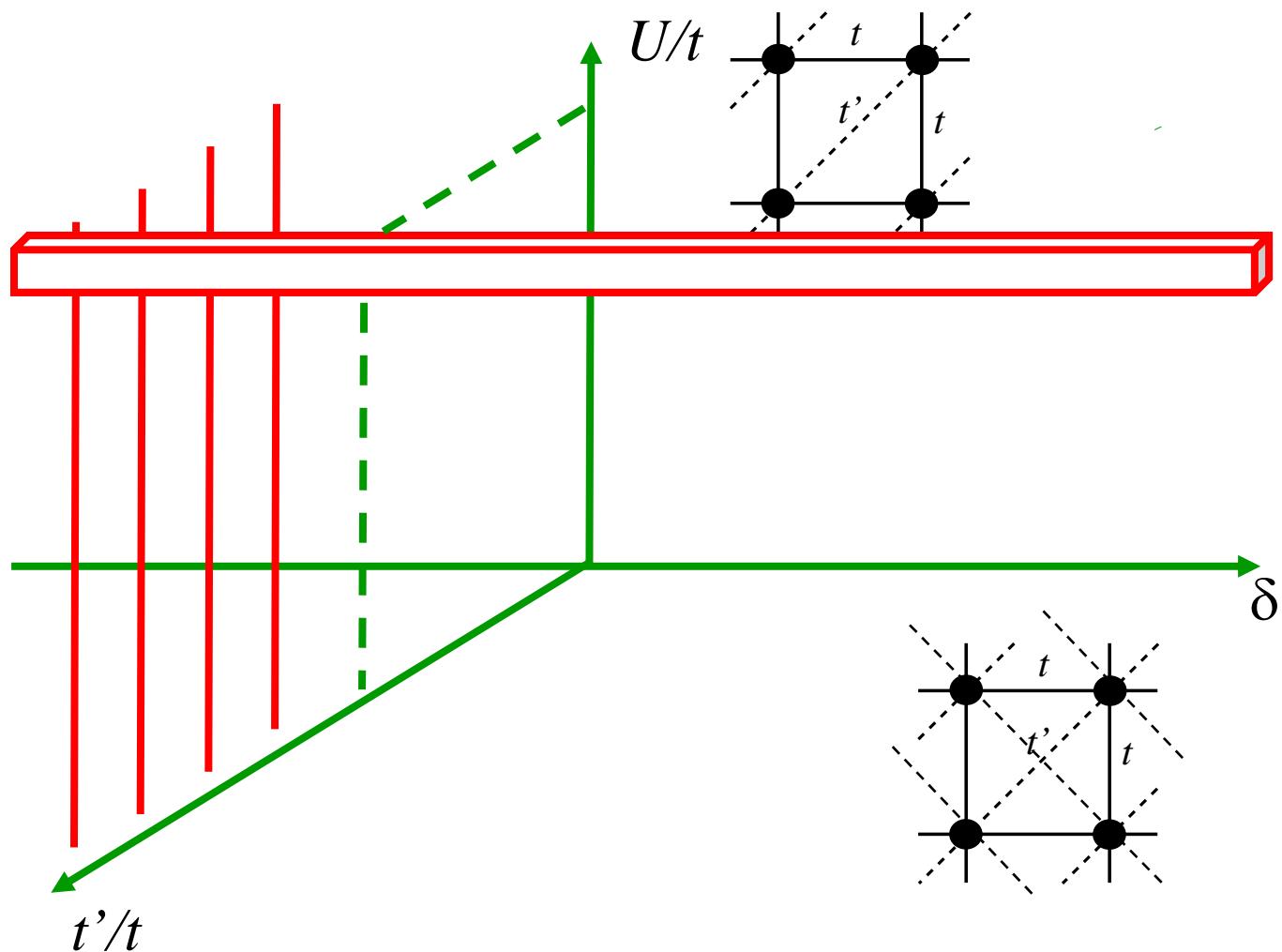
$$t \approx 50 \text{ meV}$$

$$\Rightarrow U \approx 400 \text{ meV}$$
$$t'/t \sim 0.6 - 1.1$$

$$H = \sum_{ij\sigma} (t_{ij} - \delta_{ij}\mu) c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

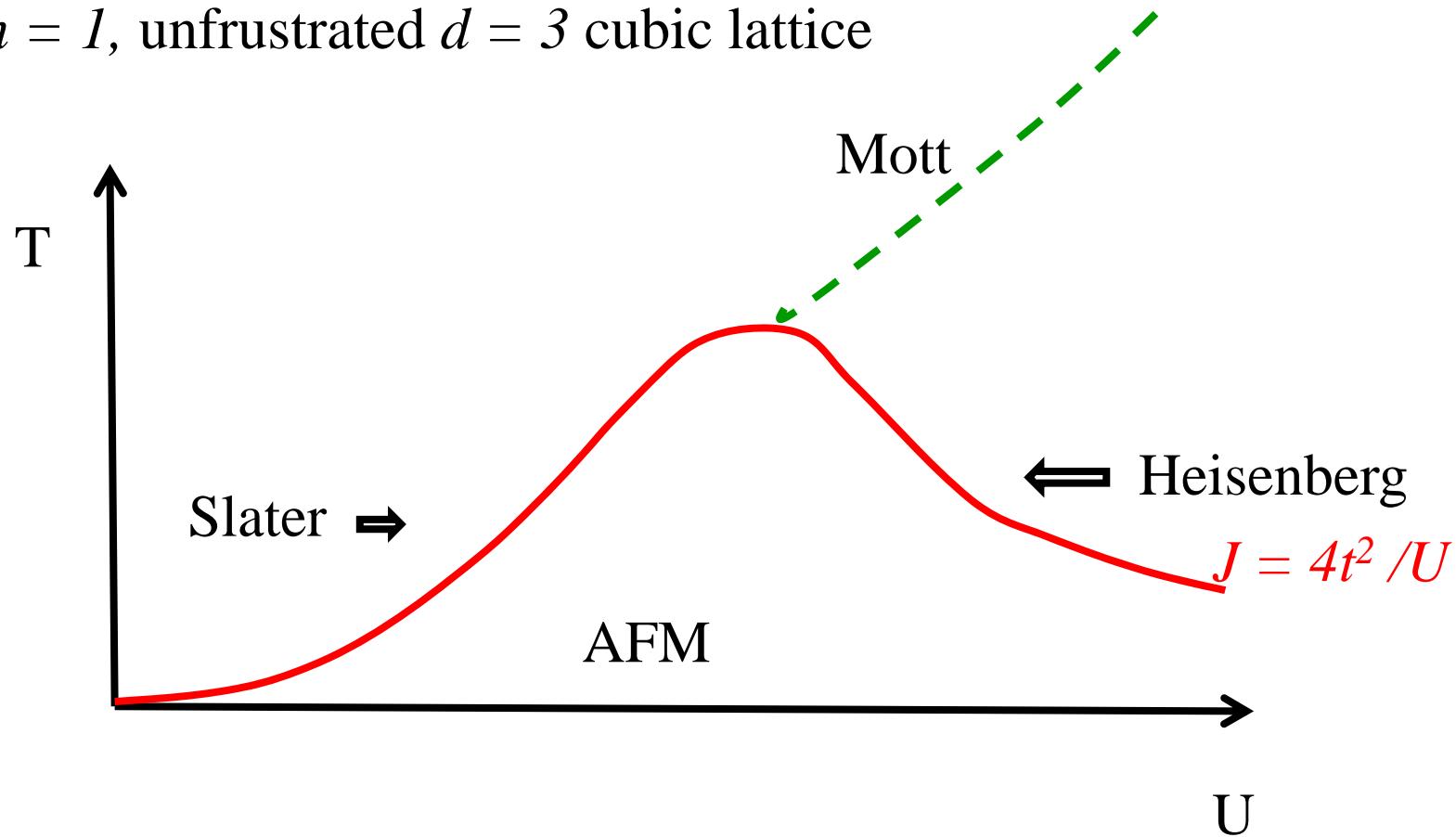


# Perspective



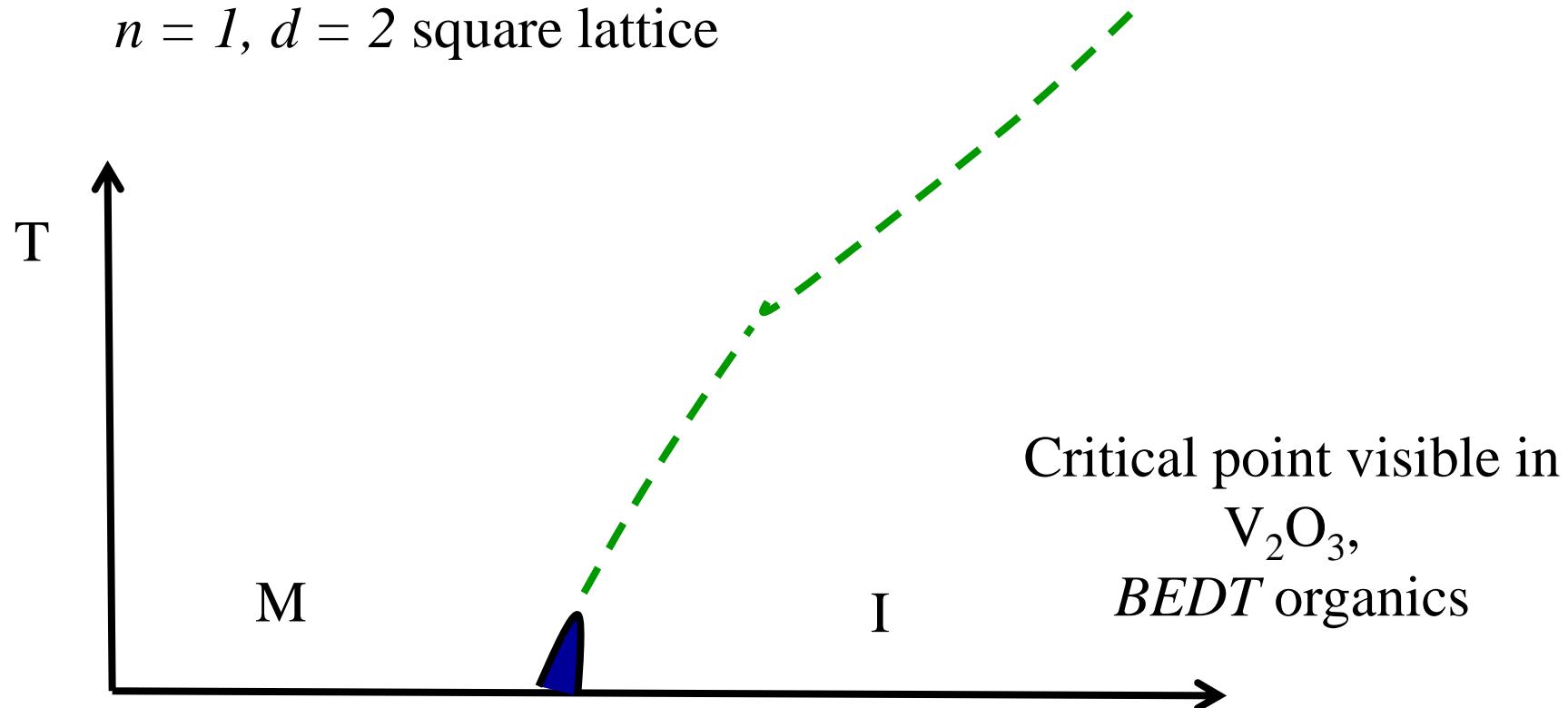
# Weak-strong coupling, and Mott transition

$n = 1$ , unfrustrated  $d = 3$  cubic lattice



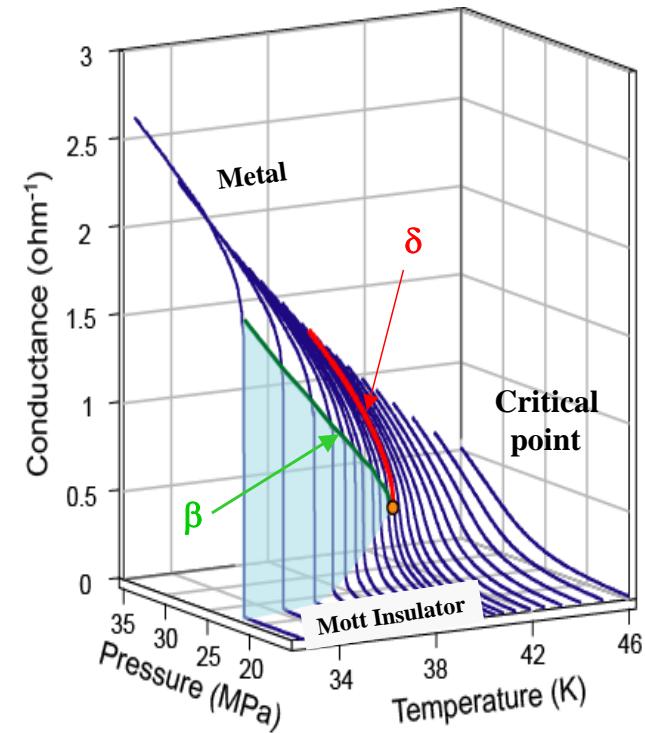
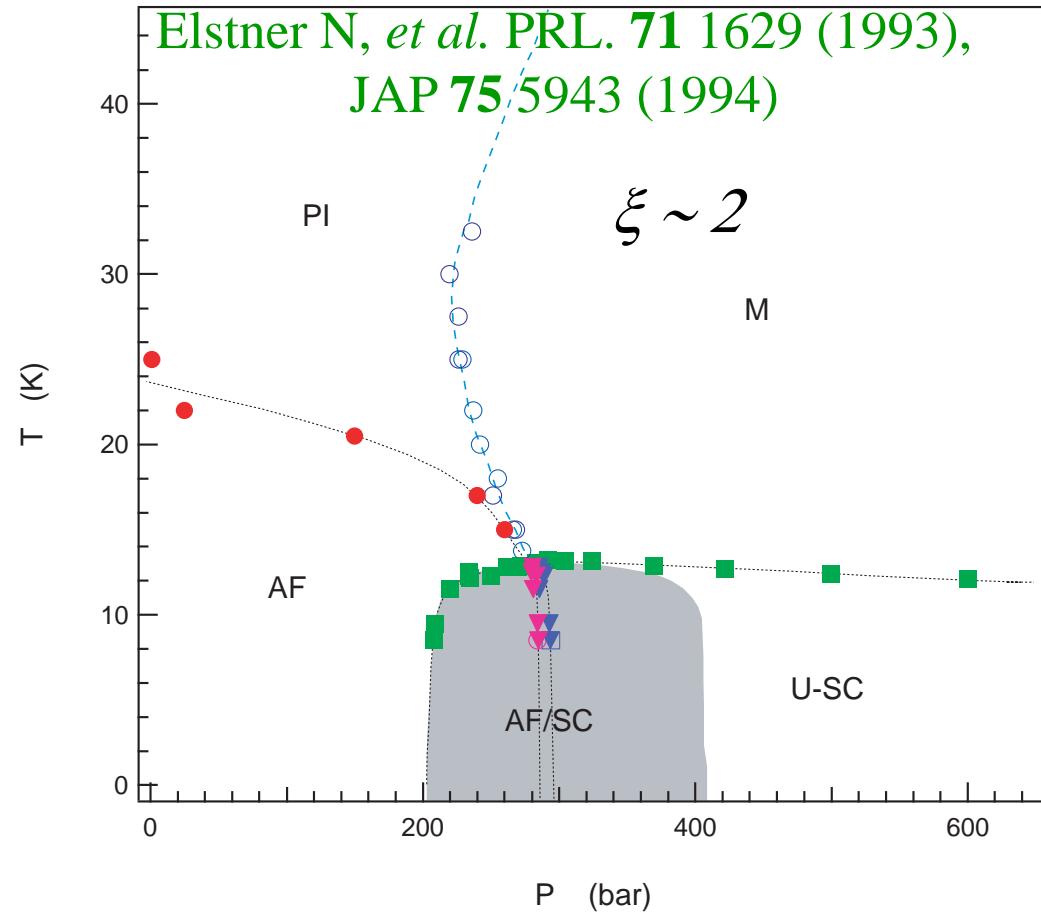
# Local moment and Mott transition

$n = 1, d = 2$  square lattice



Understanding finite temperature phase from a *mean-field theory* down to  $T = 0$

# Bare Mott critical point in organics



F. Kagawa, K. Miyagawa, + K. Kanoda  
PRB **69** (2004) +Nature **436** (2005)

Phase diagram ( $X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ )

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

# Sample litterature on the phase diagram

- H. Morita et al., J. Phys. Soc. Jpn. **71**, 2109 (2002).
- H. Kondo, T. Moriya, J.Phys.Soc.Japan **73**, 812–814 (2004)
- J. Liu et al., Phys. Rev. Lett. **94**, 127003 (2005).
- S.S. Lee et al., Phys. Rev. Lett. **95**, 036403 (2005).
- B. Powell et al., Phys. Rev. Lett. **94**, 047004 (2005). RVB
- J.Y. Gan et al., Phys. Rev. Lett. **94**, 067005 (2005).
- J. Y. Gan, Yan Chen, and F. C. Zhang Phys. Rev. B **74**, 094515 (2006). RMFT
- Watanabe T, *et al.* J. Phys. Soc. Japan **75** 074707 2006
- Galitski V and Kim Y B *Phys. Rev. Lett.* **99** 266403 (2007).
- Wrobel P and Suleja W *Phys. Rev. B* **76** 214509 (2007).
- Powell B J and McKenzie R H *Phys. Rev. Lett.* **98** 027005 (2007).
- Hunpyo Lee, Gang Li, and Hartmut Monien, *Phys. Rev. B* **78**, 205117 (2008).
- T. Watanabe et al. *Phys. Rev. B* **77**, 214505 (2008). Variational WF
- Meng, Jarrell, *et al.* triangular lattice DCA cond-mat /1304.7739v1 (2013)



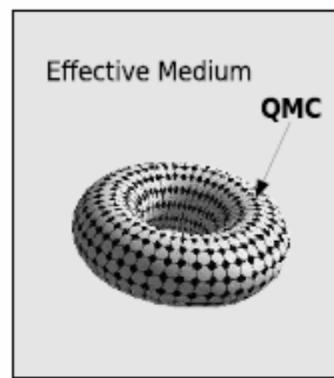
# No SC phase

- L. F. Tocchio, A. Parola, C. Gros, and F. Becca, Phys. Rev. B 80, 064419 (2009).  
T. Kashima and M. Imada, J. Phys. Soc. Jpn. 70, 2287 (2001). (PIRG)  
Clay R T, Li H and Mazumdar S Phys. Rev. Lett. **101** 166403 (2008).  
S. Dayal, R. T. Clay, and S. Mazumdar, Phys. Rev. B 85, 165141 (2012). (PIRG)  
N. Gomes, R. T. Clay, and S. Mazumdar, arXiv:1305.0843

$\frac{1}{4}$  filled model

# The method

# 2d Hubbard: Quantum cluster method

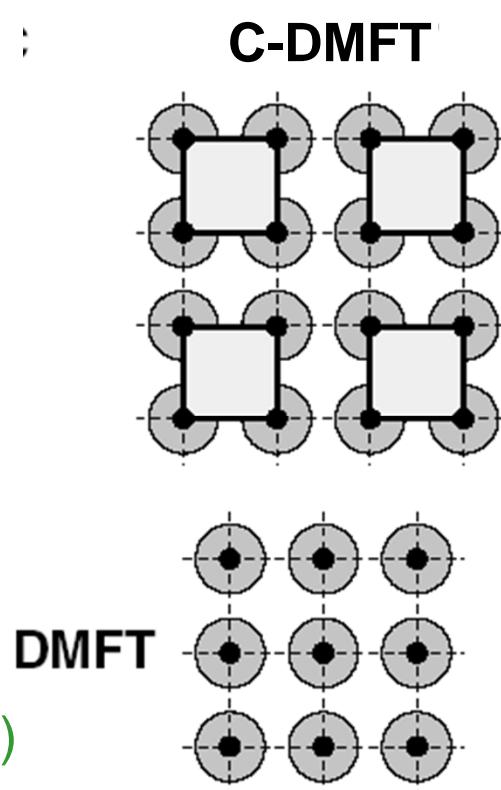


**DCA**

Hettler ... Jarrell ... Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff et al. PRL **91**, 206402 (2003).



**DMFT**

**REVIEWS**

Maier, Jarrell et al., RMP. (2005)

Kotliar et al. RMP (2006)

AMST et al. LTP (2006)



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+ and -

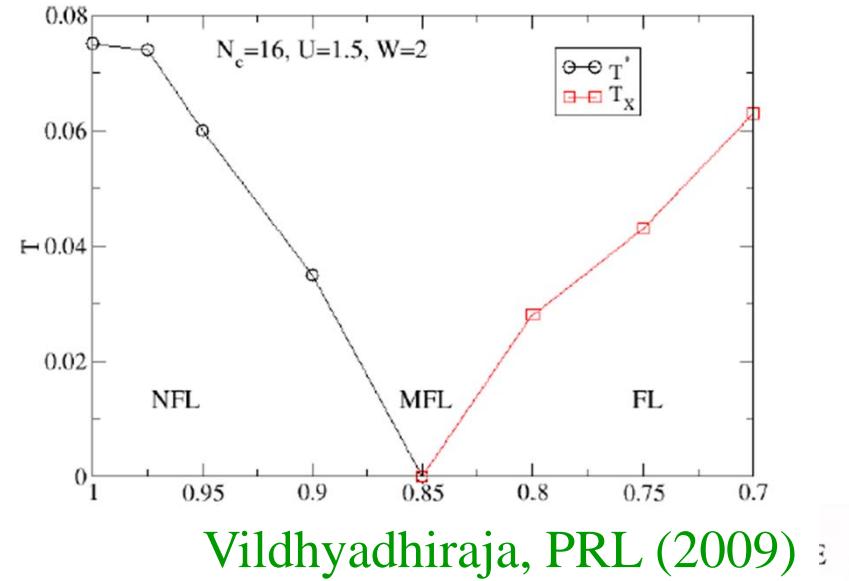
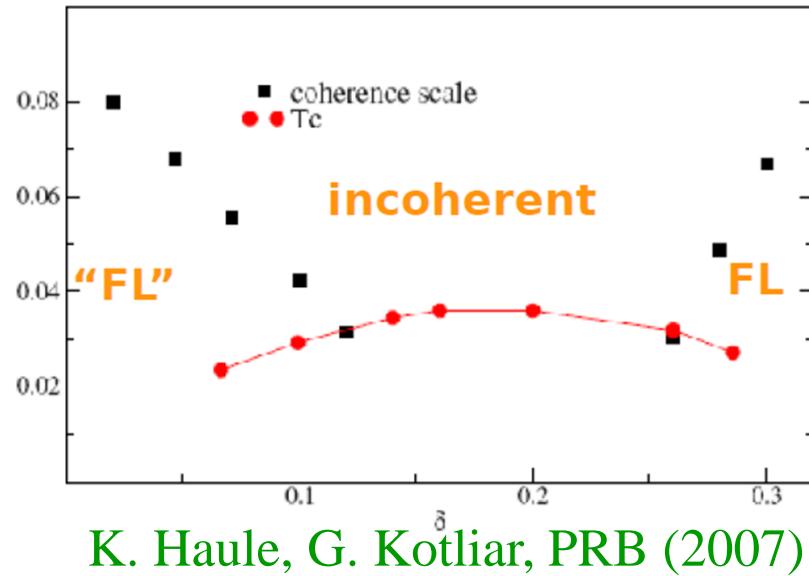
- Long range order:
  - Allow symmetry breaking in the bath (mean-field)
- Included:
  - Short-range dynamical and spatial correlations
- Missing:
  - Long wavelength p-h and p-p fluctuations



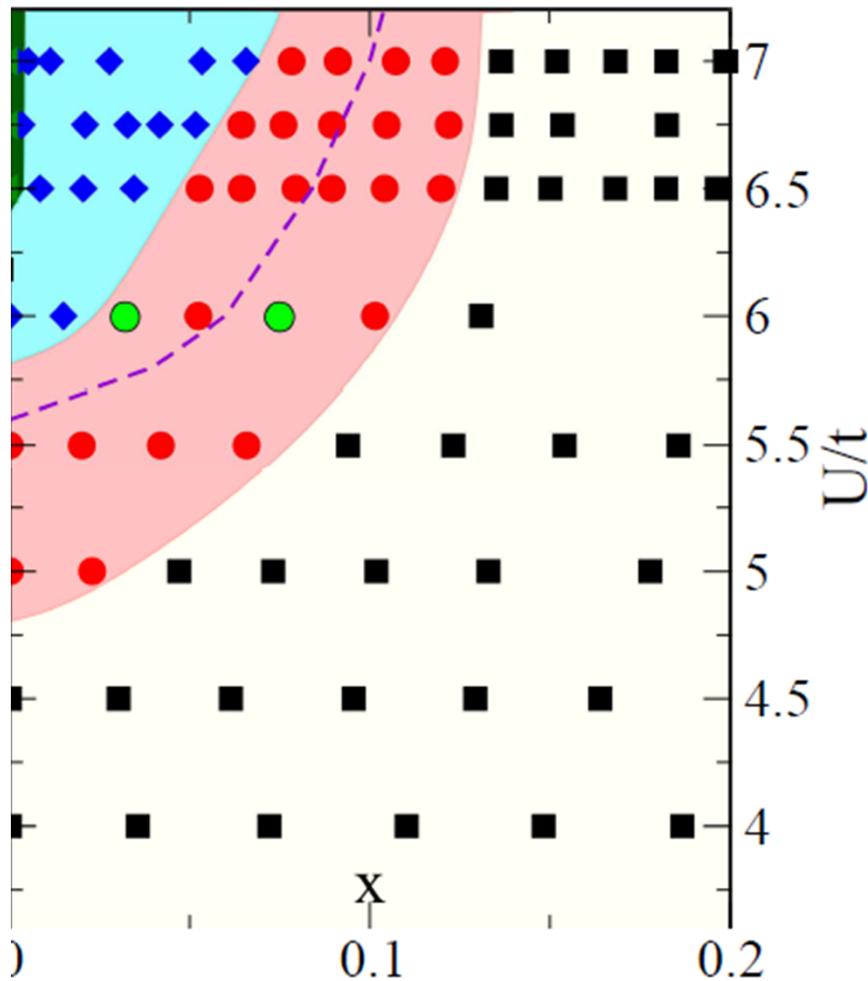
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# Doping driven Mott transition, $t' = 0$

Method	$t'$	Orbital selective	$U$	Critical point	Ref.
D+C+H 8			7		Werner et al. cond-mat (2009)
D+C+H 4					Gull et al. EPL (2008)
	-0.3		10,6		Liebsch, Merino... (2008)
					Ferrero et al. PRB (2009)
D+C+H 8			7		Gull, et al. PRB (2009)



# Doping driven Mott transition



$T = 0.25 t$

Gull, Parcollet, Millis  
arXiv:1207.2490v1

Gull, Werner, Millis, (2009)

E. Gull, M. Ferrero, O. Parcollet, A. Georges, and A. J. Millis (2009) UNIVERSITÉ DE SHERBROOKE

# Two solvers for the cluster-in-a-bath problem

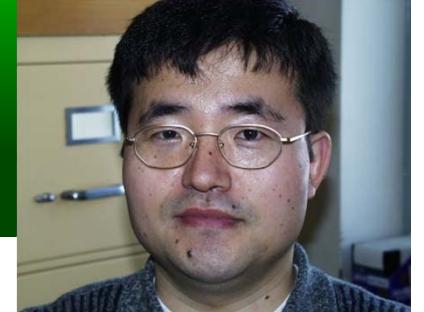


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# Competition AFM-dSC



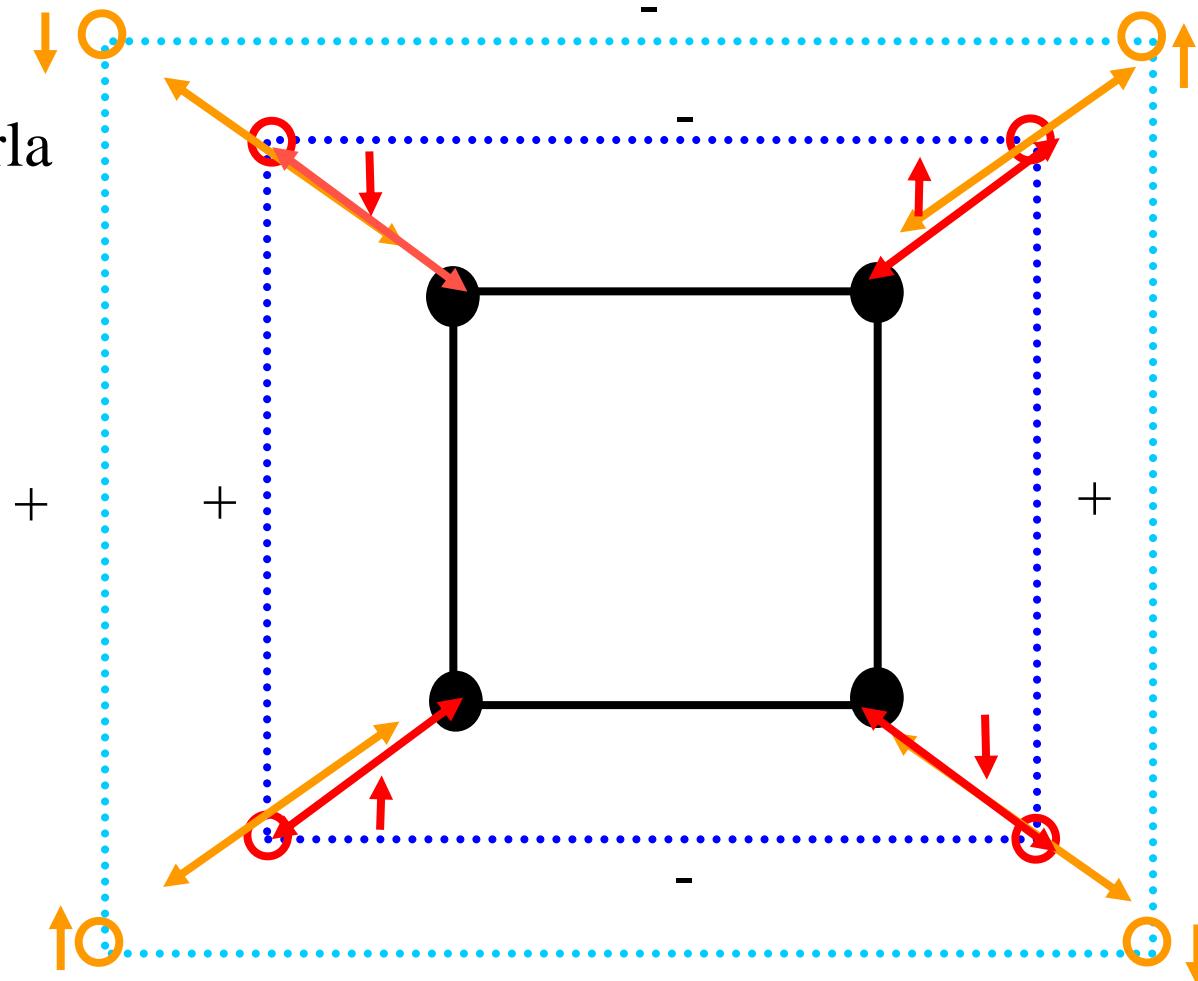
S. Kancharla



B. Kyung



David Sénéchal

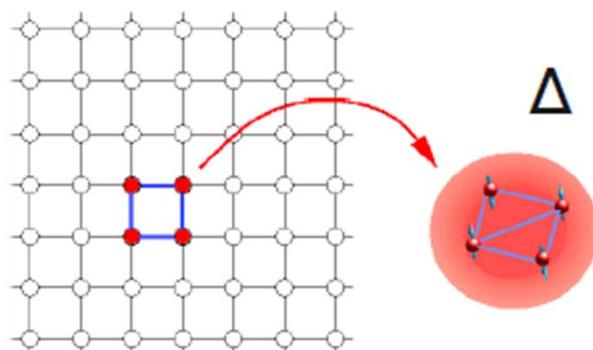


See also, Capone and Kotliar, Phys. Rev. B 74, 054513 (2006),  
Macridin, Maier, Jarrell, Sawatzky, Phys. Rev. B 71, 134527 (2005)

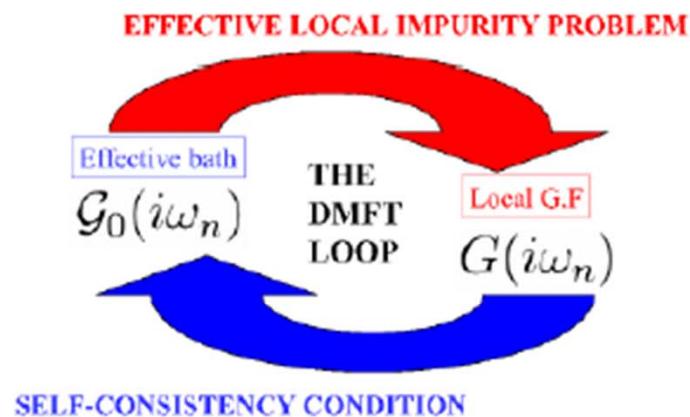


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# C-DMFT



$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$



Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

- 
- P. Werner, PRL 2006
  - P. Werner, PRB 2007
  - K. Haule, PRB 2007

$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$- \left[ \sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

# At finite T, solving cluster in a bath problem

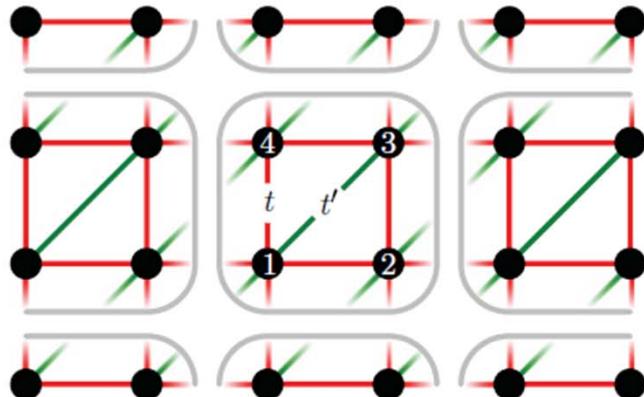
- Continuous-time Quantum Monte Carlo calculations to sum all diagrams generated from expansion in powers of hybridization.
  - P. Werner, A. Comanac, L. de' Medici, M. Troyer, and A. J. Millis, Phys. Rev. Lett. **97**, 076405 (2006).
  - K. Haule, Phys. Rev. B **75**, 155113 (2007).

# Reducing the sign problem: PII-36



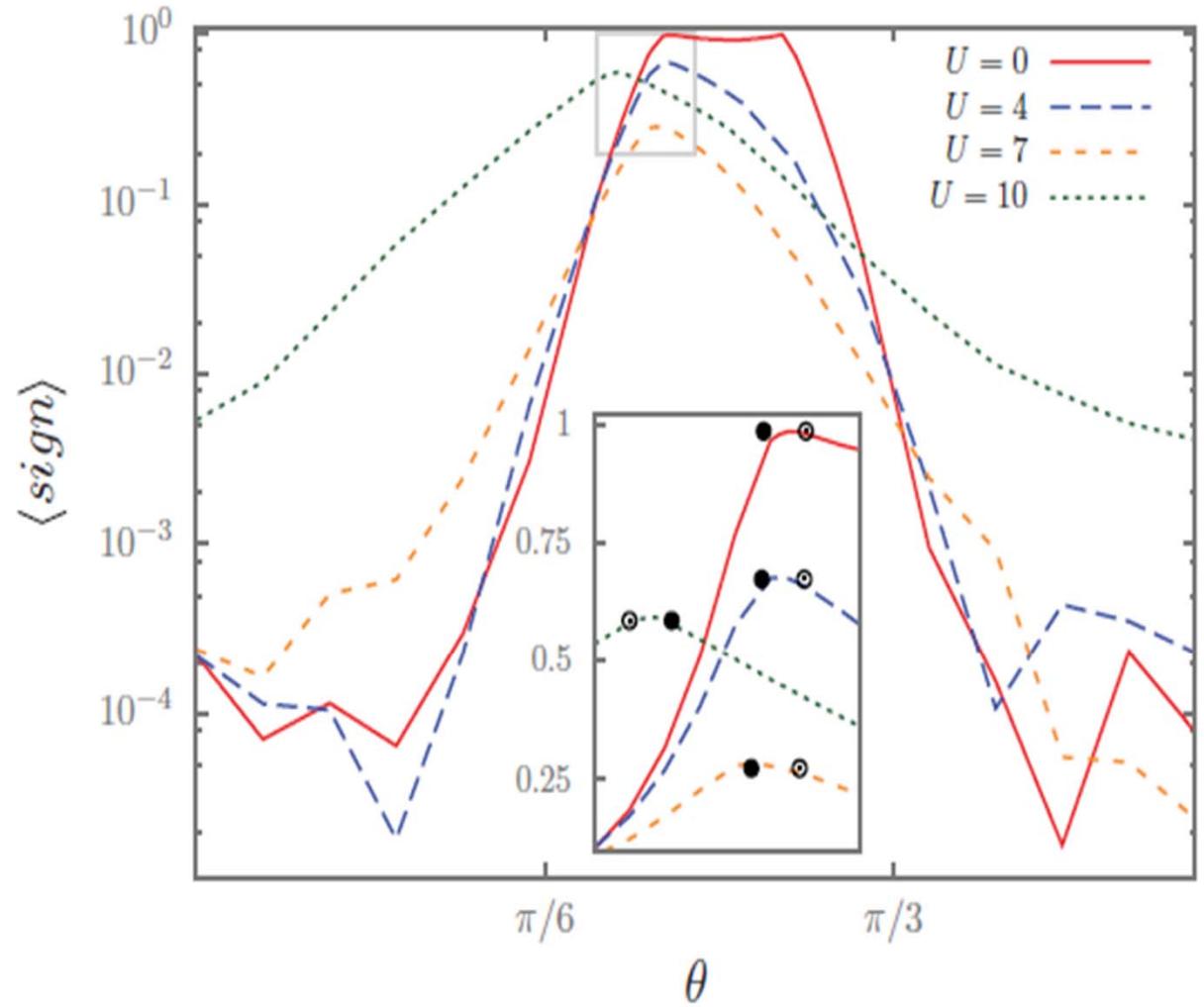
$$\cos \theta c'_{A_1\sigma} - \sin \theta c_{A_1\sigma}, \quad \sin \theta c'_{A_1\sigma} + \cos \theta c_{A_1\sigma}$$

Patrick Sémc



$$t'/t = 0.8$$

$C_{2v}$   
 $2A_1, B_1, B_2$



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- **Phase diagram: Organics and cuprates**
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PII-36

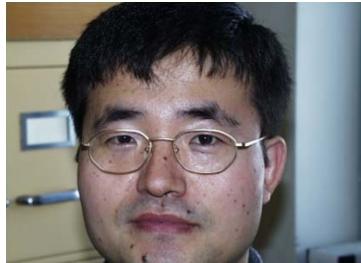


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$T = 0$  phase diagram  $n = 1$

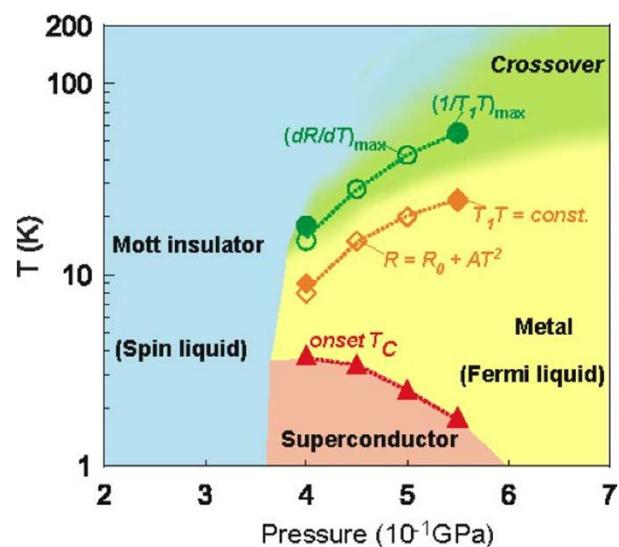
Phase diagram

Exact diagonalization as solver for  
cluster-in-a bath problem ( $T=0$ ).



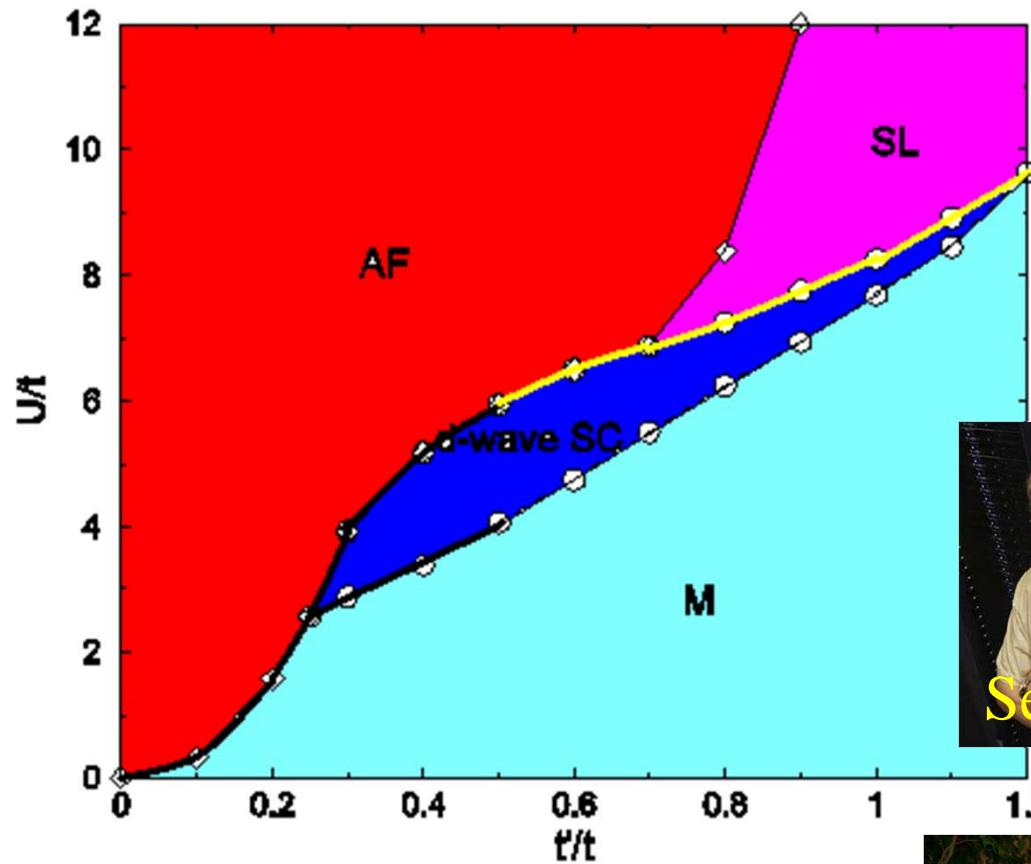
# Theoretical phase diagram BEDT

$X = \text{Cu}_2(\text{CN})_3$  ( $t' \sim t$ )



Y. Kurisaki, et al.

Phys. Rev. Lett. **95**, 177001(2005) Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)



Kyung, A.-M.S.T. PRL 97, 046402 (2006)

Sénéchal, Sahebsara, Phys. Rev. Lett. **97**, 257004

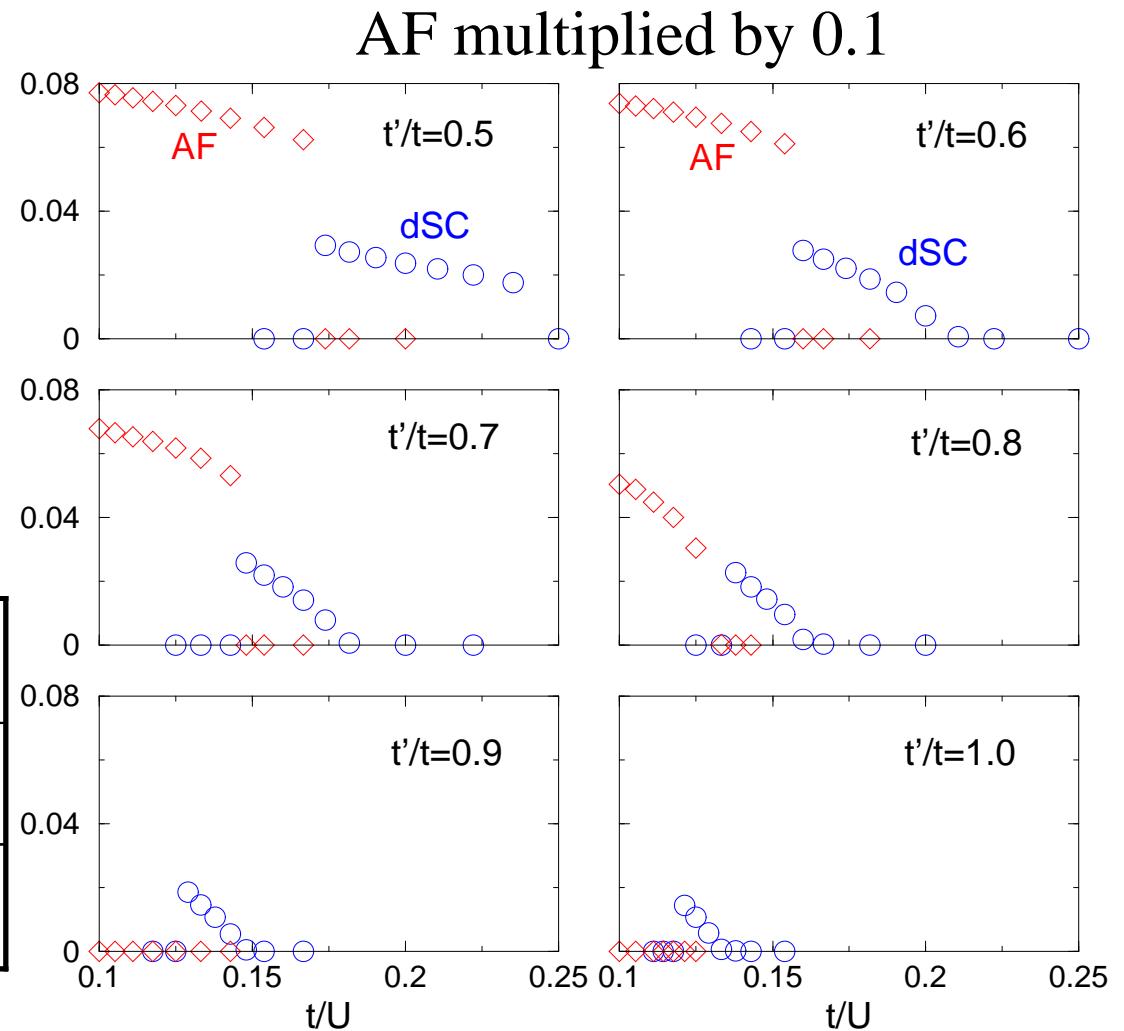


Sénéchal

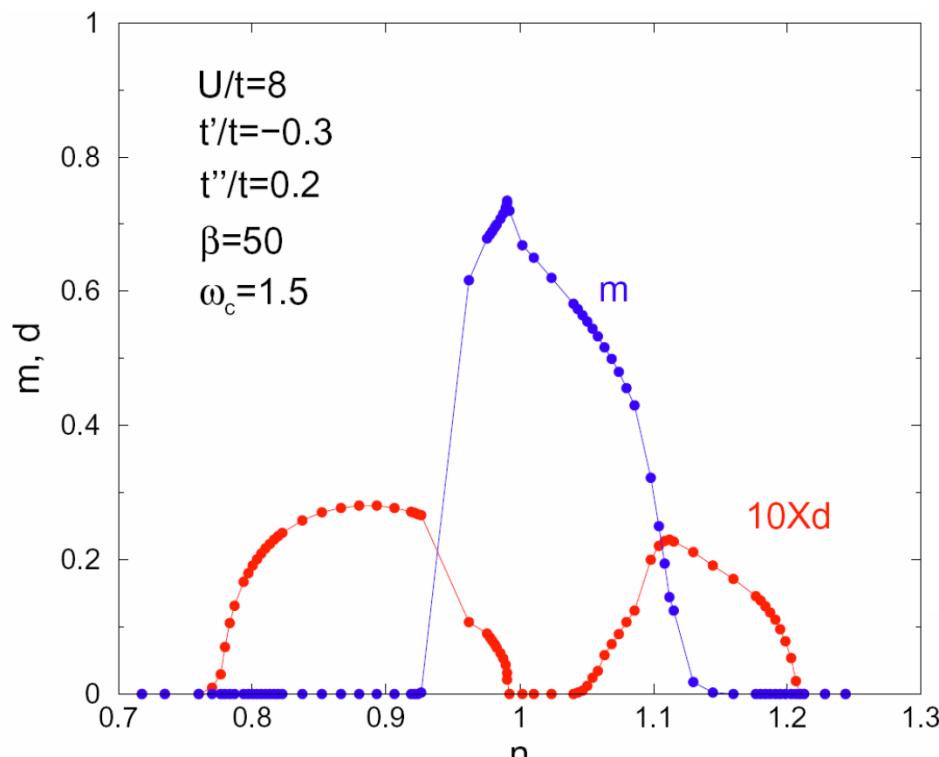
# AFM and dSC order parameters for various $t'/t$

- Discontinuous jump
- Maximum order parameter close to insulator.

X	$\text{Cu}[\text{N}(\text{CN})_2]\text{Br}$	$\text{Cu}(\text{NCS})_2$	$\text{Cu}_2(\text{CN})_3$
$t'/t$	0.68	0.84	1.06
$T_c$	11.6	10.4	3.9

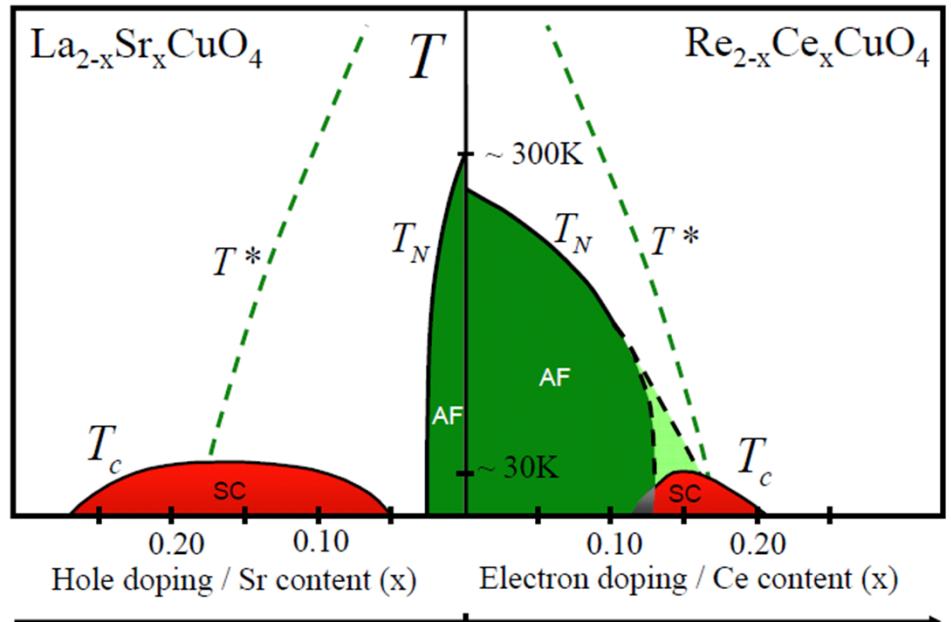


# CDMFT cuprate phase diagram



Kancharla, Kyung, Civelli,  
Sénéchal, Kotliar AMST

Phys. Rev. B (2008)  
AND Capone, Kotliar PRL (2006)



Armitage, Fournier, Greene, RMP (2009)





Giovanni Sordi

## Finite T phase diagram $n = 1$ (cuprates)

CTQMC as solver for cluster in a  
bath ( $T=0$ ).



Patrick Sémon

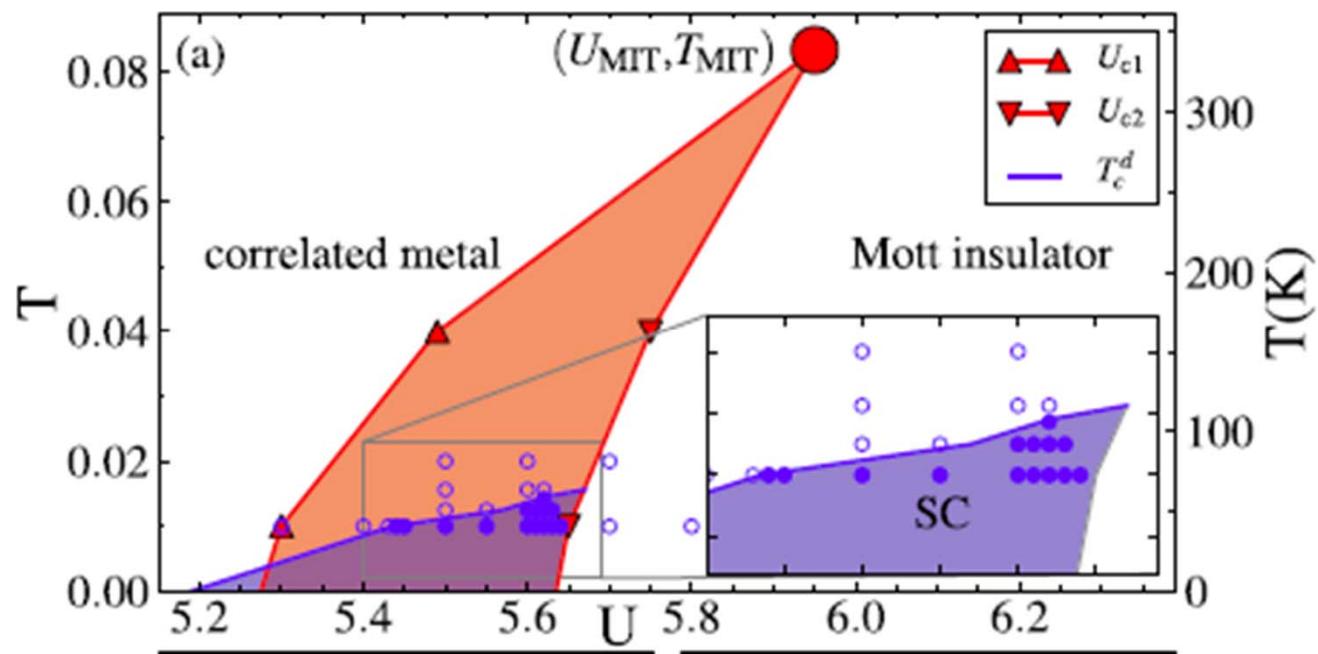


Kristjan Haule



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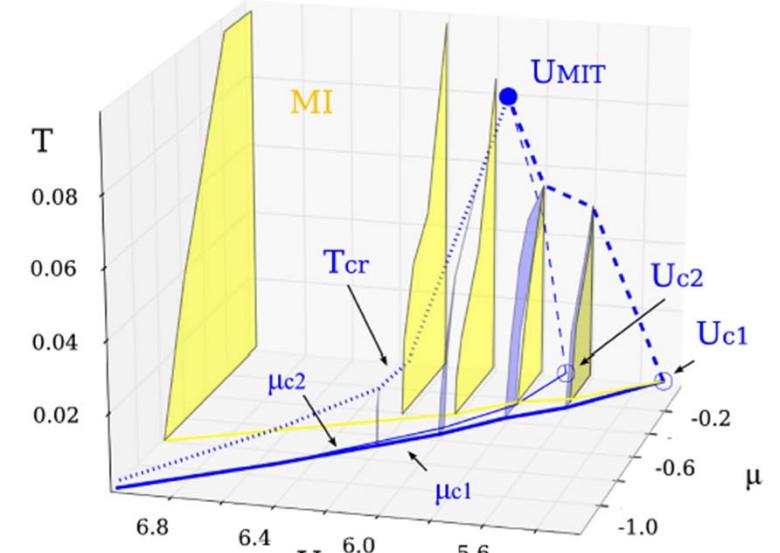
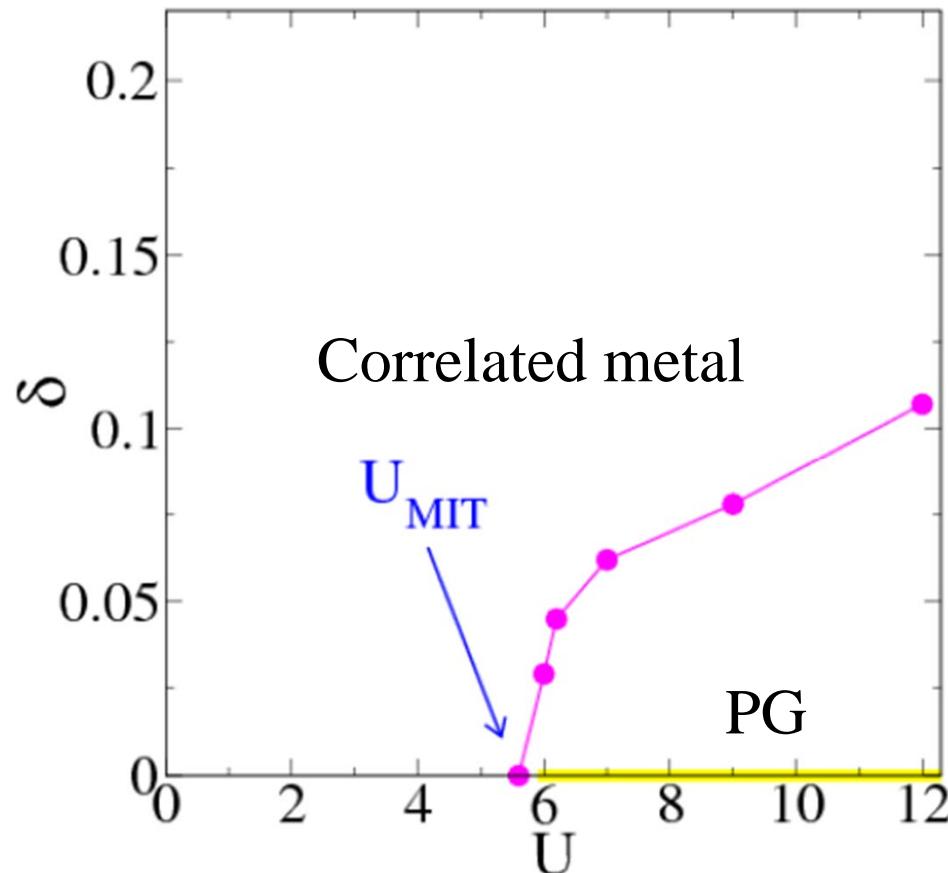
$n = 1$  unfrustrated,  $t' = 0$



Sordi et al. PRL 108, 216401 (2012)

# Link to Mott transition up to optimal doping

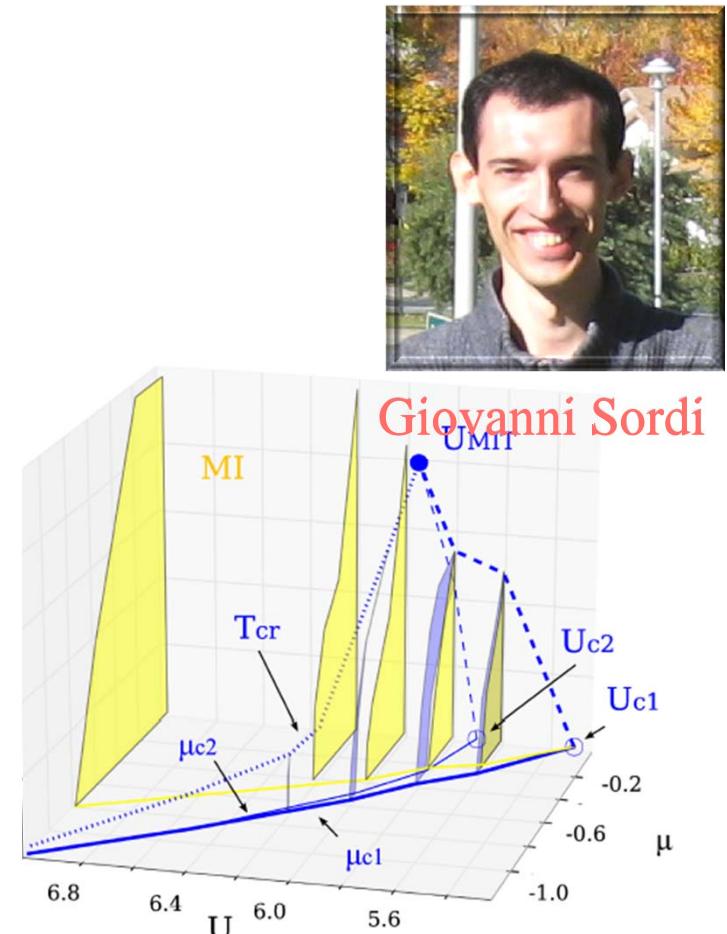
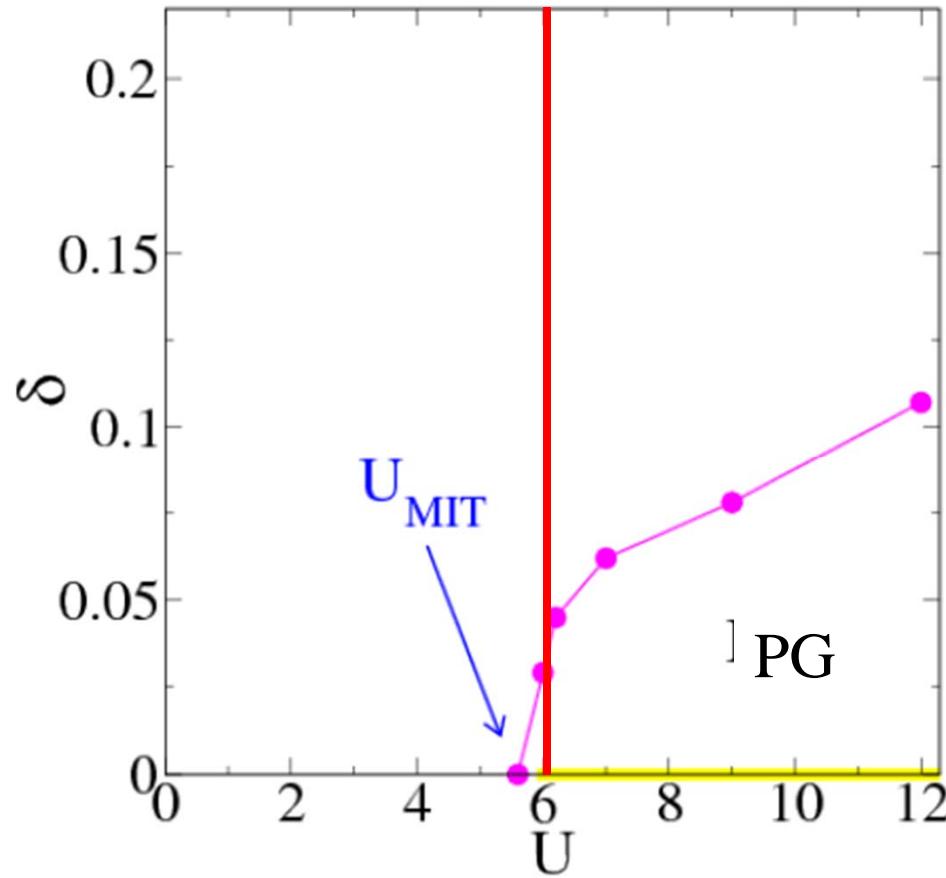
Doping dependence of critical point as a function of  $U$



G. Sordi, P. Sémon, K. Haule, and A.-M.S.T., PRL **104**, 226402 (2010);  
PRB **84**, 075161 (2011); Sci. Rep. **2**, 547 (2012); PRB **87**, 041101(R) (2013)

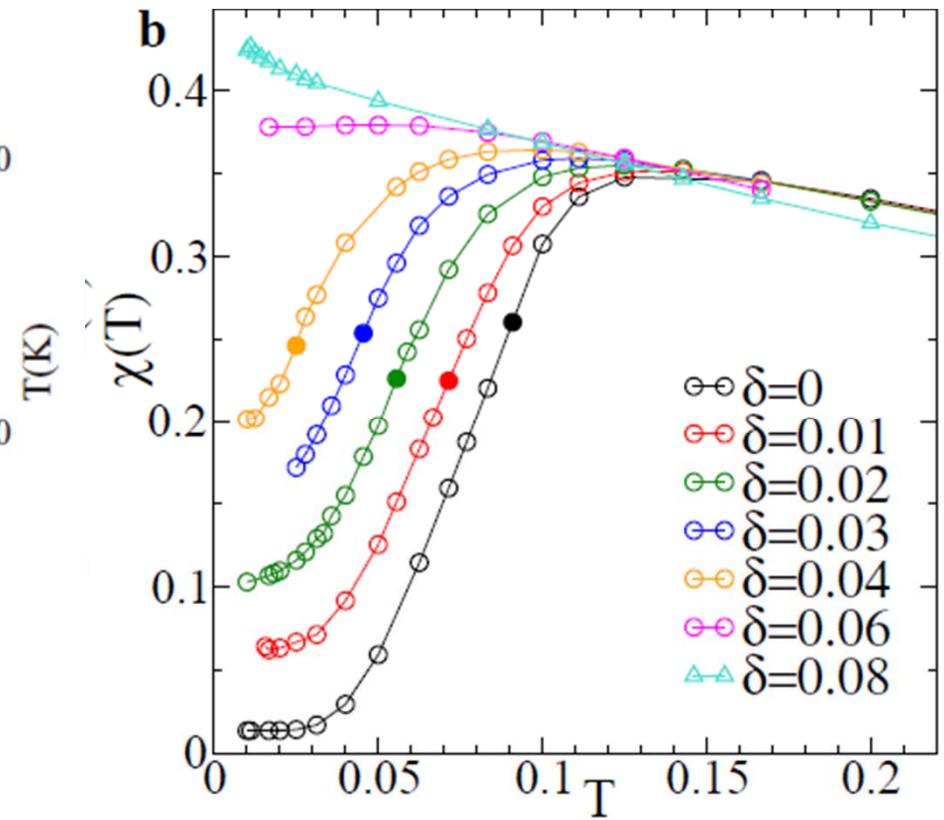
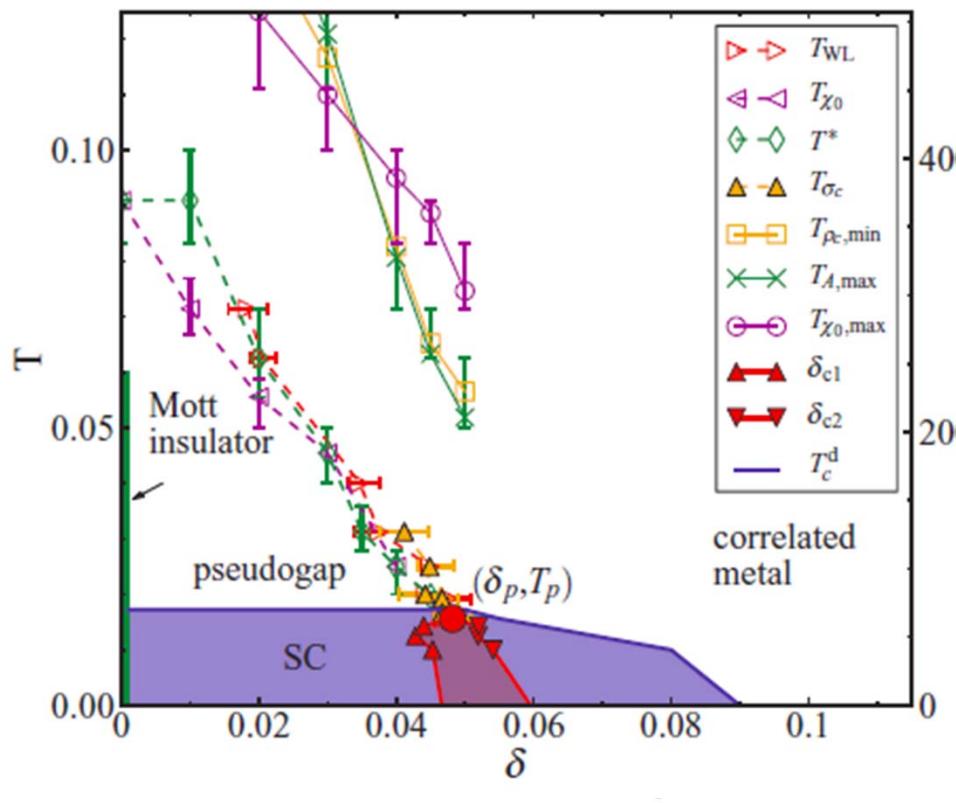
# Another phase transition related to Mott

Focus near the Mott transition



G. Sordi, P. Sémon, K. Haule, and A.-M.S.T., PRL **104**, 226402 (2010);  
PRB **84**, 075161 (2011); Sci. Rep. **2**, 547 (2012); PRB **87**, 041101(R) (2013)

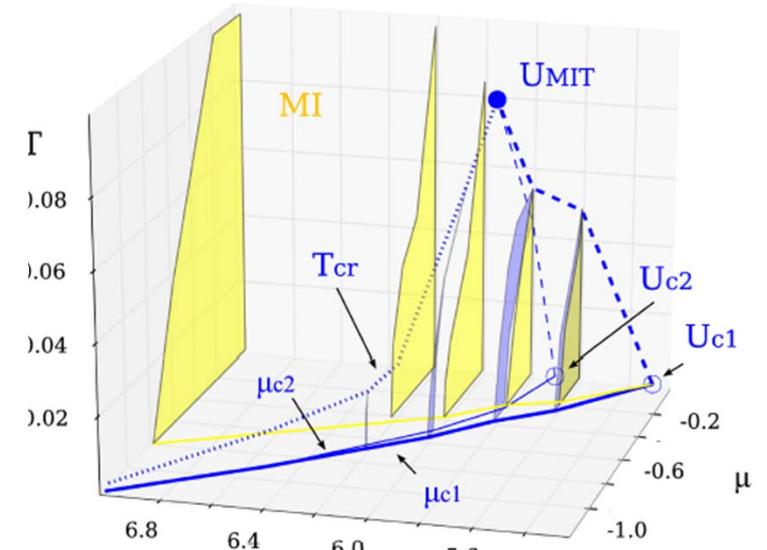
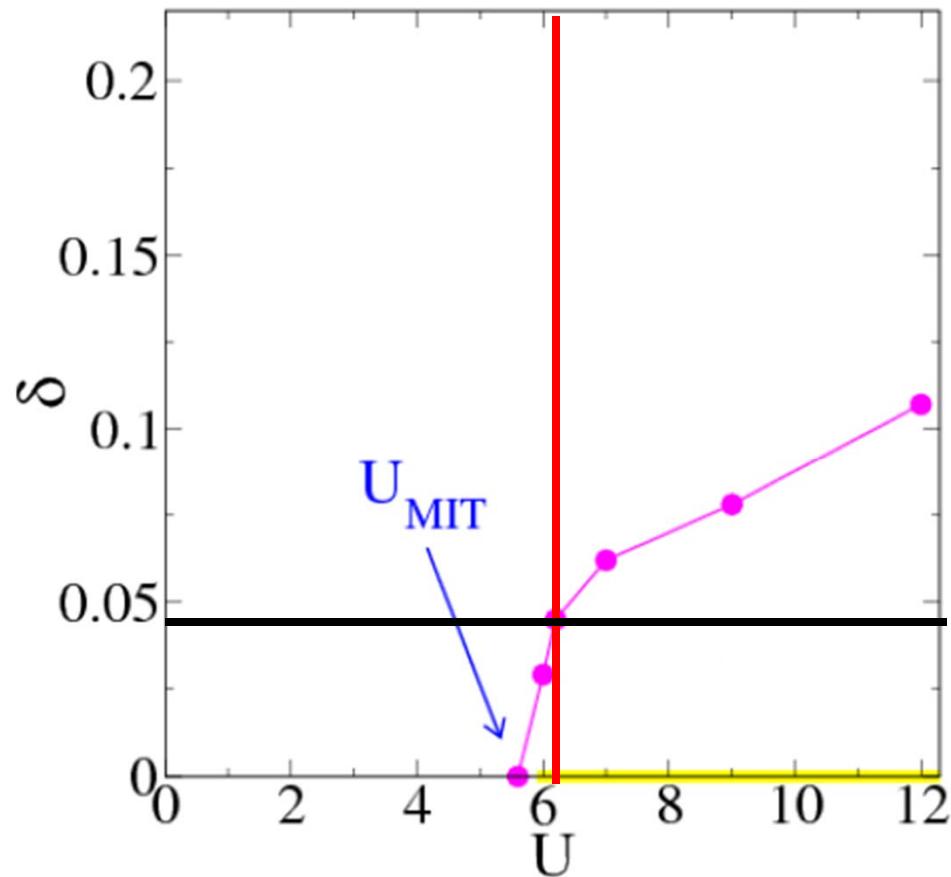
# Widom line at end of finite-doping first-order transition



Sordi et al. PRL 108, 216401 (2012)  
PRB 87, 041101(R) (2013)

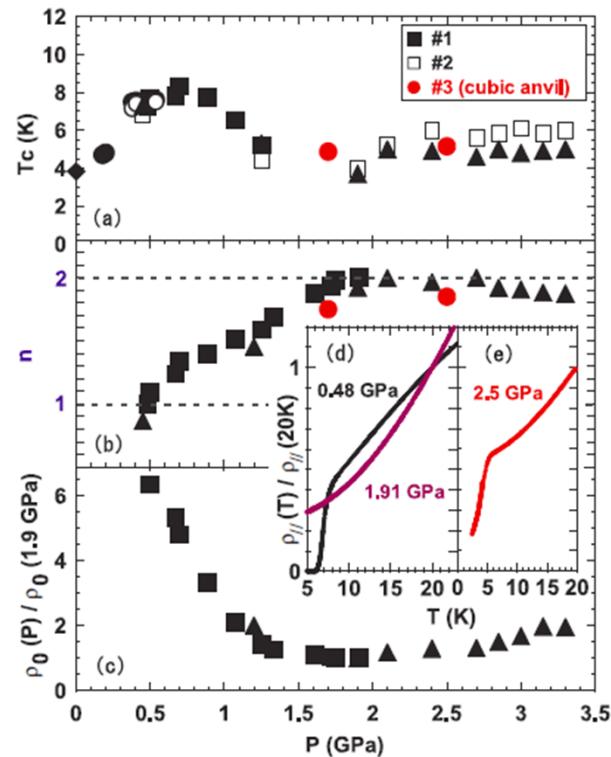
# Link to Mott transition up to optimal doping

Doping dependence of critical point as a function of  $U$

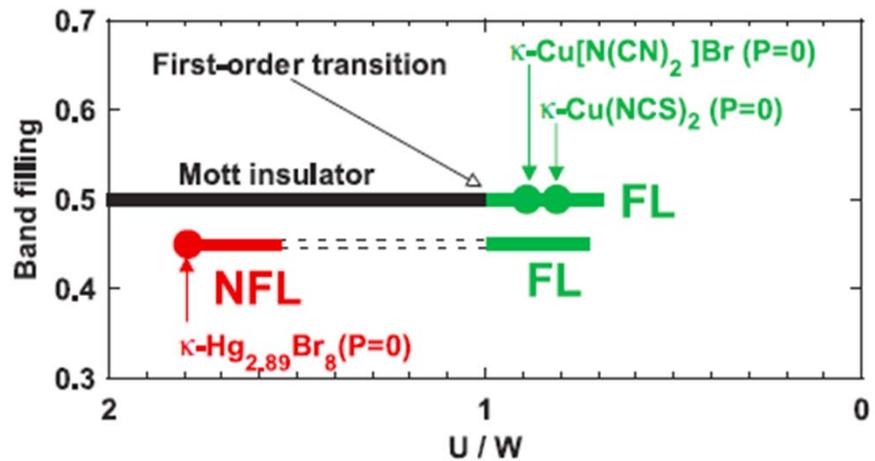


G. Sordi, P. Sémon, K. Haule, and A.-M.S.T., PRL **104**, 226402 (2010);  
PRB **84**, 075161 (2011); Sci. Rep. **2**, 547 (2012); PRB **87**, 010141 (R) (2013)

# A doped BEDT organic



	$W$ (eV)	$U$ (eV)	$U/W$	$BF$	$T_c$ (K)
$\kappa\text{-Cu}(\text{NCS})_2$ <sup>a)</sup>	0.57	0.46	0.81	0.50	10.4
$\kappa\text{-Cu}[\text{N}(\text{CN})_2]\text{Br}$ <sup>a)</sup>	0.55	0.49	0.89	0.50	11.8
$\kappa\text{-Hg}_{2.89}\text{Br}_8$ <sup>b)</sup>	0.26	0.465	1.79	0.45	4.3



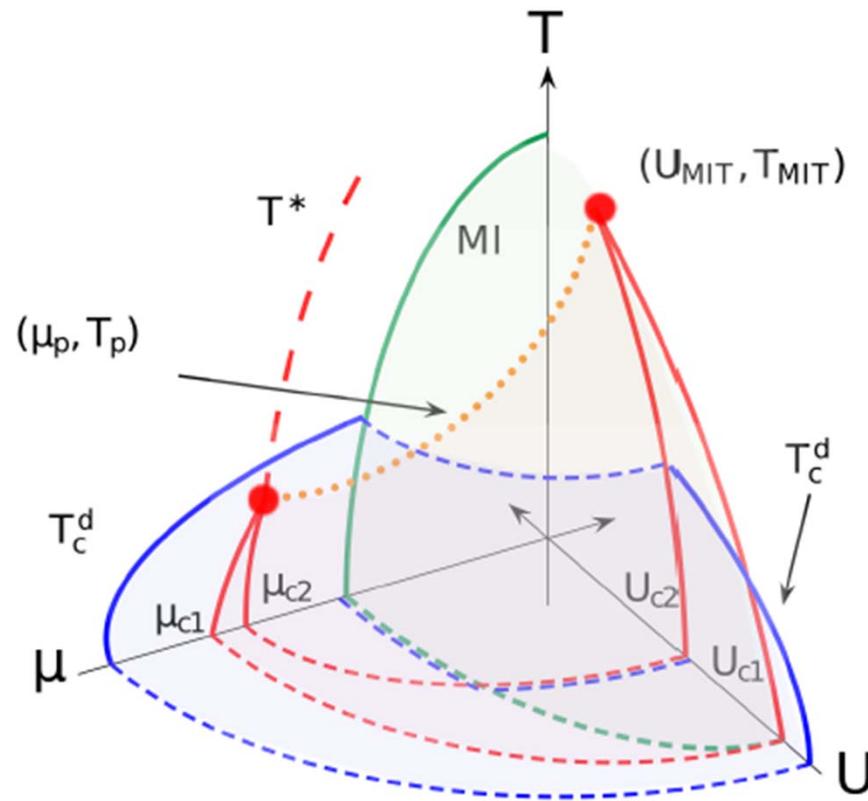
Taniguchi et al. J. Phys. Soc. Japan, **76**, 113709 (2007)

R. N. Lyubovskaya et al. JETP Lett. **45**, 530 (1987)



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# Unified phase diagram



G. Sordi, P. Sémon, K. Haule, and A.-M.S.T., PRL 108, 216401 (2012)

For SCR see: H. Kondo, T. Moriya J. Phys. Soc. Japan, **68**, 3170 (1999)

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**PII-36**



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## Mott critical point

P. Sémon and A.-M.S. Tremblay,  
PRB 85, 201101 (R) (2012)

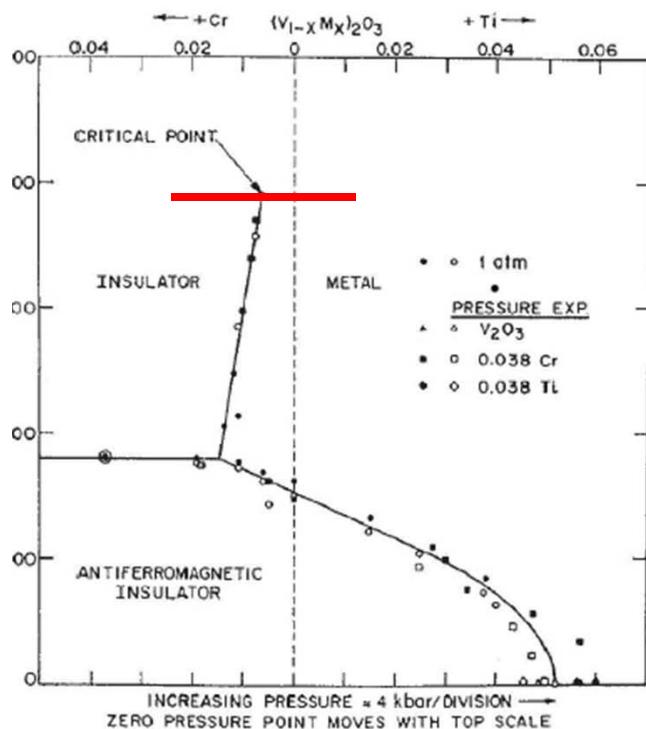


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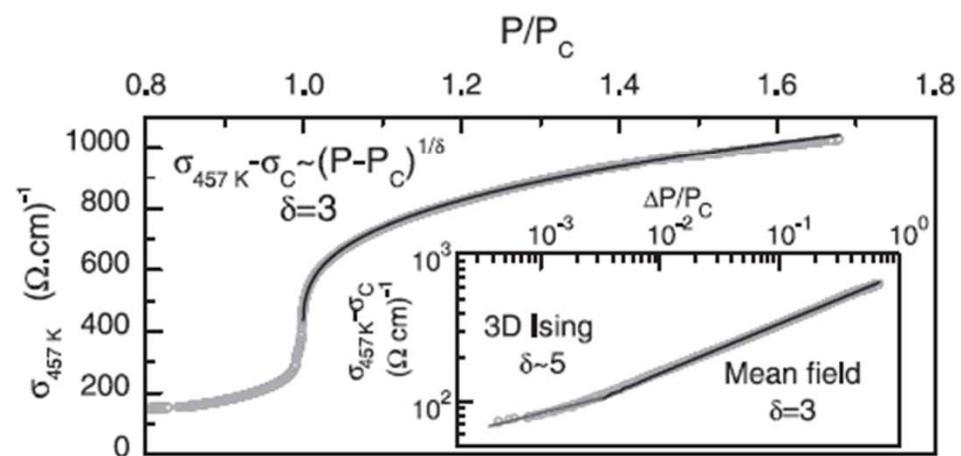
# Critical behavior



**Universality and Critical Behavior at the Mott Transition**  
P. Limelette, et al.  
*Science* **302**, 89 (2003);  
DOI: 10.1126/science.1088386



Double occupancy: Ising universality class  
C. Castellani et al., Phys. Rev. Lett. **43**, 1957 (1979).  
G. Kotliar, et al. Phys. Rev. Lett. **84**, 5180 (2000).  
Limelette et al. Science (2003)

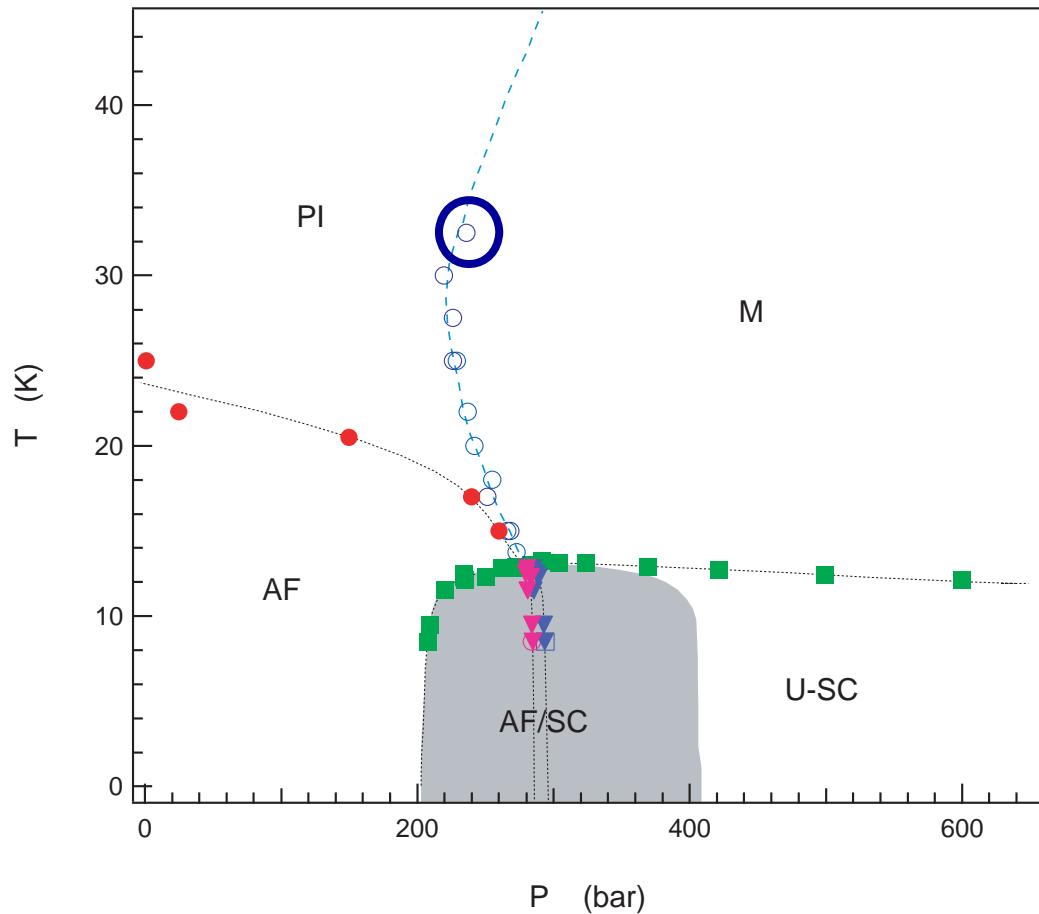


McWhan, PRB 1970; Limelette, Science 2003



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# Mott critical point in layered organics



What is the critical behavior?

Phase diagram BEDT-X  
( $X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ )

S. Lefebvre et al. PRL **85**, 5420 (2000),  
P. Limelette, et al. PRL **91** (2003)

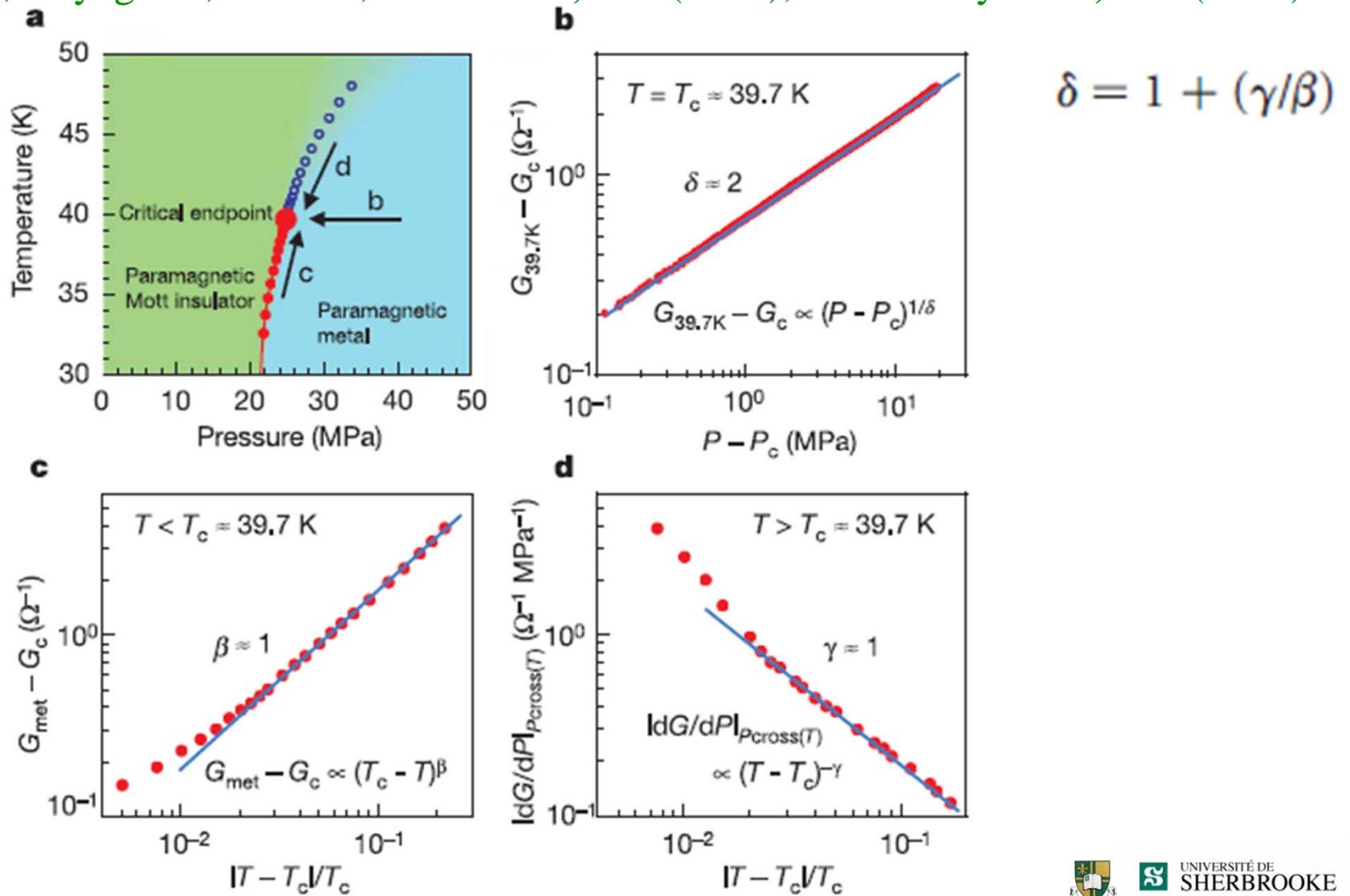
F. Kagawa, K. Miyagawa, + K. Kanoda  
PRB **69** (2004) +Nature **436** (2005)



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# Surprising critical behavior

Kagawa, Miyagawa, Kanoda, Nature 436, 534 (2005), Nature Physics 5, 880 (2009)



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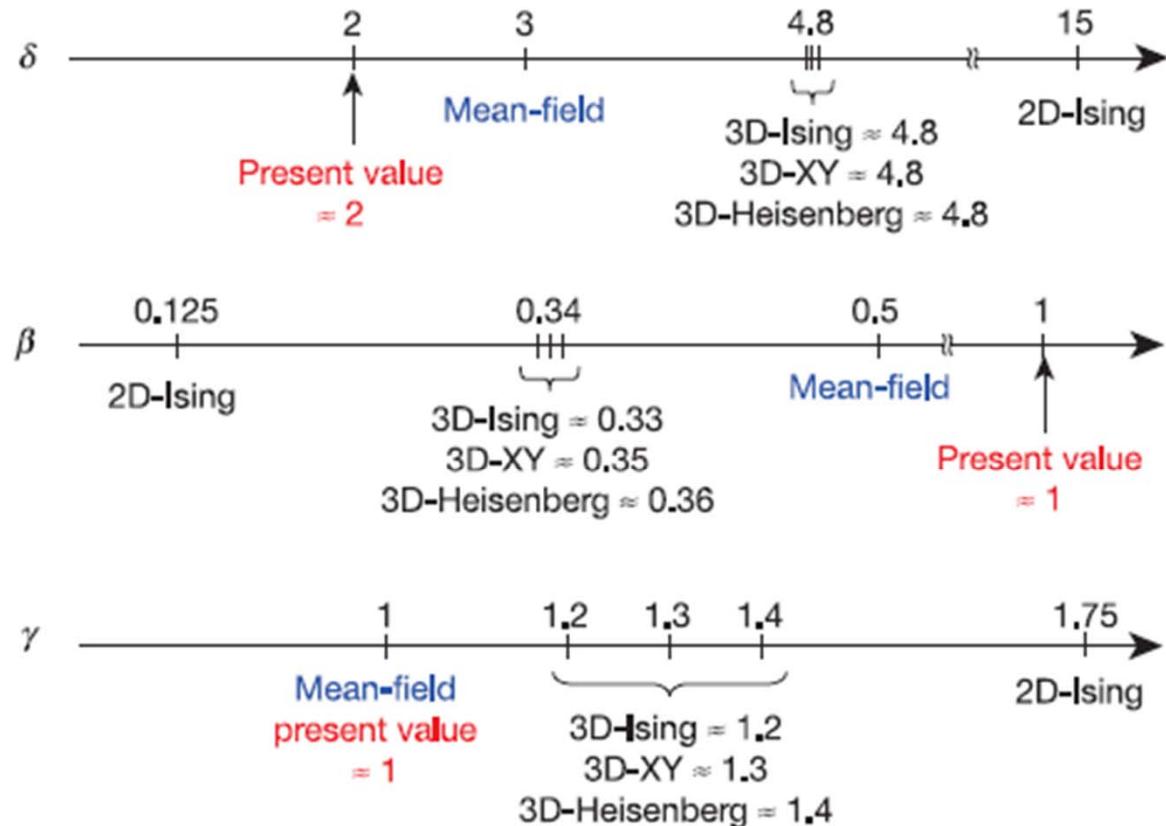
# Unconventional behavior

## Unconventional critical behaviour in a quasi-two-dimensional organic conductor

Nature

436, 534 (2005)

F. Kagawa<sup>1</sup>, K. Miyagawa<sup>1,2</sup> & K. Kanoda<sup>1,2</sup>



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# Possible explanations

M. Imada, Phys. Rev. B **72**, 075113 (2005).

M. Imada, et al. J. Phys.: Condens.Matter **22**, 164206 (2010).

S. Papanikolaou, R. M. Fernandes, E. Fradkin, P. W. Phillips,  
J. Schmalian, and R. Sknepnek, Phys. Rev. Lett. **100**, 026408 (2008).



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# Numerical results

P. Sémon and A.-M.S.T. PRB **85**, 201101(R)



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P. Sémon and A.-M.S.T. PRB **85**, 201101(R)

## The method

# Cellular dynamical mean-field theory Continuous-time quantum Monte Carlo Hybridization expansion

P. Werner, et al., Phys. Rev. Lett. **97**, 076405 (2006).

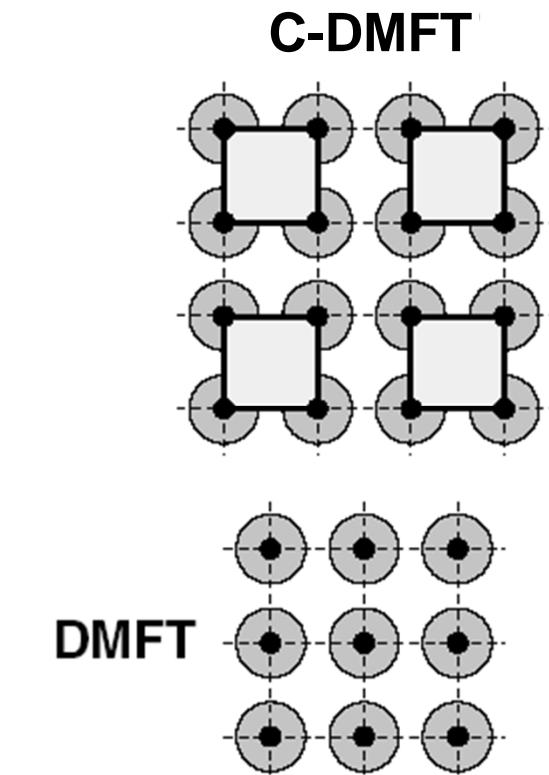
P. Werner and A. J. Millis, Phys. Rev. B **74**, 155107 (2006).

E. Gull, et al., Rev. Mod. Phys. **83**, 349 (2011).

K. Haule, Phys. Rev. B **75**, 155113 (2007).

# 2d Hubbard: Quantum cluster method physics

- Observed behavior is a transient from a QCP?
- Quantum fluctuations
- Cluster necessary in  $d = 2$  for
- Short-range spatial fluctuations
- Disentangle effects of  $J$
- No low  $q$  spatial fluctuations



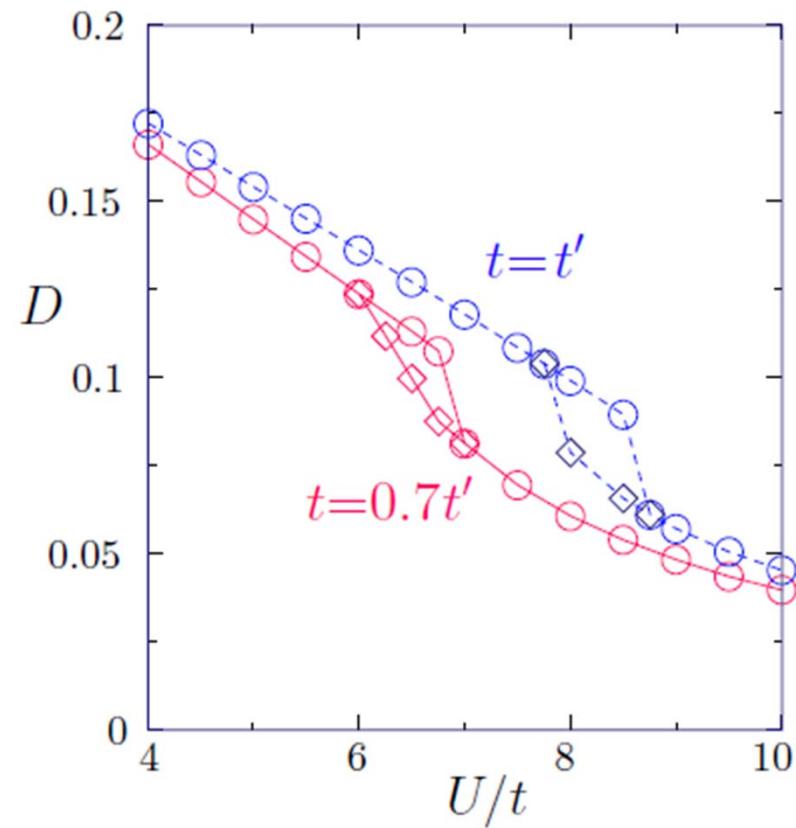
## REVIEWS

Maier, Jarrell et al., RMP. (2005)  
Kotliar et al. RMP (2006)  
AMST et al. LTP (2006)



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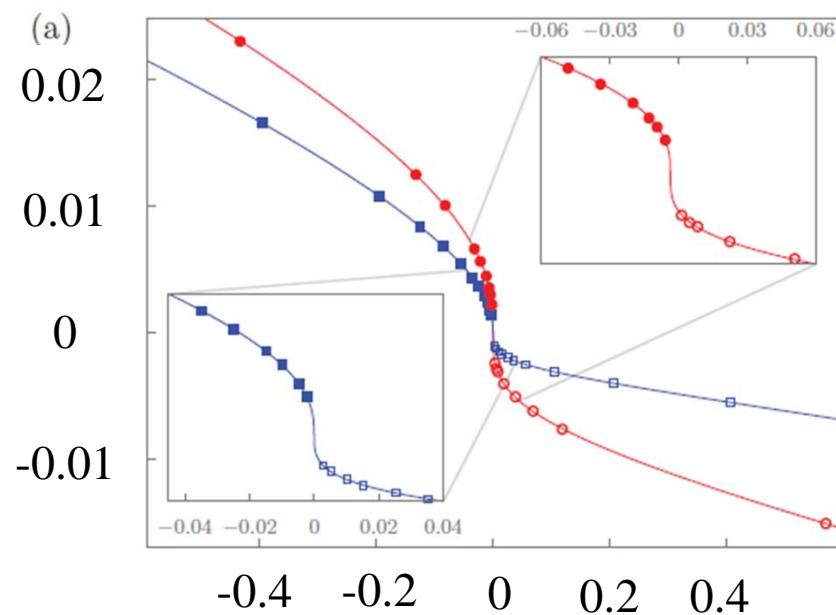
# Mott transition, jump in double occupancy



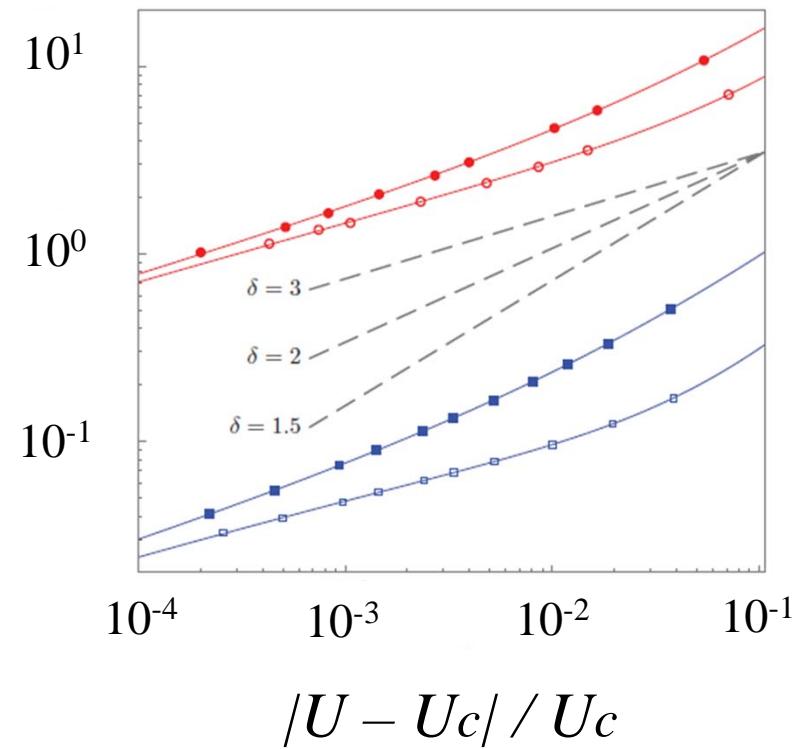
# Double occupancy ( $\delta$ )

- P. Sémond and A.-M.S. Tremblay, PRB 85, 201101 (R) (2012)

$D - D_c$



$|D - D_c| / D_c$



Red circles: CDMFT

Blue squares: single-site DMFT

# Subleading corrections to mean-field

$$a\eta + c\eta^3 = h \quad \delta = 3 \quad \text{Kotliar et al. PRL } \mathbf{84}, \text{ (2000)}$$

$$\alpha = \alpha_1(U - U_c) + \alpha_2(T - T_c)$$

$$\alpha_1(U - U_c)\eta + c\eta^3 = h_1(U - U_c)$$

$$\eta = \sum_{i=1}^{\infty} \delta U^{i/3} \eta_i$$

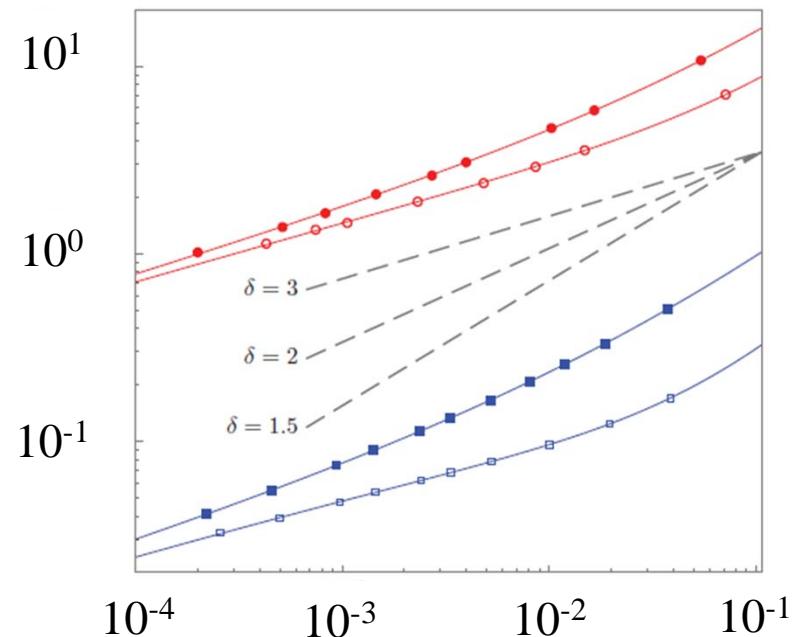
$$D = D_c + a_1\eta + a_2\eta^2$$

$$D - D_c = c_1 \operatorname{sgn}(\delta U) |\delta U|^{1/\delta} + c_2 |\delta U|^{2/\delta} + c_3 \delta U$$

- P. Sémond and A.-M.S. Tremblay, PRB 85, 201101 (R) (2012)

# Double occupancy ( $\delta$ )

$$|D - D_c| / D_c$$



Kotliar et al. PRL **84**, (2000)

$$D - D_c = c_1 \operatorname{sgn}(\delta U) |\delta U|^{1/\delta} + c_2 |\delta U|^{2/\delta} + c_3 \delta U$$

$$|U - U_c| / U_c$$

Red circles: CDMFT  
Blue squares: single-site DMFT

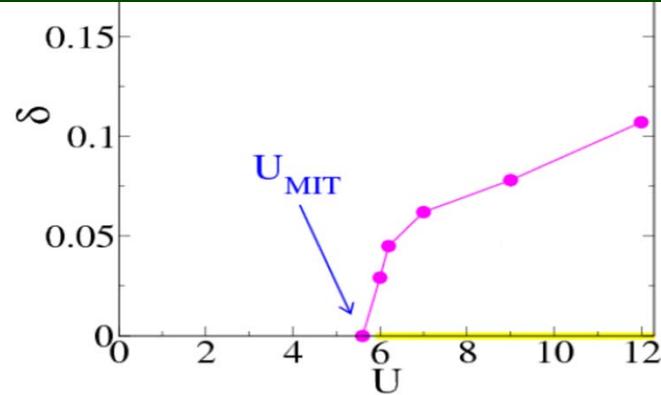
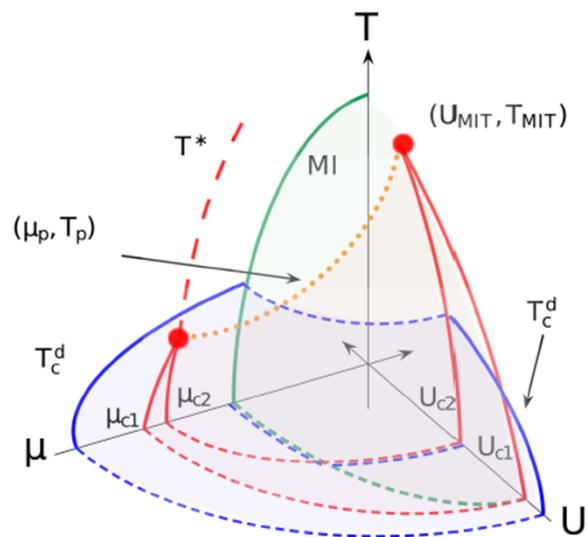
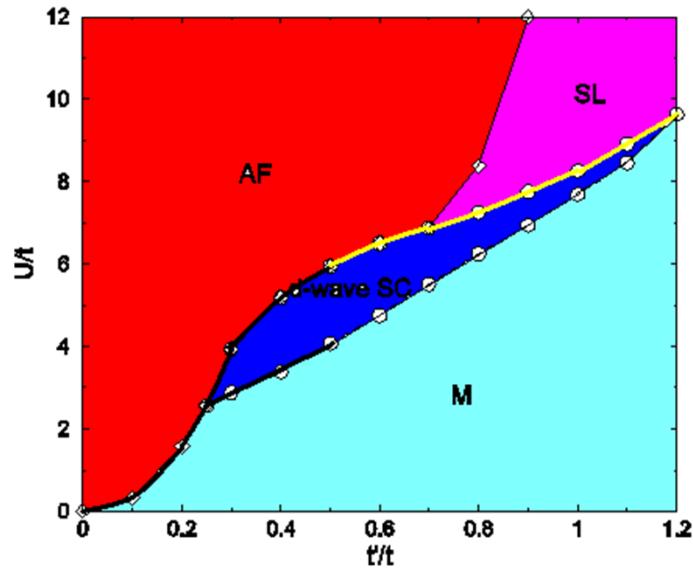
$$\delta = 3.04 \pm 0.25$$

$$\delta = 2.93 \pm 0.15$$

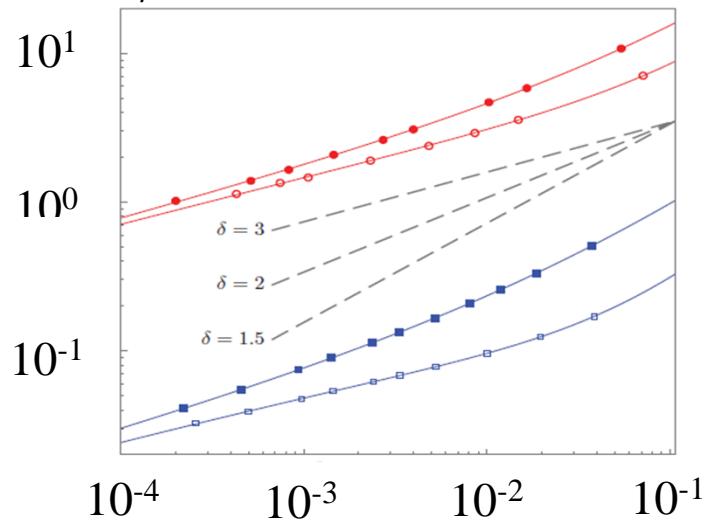


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# Summary



$|D - D_c| / D_c$



$|U - U_c| / U_c$



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# Main collaborators



Giovanni Sordi



Kristjan Haule



David Sénéchal



Bumsoo Kyung



Patrick Sémon



Massimo Capone



Sarma Kancharla



Marcello Civelli



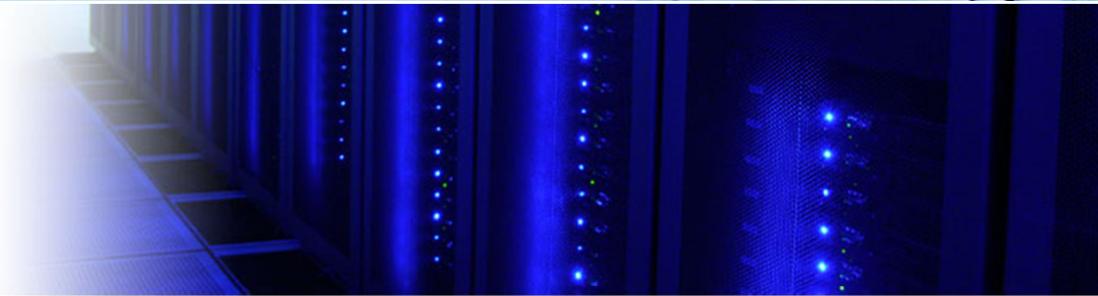
Gabriel Kotliar



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André-Marie Tremblay



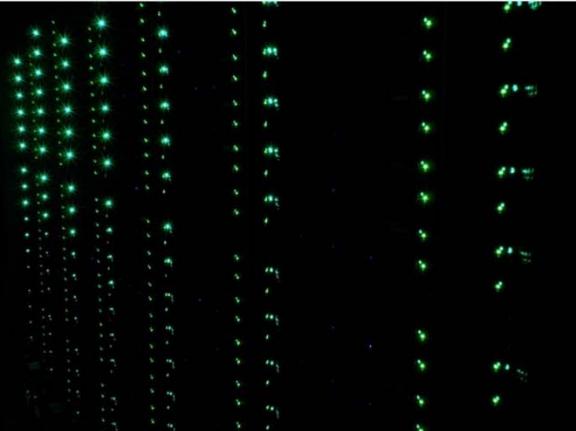
Le regroupement québécois sur les matériaux de pointe



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merci

thank you