

# Insulators, metals, pseudogaps and cuprate superconductivity

A.-M. Tremblay



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for ADVANCED RESEARCH



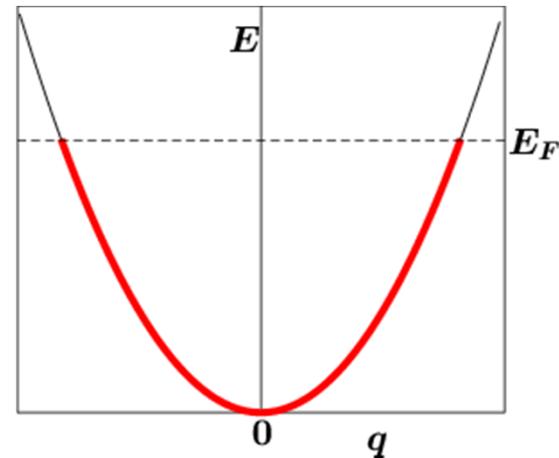
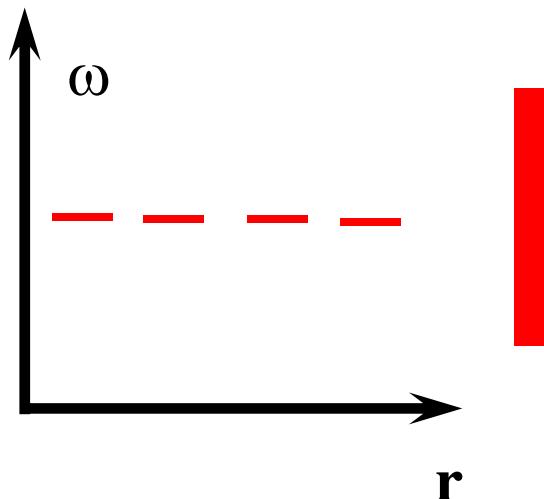
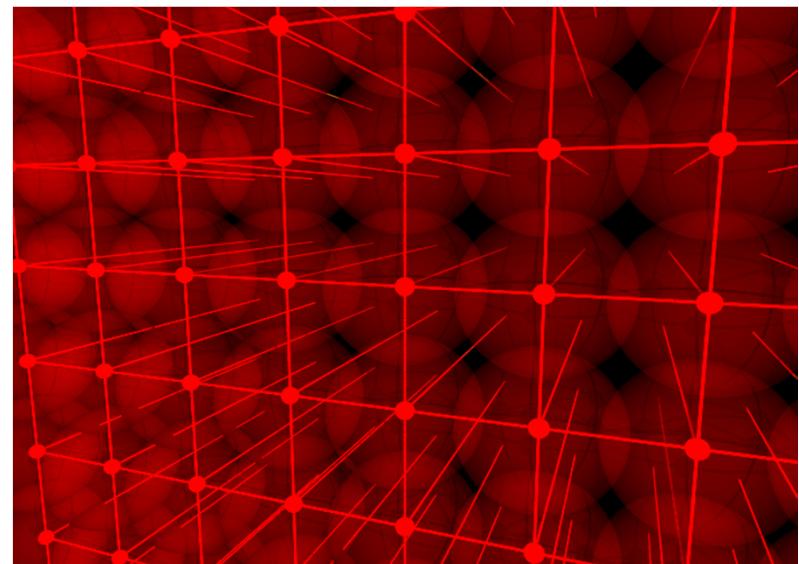
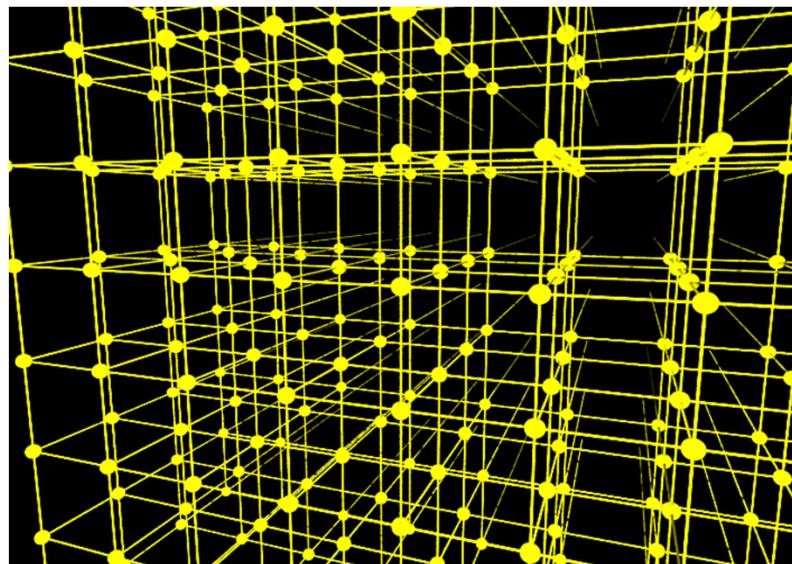
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Rutherford, June 20th, 2013



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# How to make a metal



Courtesy, S. Julian



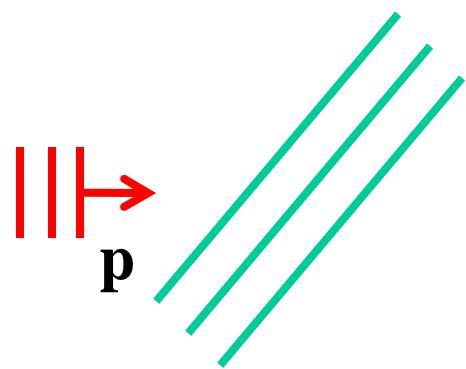
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# Superconductivity



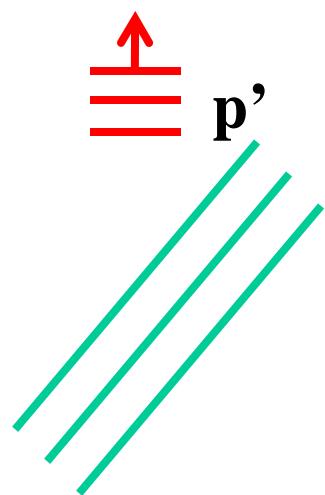
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# Attraction mechanism in the metallic state



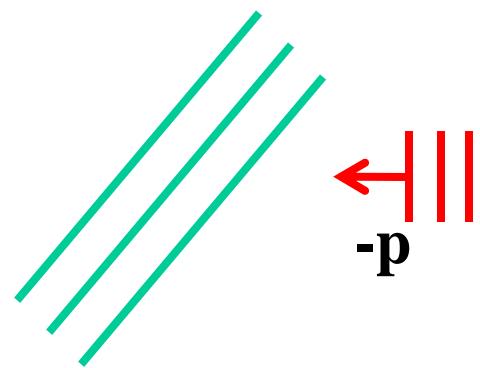
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# Attraction mechanism in the metallic state



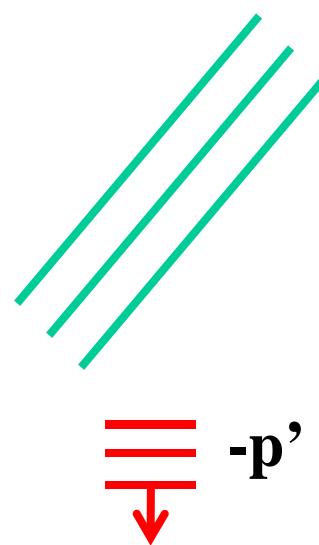
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# Attraction mechanism in the metallic state

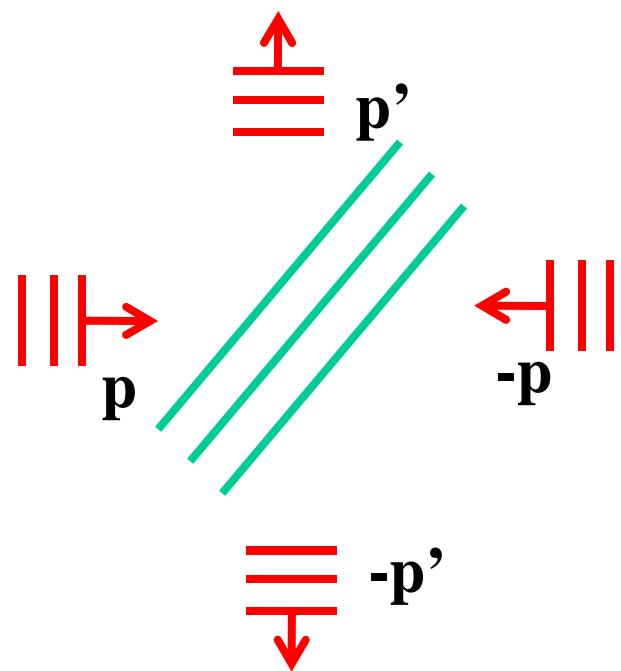


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# Attraction mechanism in the metallic state



# Attraction mechanism in the metallic state



# #1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^*$$

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left( \langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* + \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \right)$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$

# Breakdown of band theory Half-filled band is metallic?



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# Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937



Mott, 1949



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# « Conventional » Mott transition

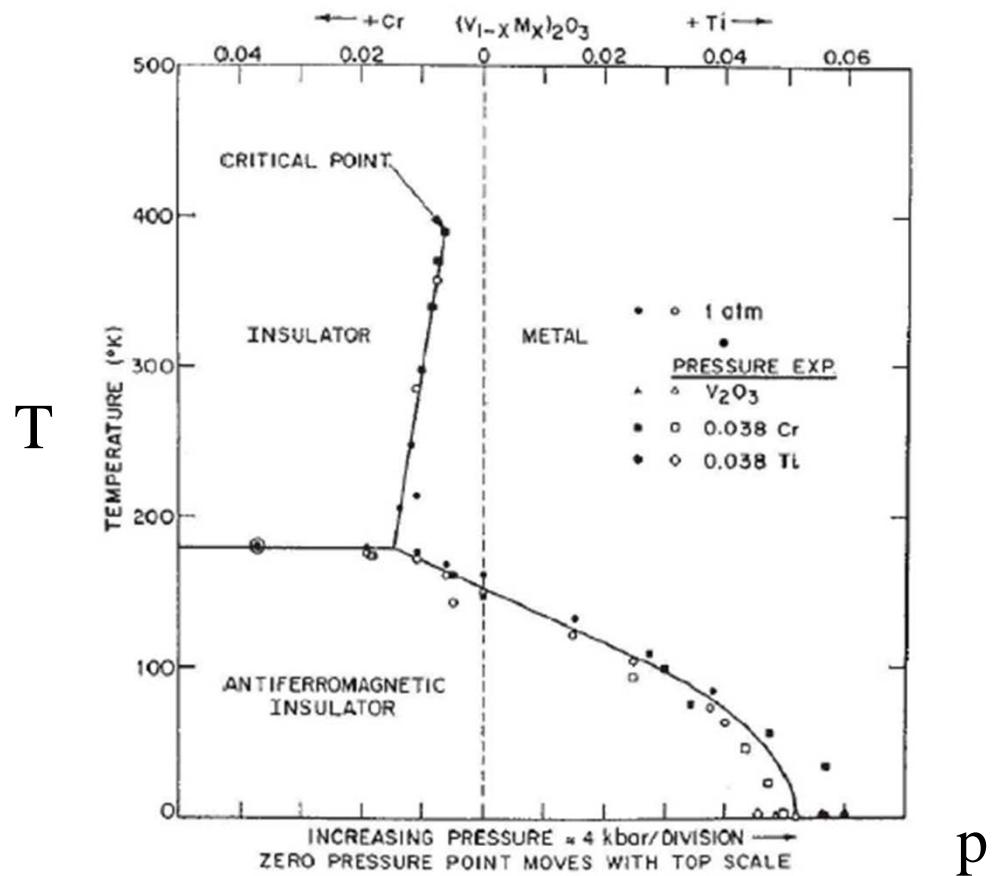


Figure: McWhan, PRB 1970; Limelette, Science 2003

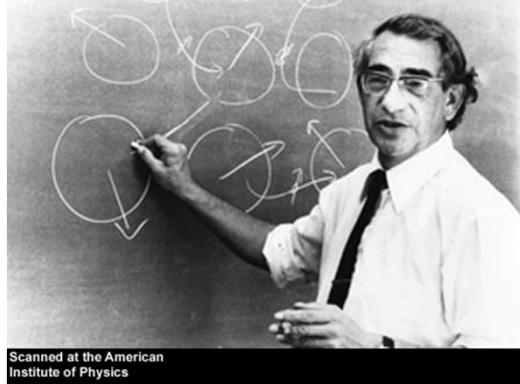
# Model

$$H = -\sum_{<ij>\sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



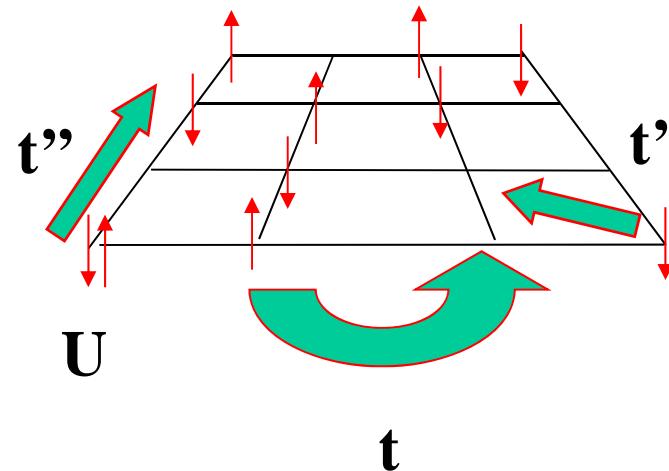
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# Hubbard model



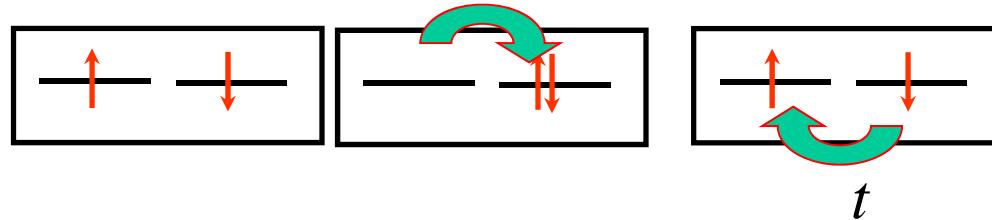
Scanned at the American Institute of Physics

$\mu$



1931-1980

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$t = 1$

Effective model, Heisenberg:  $J = 4t^2 / U$



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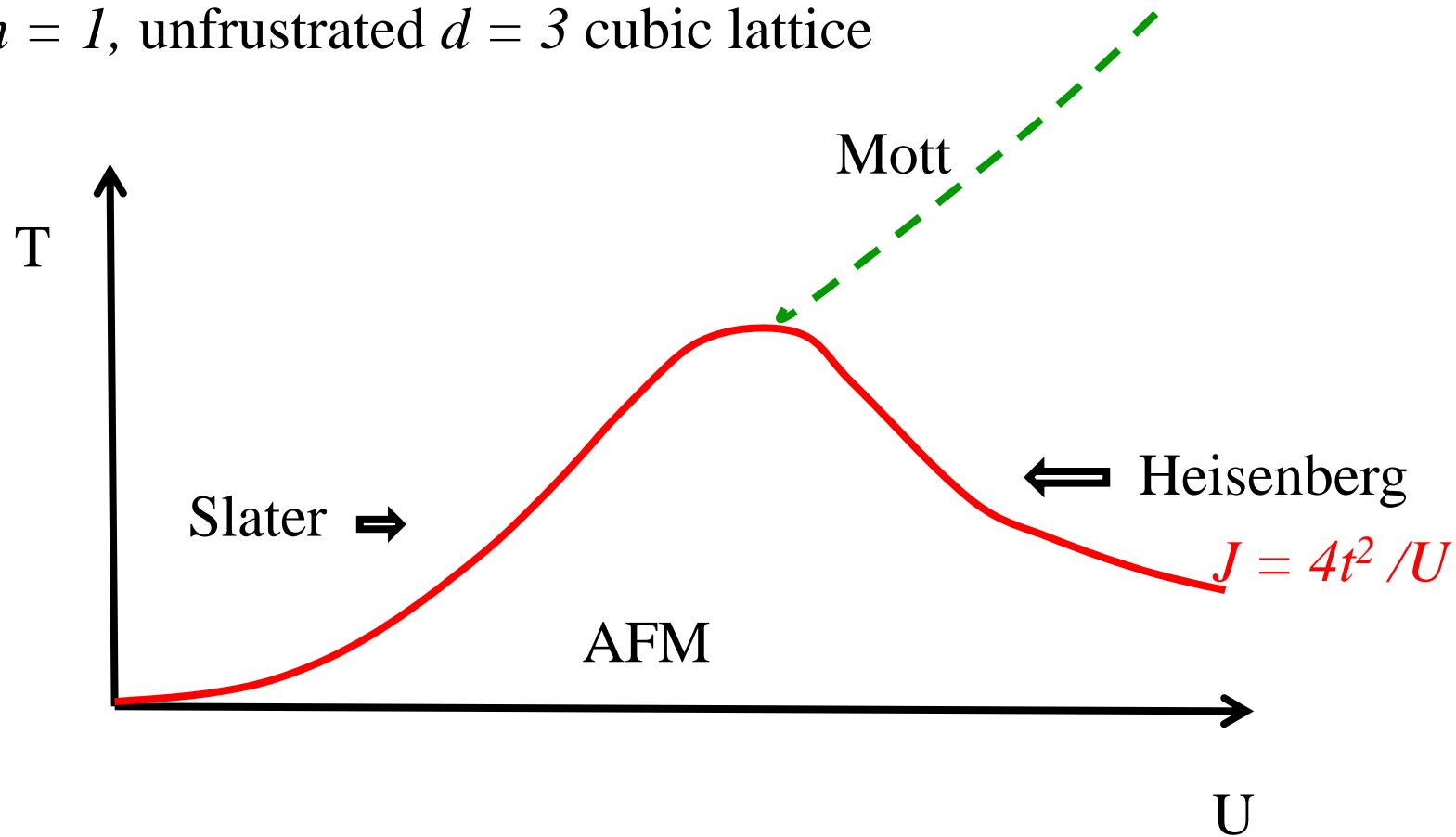
# Strongly vs weakly correlated



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# Local moment and Mott transition

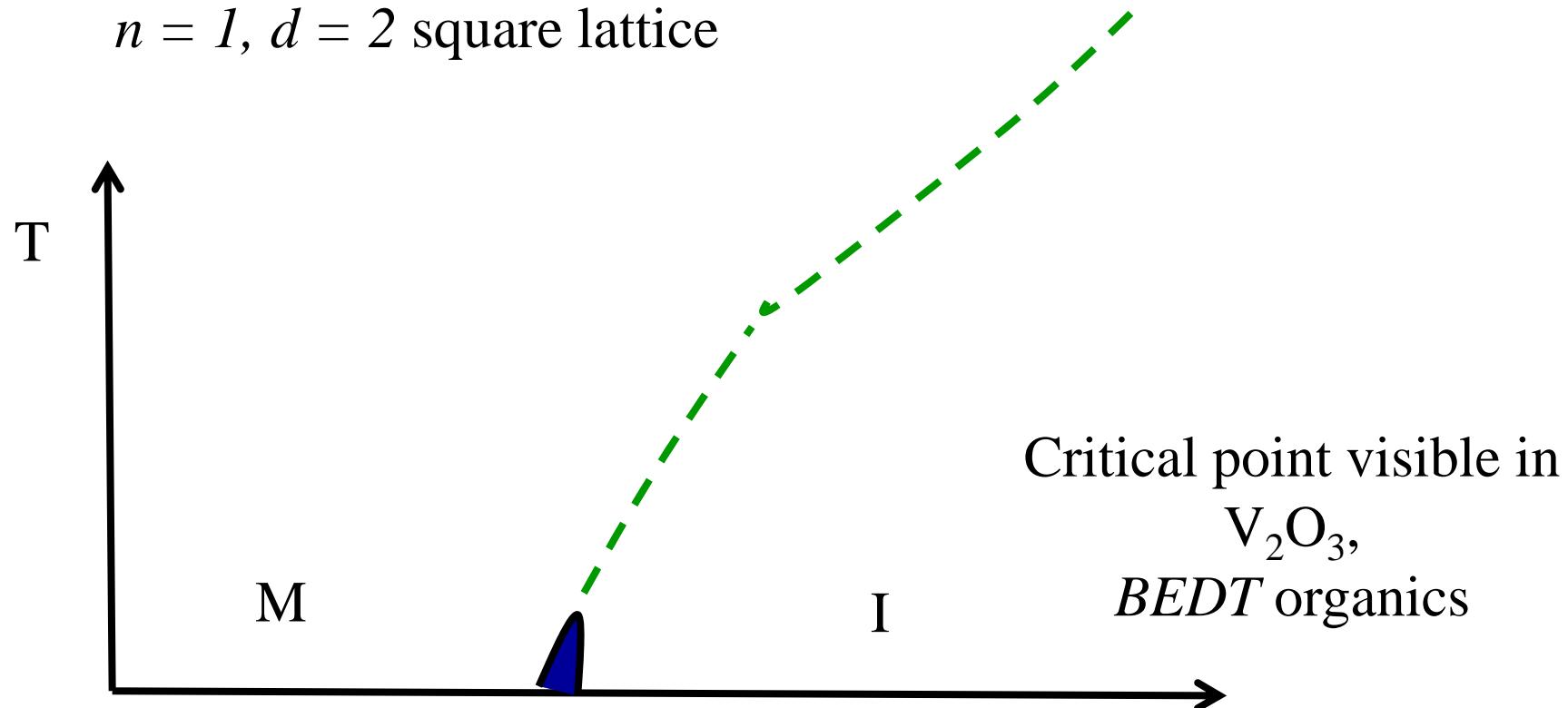
$n = 1$ , unfrustrated  $d = 3$  cubic lattice



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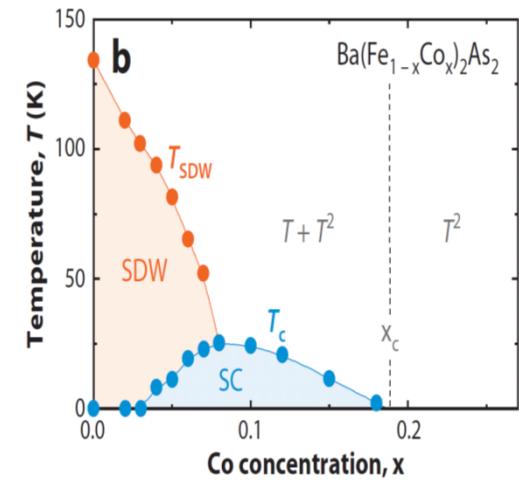
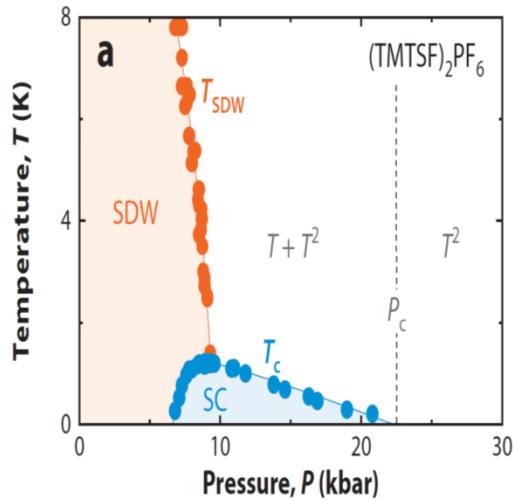
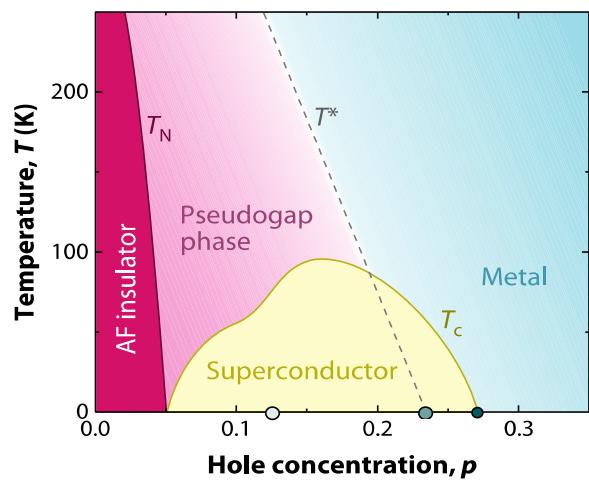
# Local moment and Mott transition

$n = 1, d = 2$  square lattice



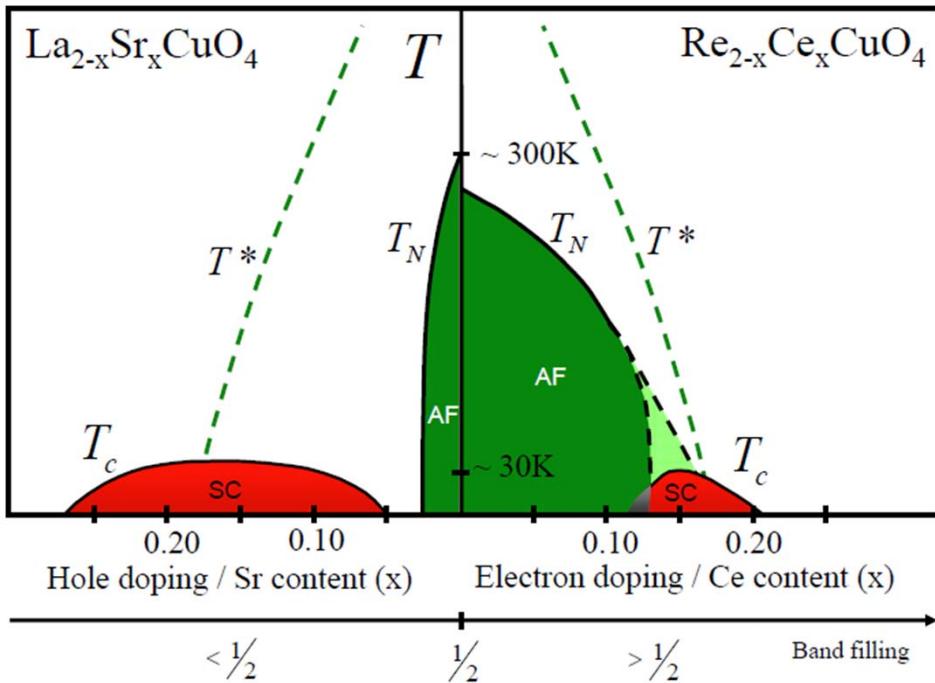
Understanding finite temperature phase from a *mean-field theory* down to  $T = 0$

# Superconductivity and strong repulsion



# High-temperature superconductors

Armitage, Fournier, Greene, RMP (2009)

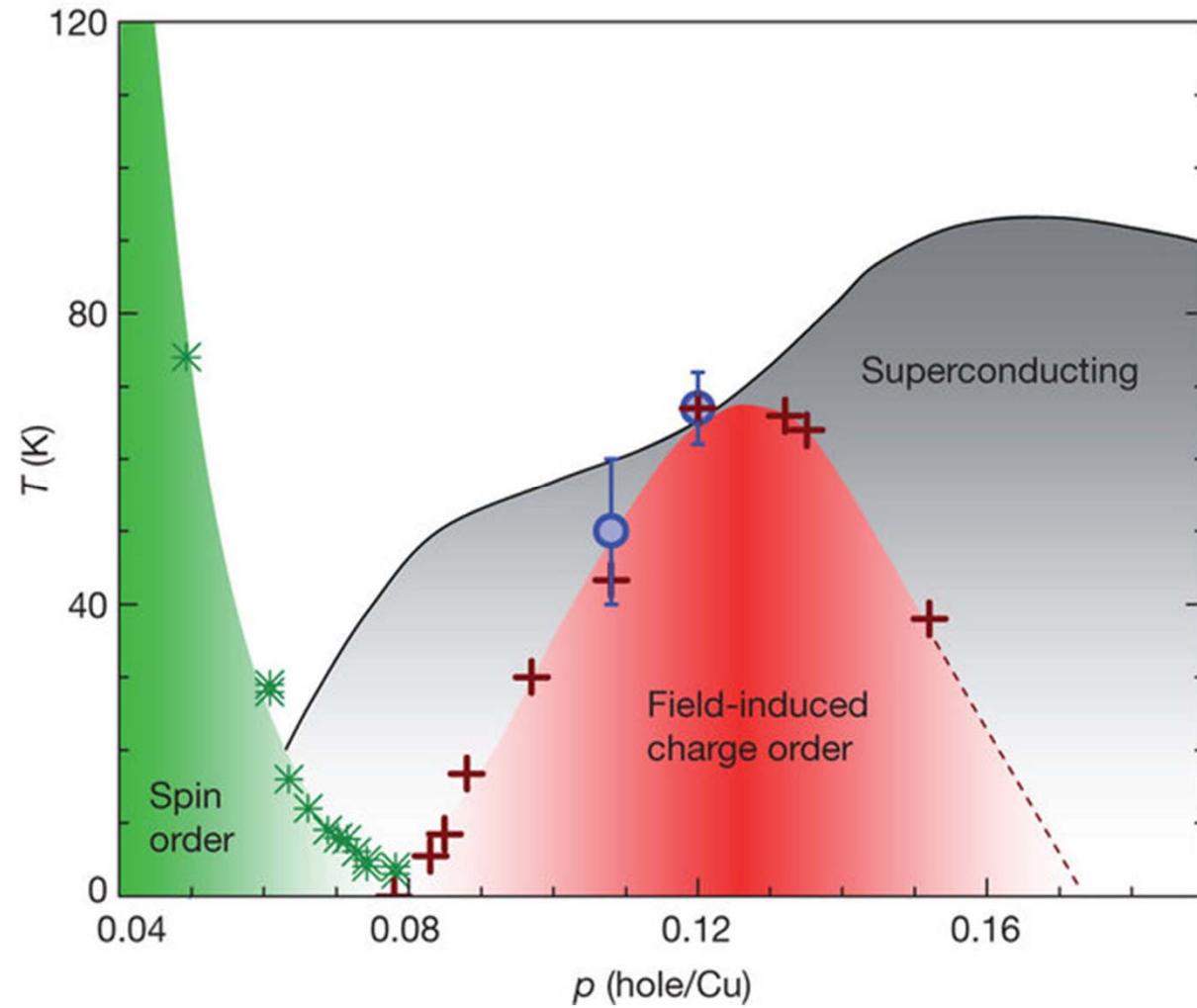


- Competing order
  - Current loops: Varma, PRB **81**, 064515 (2010)
  - Stripes or nematic: Kivelson et al. RMP **75** 1201(2003); J.C.Davis
  - d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
  - SDW: Sachdev PRB **80**, 155129 (2009) ...

- Or Mott Physics?
  - RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

What is under the dome?  
Mott Physics away from  $n = 1$

# Stripes and reconstructed Fermi surface



Wu et al. Julien, Nature **477**, 191–194 (2011)

# Competing CDW order

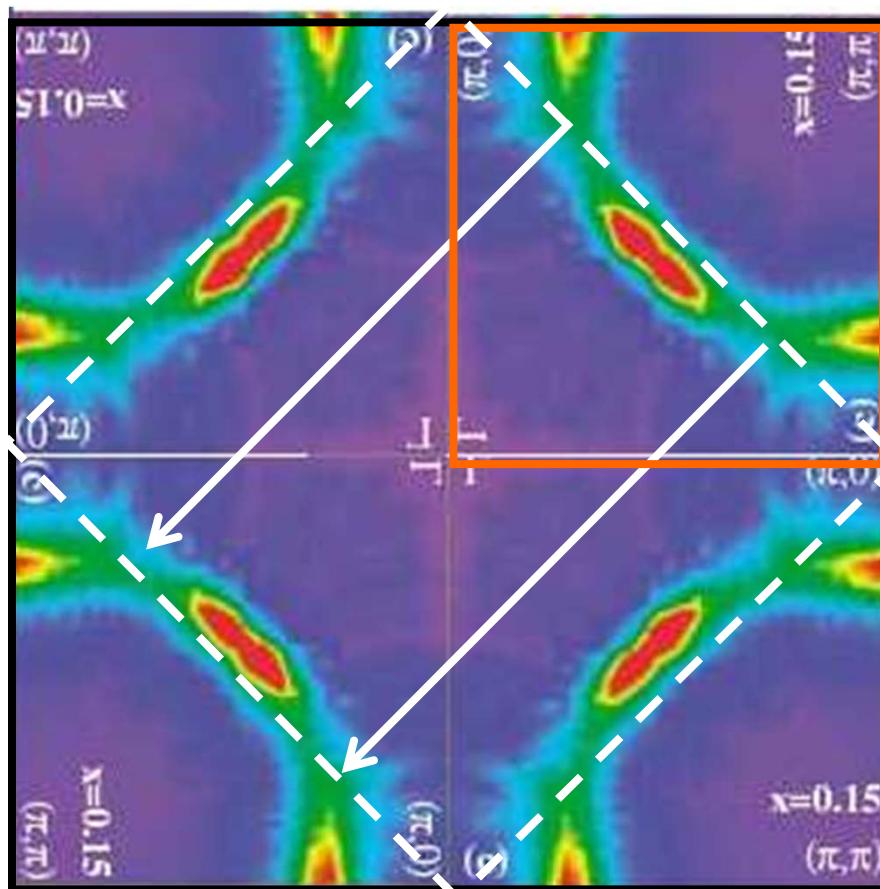
- Wise, W. D. et al. Charge-density-wave origin of cuprate checkerboard visualized by scanning tunnelling microscopy. *Nature Phys.* 4, 696699 (2008).
- Lawler, M. J. et al. Intra-unit-cell electronic nematicity of the high-T<sub>c</sub> copper-oxide pseudogap states. *Nature* 466, 347351 (2010).
- Parker, C. V. et al. Fluctuating stripes at the onset of the pseudogap in the high-T<sub>c</sub> superconductor B<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>8</sub>C<sub>x</sub>. *Nature* 468, 677680 (2010).
- Chang, J. et al. Direct observation of competition between superconductivity and charge density wave order in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub>:67. *Nature Phys.* 8, 871876 (2012).
- Ghiringhelli, G. et al. Long-range incommensurate charge fluctuations in (Y;Nd)Ba<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub>C<sub>x</sub>. *Science* 337, 821825 (2012).
- Achkar, A. J. et al. Distinct charge orders in the planes and chains of ortho-III-ordered YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6</sub>C superconductors identified by resonant elastic X-ray scattering. *Phys. Rev. Lett.* 109, 167001 (2012).
- Wu, T. et al. Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa<sub>2</sub>Cu<sub>3</sub>O<sub>y</sub>. *Nature* 477, 192194 (2011).
- LeBoeuf, D. et al. Thermodynamic phase diagram of static charge order in underdoped YBa<sub>2</sub>Cu<sub>3</sub>O<sub>y</sub>. *Nature Phys.* 9, 7983 (2013).

# Three broad classes of mechanisms for pseudogap

- Rounded first order transition
- $d = 2$  precursor to a lower temperature broken symmetry phase
- Mott physics
  - Competing order
    - Current loops: Varma, PRB **81**, 064515 (2010)
    - Stripes or nematic: Kivelson et al. RMP 75 1201(2003); J.C.Davis
    - d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
    - SDW: Sachdev PRB **80**, 155129 (2009) ...
  - Or Mott Physics?
    - RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

# Hot spots from AFM quasi-static scattering

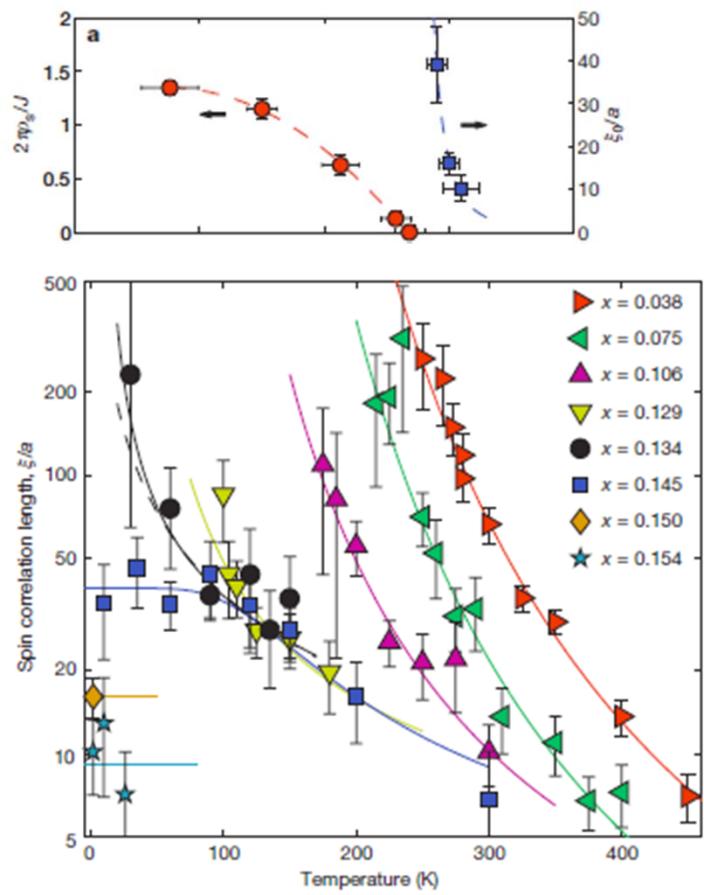
$d = 2$



Armitage et al. PRL 2001

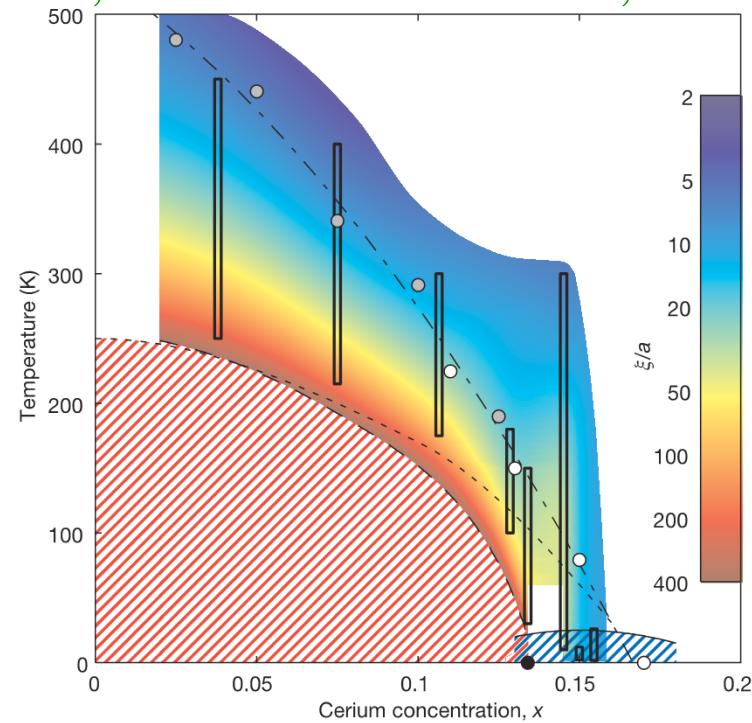
# e-doped cuprates: precursors

NCCO

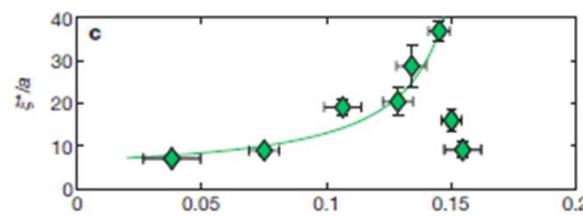


$$Z = 1$$

Motoyama, E. M. et al.. Nature 445, 186–189 (2007).



Vilk, A.-M.S.T (1997)  
Kyung, Hankevych, A.-M.S.T., PRL, 2004



$$\xi^* = 2.6(2)\xi_{\text{th}}$$



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# Back to hole-doped



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# What is the minimal model?

H. Alloul arXiv:1302.3473

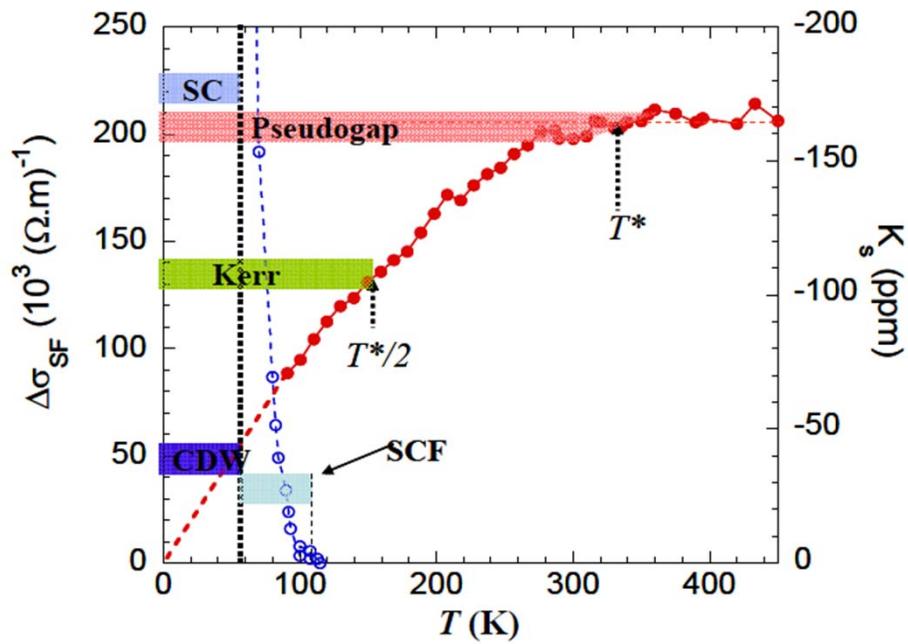
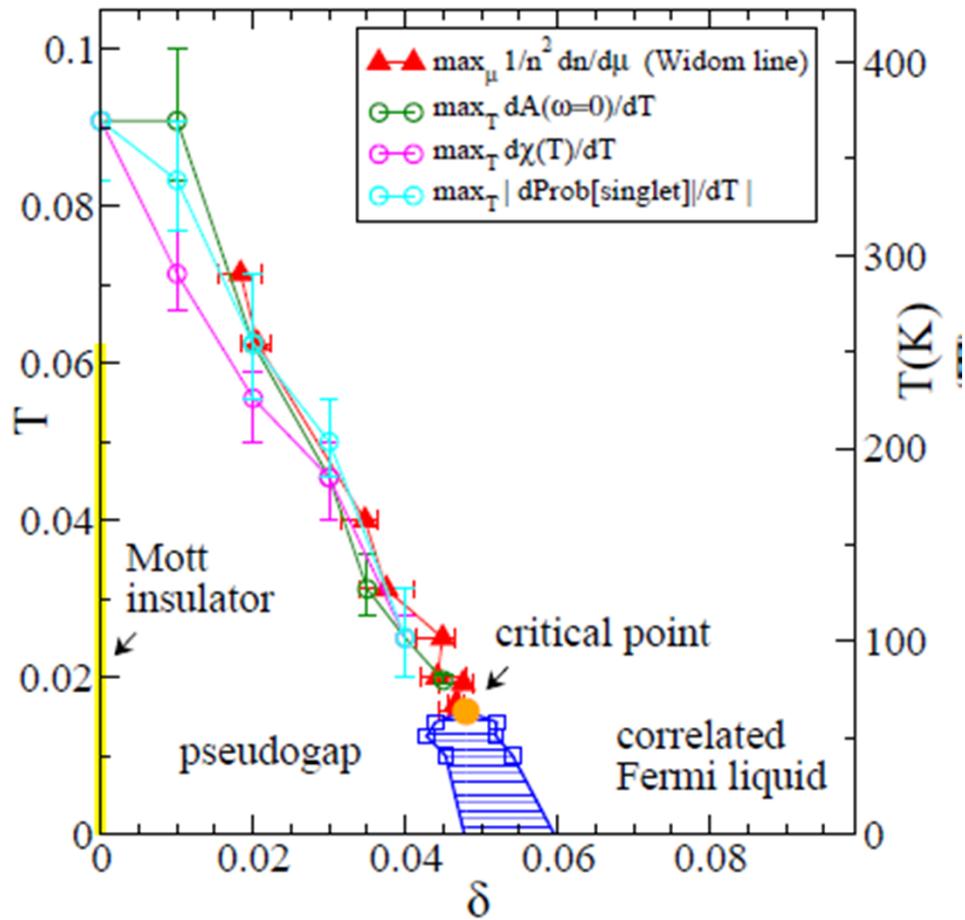


Fig 1 Spin contribution  $K_s$  to the  $^{89}\text{Y}$  NMR Knight shift [11] for  $\text{YBCO}_{6.6}$  permit to define the PG onset  $T^*$ . Here  $K_s$  is reduced by a factor two at  $T \sim T^*/2$ . The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).

# Pseudogap from Mott physics



G. Sordi, *et al.* Scientific Reports 2, 547 (2012)

Competing order is a consequence of the pseudogap, not its cause:

Parker et al. Nature 468, 677 (2010)

# Hole-doped cuprates as Mott insulators

# Mott-Ioffe-Regel limit

$$\sigma = \frac{ne^2\tau}{m}$$

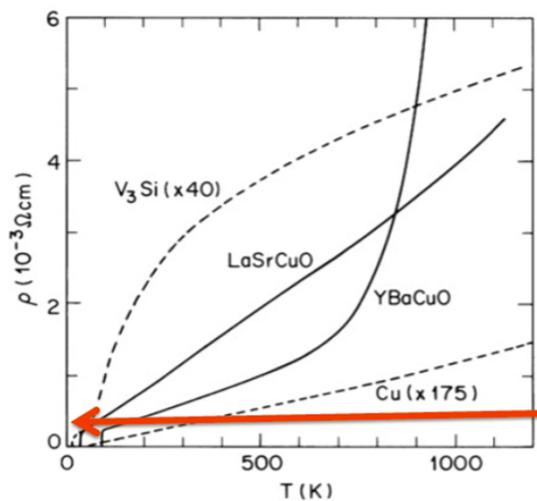
$$k_F\ell = \frac{2\pi}{\lambda_F}\ell \sim 2\pi$$

$$\sigma_{MIR} = \frac{e^2}{\hbar d}$$



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# Hole-doped cuprates and MIR limit



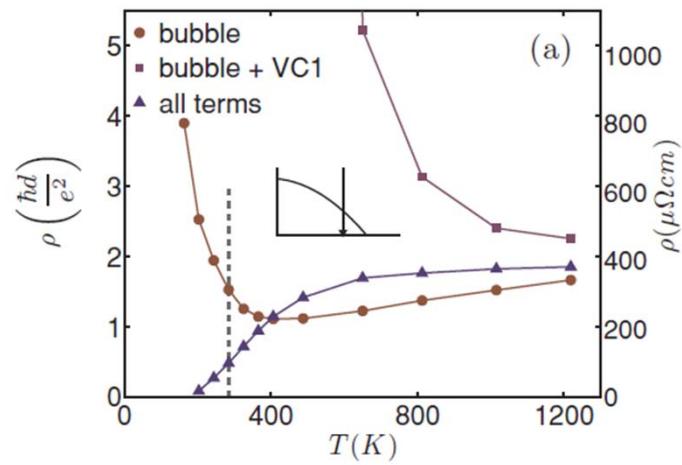
Gurvitch & Fiory  
PRL 59, 1337  
(1987)

MIR limit  
Mean-free path  
~ Fermi wavelength

LSCO 17%, YBCO optimal

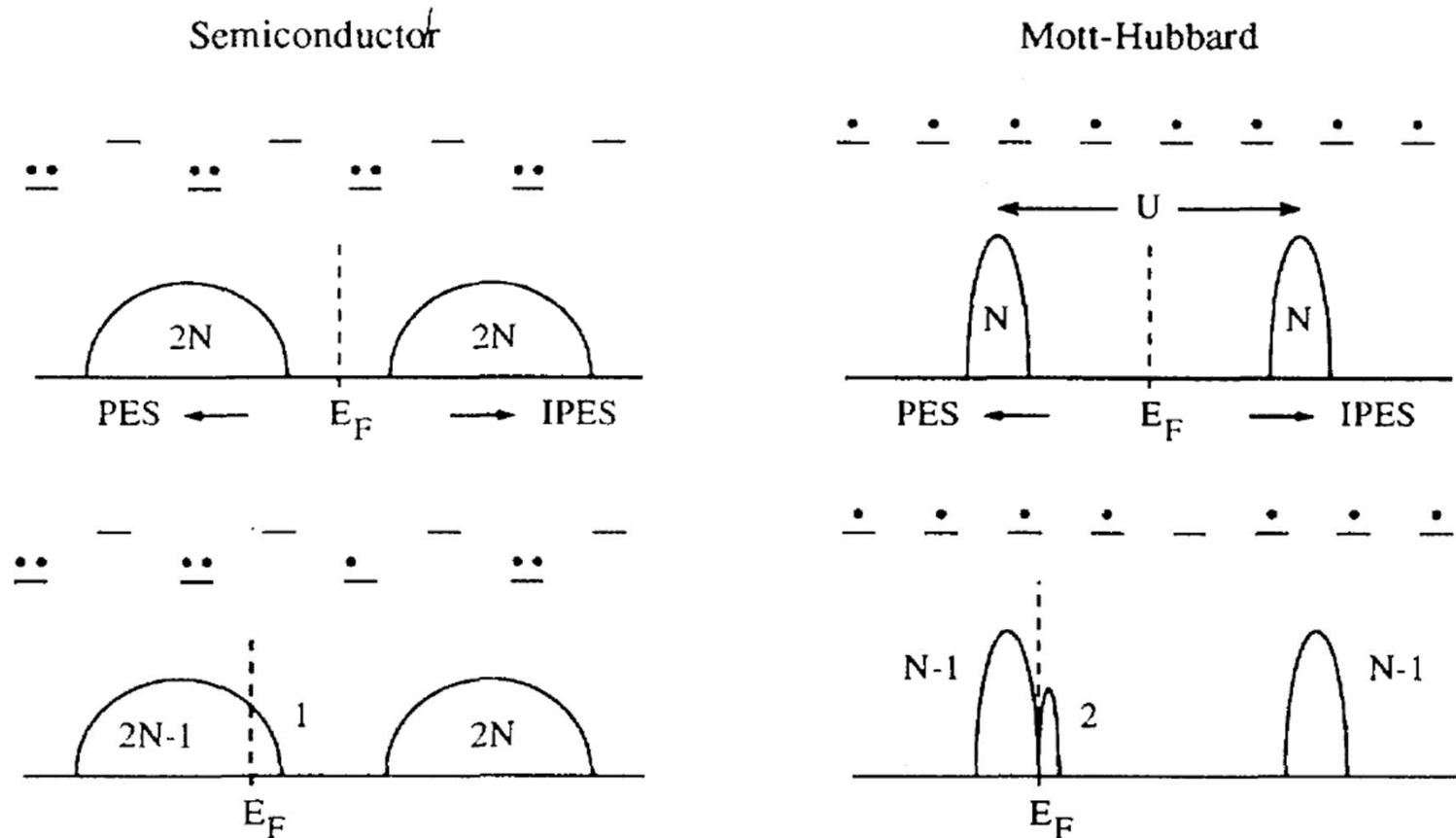
PHYSICAL REVIEW B 84, 085128 (2011)

Optical and dc conductivity of the two-dimensional Hubbard model in the pseudogap regime and across the antiferromagnetic quantum critical point including vertex corrections



Dominic Bergeron et al. TPSC  
PRB 84, 085128 (2011)

# Spectral weight transfer

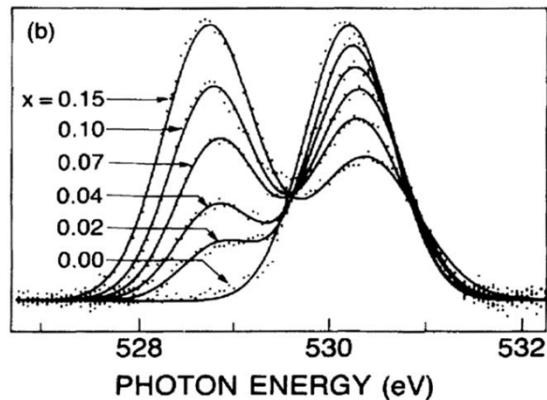


Meinders *et al.* PRB **48**, 3916 (1993)

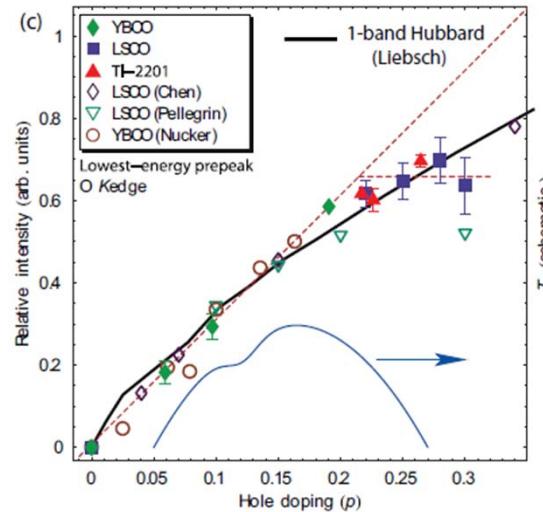


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# Experiment: X-Ray absorption



Chen et al. PRL **66**, 104 (1991)



Peets et al. PRL **103**, (2009),  
Phillips, Jarrell PRL , vol. **105**, 199701 (2010)

Number of low energy states above  $\omega = 0$  scales as  $2x +$   
Not as  $1+x$  as in Fermi liquid

Meinders *et al.* PRB **48**, 3916 (1993)

# Outline

- Method
- Finite  $T$  phase diagram
  - Normal state (no LRO, what is below the dome)
    - First order transition
    - Widom line and pseudogap
  - Superconductivity



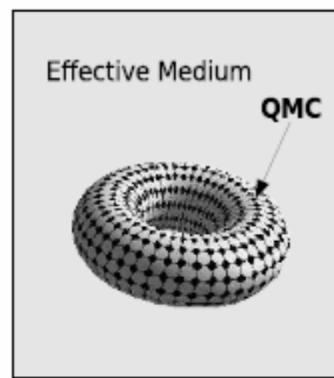
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# Method



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# *2d Hubbard: Quantum cluster method*

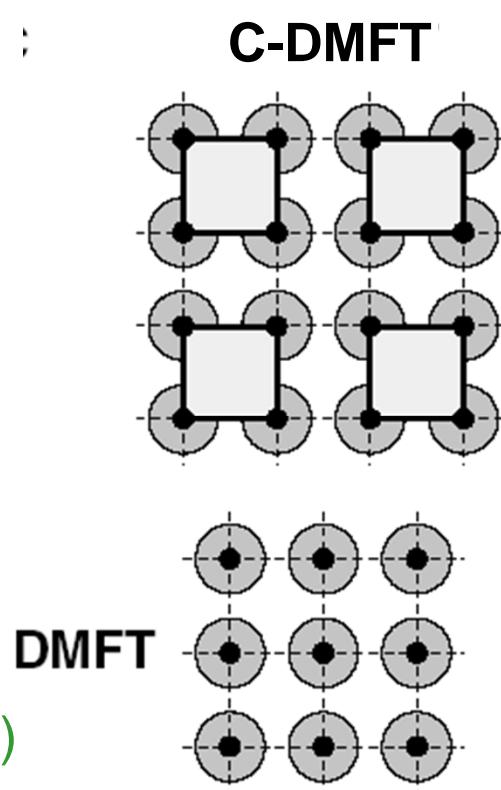


**DCA**

Hettler ... Jarrell ... Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff et al. PRL **91**, 206402 (2003).



**DMFT**

**REVIEWS**

Maier, Jarrell et al., RMP. (2005)

Kotliar et al. RMP (2006)

AMST et al. LTP (2006)



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## Another way to look at this (Potthoff)

$$\Omega_{\mathbf{t}}[G] = \Phi[G] - Tr[(G_{0\mathbf{t}}^{-1} - G^{-1})G] + Tr \ln(-G)$$

$$\frac{\delta \Phi[G]}{\delta G} = \Sigma$$

$$\Omega_{\mathbf{t}}[\Sigma] = \boxed{\Phi[G] - Tr[\Sigma G]} - Tr \ln(-G_{0\mathbf{t}}^{-1} + \Sigma)$$

Still stationary (chain rule)

$$\Omega_{\mathbf{t}}[\Sigma] = \boxed{F[\Sigma]} - Tr \ln(-G_{0\mathbf{t}}^{-1} + \Sigma)$$

# SFT : Self-energy Functional Theory

With  $F[\Sigma]$  Legendre transform of Luttinger-Ward funct.

$$\Omega_t[\Sigma] = F[\Sigma] + \text{Tr} \ln(-(G_0^{-1} - \Sigma)^{-1})$$

is stationary with respect to  $\Sigma$  and equal to grand potential there.

$$\Omega_t[\Sigma] = \Omega_{t'}[\Sigma] - \text{Tr} \ln(-(G_0'^{-1} - \Sigma)^{-1}) + \text{Tr} \ln(-(G_0^{-1} - \Sigma)^{-1}).$$

Vary with respect to parameters of the cluster (including Weiss fields)

Variation of the self-energy, through parameters in  $H_0(t')$

+ and -

- Long range order:
  - Allow symmetry breaking in the bath (mean-field)
- Included:
  - Short-range dynamical and spatial correlations
- Missing:
  - Long wavelength p-h and p-p fluctuations



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# Finite $T$ phase diagram

Normal state of the cuprates



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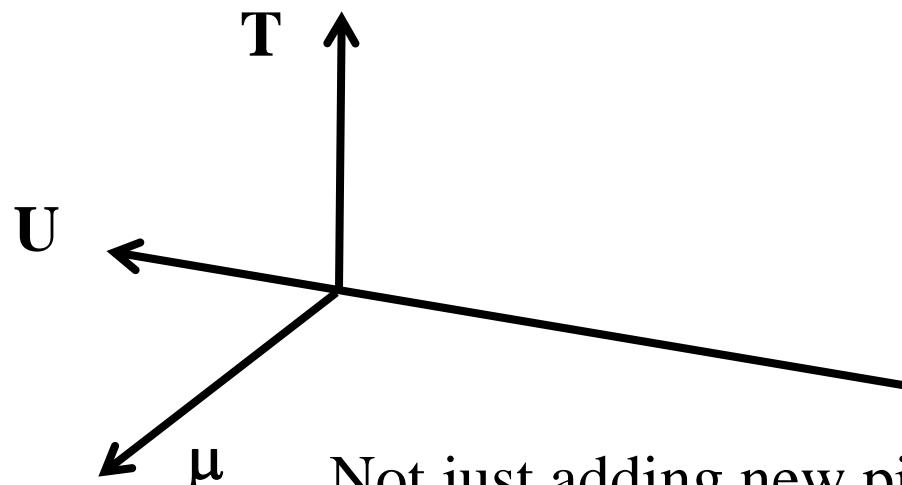


Giovanni Sordi

G. Sordi, K. Haule, A.-M.S.T  
PRL, **104**, 226402 (2010)  
and

Phys. Rev. B, **84**, 075161 (2011)

## Doping-induced Mott transition ( $t'=0$ )



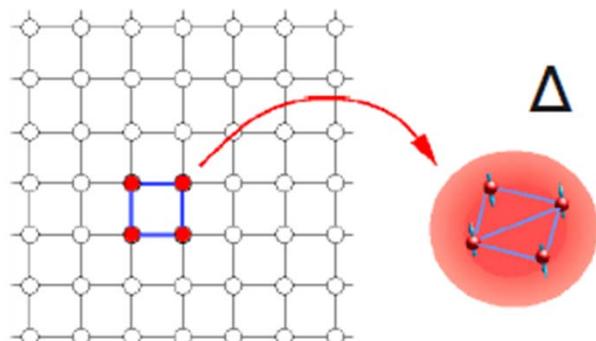
Not just adding new piece:

Lesson from DMFT, first order transition + critical  
point governs phase diagram



Kristjan Haule

# C-DMFT



Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006

P. Werner, PRB 2007

K. Haule, PRB 2007

$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$

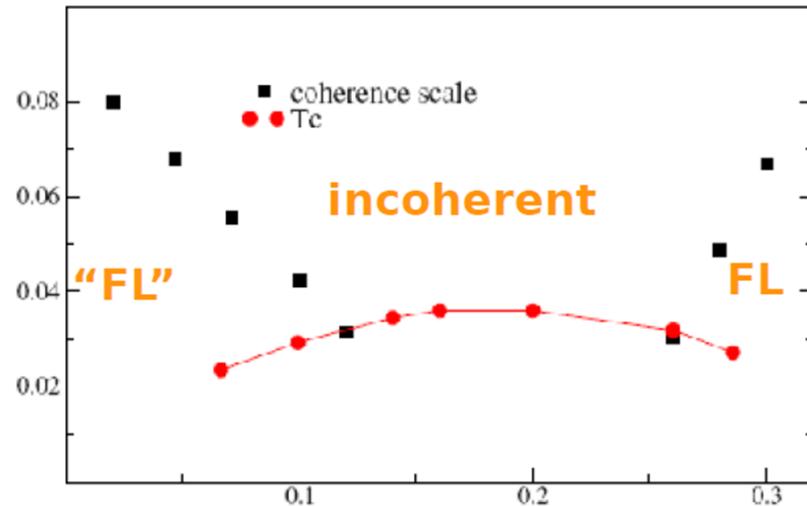
Continuous-time Quantum Monte Carlo calculations to sum all diagrams generated from expansion in powers of hybridization.

P. Werner, A. Comanac, L. de' Medici, M. Troyer, and A. J. Millis, Phys. Rev. Lett. **97**, 076405 (2006).

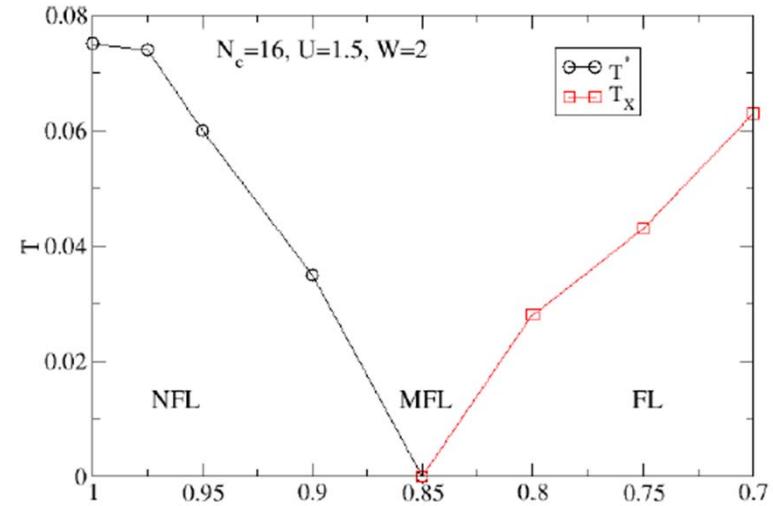
K. Haule, Phys. Rev. B **75**, 155113 (2007).

# Doping driven Mott transition, $t' = 0$

Method	$t'$	Orbital selective	$U$	Critical point	Ref.
D+C+H 8			7		Werner et al. cond-mat (2009)
D+C+H 4					Gull et al. EPL (2008)
	-0.3		10,6		Liebsch, Merino... (2008)
					Ferrero et al. PRB (2009)
D+C+H 8			7		Gull, et al. PRB (2009)

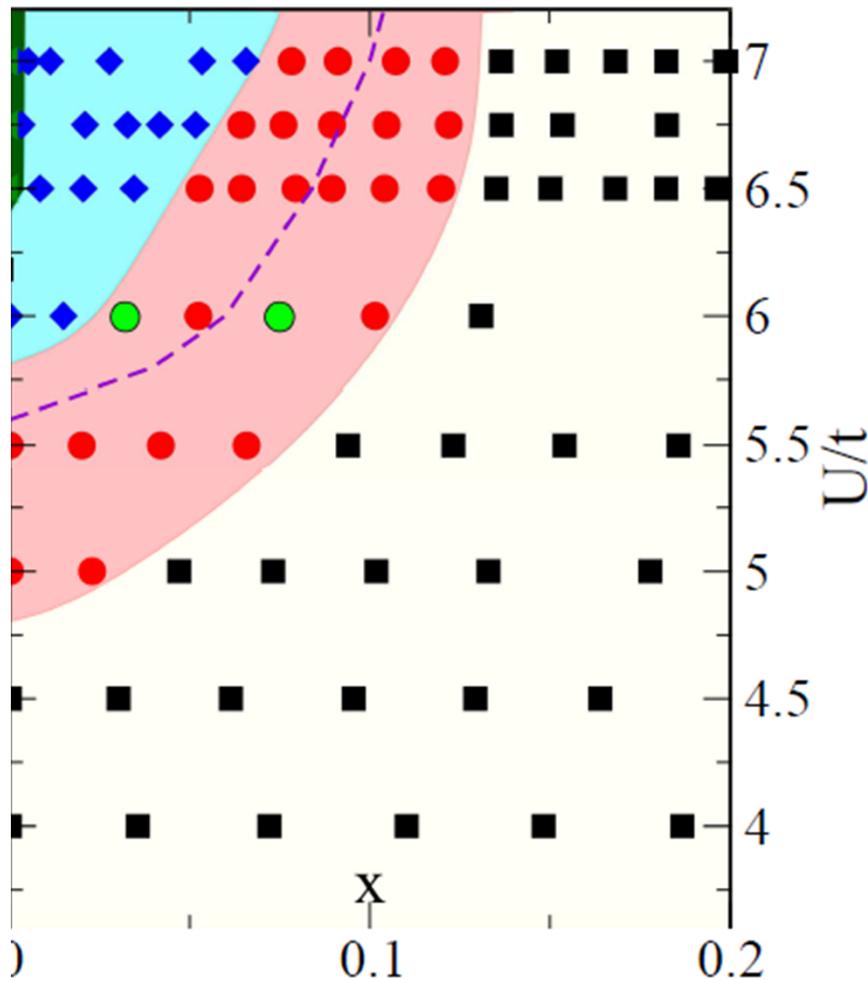


K. Haule, G. Kotliar, PRB (2007)



Vildhyadhiraja, PRL (2009)

# Doping driven Mott transition



$T = 0.25 t$

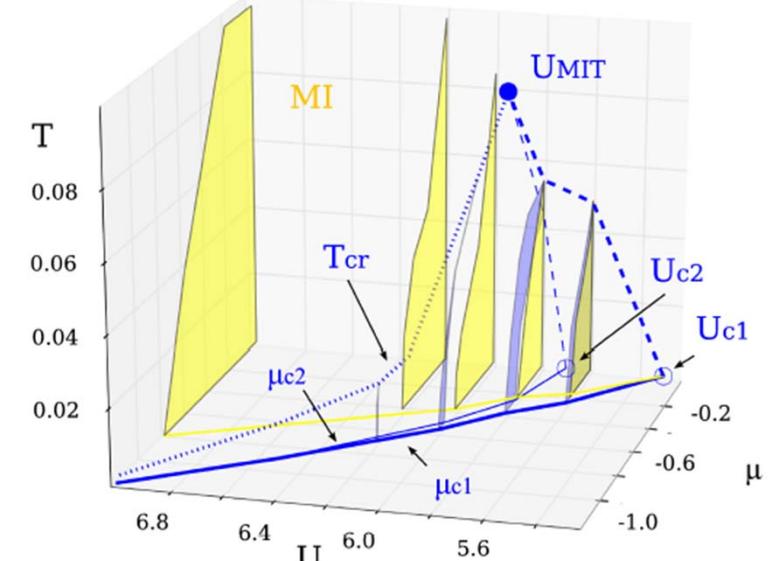
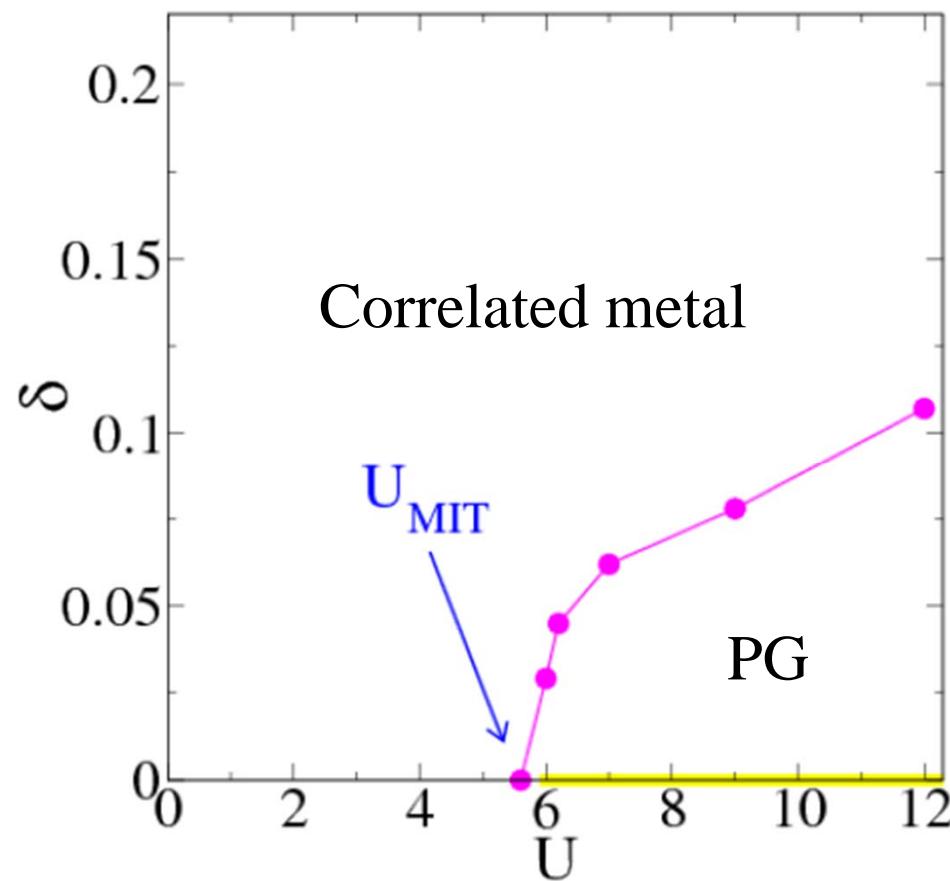
Gull, Parcollet, Millis  
arXiv:1207.2490v1

Gull, Werner, Millis, (2009)

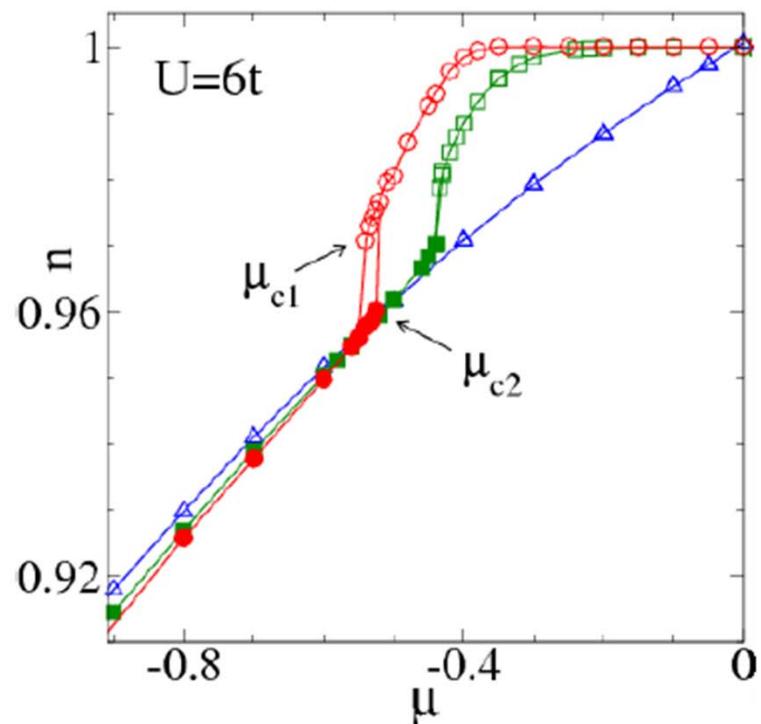
E. Gull, M. Ferrero, O. Parcollet, A. Georges, and A. J. Millis (2009) UNIVERSITÉ DE SHERBROOKE

# Link to Mott transition up to optimal doping

Doping dependence of critical point as a function of  $U$



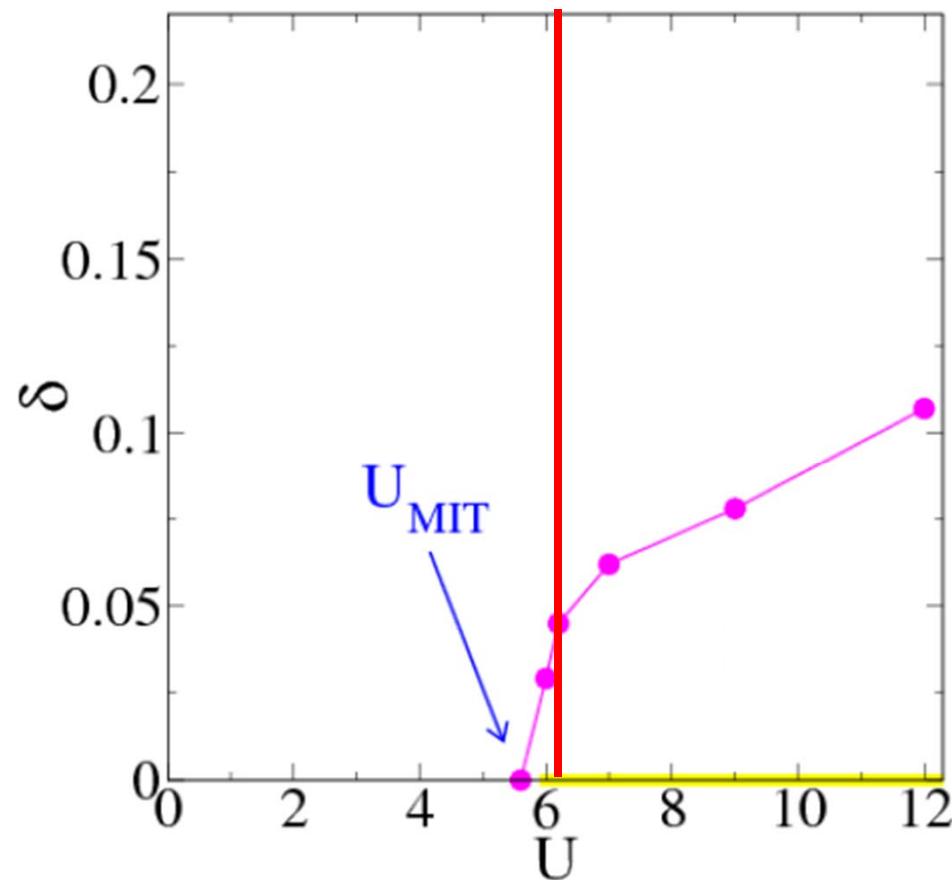
# First order transition at finite doping



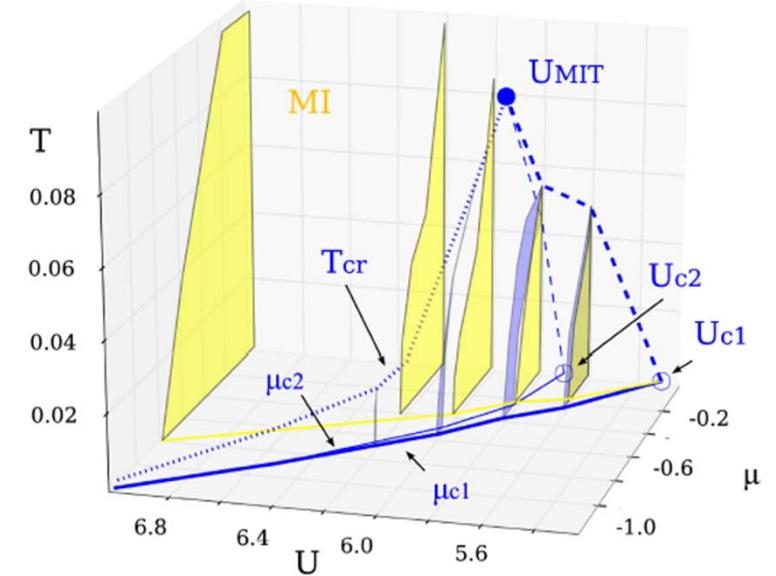
$n(\mu)$  for several temperatures:  
 $T/t = 1/10, 1/25, 1/50$

# Link to Mott transition up to optimal doping

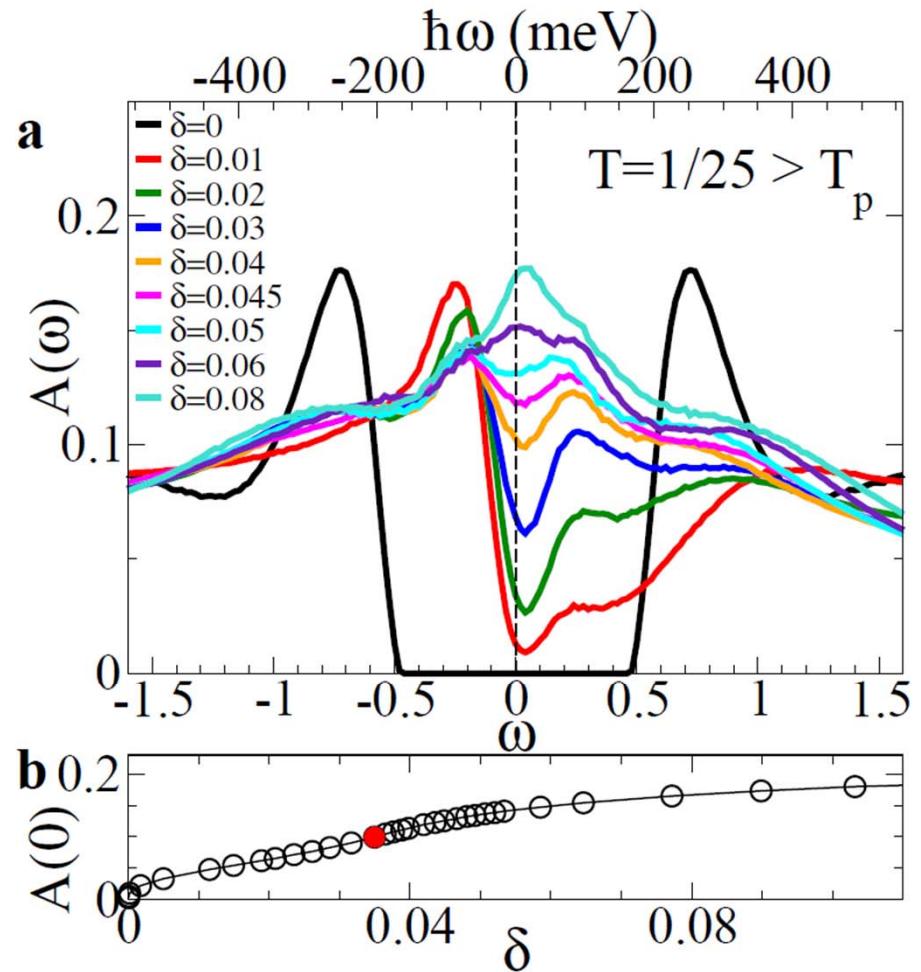
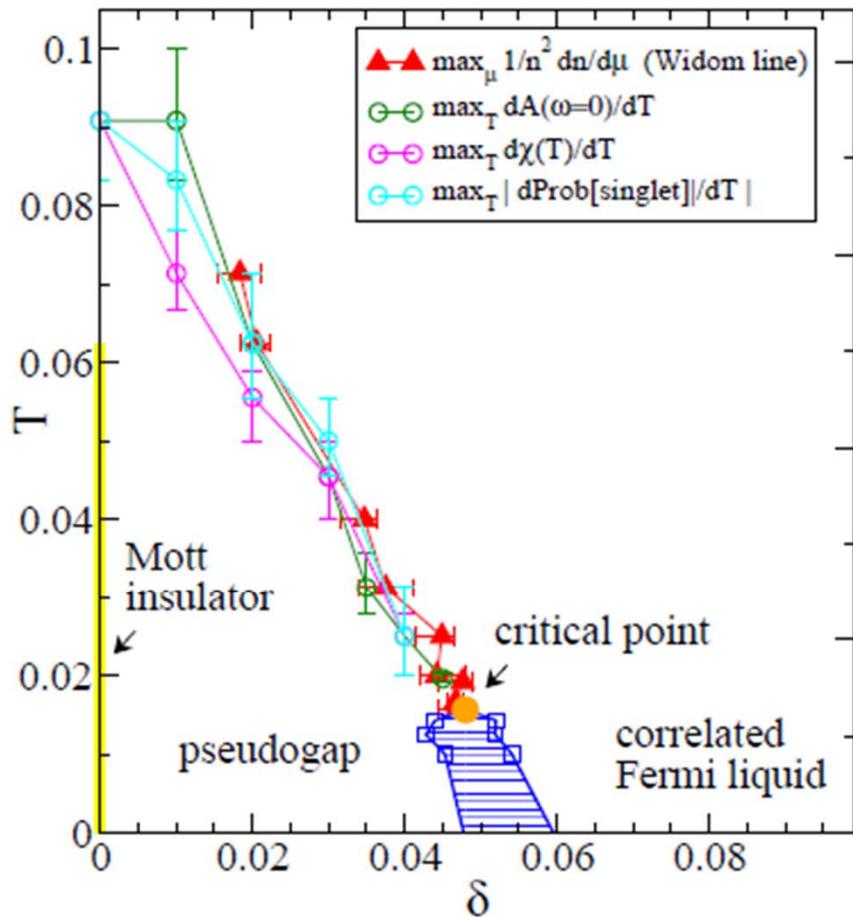
Doping dependence of critical point as a function of  $U$



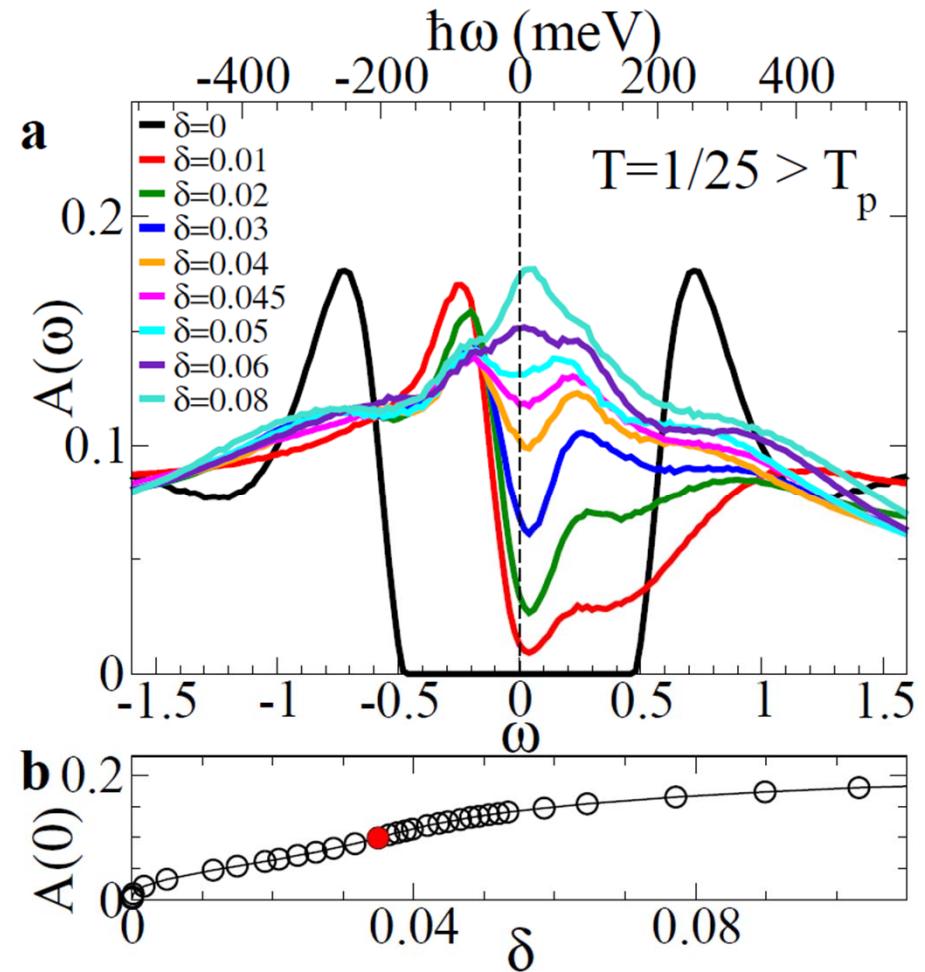
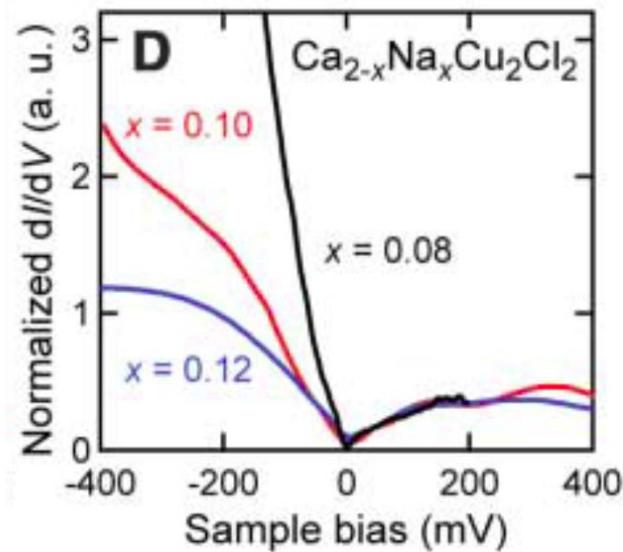
Smaller  $D$  and  $S$



# Density of states



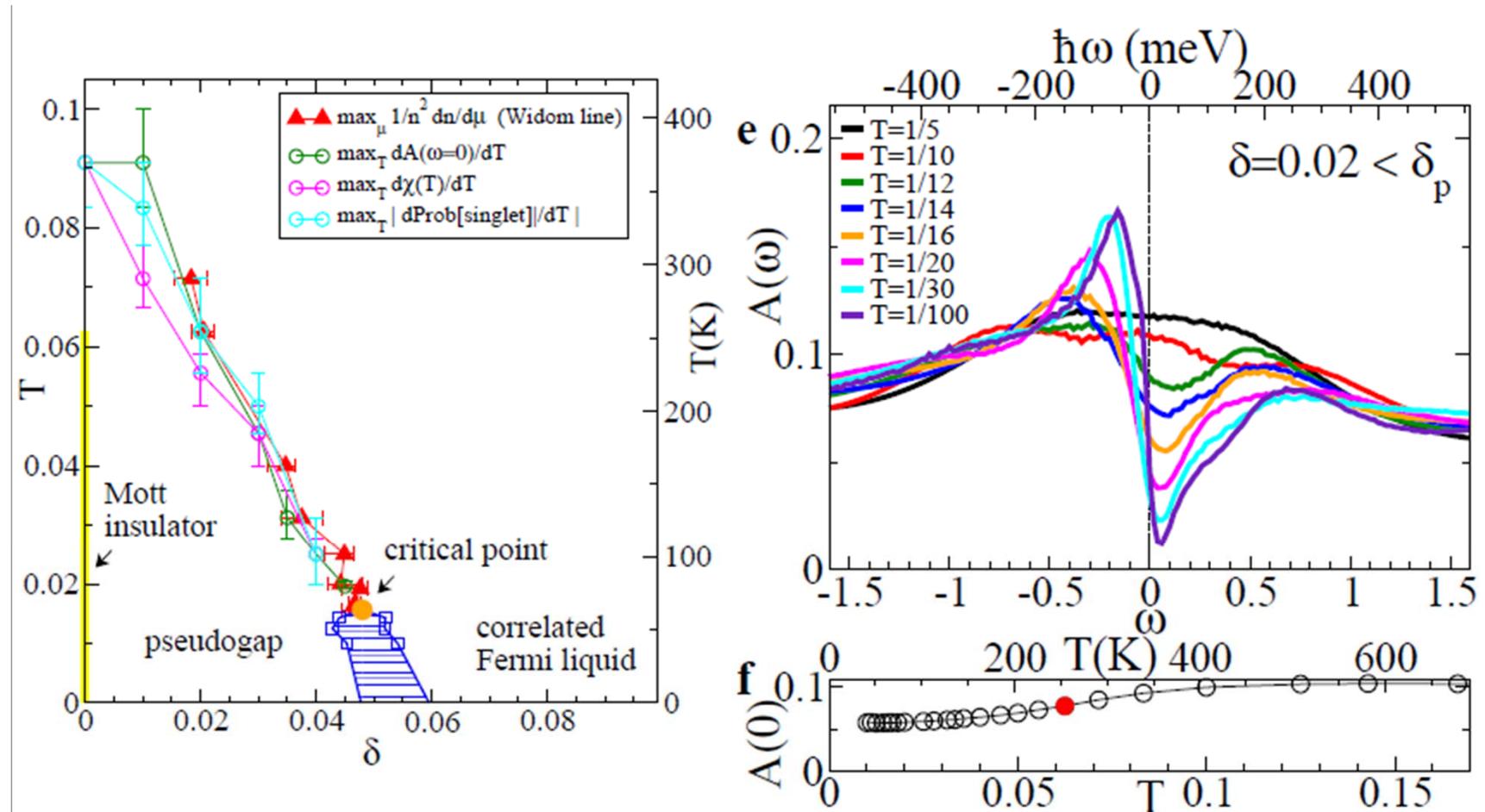
# Density of states



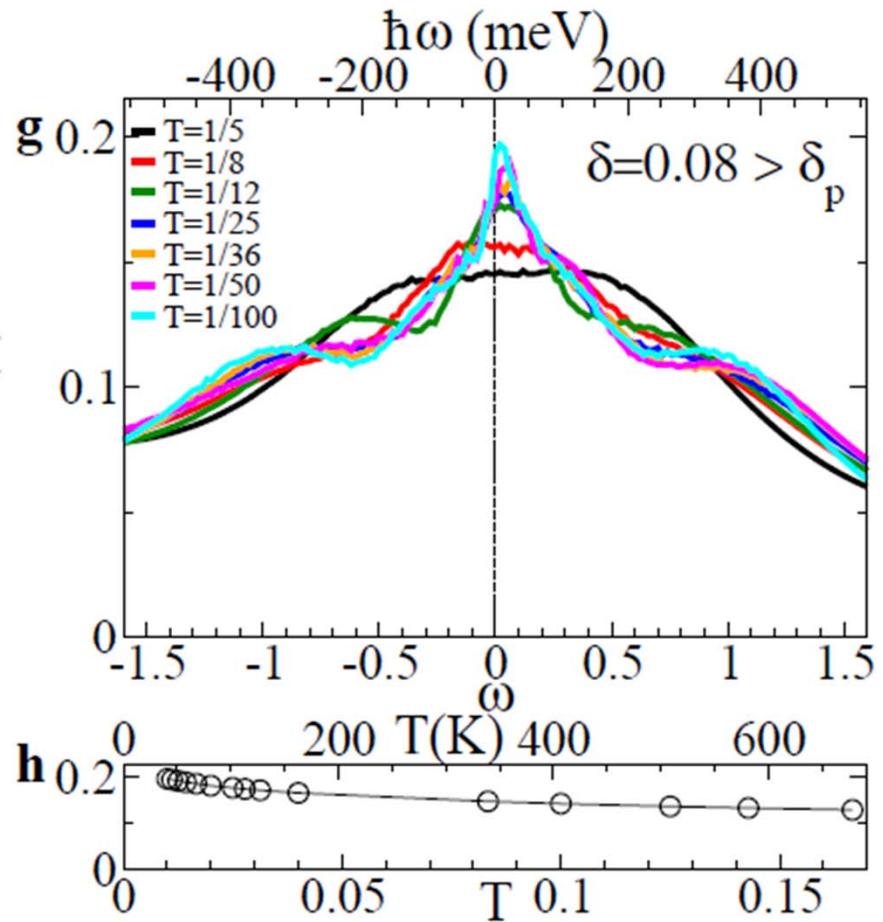
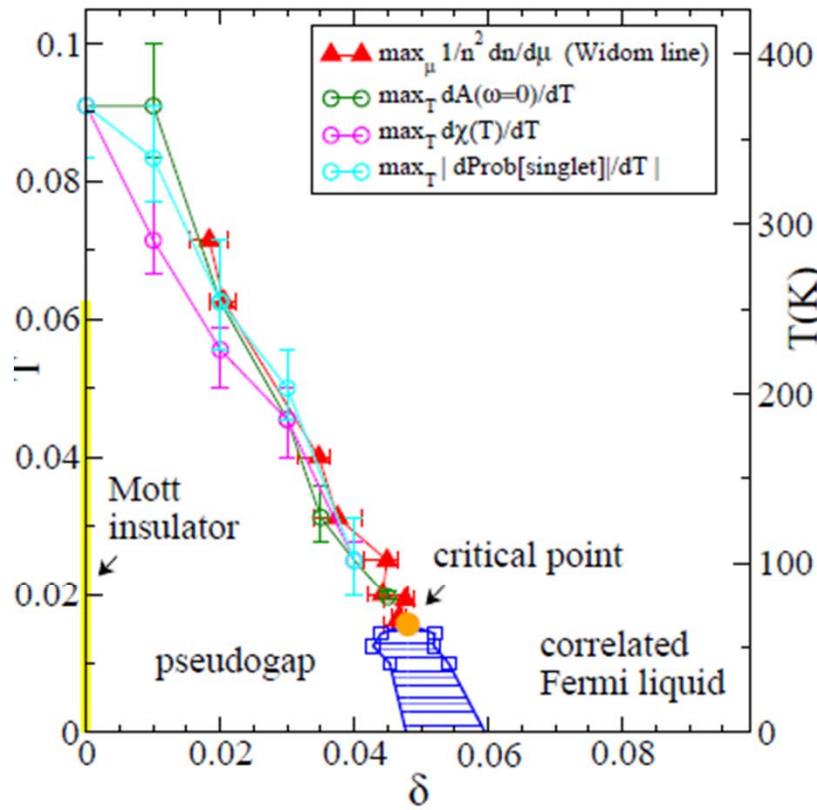
Khosaka et al. *Science* **315**, 1380 (2007);



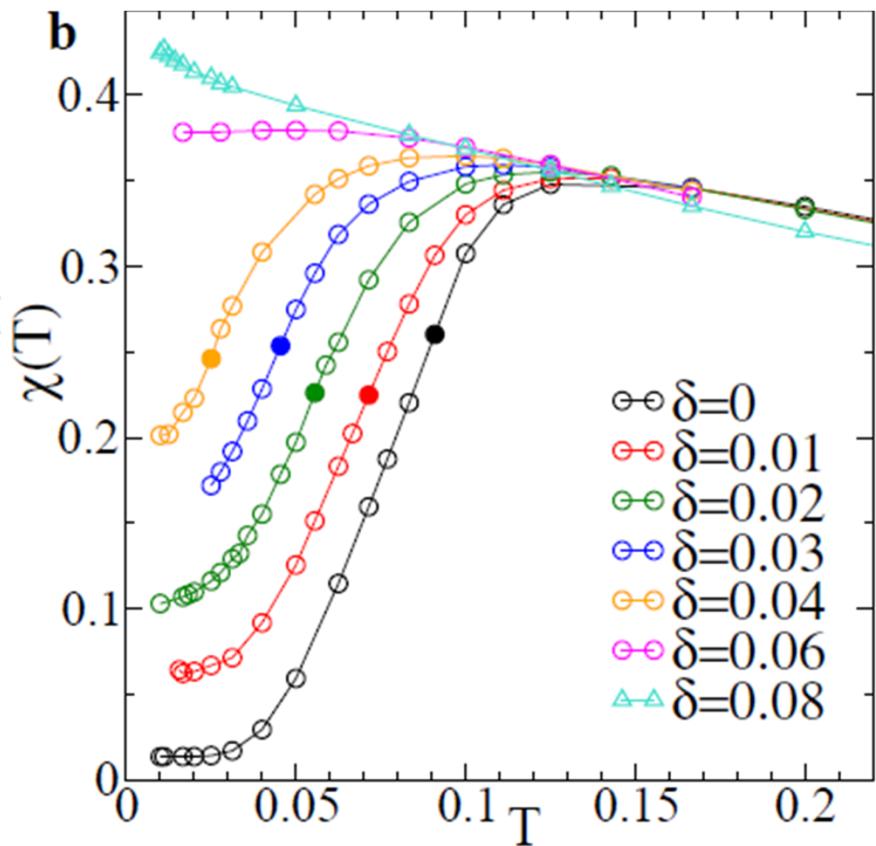
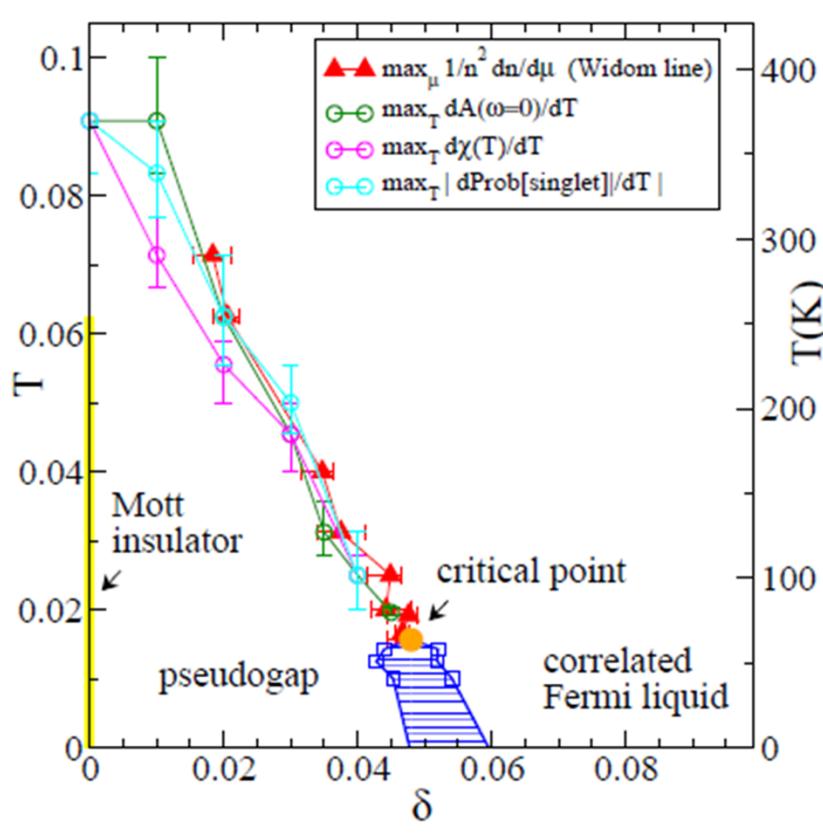
# Density of states



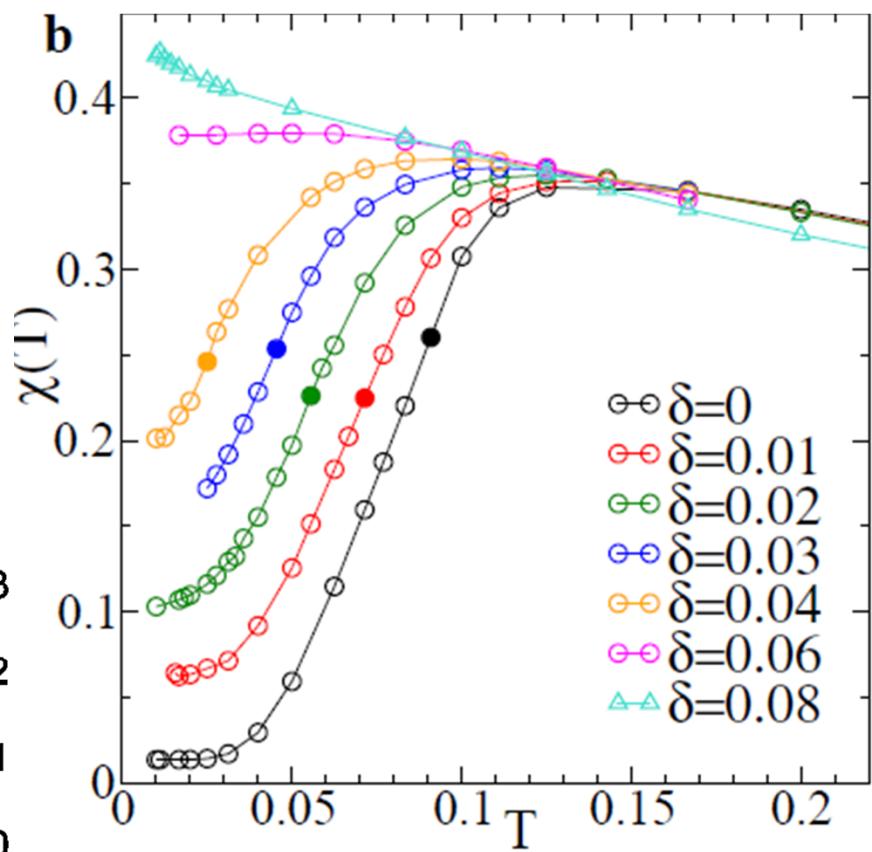
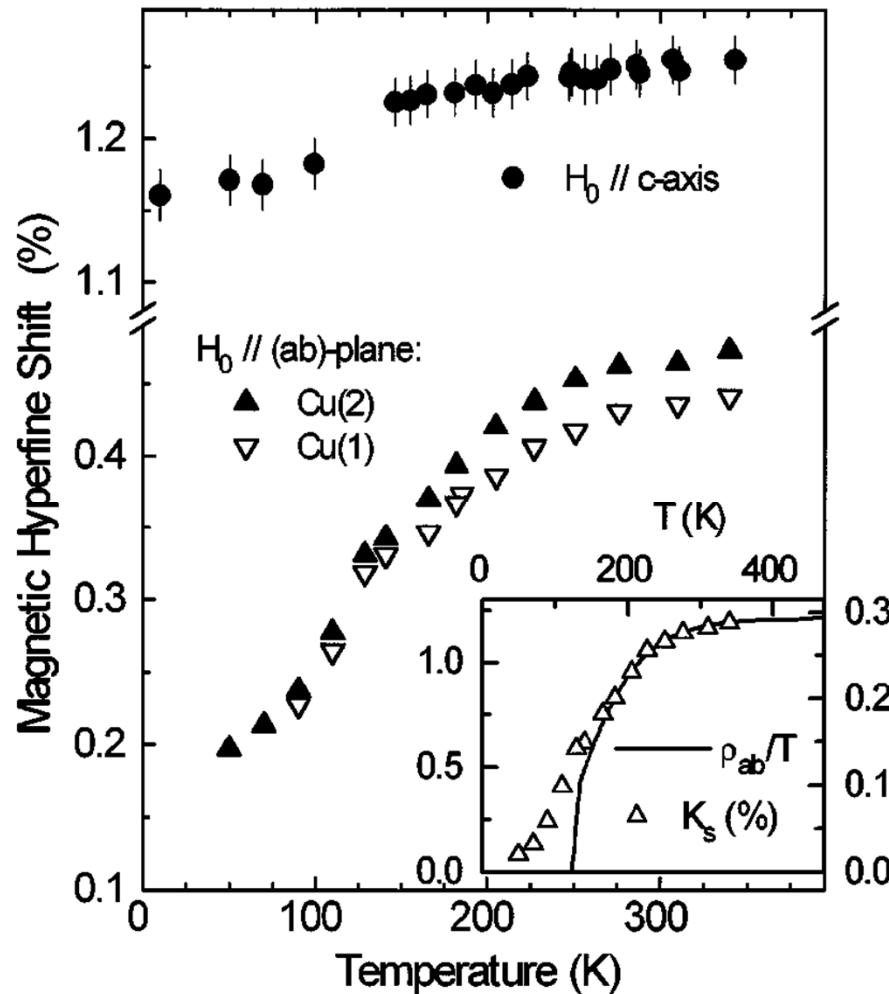
# Density of states



# Spin susceptibility



# Spin susceptibility



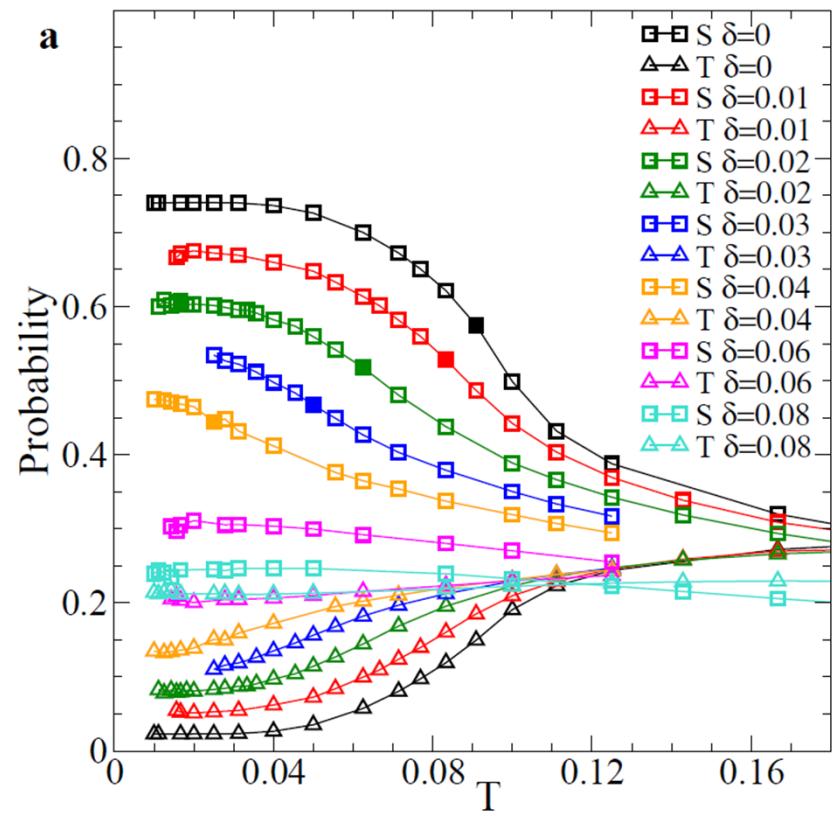
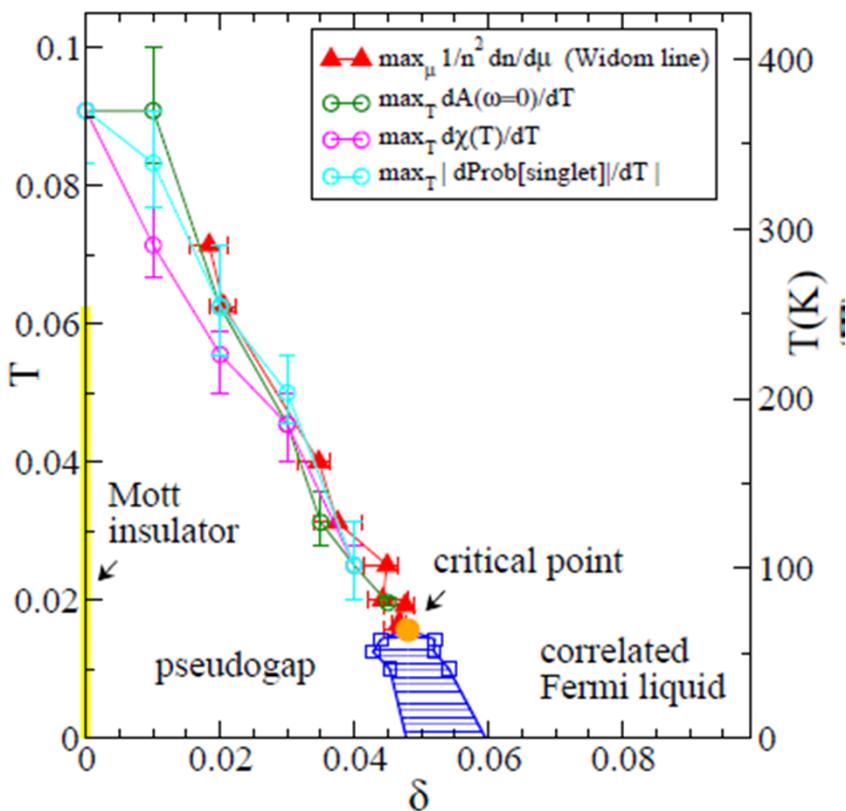
Underdoped Hg1223

Julien et al. PRL 76, 4238 (1996)



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# Plaquette eigenstates



# What is the minimal model?

H. Alloul arXiv:1302.3473

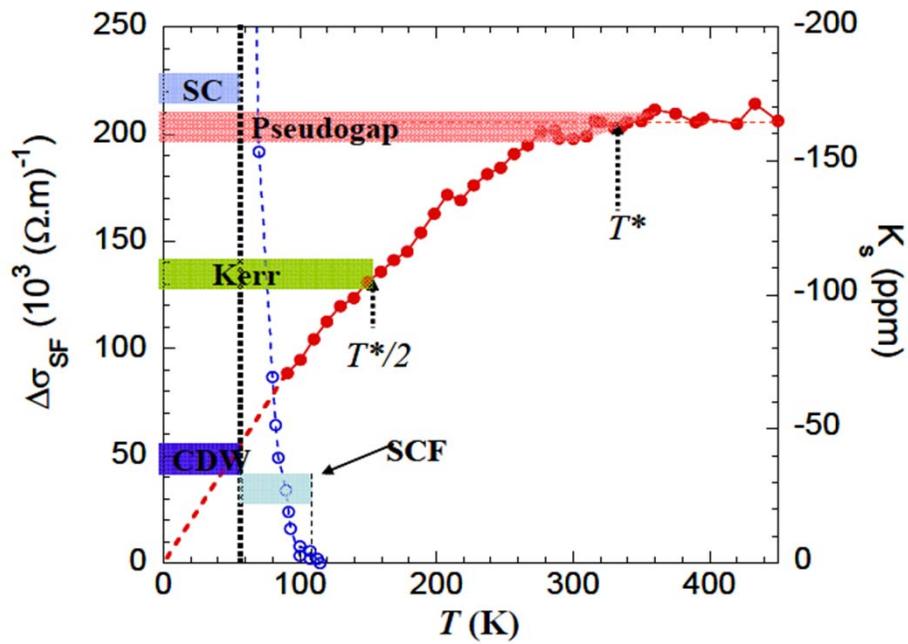
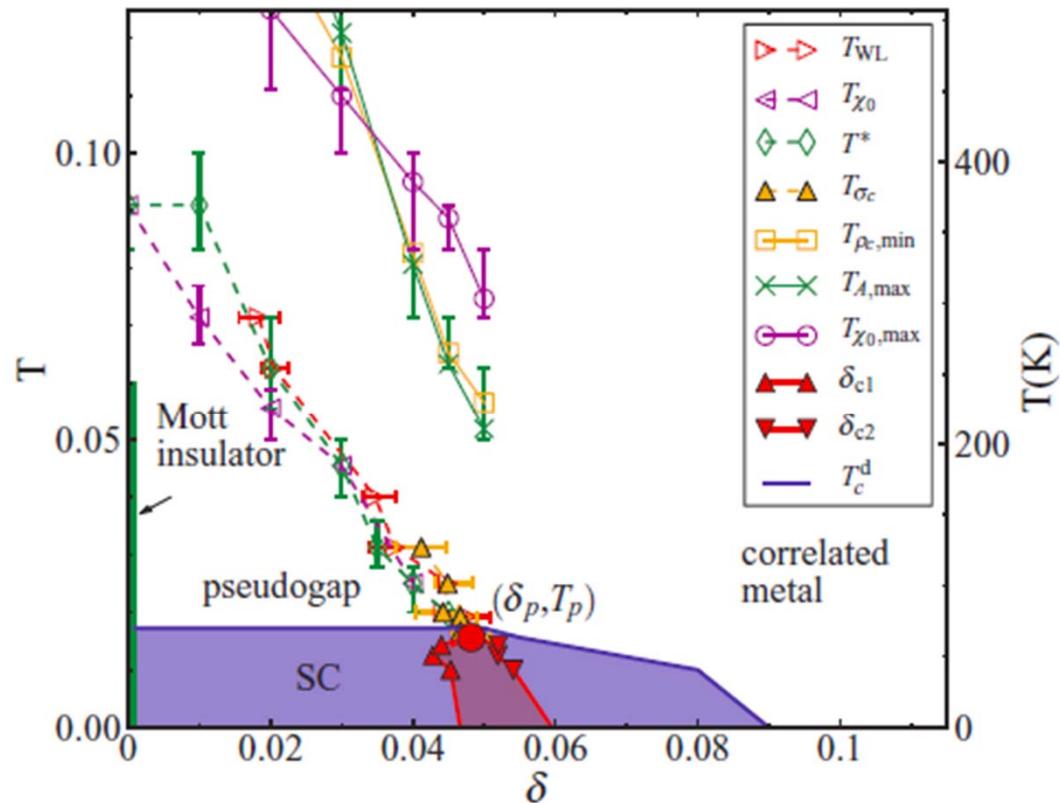


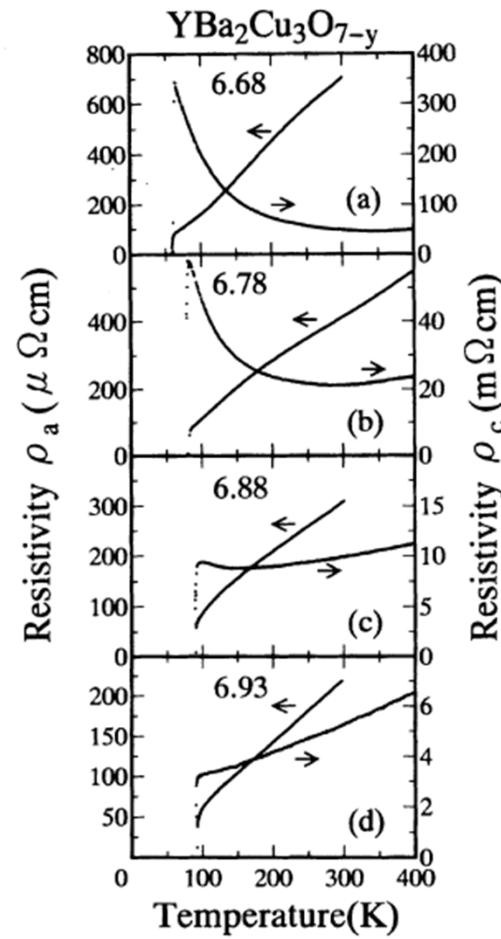
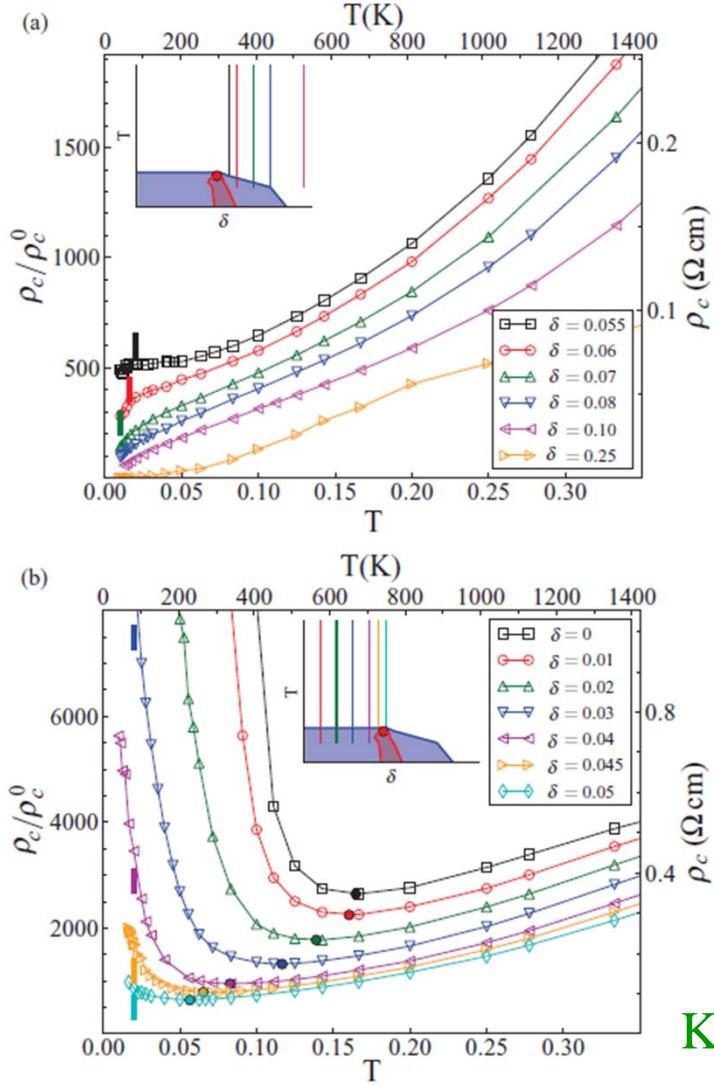
Fig 1 Spin contribution  $K_s$  to the  $^{89}\text{Y}$  NMR Knight shift [11] for  $\text{YBCO}_{6.6}$  permit to define the PG onset  $T^*$ . Here  $K_s$  is reduced by a factor two at  $T \sim T^*/2$ . The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).

# Two crossover lines



Sordi et al. PRL 108, 216401 (2012)  
PRB 87, 041101(R) (2013)

# C-axis resistivity

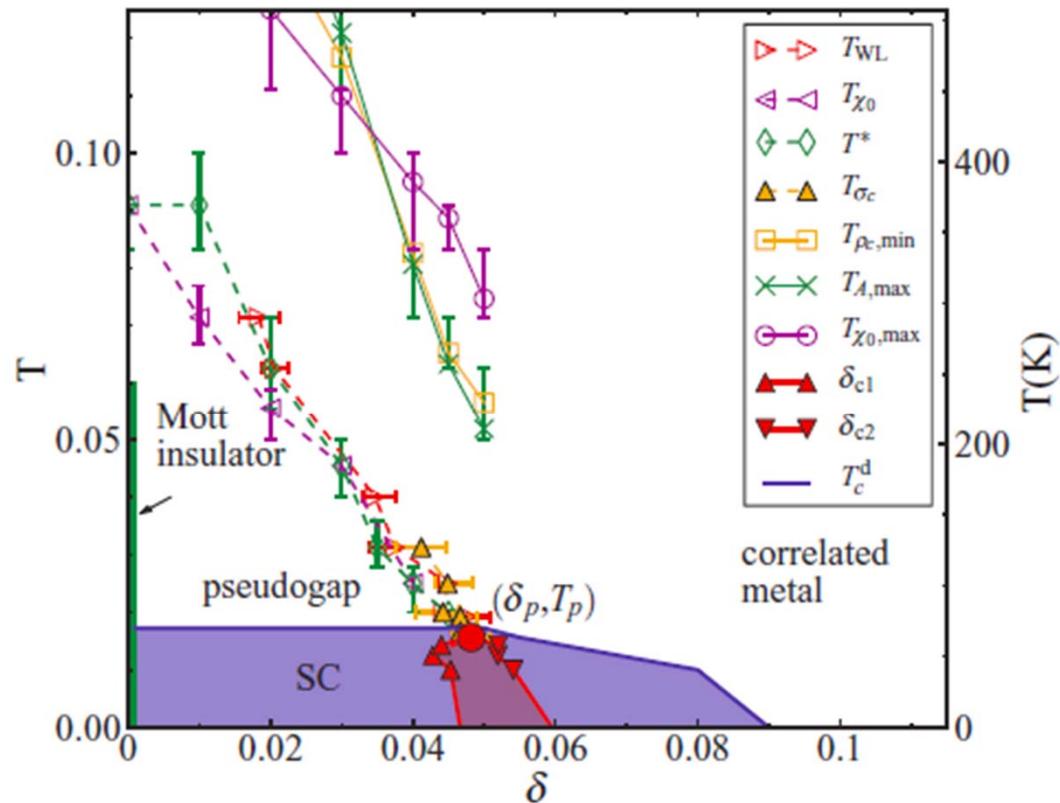


K. Takenaka, K. Mizuhashi, H. Takagi, and S. Uchida,  
Phys. Rev.B 50, 6534 (1994).



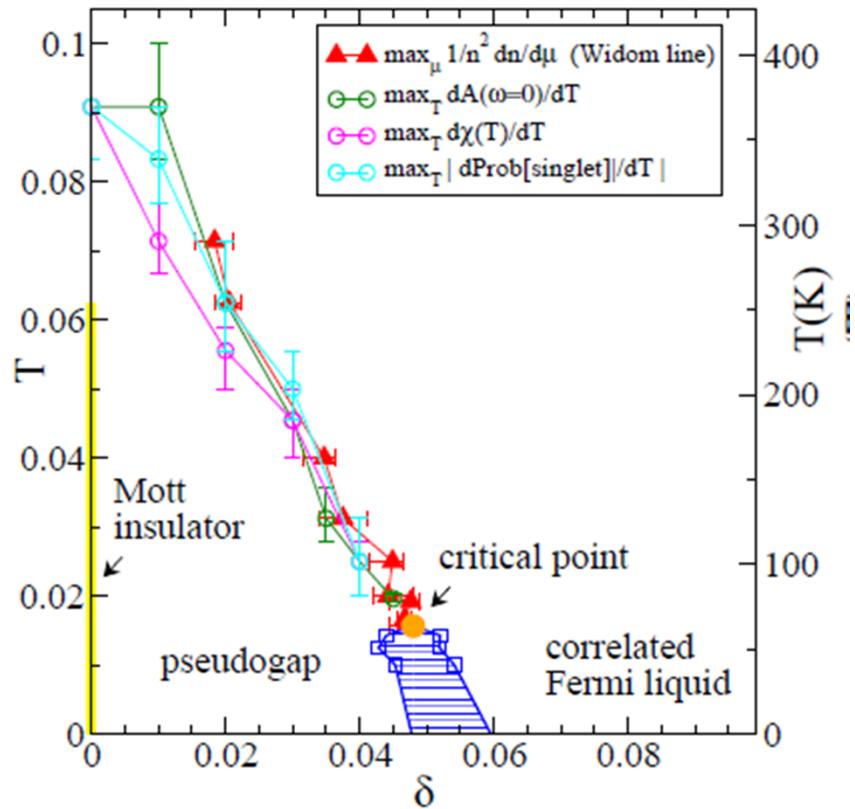
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# Two crossover lines



Sordi et al. PRL 108, 216401 (2012)  
PRB 87, 041101(R) (2013)

# Pseudogap $T^*$ along the Widom line



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Giovanni Sordi



Patrick Sémon



Kristjan Haule

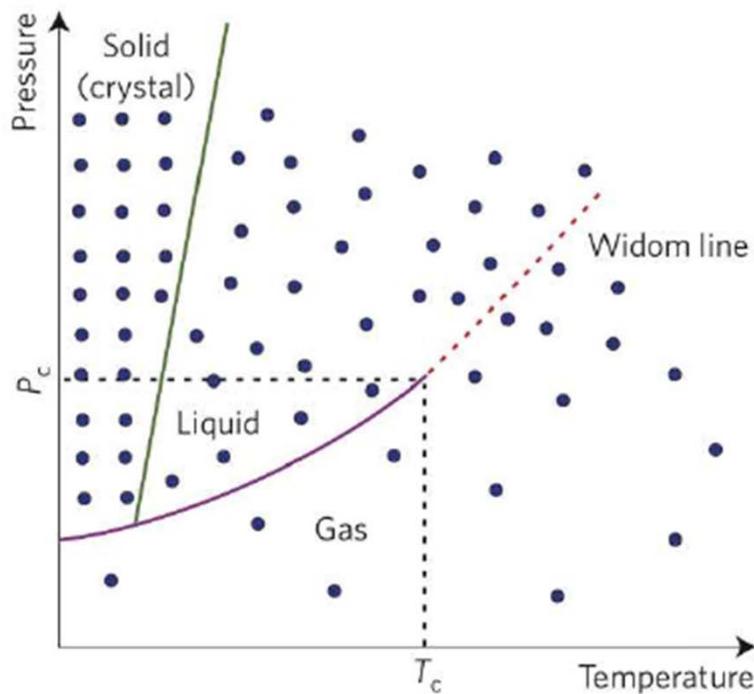
# The Widom line

G. Sordi, *et al.* Scientific Reports 2, 547 (2012)



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# What is the Widom line?

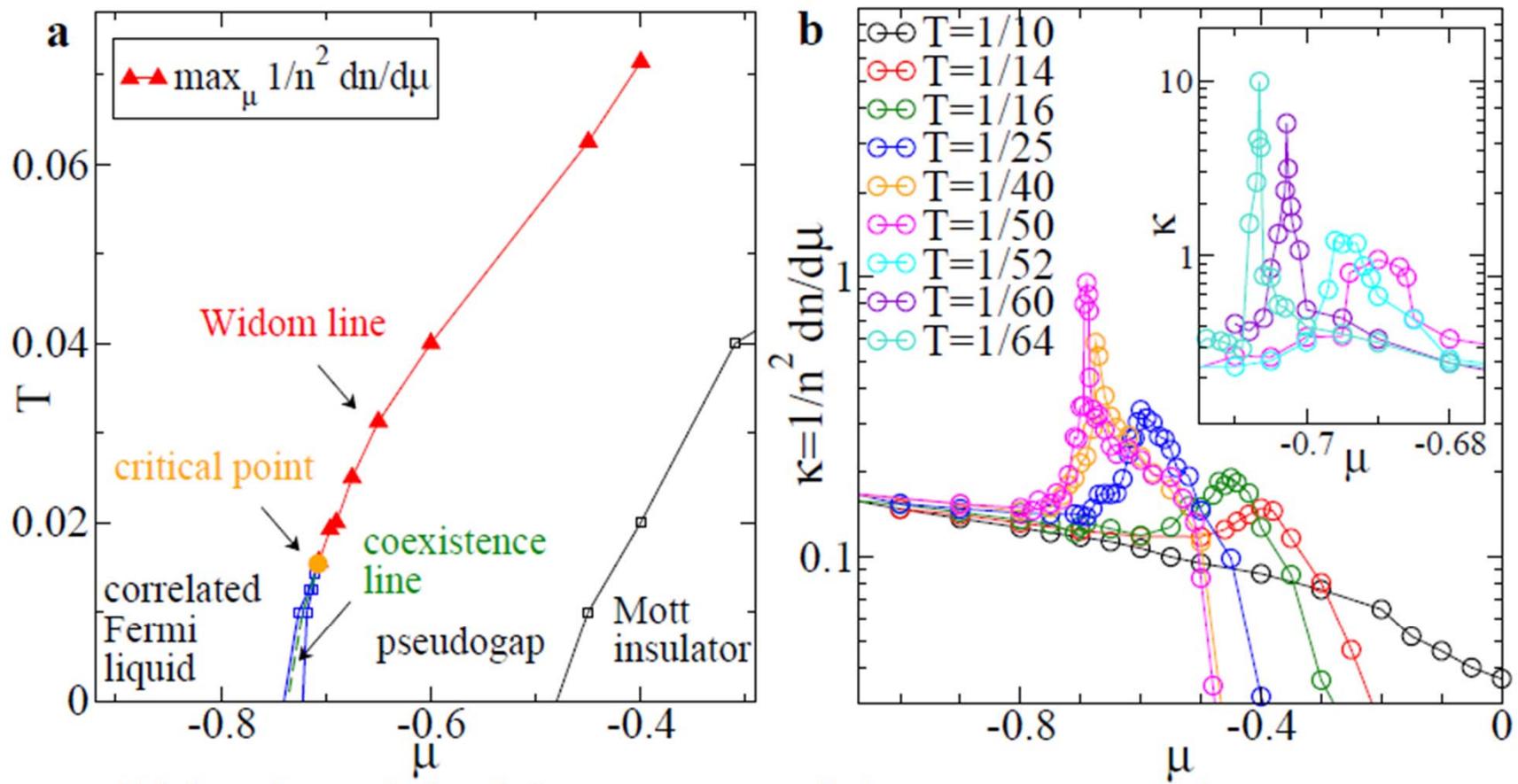


McMillan and Stanley, Nat Phys 2010

- ▶ it is the continuation of the coexistence line in the supercritical region
- ▶ line where the **maxima of different response functions** touch each other asymptotically as  $T \rightarrow T_p$
- ▶ liquid-gas transition in water: max in isobaric heat capacity  $C_p$ , isothermal compressibility, isobaric heat expansion, etc

- ▶ **DYNAMIC crossover arises from crossing the Widom line!**  
water: Xu et al, PNAS 2005,  
Simeoni et al Nat Phys 2010

# Pseudogap $T^*$ along the Widom line



Widom line: defined from maxima of charge compressibility

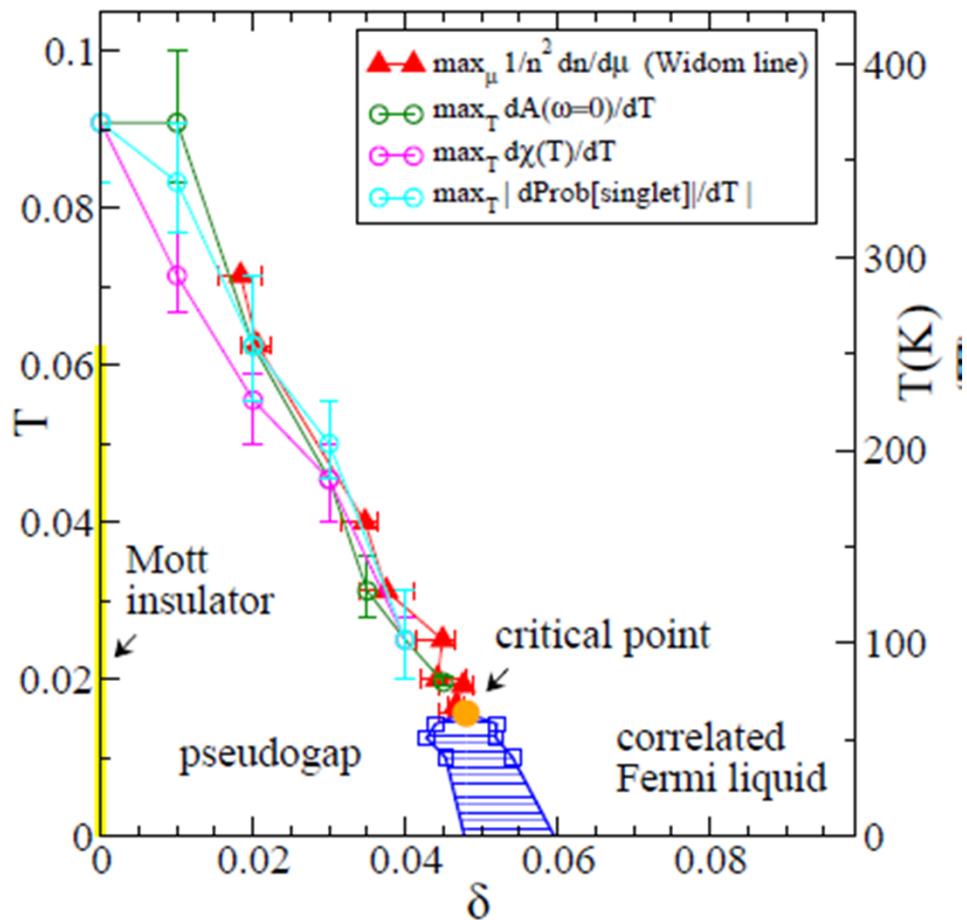
$$\kappa = 1/n^2(dn/d\mu)_T$$

divergence of  $\kappa$  at the (classical) critical point!



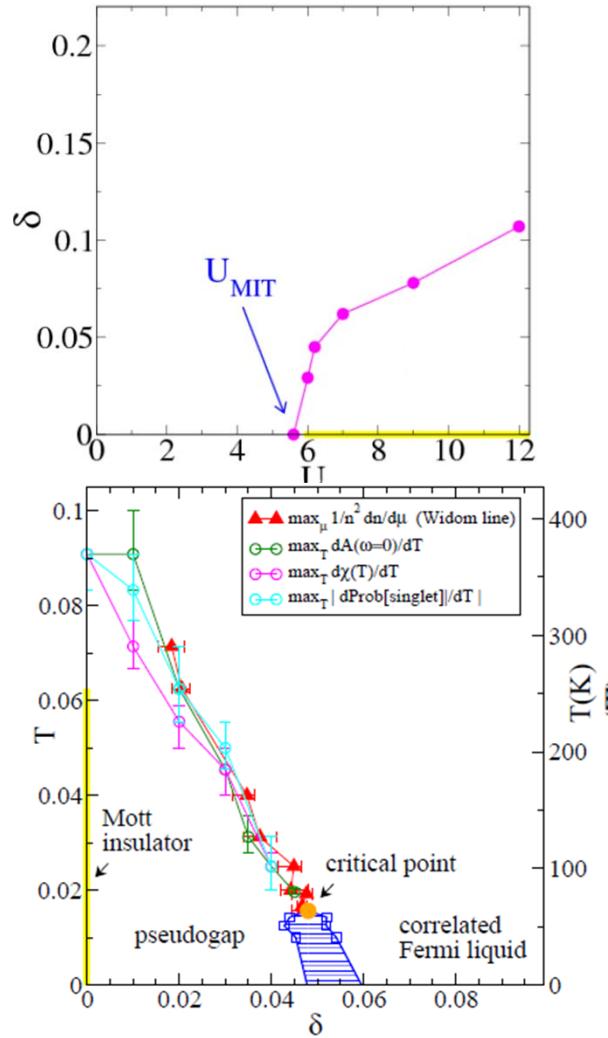
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# Phase diagram



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# Summary: normal state



- Mott physics extends way beyond half-filling
- Pseudogap is a phase
- Pseudogap  $T^*$  is a Widom line
- High compressibility (stripes?)



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Giovanni Sordi



Patrick Sémon



Kristjan Haule

# Finite $T$ phase diagram Superconductivity

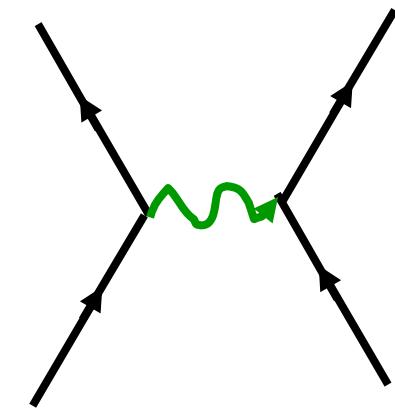
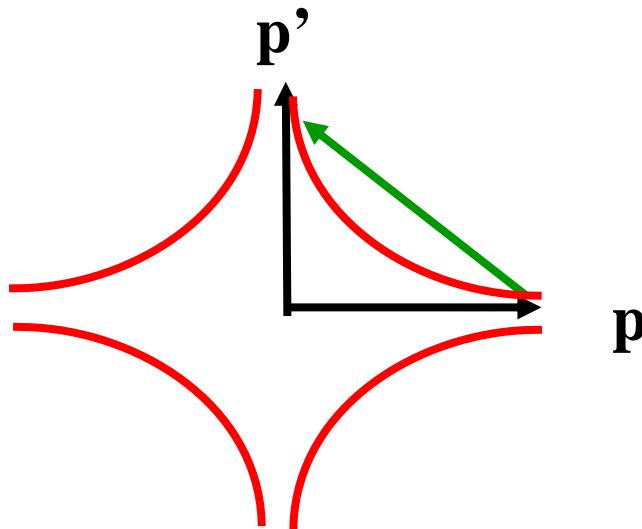
Sordi et al. PRL **108**, 216401 (2012)



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# Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery  
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?  
Kohn-Luttinger  
 $T_c$  with pressure

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch  
P.R. B **34**, 8190-8192 (1986).

Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)

# A cartoon strong coupling picture

P.W. Anderson Science 317, 1705 (2007)

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left( \frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left( \frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow}^\dagger c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

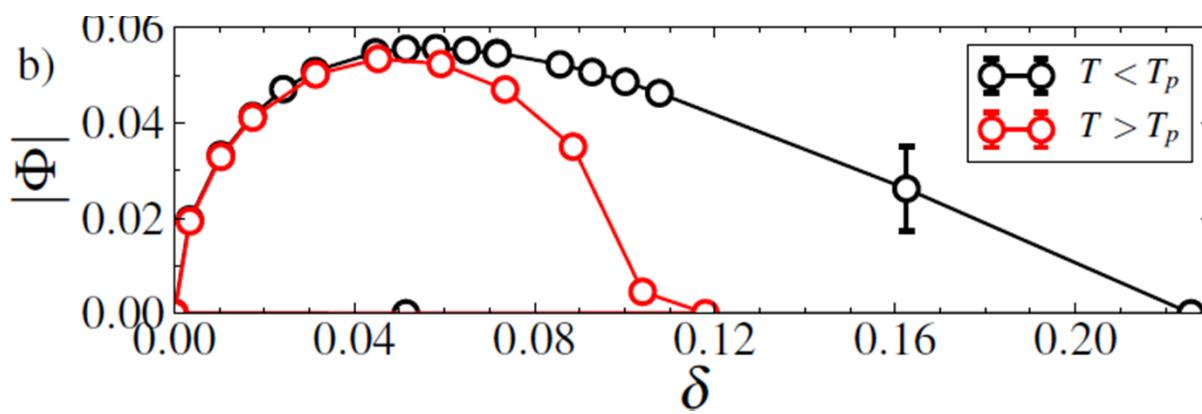
Pair state orthogonal to repulsive core of Coulomb interaction

Miyake, Schmitt–Rink, and Varma  
P.R. B 34, 6554-6556 (1986)



# Cuprates (doping driven transition)

Giovanni Sordi



Patrick Sémon

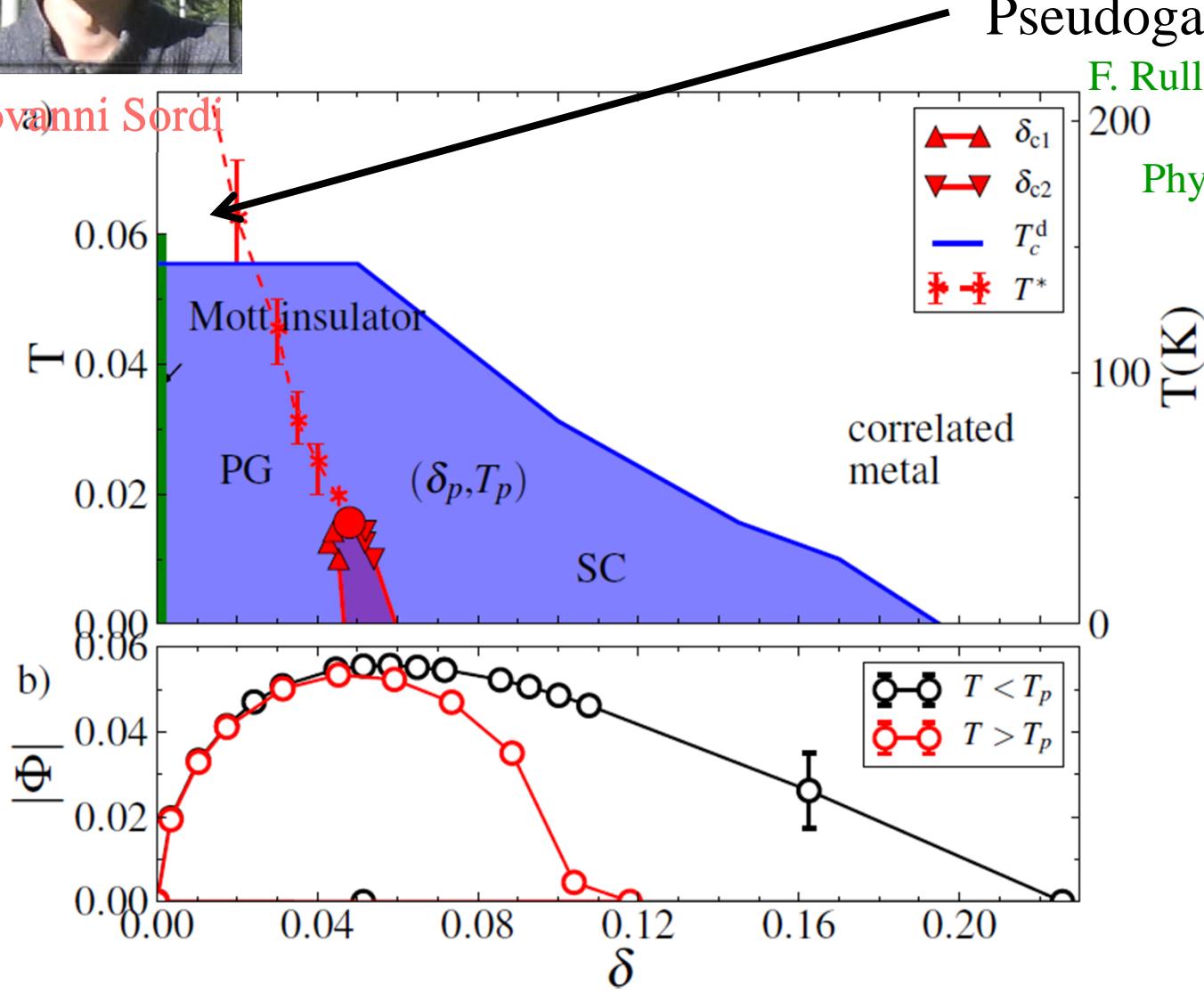


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# Cuprates (doping driven transition)

Giovanni Sordi



Pseudogap vs pair

F. Rullier-Albenque, H. Alloul,  
and G.Rikken,  
Phys. Rev. B **84**, 014522  
(2011).

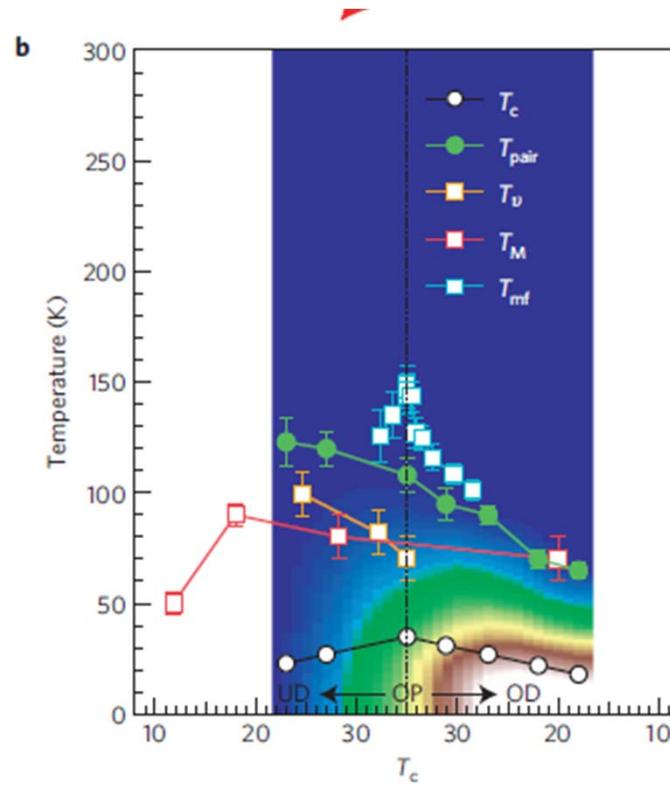


Patrick Sémon



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# T<sub>pair</sub>



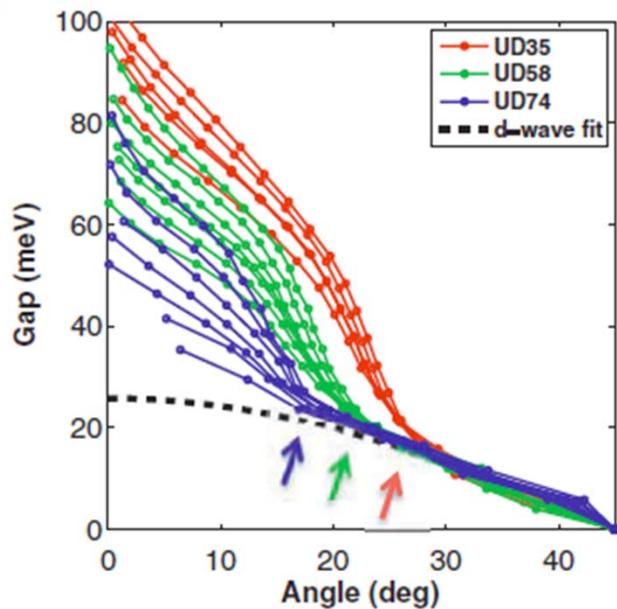
ARPES  
Bi2212

Kondo, Takeshi, et al. Kaminski Nature  
Physics 2011, 7, 21-25



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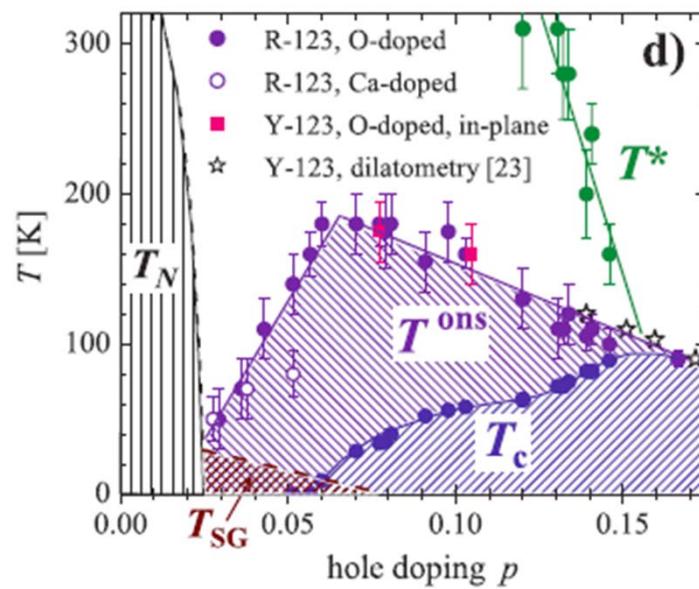
# Meaning of $T_c^d$ : Local pair formation



A. Pushp, Parker, ... A. Yazdani,  
Science **364**, 1689 (2009)

However, our measurements demonstrate that the nodal gap does not change with reduced doping. The pairing strength does not get weaker or stronger as the Mott insulator is approached; rather, it saturates.

# Fluctuating region



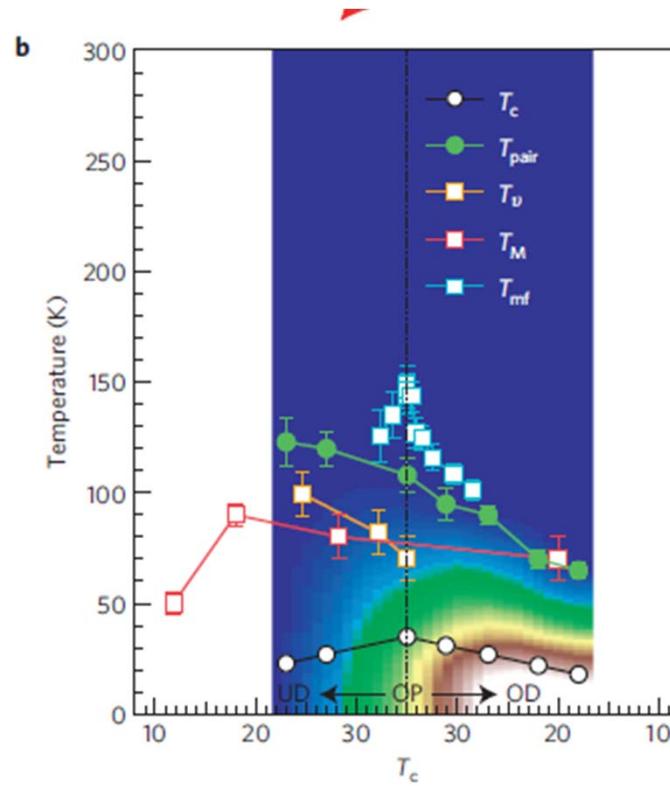
Infrared response

Dubroka et al. PRL 106, 047006 (2011)



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# T<sub>pair</sub>



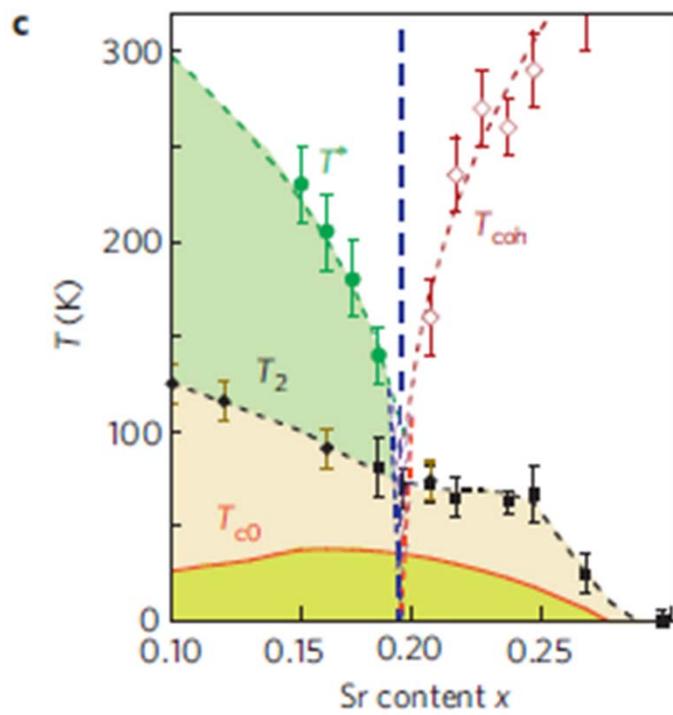
ARPES  
Bi2212

Kondo, Takeshi, et al. Kaminski Nature  
Physics 2011, 7, 21-25



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$T_2$



Magnetoresistance, LSCO  
Fluctuating vortices

Patrick M. Rourke, et al. Hussey Nature Physics 7, 455–458 (2011)



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# Giant proximity effect

$$\begin{aligned}T_c &= 32 \text{ K} \\T_c &< 5 \text{ K}\end{aligned}$$

Morenzoni et al.,  
Nature Comms. 2 (2011)

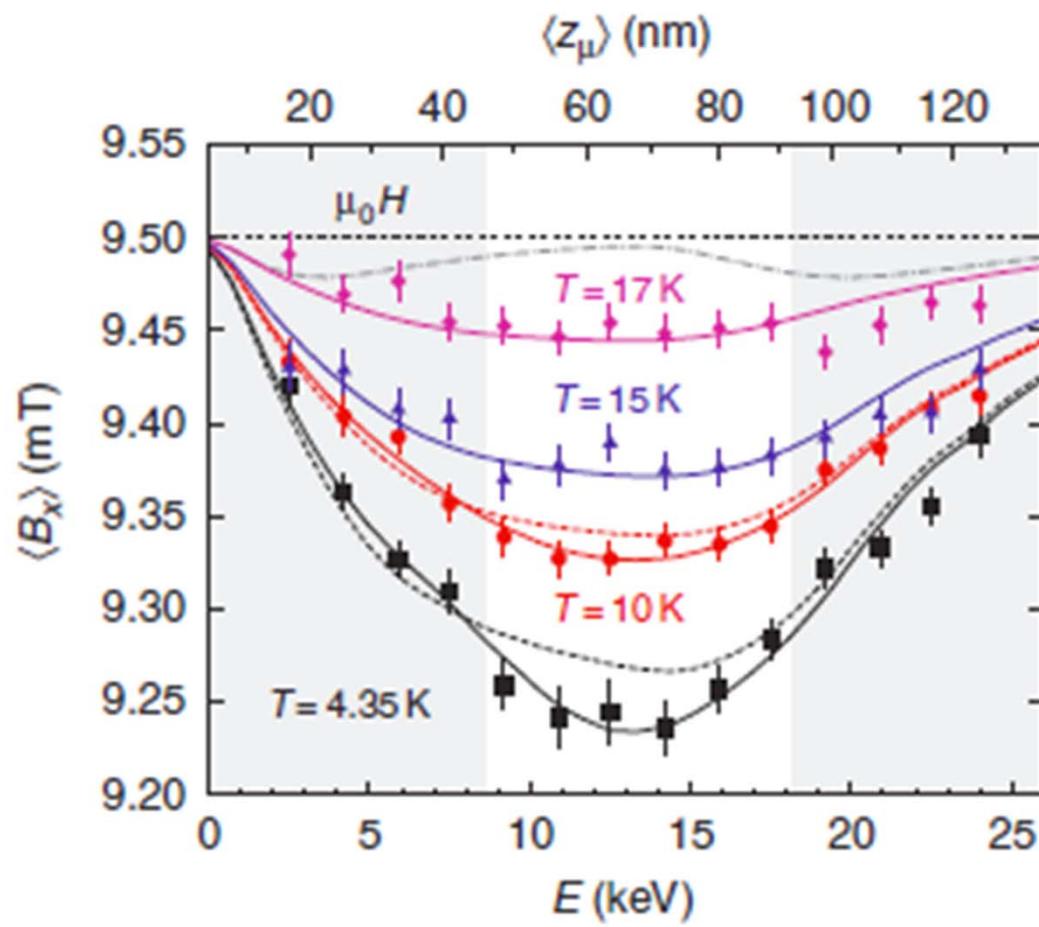


Figure 6 | Depth profile of the local field at different temperatures. The

# Actual $T_c$ in underdoped

- Quantum and classical phase fluctuations
  - V. J. Emery and S. A. Kivelson, Phys. Rev. Lett. **74**, 3253 (1995).
  - V. J. Emery and S. A. Kivelson, Nature **374**, 474 (1995).
  - D. Podolsky, S. Raghu, and A. Vishwanath, Phys. Rev. Lett. **99**, 117004 (2007).
  - Z. Tesanovic, Nat Phys **4**, 408 (2008).
- Magnitude fluctuations
  - I. Ussishkin, S. L. Sondhi, and D. A. Huse, Phys. Rev. Lett. **89**, 287001 (2002).
- Competing order
  - E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, Annual Review of Condensed Matter Physics **1**, 153 (2010).
- Disorder
  - F. Rullier-Albenque, H. Alloul, F. Balakirev, and C. Proust, EPL (Europhysics Letters) **81**, 37008 (2008).
  - H. Alloul, J. Bobro, M. Gabay, and P. J. Hirschfeld, Rev. Mod. Phys. **81**, 45 (2009).

# Larger clusters

- Is there a minimal size cluster where  $T_c$  vanishes before half-filling?
- Learn something from small clusters as well
- Local pairs in underdoped



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# Larger cluster 8 site DCA

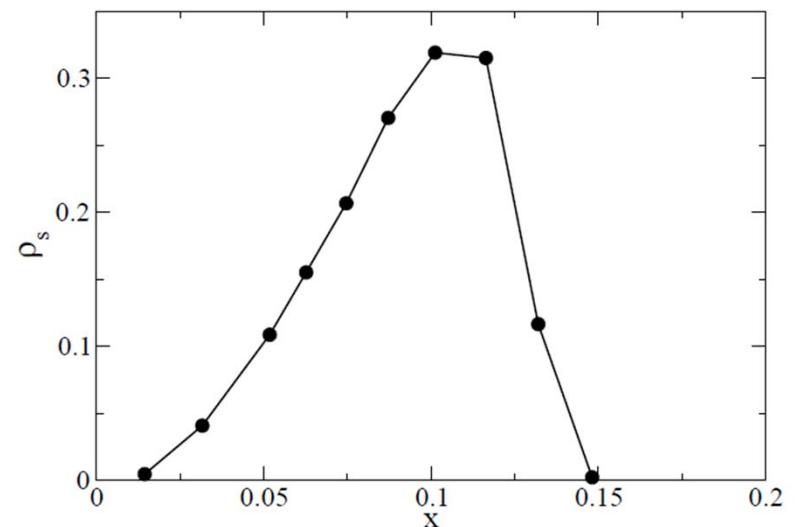
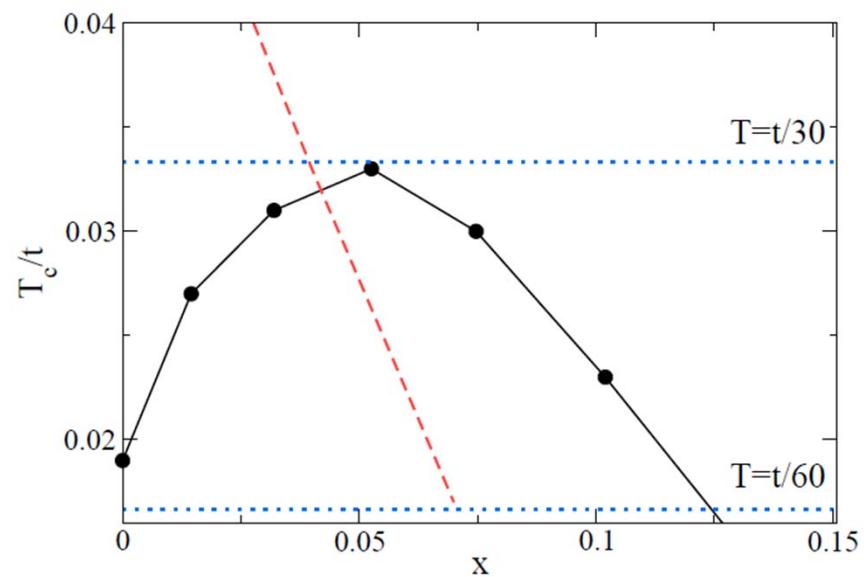
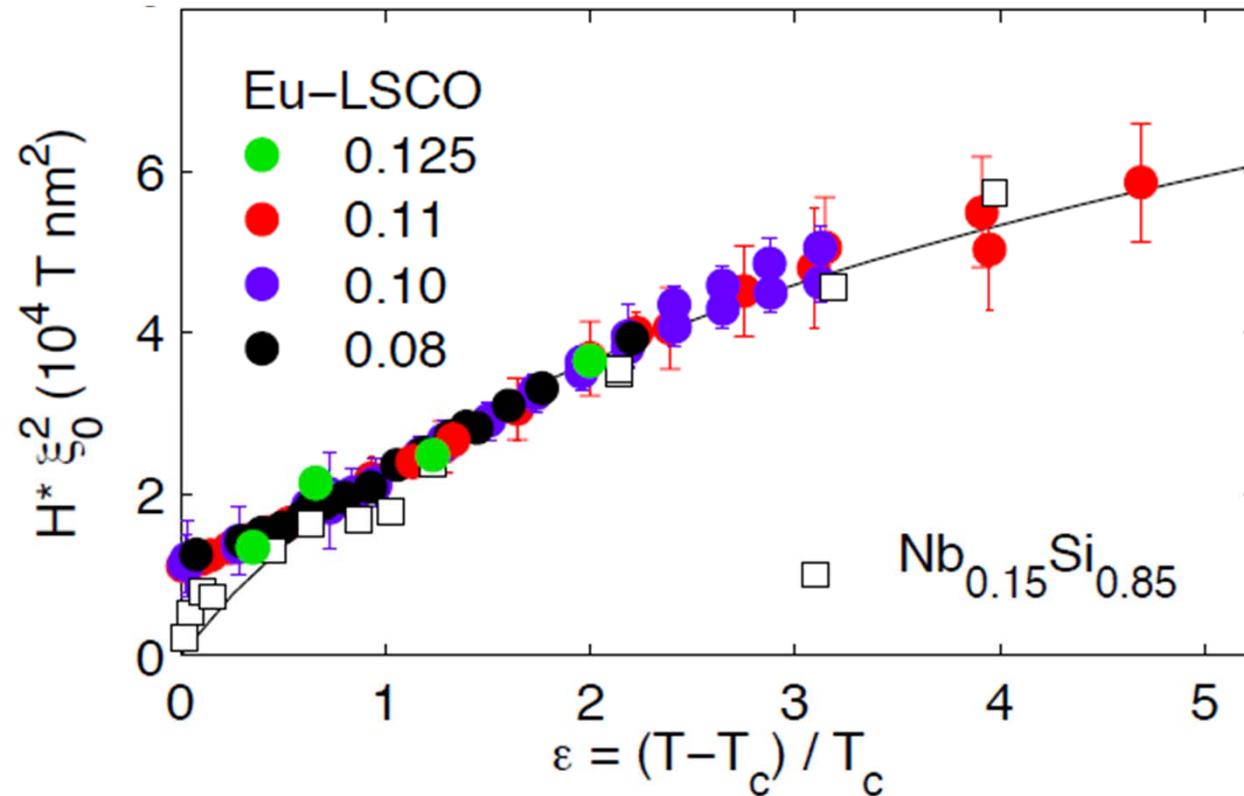


FIG. 8. Superfluid stiffness  $\rho_s$  determined in the superconducting state at  $T = t/60$  from Eq. 15, as a function of doping.

Gull, Millis, arxiv.org:304.6406

# Gaussian amplitude fluctuations in Eu-LSCO

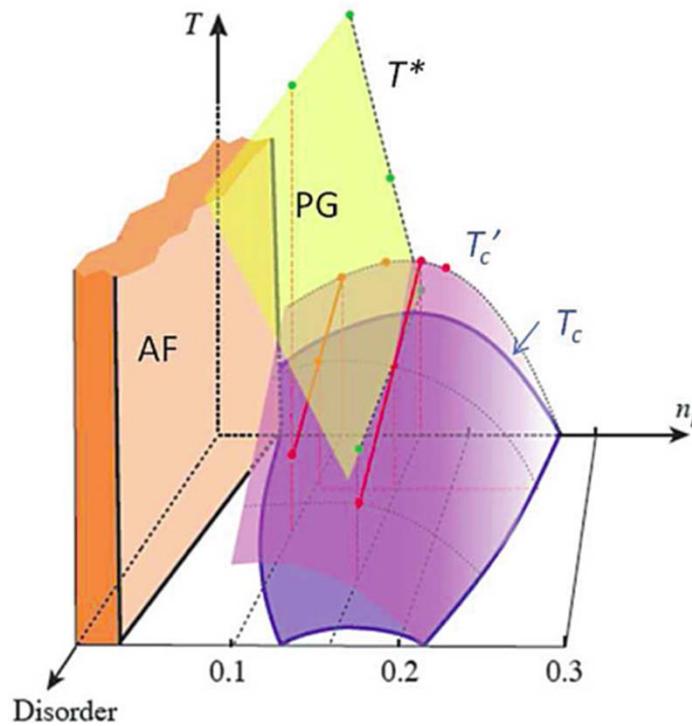


Chang, Doiron-Leyraud et al.



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# Effect of disorder



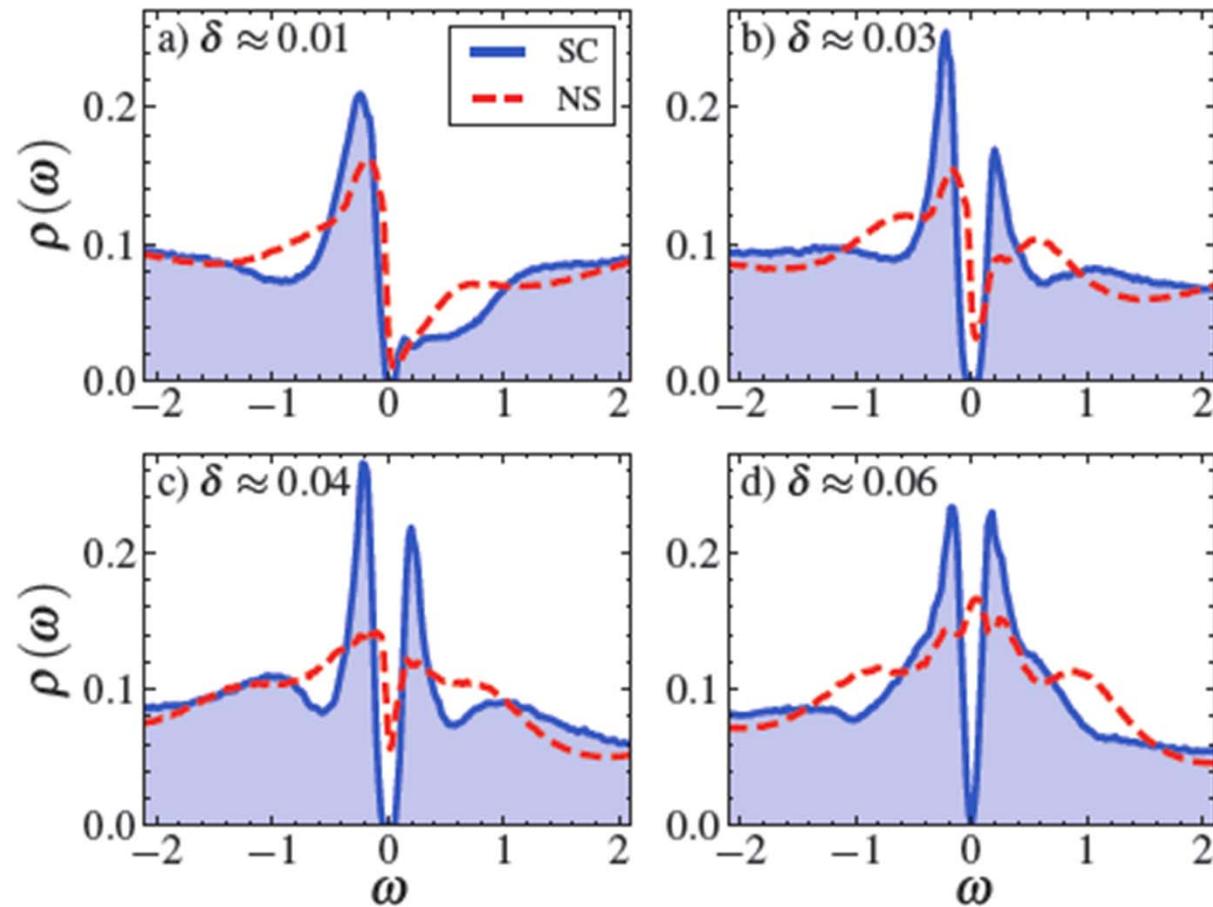
F. Rullier-Albenque, H. Alloul, and G.Rikken,  
Phys. Rev. B **84**, 014522 (2011).

One last remark: Strongly correlated  
superconductivity



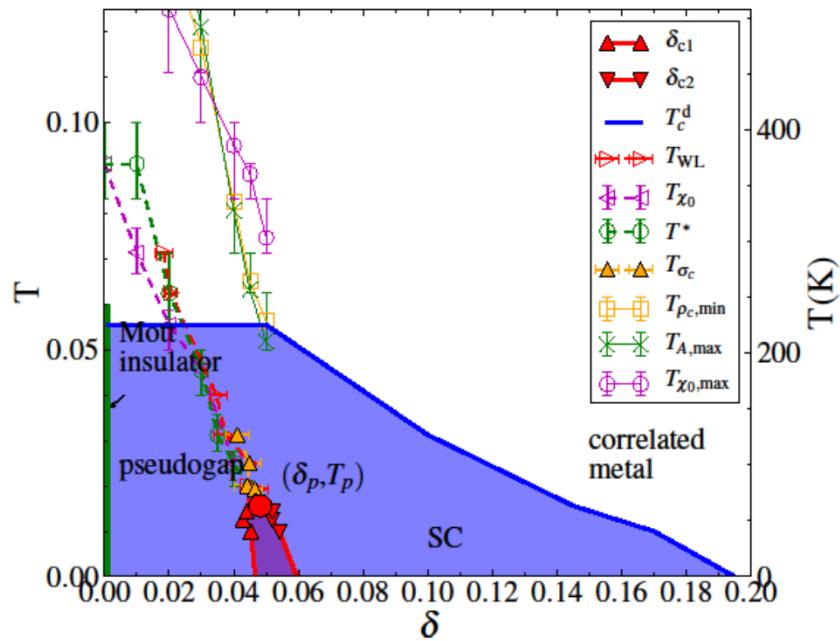
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# First-order transition leaves its mark



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# Summary



- Below the dome finite  $T$  critical point (not QCP) controls normal state
- First-order transition destroyed but traces in the dynamics
- $T^*$  different from  $T_c^d$
- Actual  $T_c$  in underdoped
  - Competing order
  - Long wavelength fluctuations (see O.P.)
  - Disorder

# Conclusions

- Tools for Hubbard model, high Tc.
- The influence of Mott Physics extends way beyond half-filling
  - Pseudogap as a phase
  - Effects of critical point at high temperature (Widom line)
  - Superconductivity
    - Retardation effects in pairing
    - Pseudogap vs superconductivity



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# Main collaborators



Giovanni Sordi



Kristjan Haule



David Sénéchal



Bumsoo Kyung



Patrick Sémon



Dominic Bergeron



Sarma Kancharla



Marcello Civelli



Massimo Capone

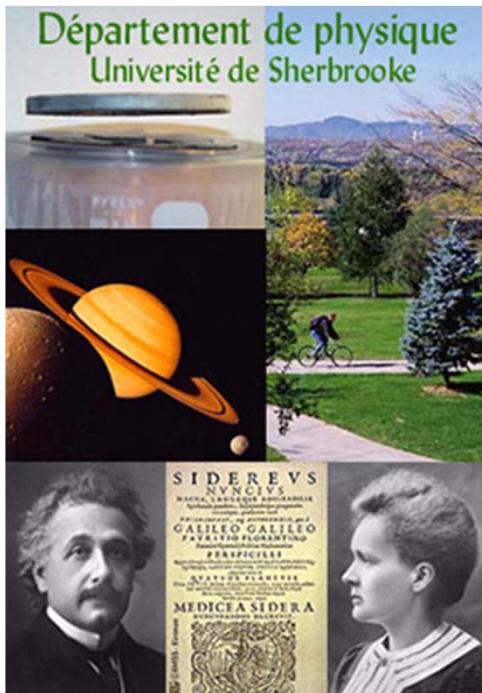


Gabriel Kotliar



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# André-Marie Tremblay



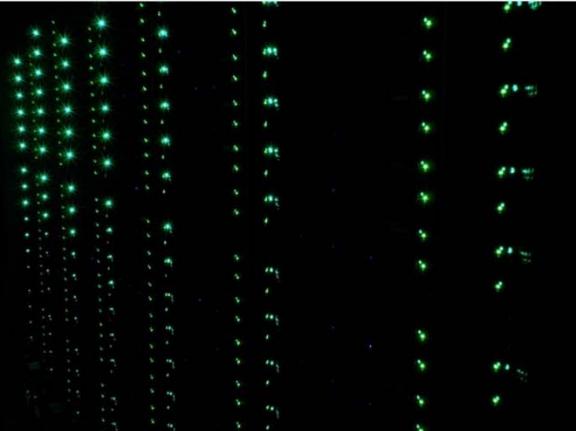
Le regroupement québécois sur les matériaux de pointe



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Le calcul de haute performance

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merci

thank you