

High-temperature superconductors: Where is the mystery?

A.-M. Tremblay



CIFAR
CANADIAN INSTITUTE
for ADVANCED RESEARCH

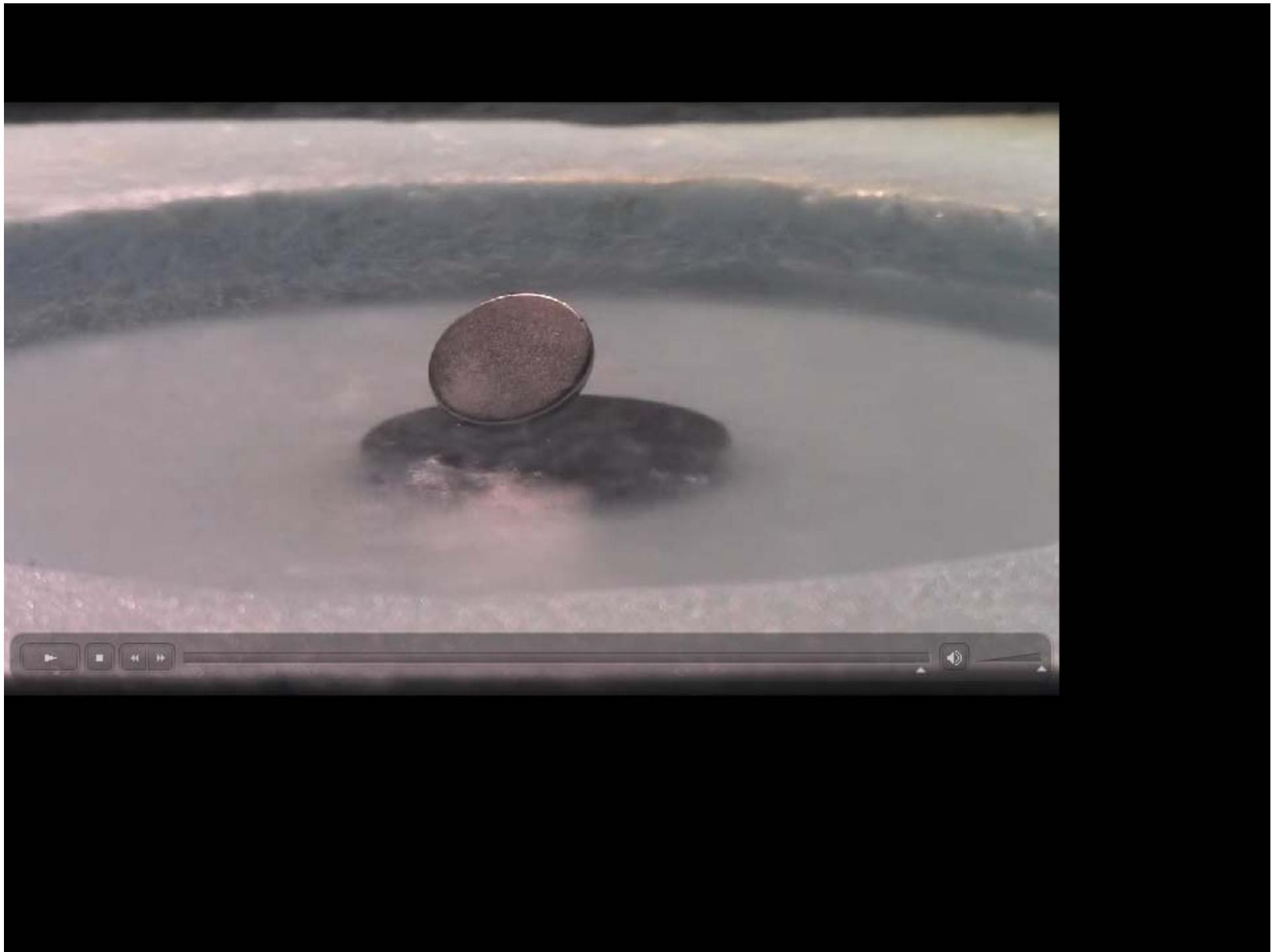
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Strongly correlated systems: from models to materials
9 January 2014



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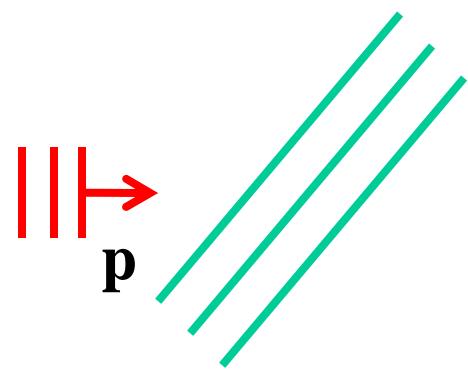


Superconductivity



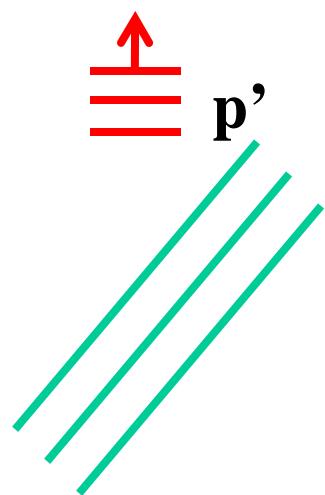
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Attraction mechanism in the metallic state



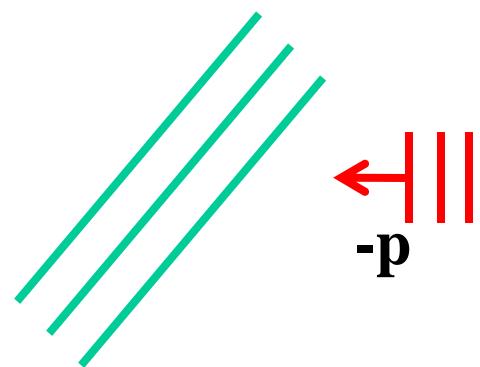
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Attraction mechanism in the metallic state



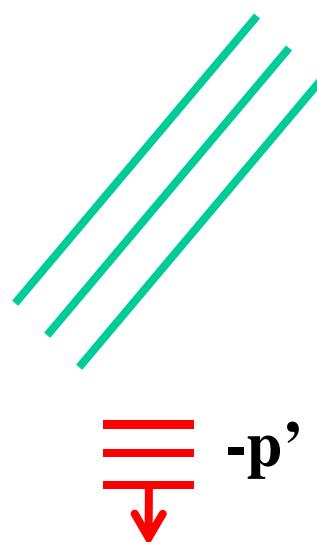
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Attraction mechanism in the metallic state



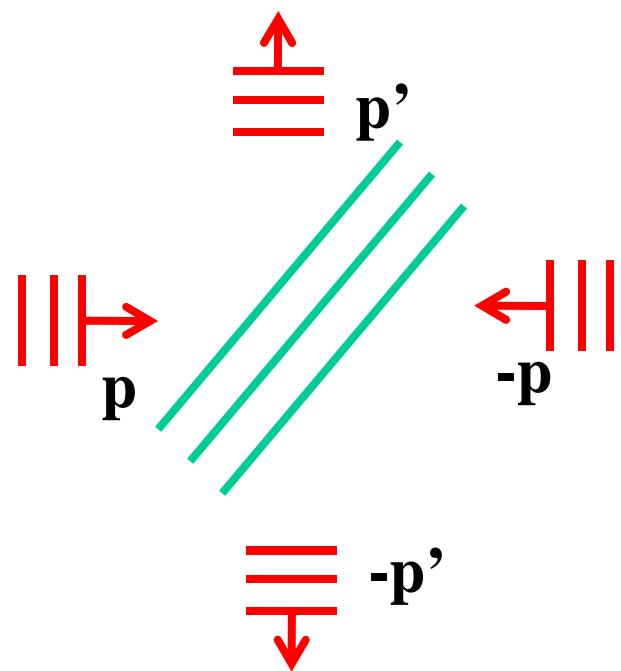
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Attraction mechanism in the metallic state



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Attraction mechanism in the metallic state



#1 Cooper pair, #2 Phase coherence

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^*$$

$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \left(\langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* + \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \right)$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$

Breakdown of band theory Half-filled band is metallic?



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Half-filled band: Not always a metal

NiO, Boer and Verway



Peierls, 1937



Mott, 1949



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« Conventional » Mott transition

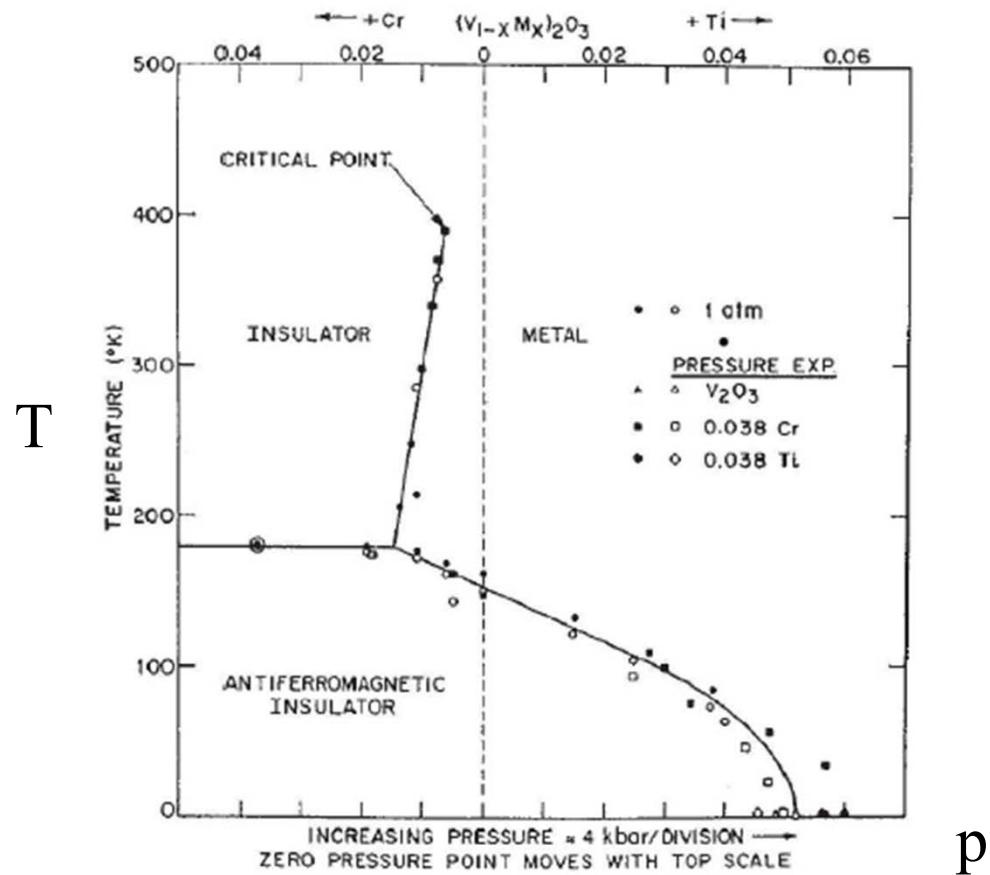
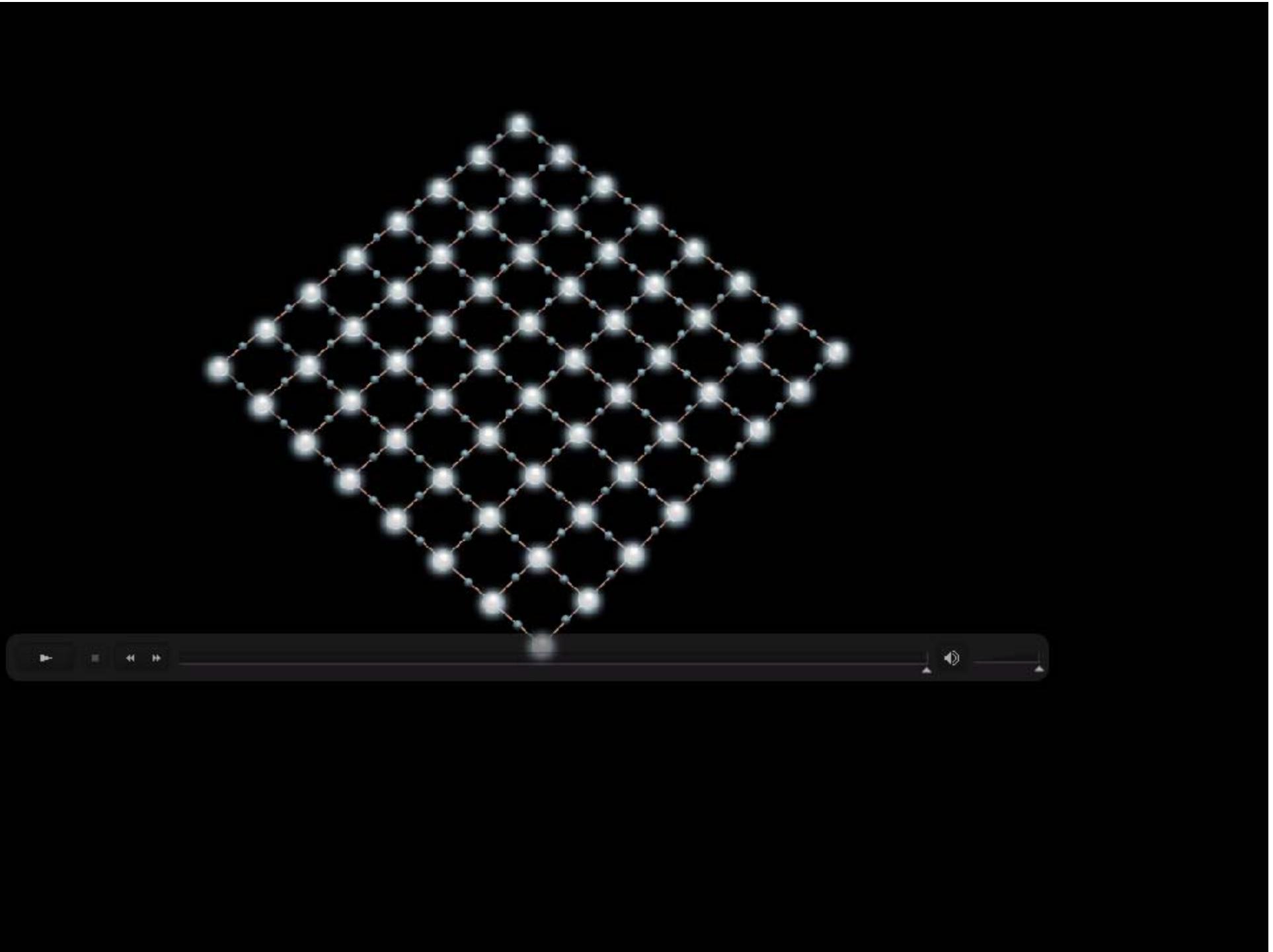


Figure: McWhan, PRB 1970; Limelette, Science 2003



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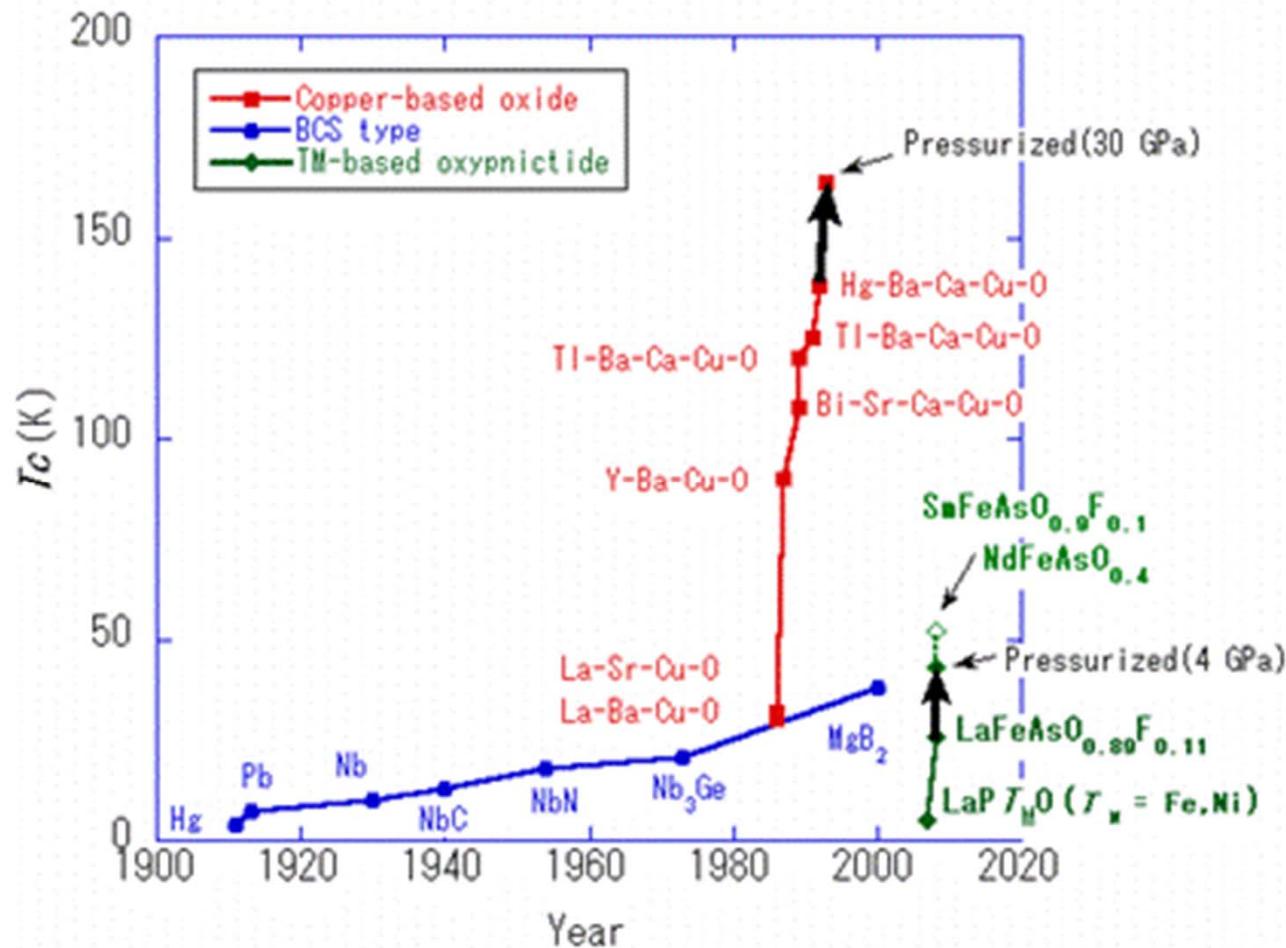
High temperature superconductors

Failure of
BCS theory
Band structure
and more



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New and old superconductors



H. Takahashi: JPSJ Online—News and Comments [June 10, 2008]

Atomic structure

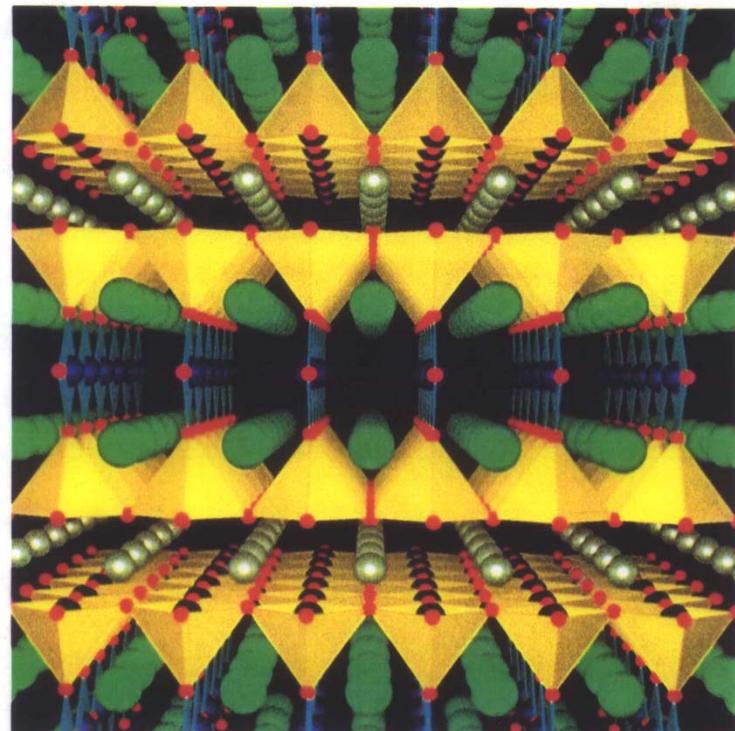
SCIENTIFIC AMERICAN

How nonsense is deleted from genetic messages.

Rx for economic growth: aggressive use of new technology.

Can particle physics test cosmology?

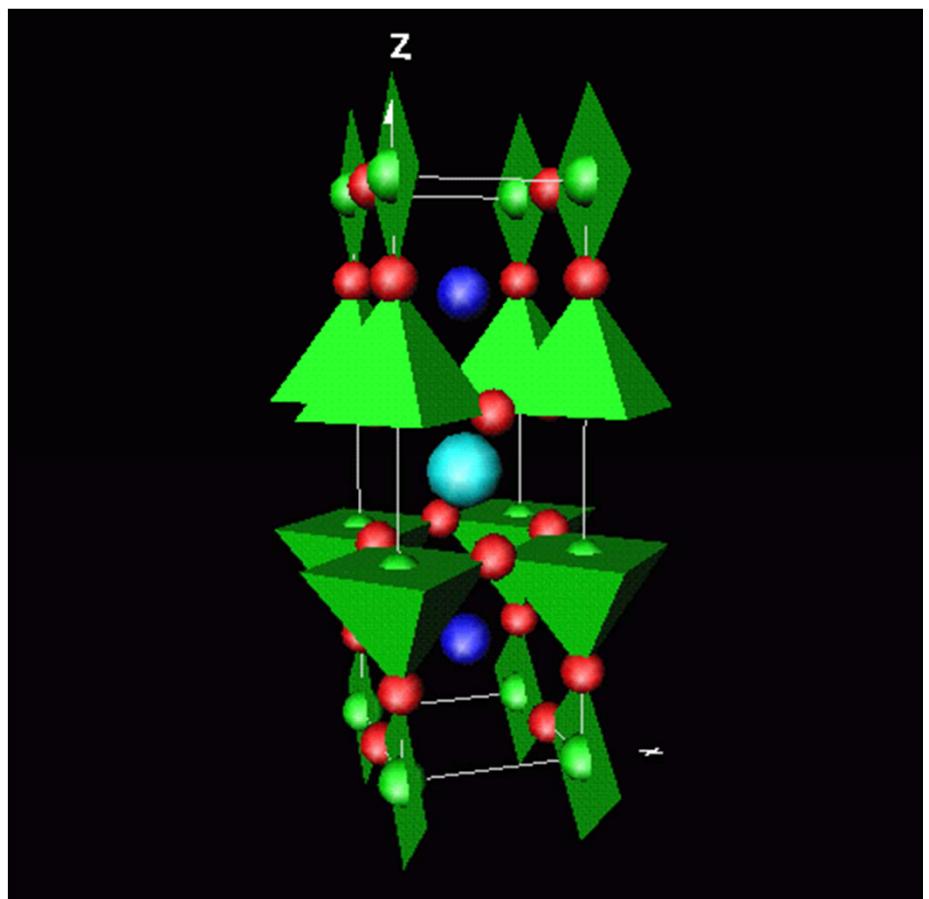
JUNE 1988
\$3.50



High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.



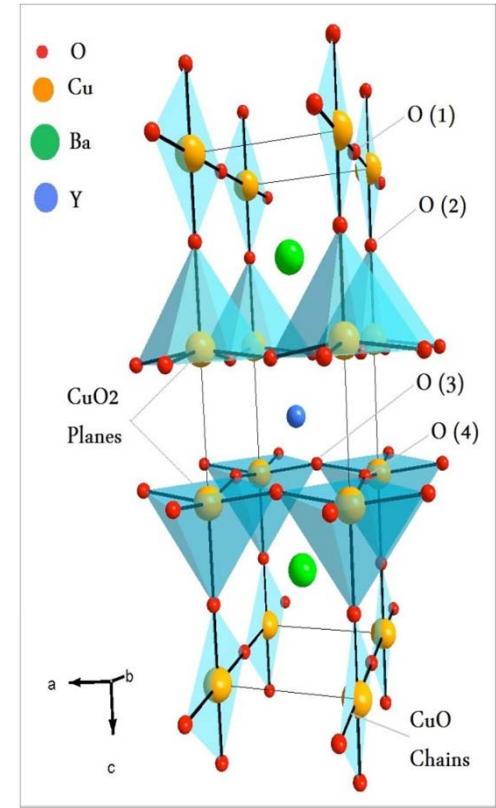
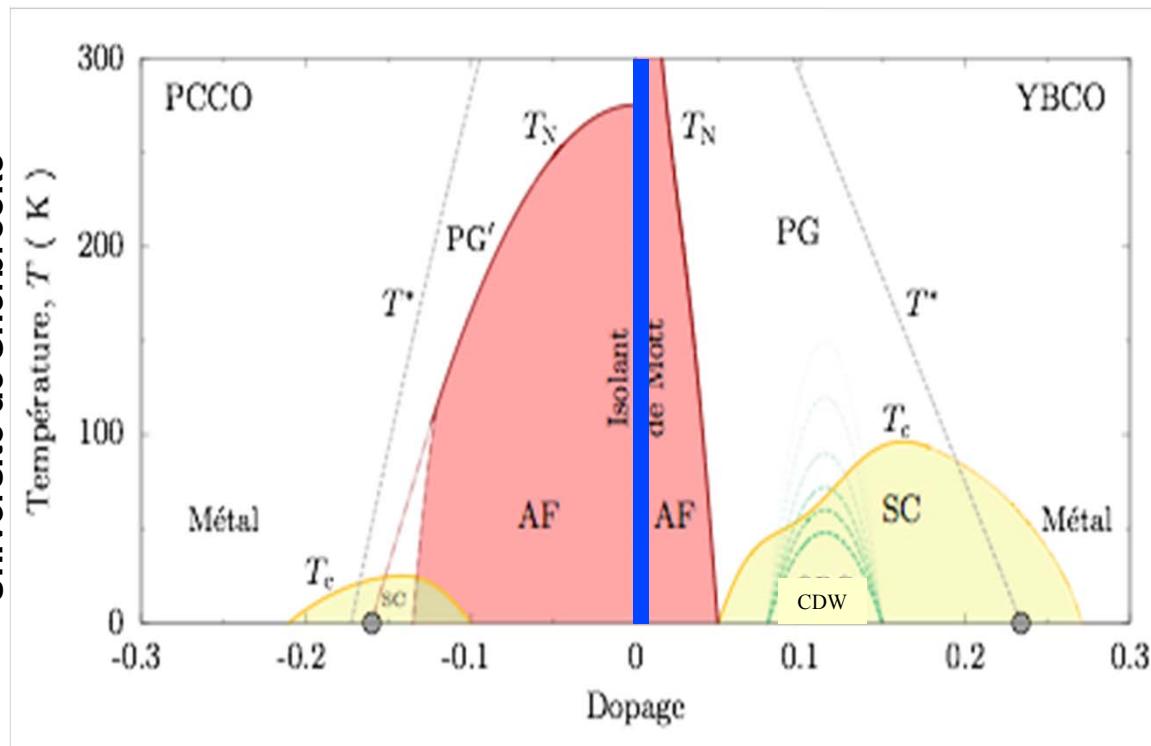
92-37



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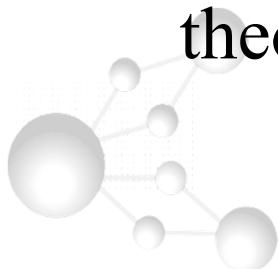
Our road map

Thèse de Francis Laliberté,
Université de Sherbrooke



Conventional wisdom vs high T_c

- Transition metals
- Cubic
- Stay away from
 - O
 - Magnets
 - Insulators
- Stay away from theorists
- Cu
- Layered
- Stay close to
 - O
 - Magnets
 - Insulators



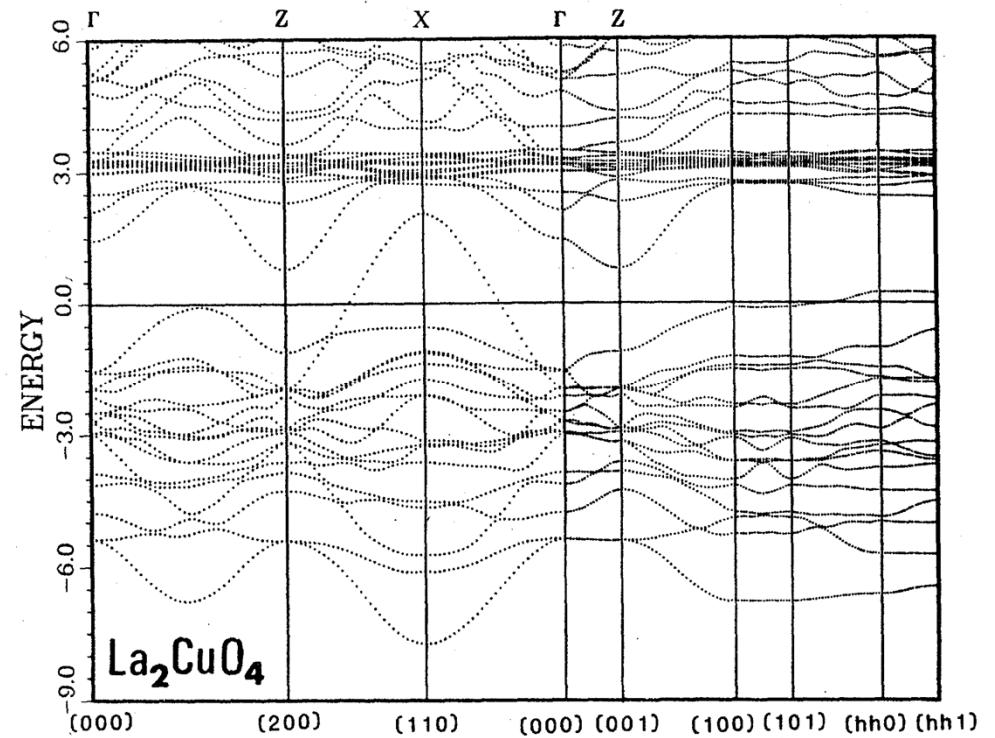
2. The model

$$H = -\sum_{<ij>\sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



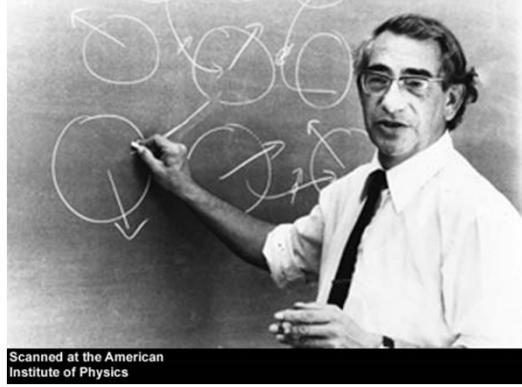
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Band structure for high T_c

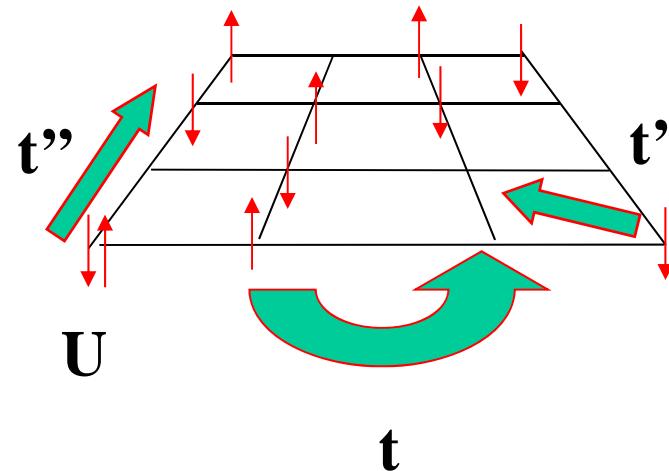


W. Pickett, Rev. Mod. Phys. 1989

Hubbard model



μ



1931-1980

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Attn: Charge transfer insulator



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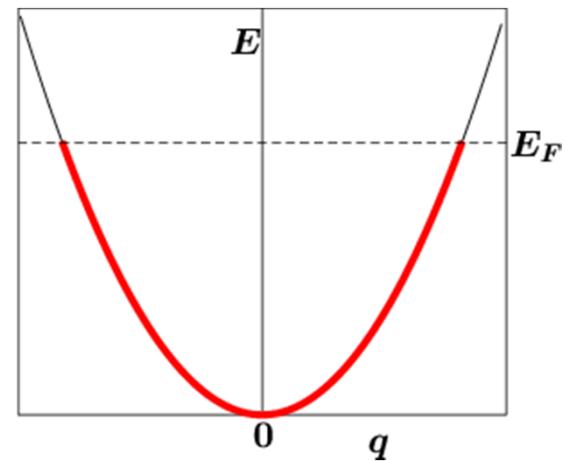
$$U=0$$

$$H = -\sum_{<ij>\sigma} t_{i,j} \left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right)$$

$$c_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_i} c_{\mathbf{k}\sigma}$$

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

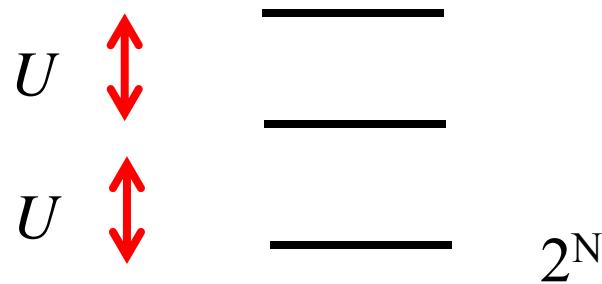
$$|\Psi\rangle=\prod_{\mathbf{k},\sigma} c_{\mathbf{k}\sigma}^\dagger |0\rangle$$



$$t_{ij} = 0$$

$$H =$$

⋮



$$|\Psi\rangle = \prod_{\mathbf{i}} c_{\mathbf{i}\uparrow}^\dagger \prod_{\mathbf{j}} c_{\mathbf{j}\downarrow}^\dagger |0\rangle$$

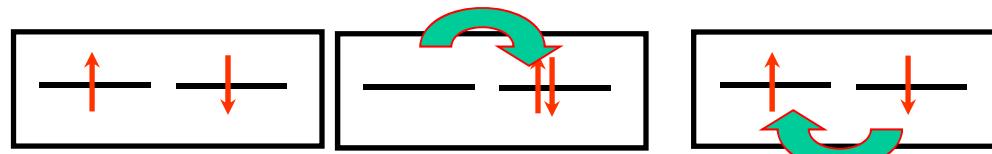
$$2^N$$



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Interesting in the general case

$$H = -\sum_{<ij>\sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



t

Effective model, Heisenberg: $J = 4t^2 / U$

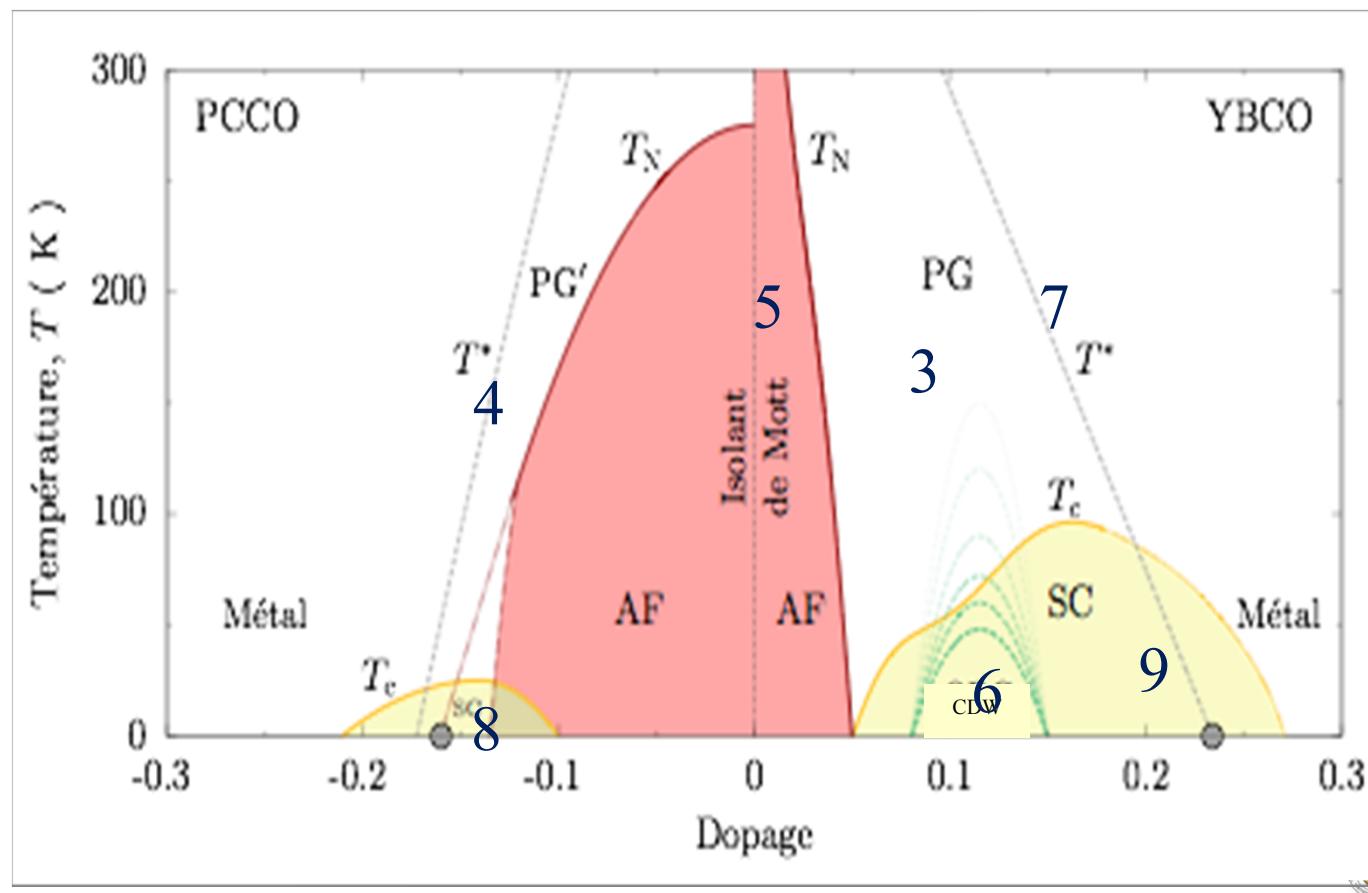


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Outline

For references, September 2013 Julich summer school
Strongly Correlated Superconductivity

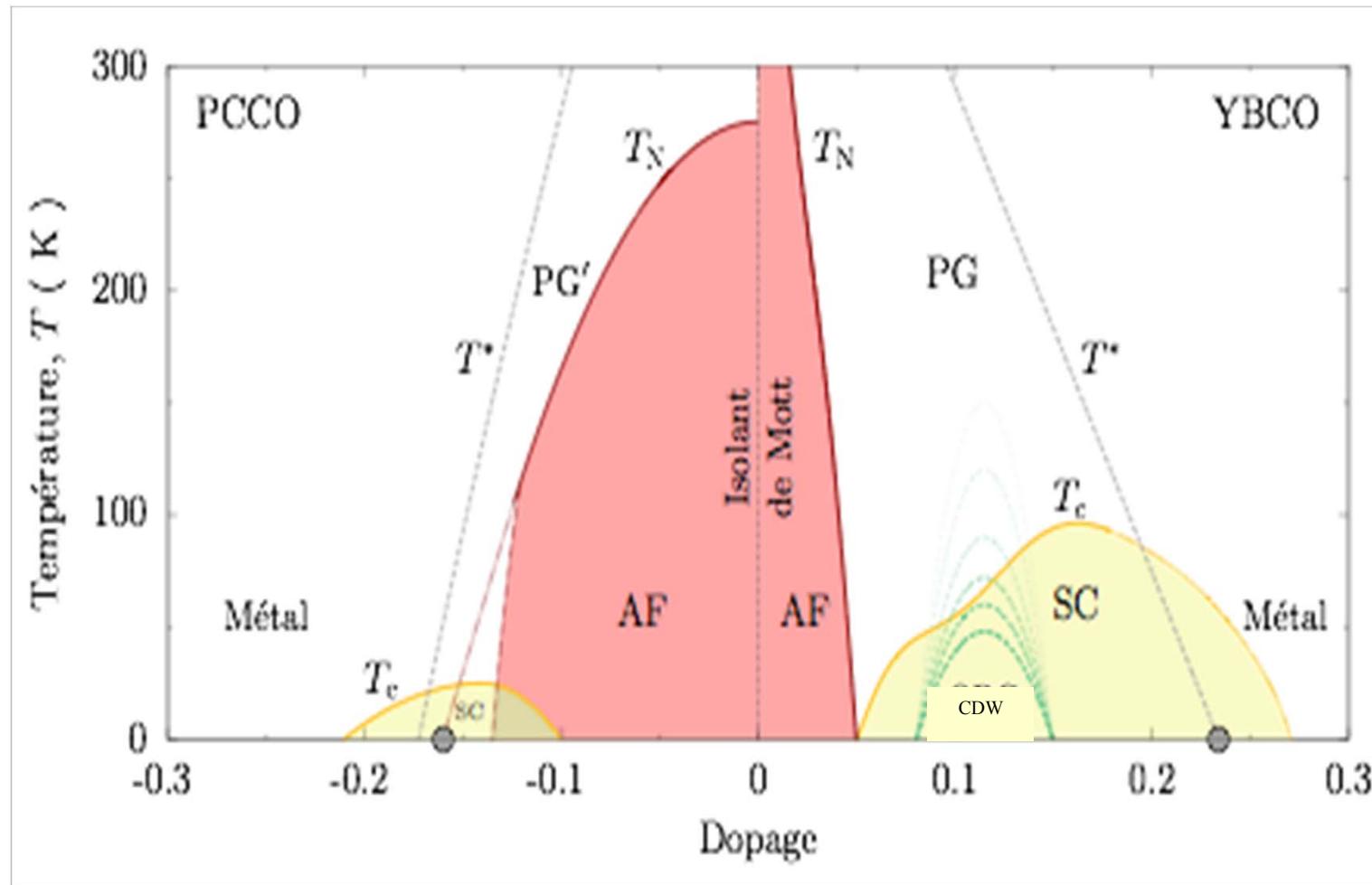
<http://www.cond-mat.de/events/correl13/manuscripts/tremblay.pdf>



3. A normal, normal state?

Our road map

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« Phase » and emergent properties

- Emergent properties
 - e.g. Fermi surface
 - Shiny
 - Quantum oscillations (in B field)
- Many microscopic models will do the same
 - Electrons in box or atoms in solid, Fermi surface
 - Concept of Fermi liquid
 - Often hard to « derive » from first principles (fractionalization - gauge theories)



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h -doped are strongly correlated:
evidence from the normal state



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Mott-Ioffe-Regel limit

$$\sigma = \frac{ne^2\tau}{m}$$

$$n = \frac{1}{2\pi d} k_F^2$$

$$\sigma = \left(\frac{1}{2\pi d} k_F^2 \right) \frac{e^2 \tau}{m}$$

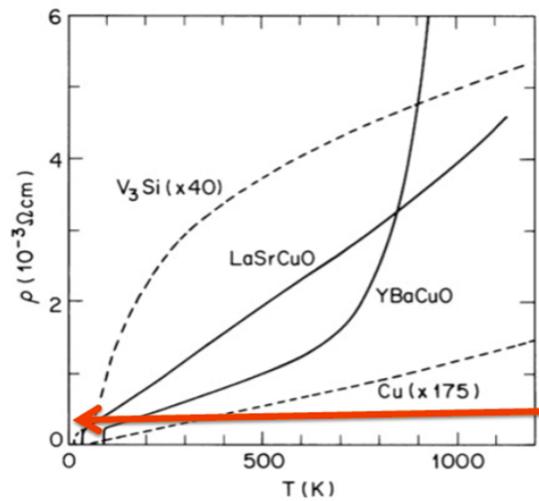
$$\ell = \left(\frac{\hbar k_F}{m} \right) \tau$$

$$\sigma = \frac{1}{2\pi d} k_F e^2 \left(\frac{\ell}{\hbar} \right)$$

$$k_F \ell = \frac{2\pi}{\lambda_F} \ell \sim 2\pi$$

$$\sigma_{MIR} = \frac{e^2}{\hbar d}$$

Hole-doped cuprates and MIR limit



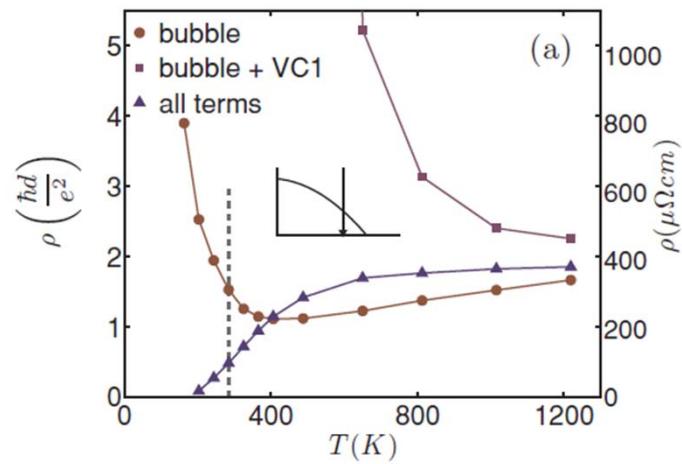
Gurvitch & Fiory
PRL 59, 1337
(1987)

MIR limit
Mean-free path
~ Fermi wavelength

LSCO 17%, YBCO optimal

PHYSICAL REVIEW B 84, 085128 (2011)

Optical and dc conductivity of the two-dimensional Hubbard model in the pseudogap regime and across the antiferromagnetic quantum critical point including vertex corrections

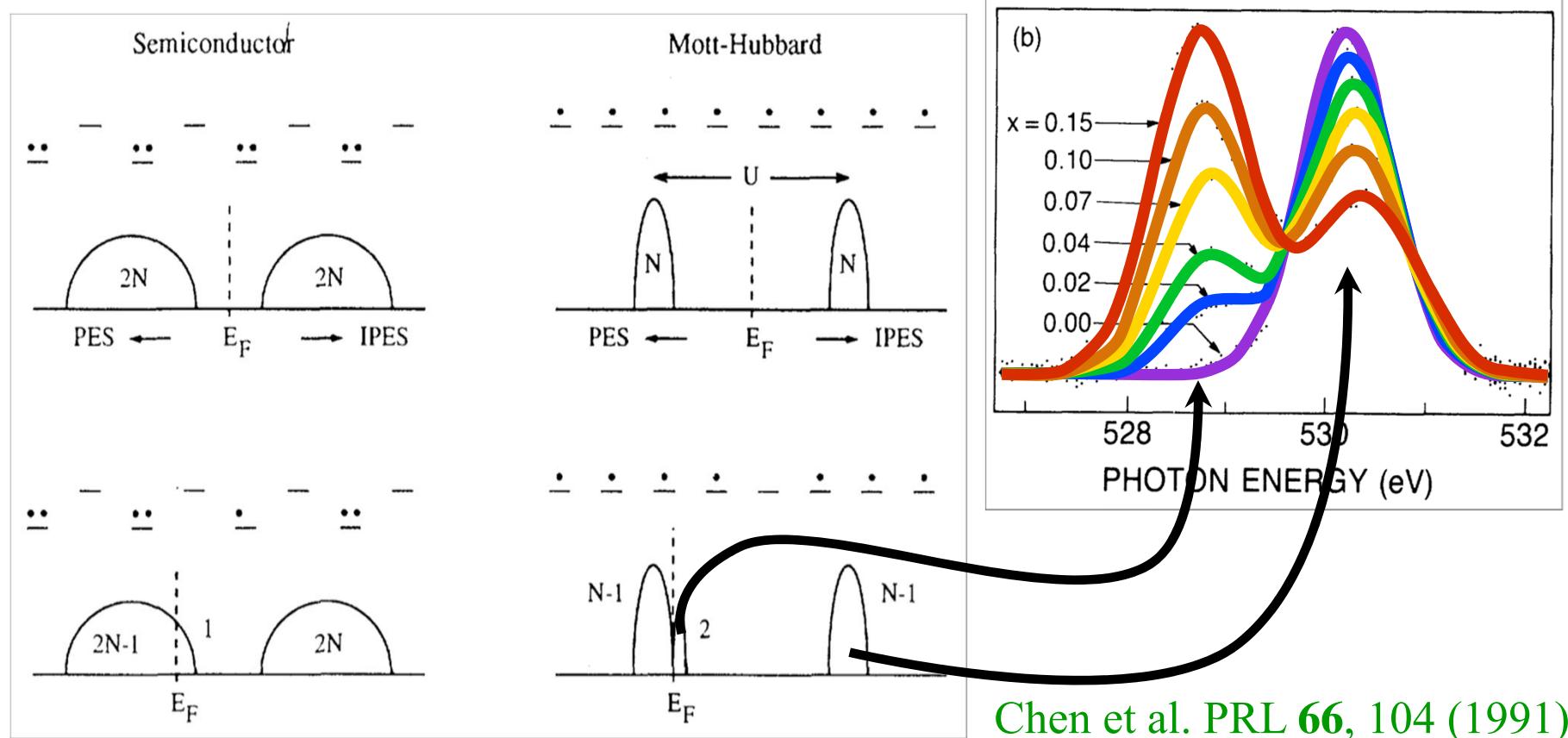


Dominic Bergeron & AMST
PRB 2011
TPSC



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Experiment, X-Ray absorption

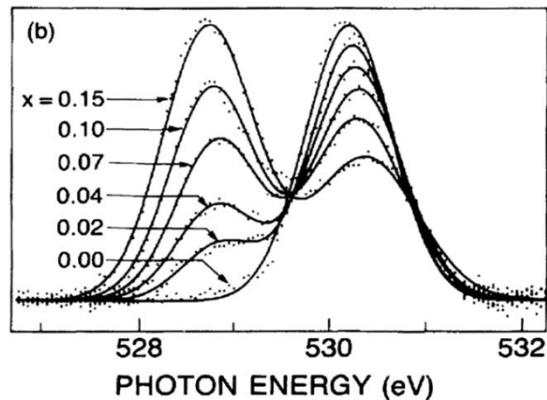


Chen et al. PRL **66**, 104 (1991)

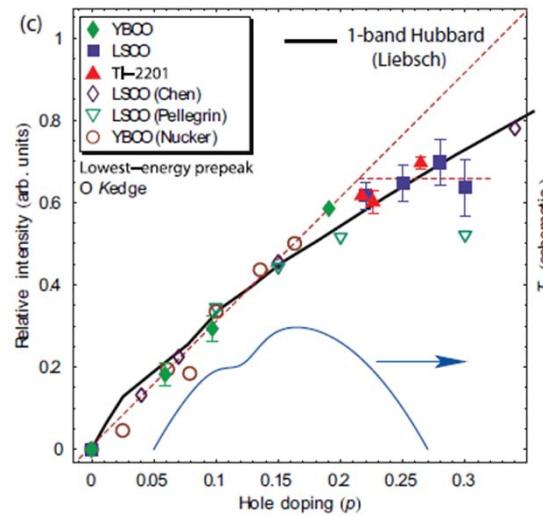


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Experiment: X-Ray absorption



Chen et al. PRL **66**, 104 (1991)

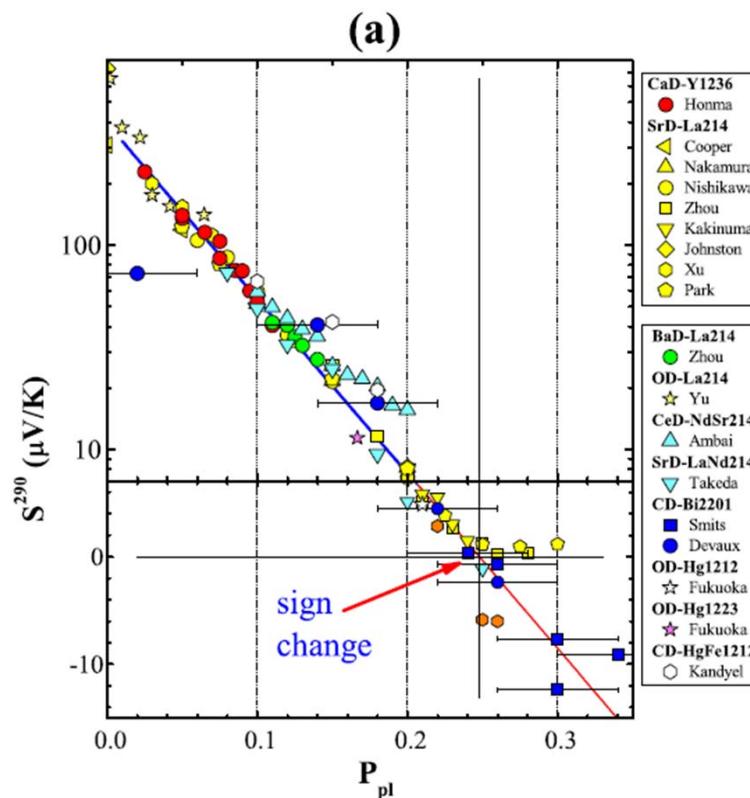


Peets et al. PRL **103**, (2009),
Phillips, Jarrell PRL , vol. **105**, 199701 (2010)

Number of low energy states above $\omega = 0$ scales as $2x +$
Not as $1+x$ as in Fermi liquid

Meinders *et al.* PRB **48**, 3916 (1993)

Thermopower

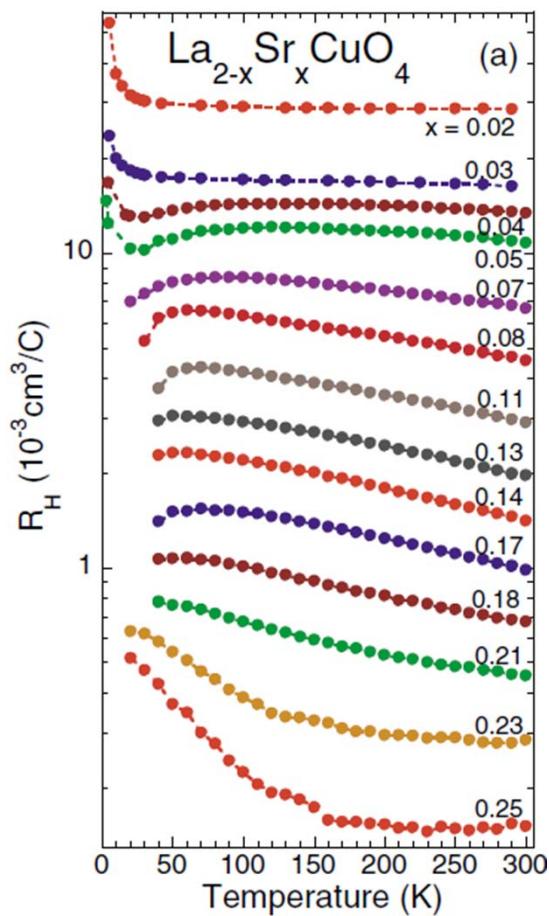
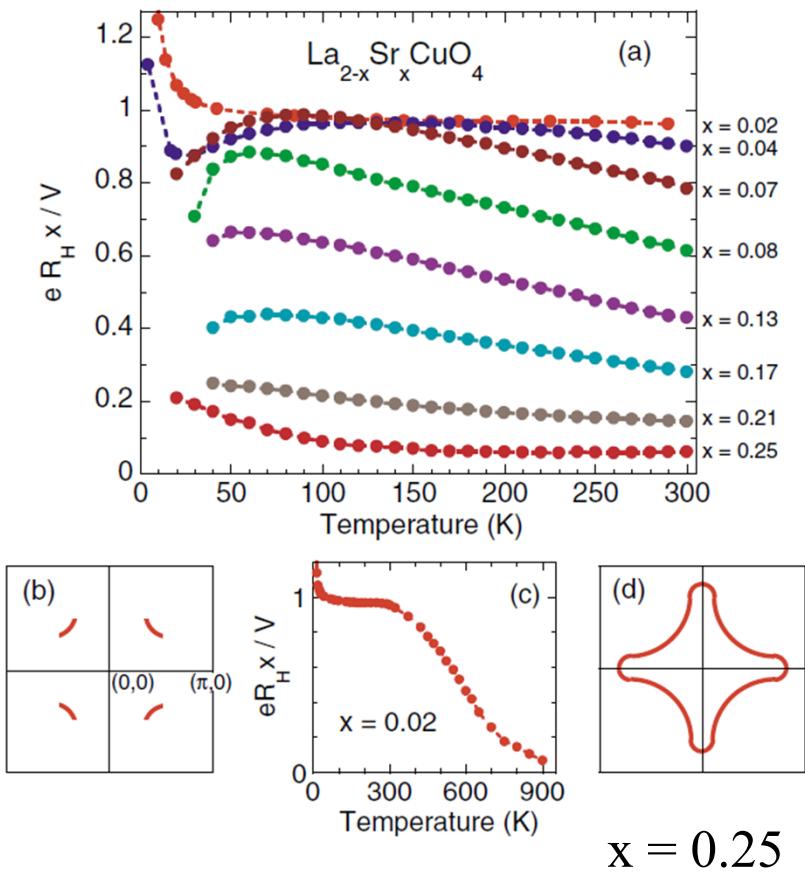


T. Honma and P. H. Hor, Phys. Rev. B **77**,
184520 (2008).



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Hall coefficient

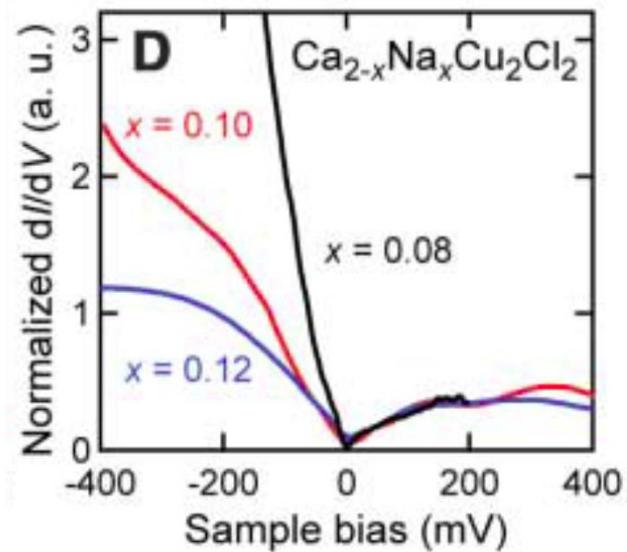


Ando et al. PRL 92, 197001 (2004)



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Density of states (STM)



Khosaka et al. *Science* **315**, 1380 (2007);

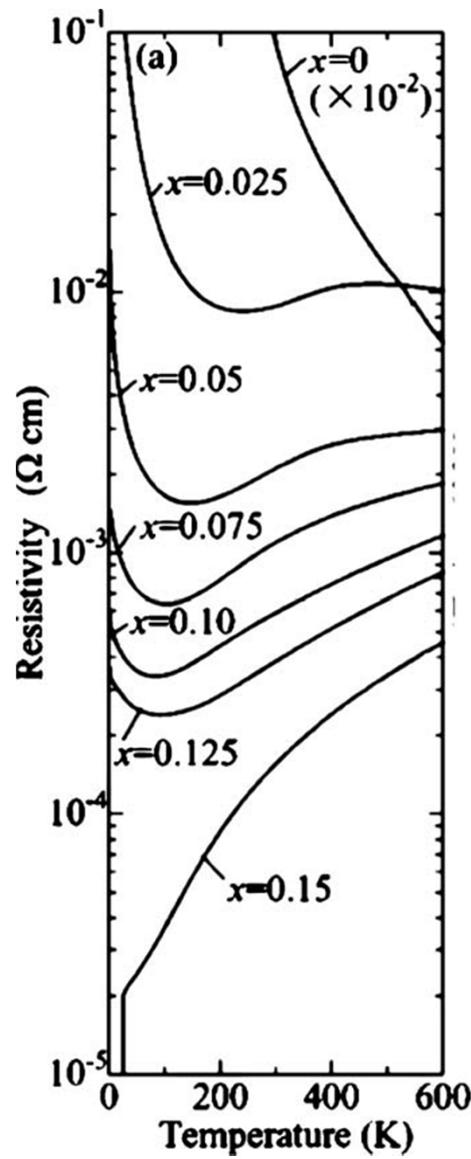
4. e-doped cuprates

Less strongly coupled: evidence from
the normal state

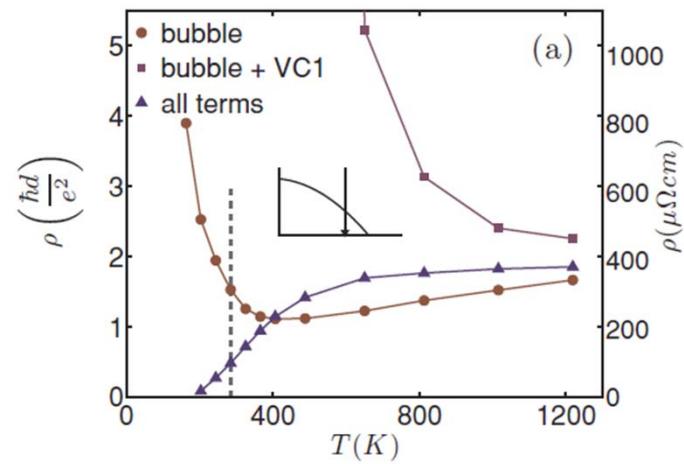


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Electron-doped and MIR limit



NCCO



Dominic Bergeron et al. TPSC
PRB **84**, 085128 (2011)

Onose et al. 2004

Quantum critical points

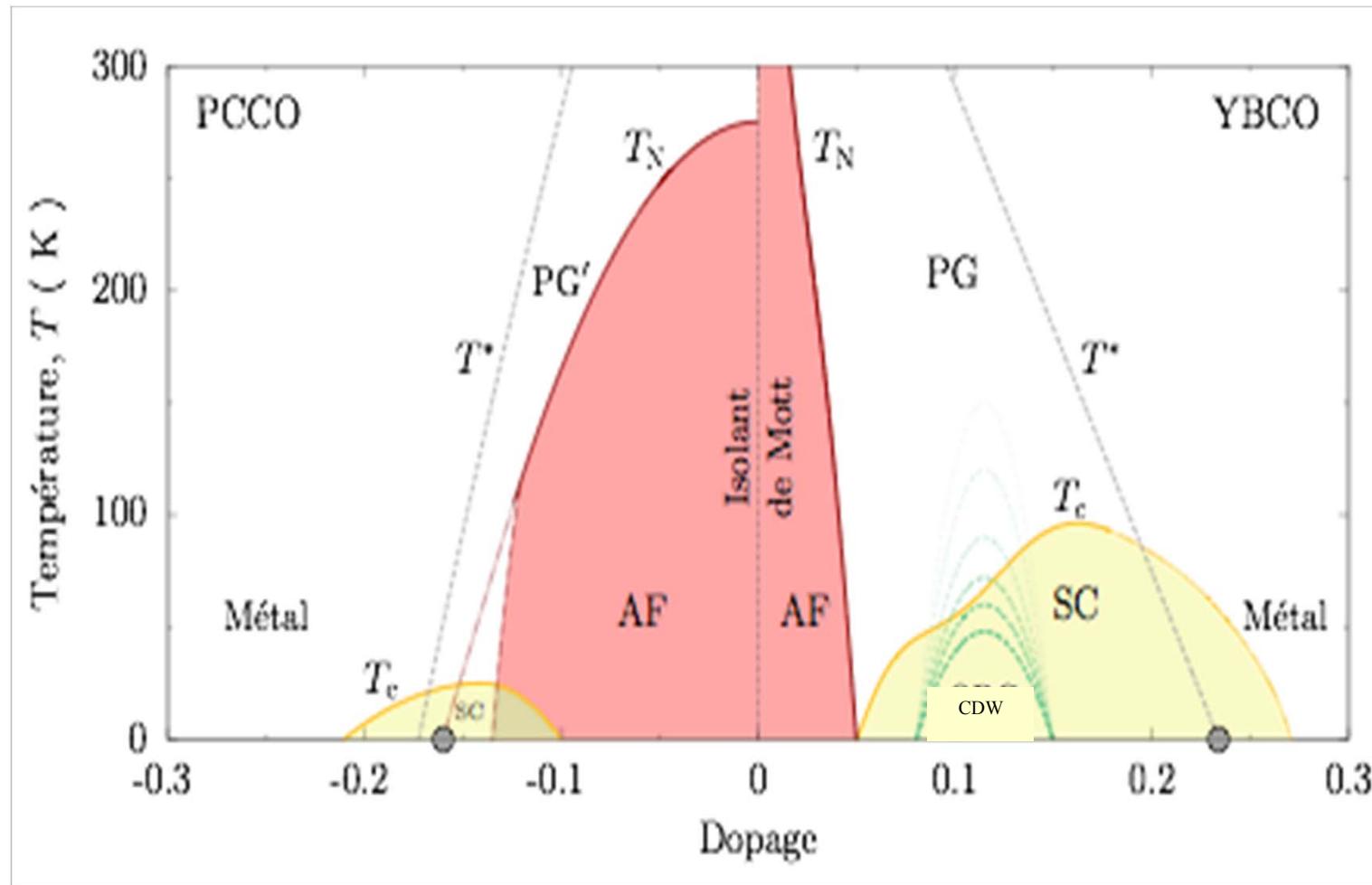
e-doped



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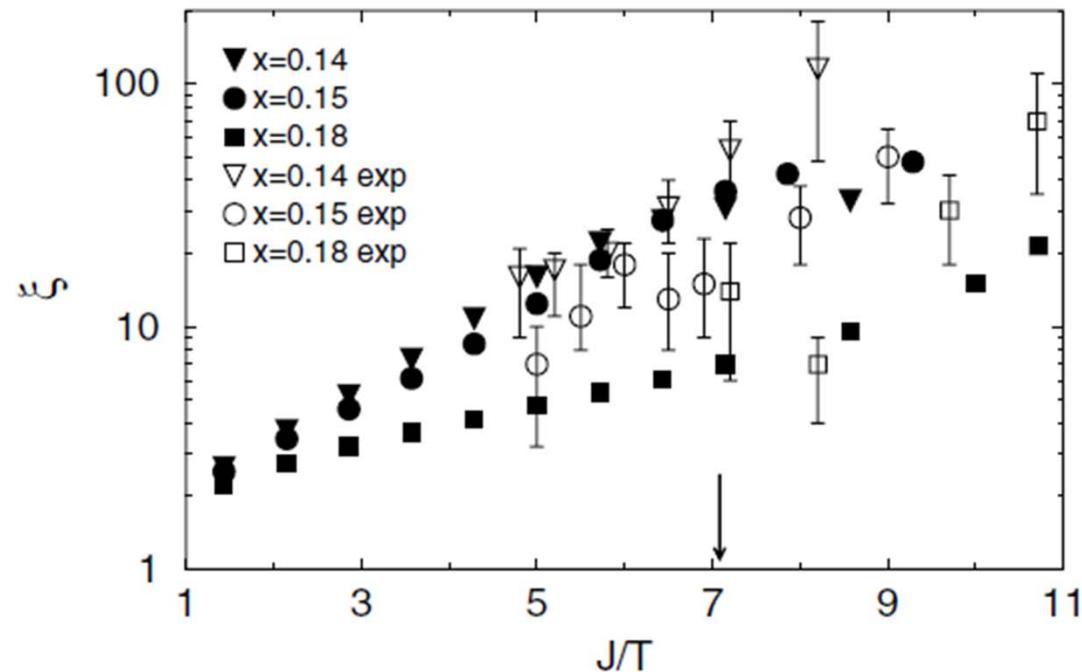
Our road map

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Université de Sherbrooke



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TPSC vs experiment for ξ

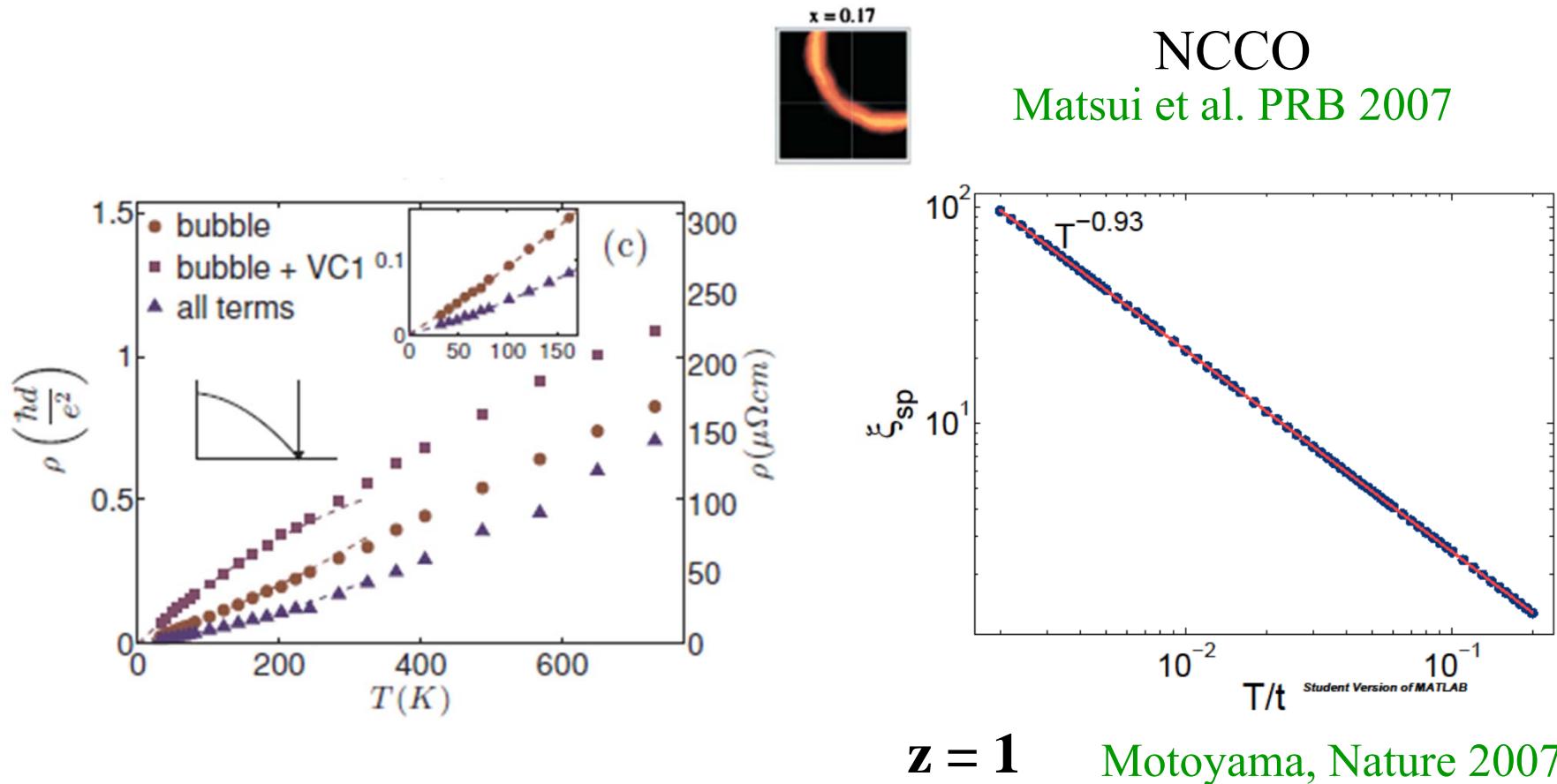


Kyung et al. PRL 93, 147004 (2004)

P. K. Mang et al., Phys. Rev. Lett. 93, 027002 (2004).

M. Matsuda et al., Phys. Rev. B 45, 12 548 (1992).

$\xi(T)$ at the QCP



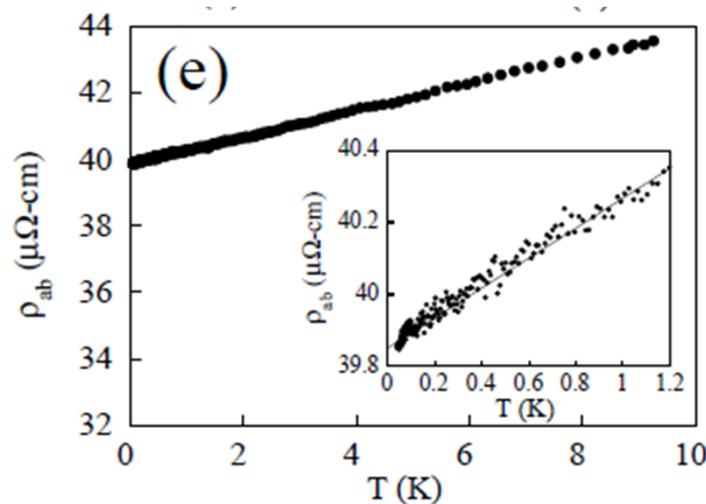
$$U=6, t'=-0.175, t''=0.05, n=1.2007$$

Dominic Bergeron TPSC



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Linear resistivity

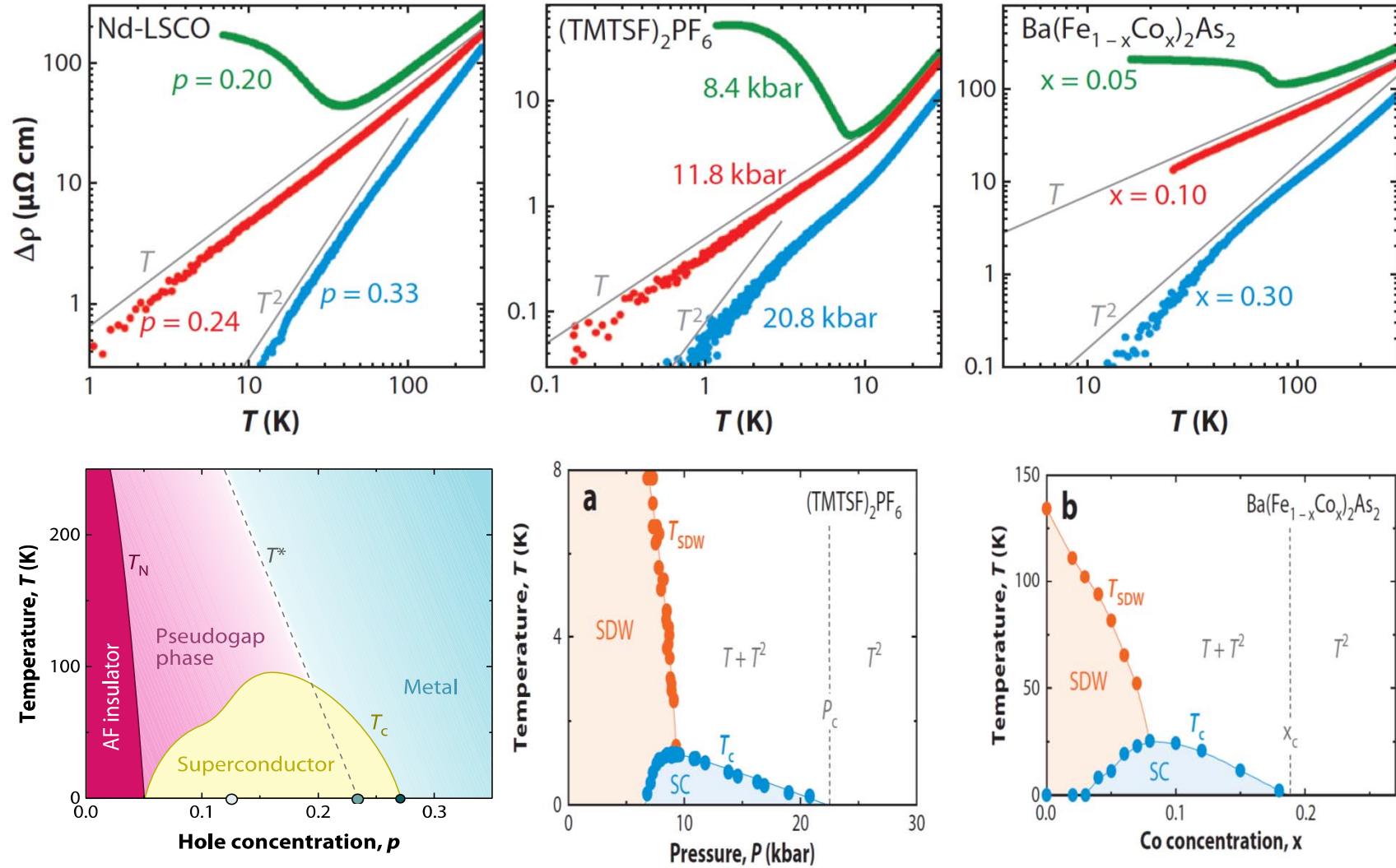


Fournier et al. PRL 1998



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QCP and linear T dependence of R

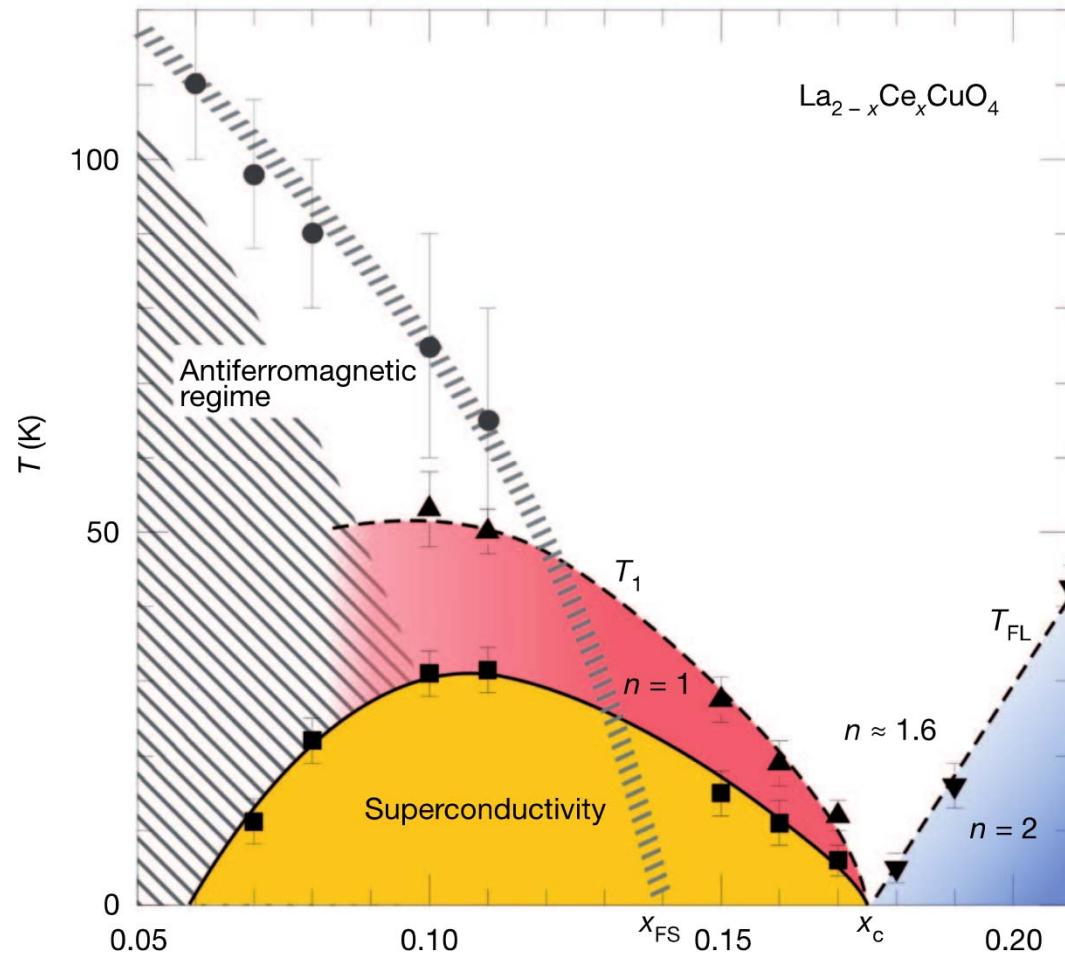


L. Taillefer, Annual Reviews of CMP 2010



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Another QCP



Jin et al. Nature 2011



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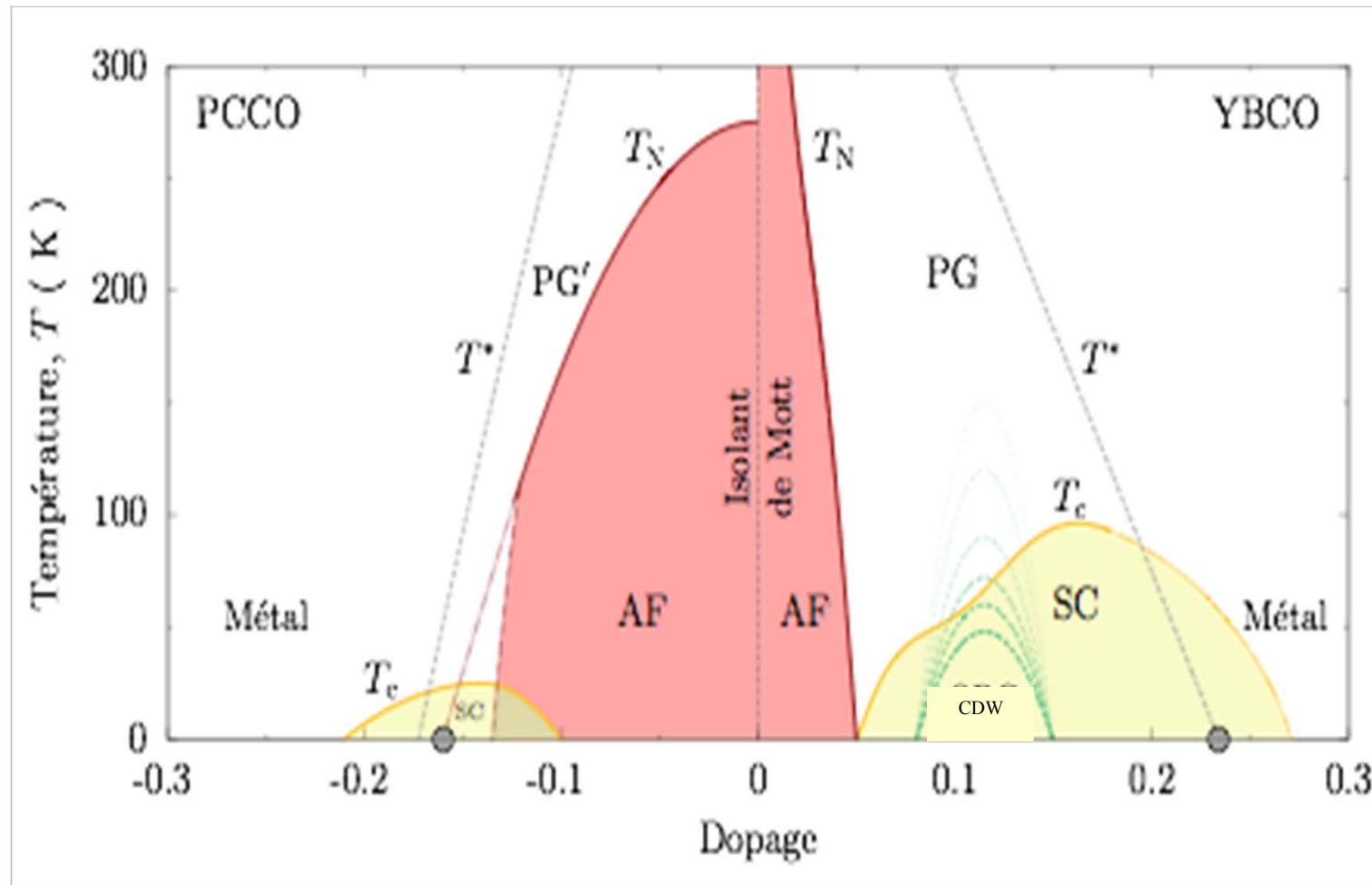
Pseudogap

e-doped



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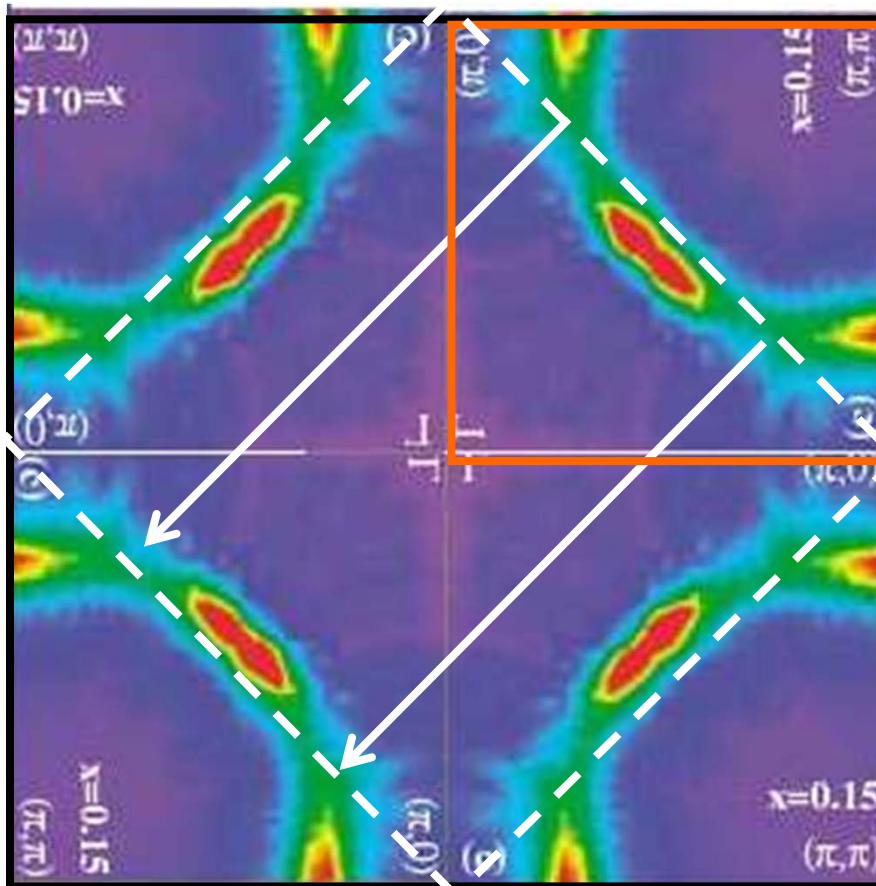
Our road map



Hot spots from AFM quasi-static scattering

Mermin-Wagner

$d = 2$



Armitage et al. PRL 2001

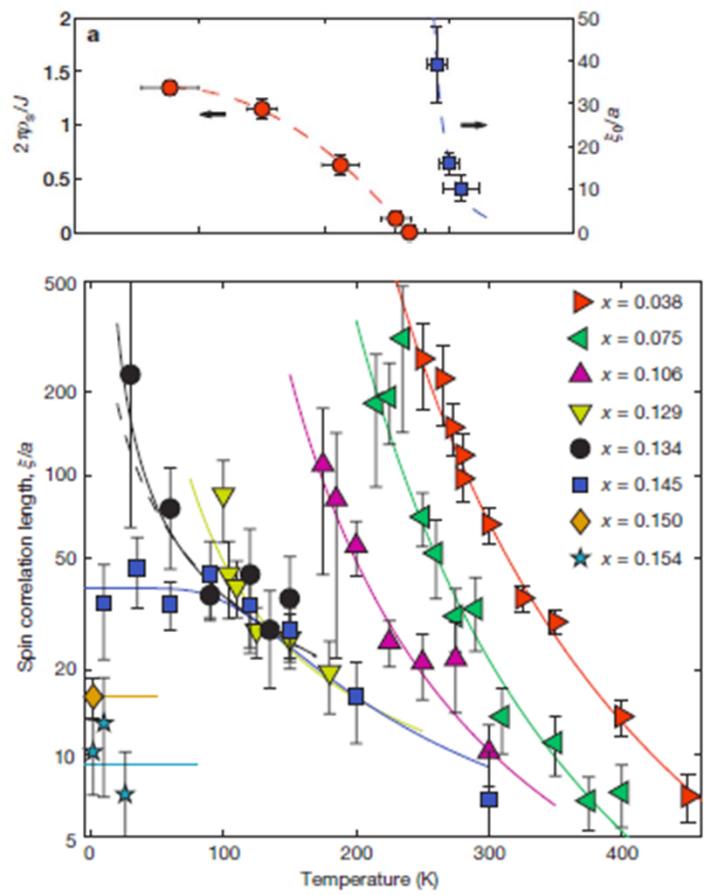
Vilk, A.-M.S.T (1997)
Kyung, Hankevych,
A.-M.S.T., PRL, 2004

$$\xi^* = 2.6(2) \xi_{\text{th}}$$

Motoyama, E. M. et al..
445, 186–189 (2007).

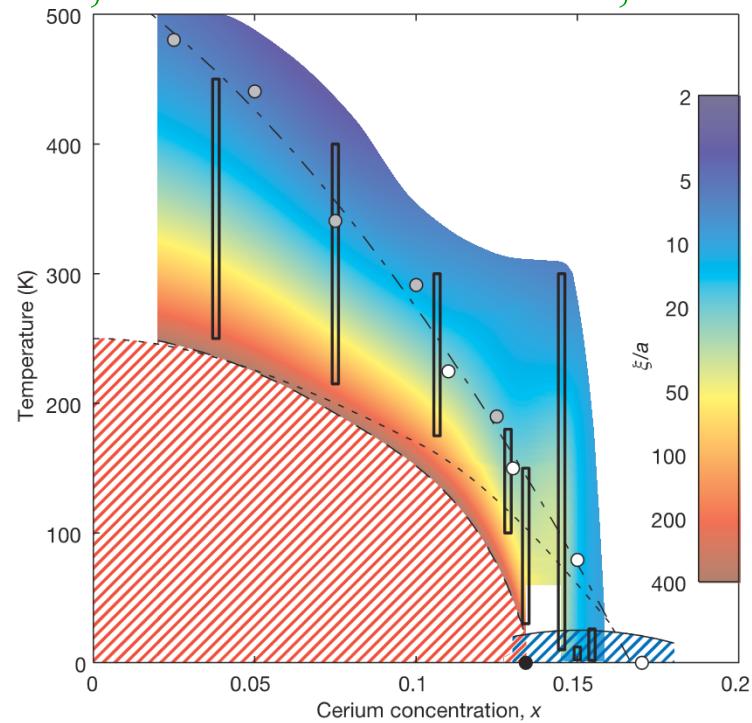
e-doped cuprates: precursors

NCCO

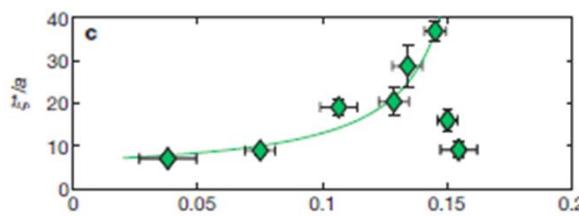


$$Z = 1$$

Motoyama, E. M. et al.. Nature 445, 186–189 (2007).



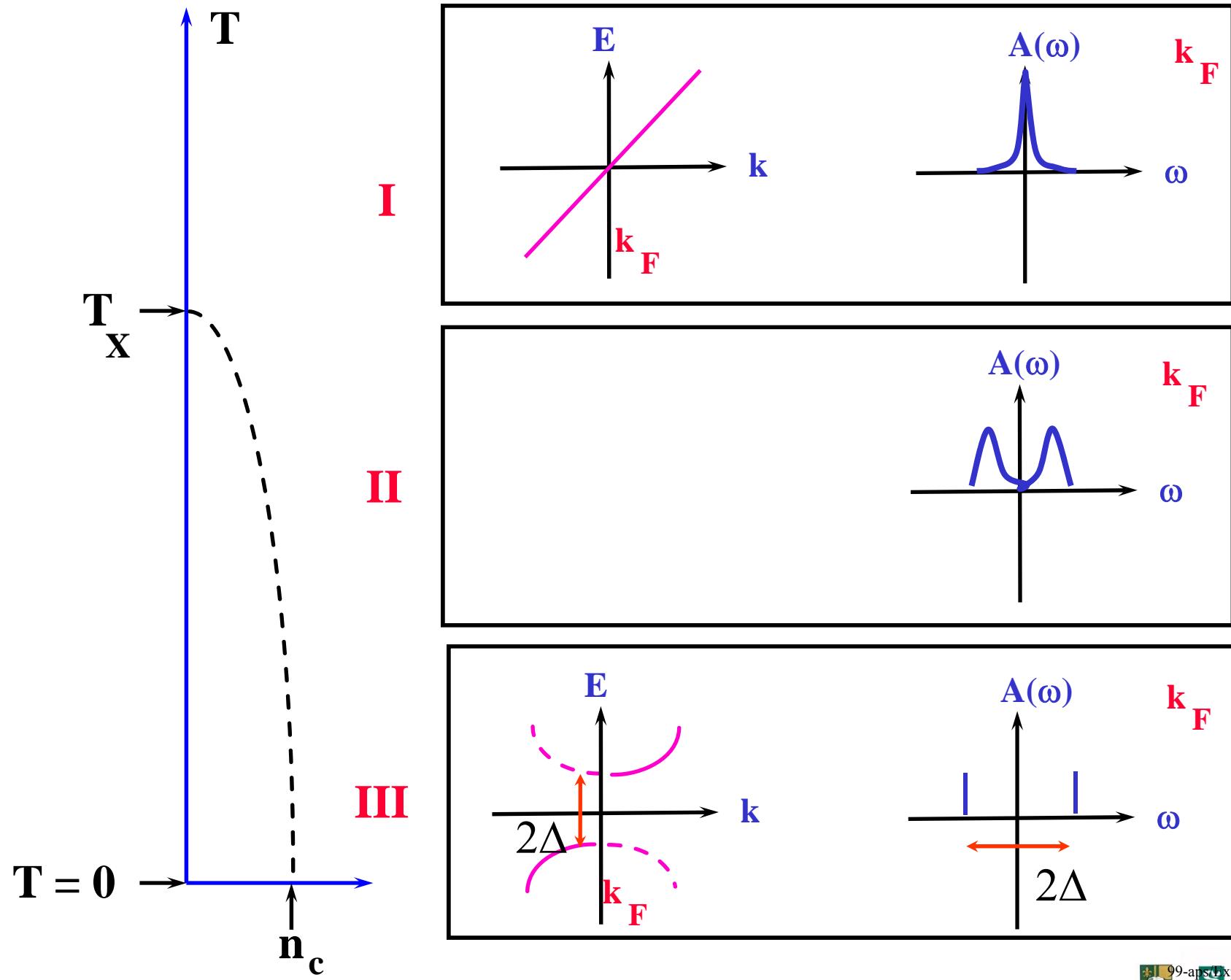
Vilk, A.-M.S.T (1997)
Kyung, Hankevych, A.-M.S.T., PRL, 2004



$$\xi^* = 2.6(2)\xi_{\text{th}}$$

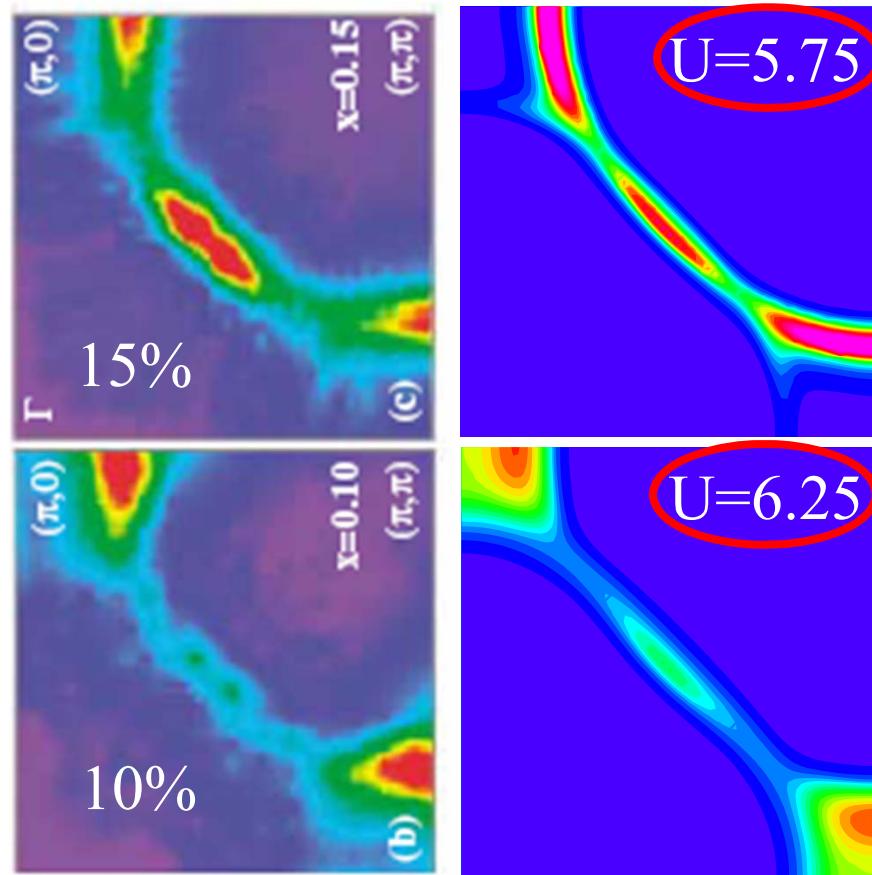


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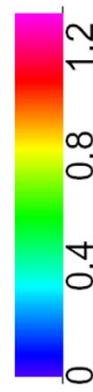


Fermi surface plots

Hubbard repulsion U has to...



be not too large



increase for
smaller doping

Hankevych, Kyung, A.-M.S.T., PRL, sept. 2004

B.Kyung *et al.*, PRB **68**, 174502 (2003)

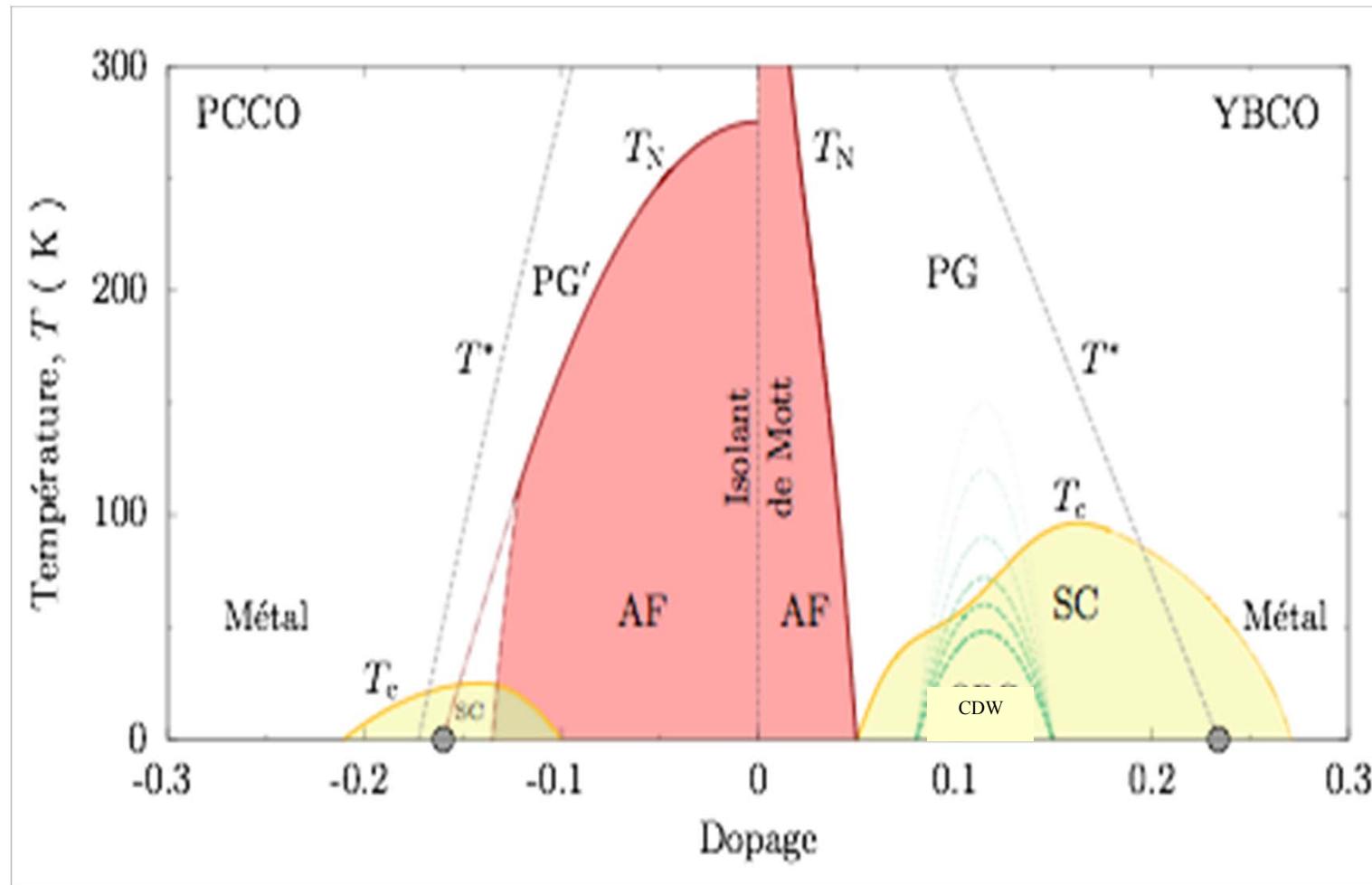
5. Weakly and strongly correlated antiferromagnets

What is a phase?



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Our road map



Antiferromagnetic phase: emergent properties

- Some broken symmetries
 - Time reversal symmetry
 - Translation by one lattice spacing
 - Unbroken Time-reversal times translation by lattice vector \mathbf{a}
 - Spin waves
 - Single-particle gap



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Differences between weakly and strongly correlated

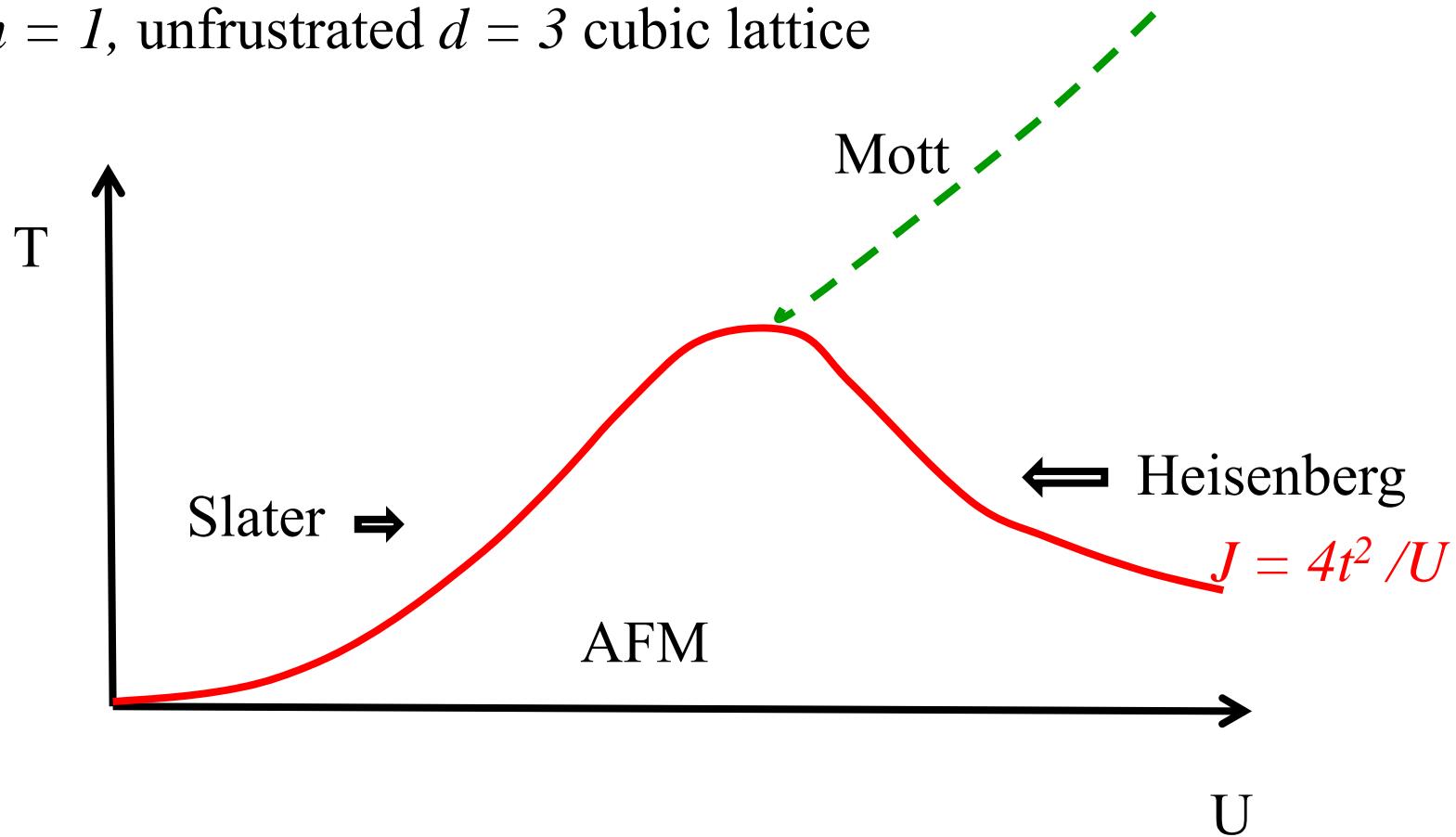
- Different in ordered phase (finite frequency)
 - Ordered moment
 - Landau damping
 - Spin waves all the way or not to J
- Different, even more, in the normal state:
 - metallic in $d = 3$ if weakly correlated
 - Insulating if strongly correlated
 - Pressure dependence of T_N



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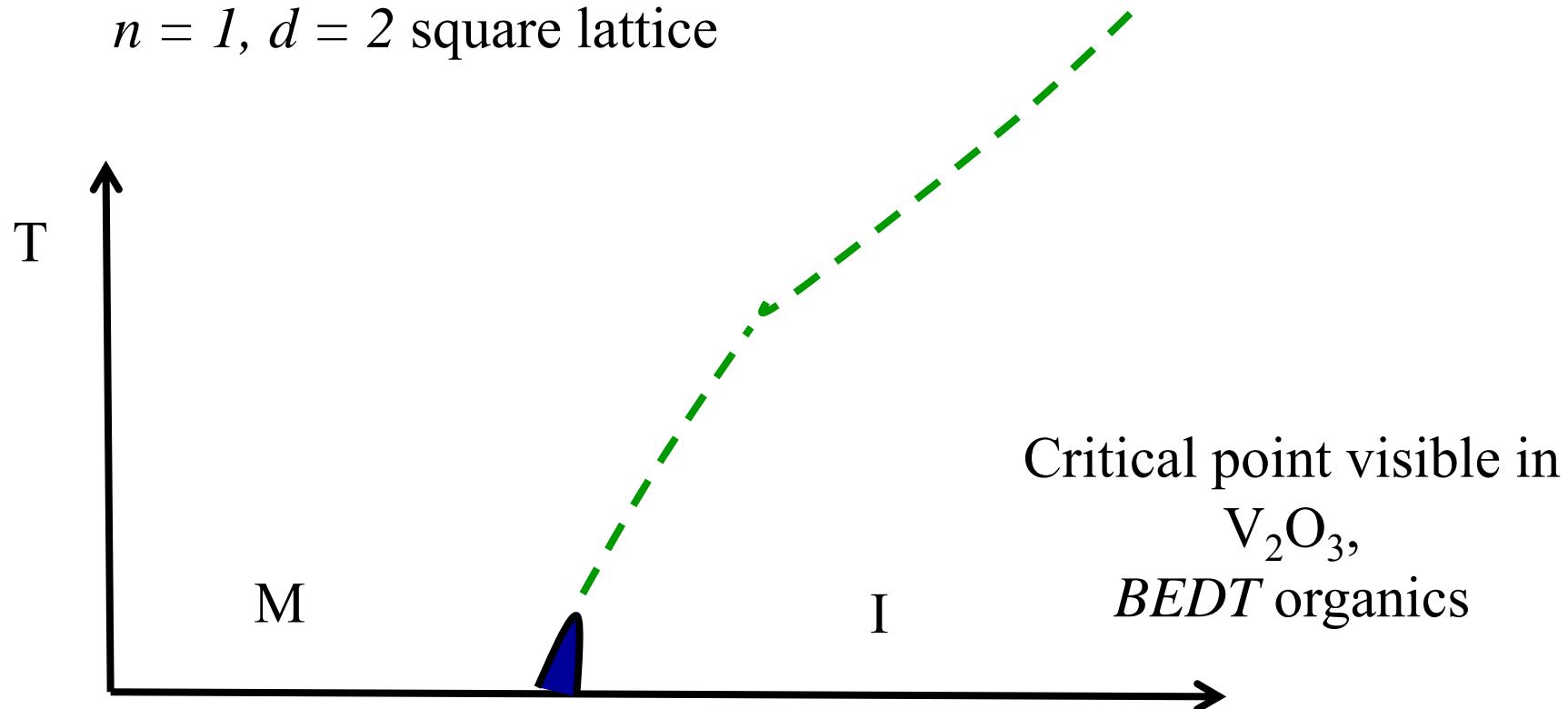
Local moment and Mott transition

$n = 1$, unfrustrated $d = 3$ cubic lattice



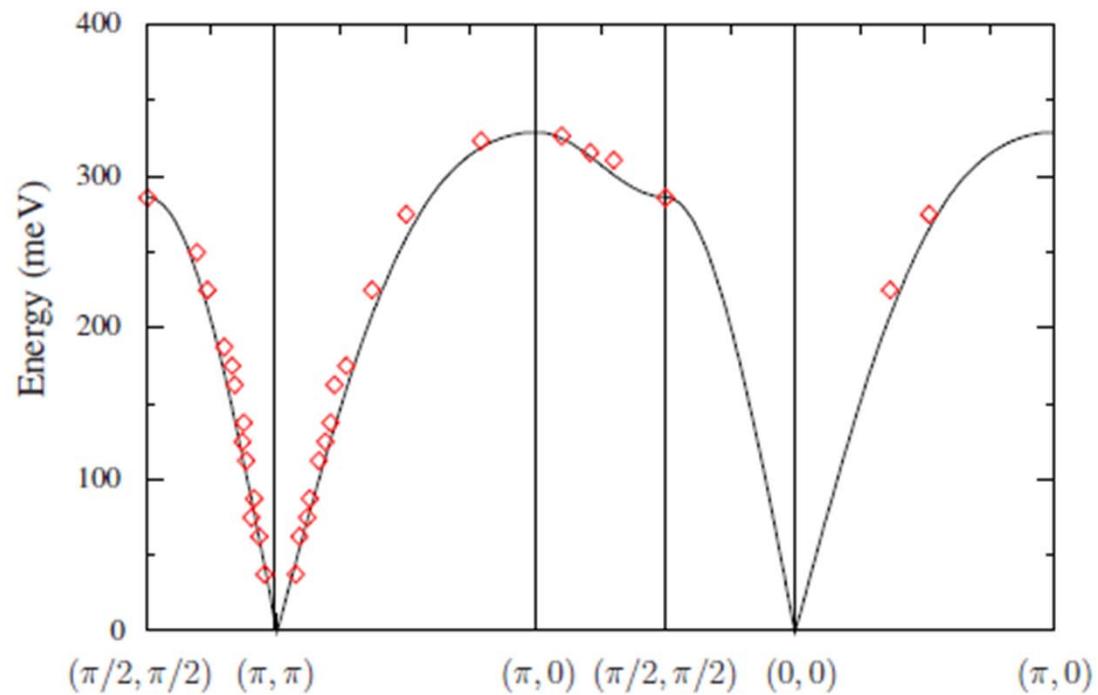
Local moment and Mott transition

$n = 1, d = 2$ square lattice



Understanding finite temperature phase from a *mean-field theory* down
to $T = 0$

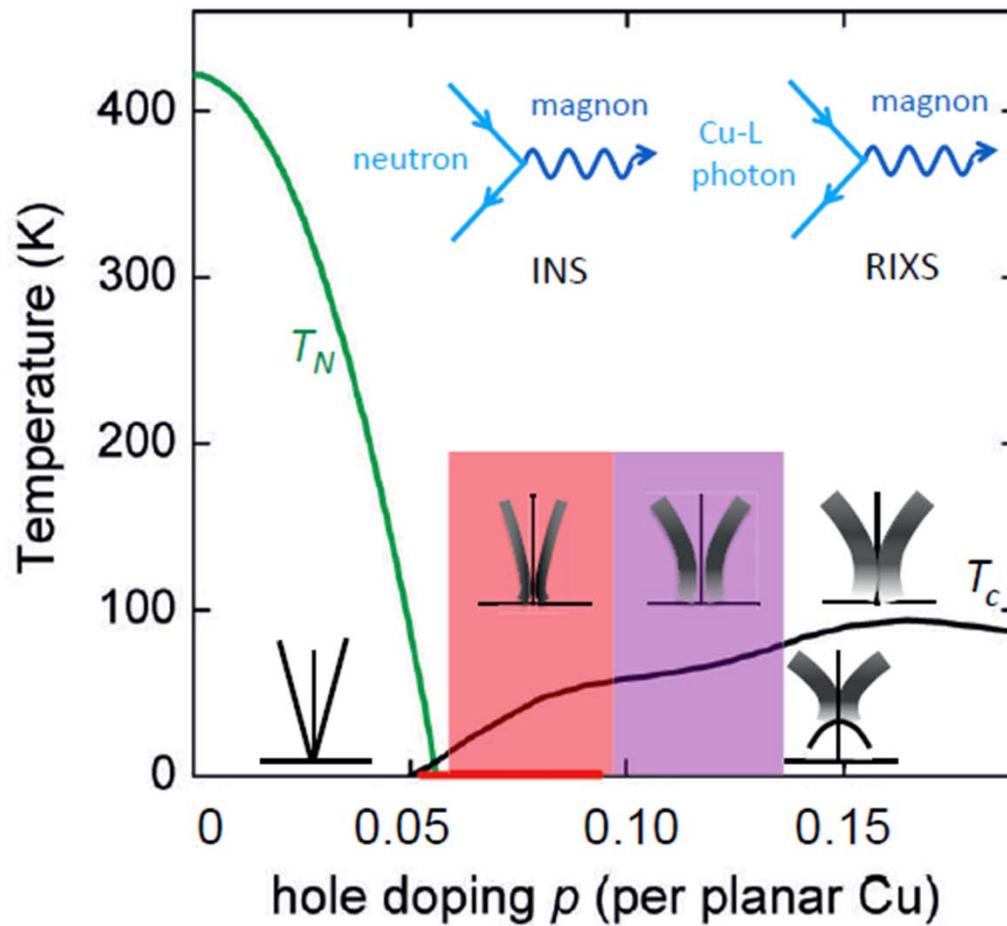
High T_c: almost Heisenberg model



Delannoy et al. PRB **79** (2009)
Dalla Piazza, Phys. Rev. B **85** (2012)

$U \sim 8$ to $10t$

Summary, magnetic excitation spectrum

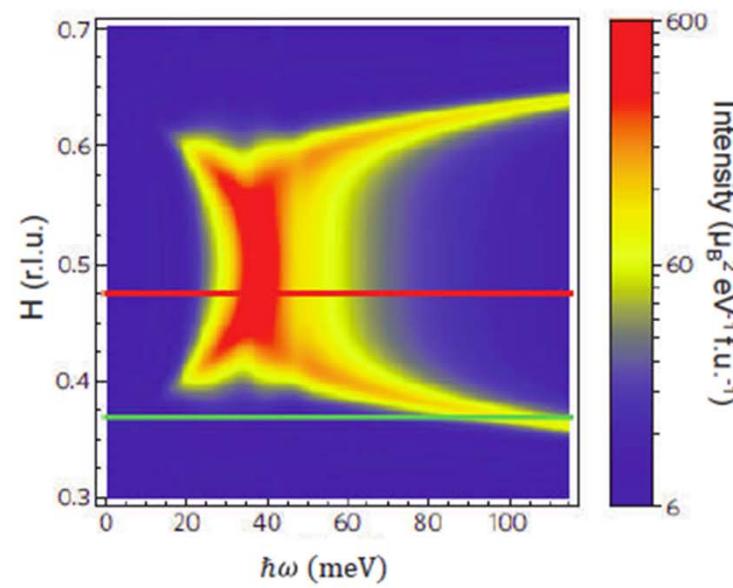
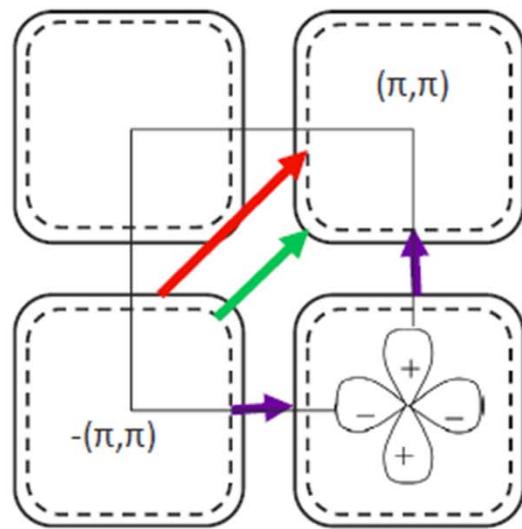


Keimer, Julich summer school 2013



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Spin fluctuations, energy momentum



Note resonance in superconducting state
Above 100 meV almost doping independent (RIXS)

Keimer, Julich summer school 2013



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6. Charge Density Wave

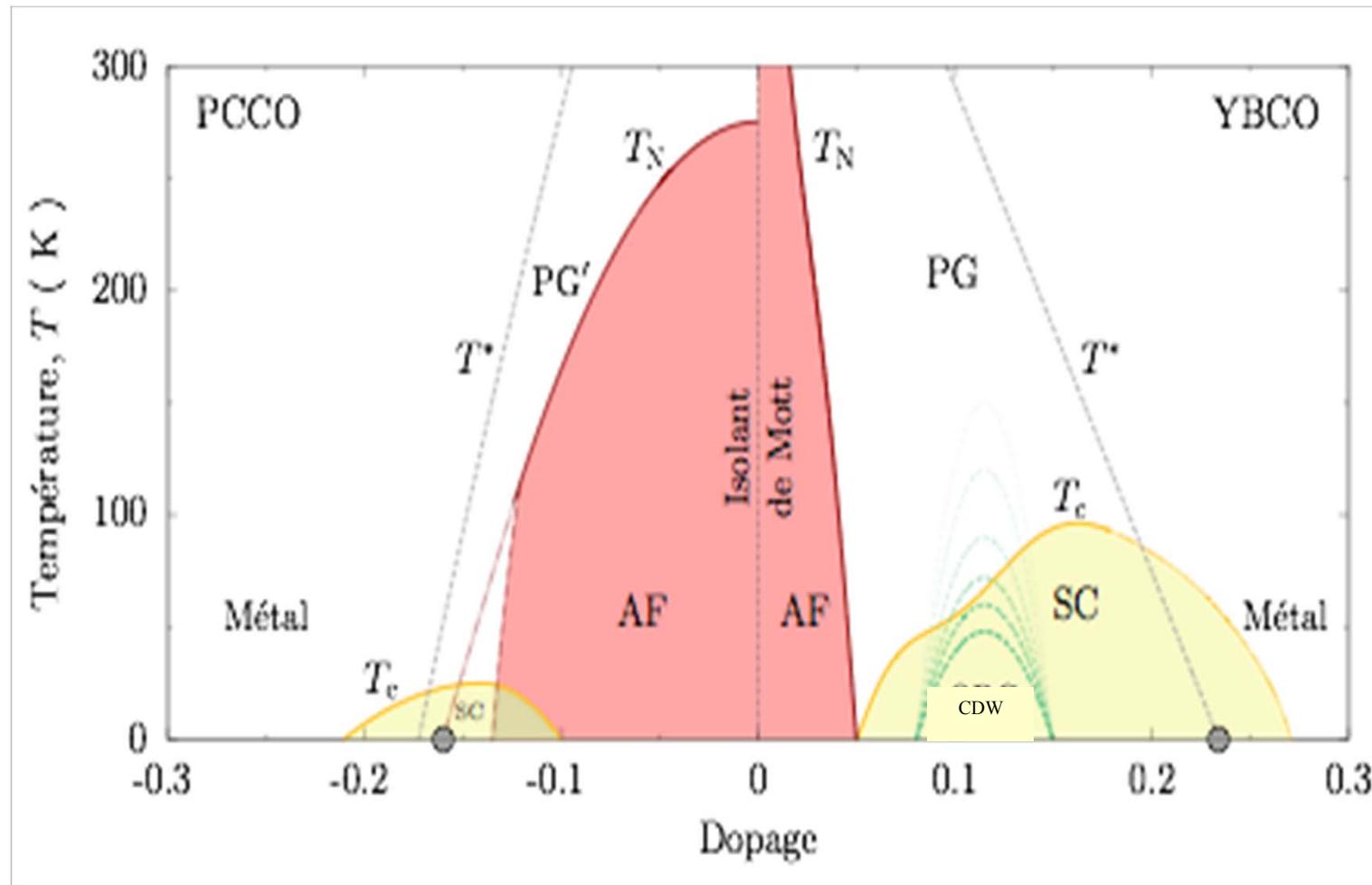
h-doped



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Our road map

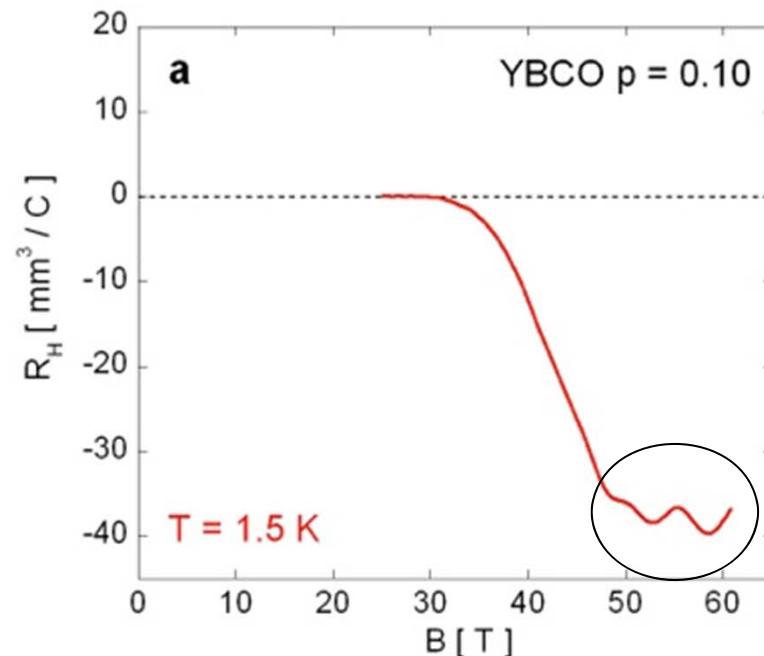
Thèse de Francis Laliberté,
Université de Sherbrooke



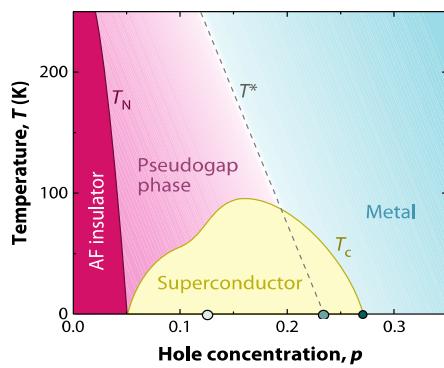
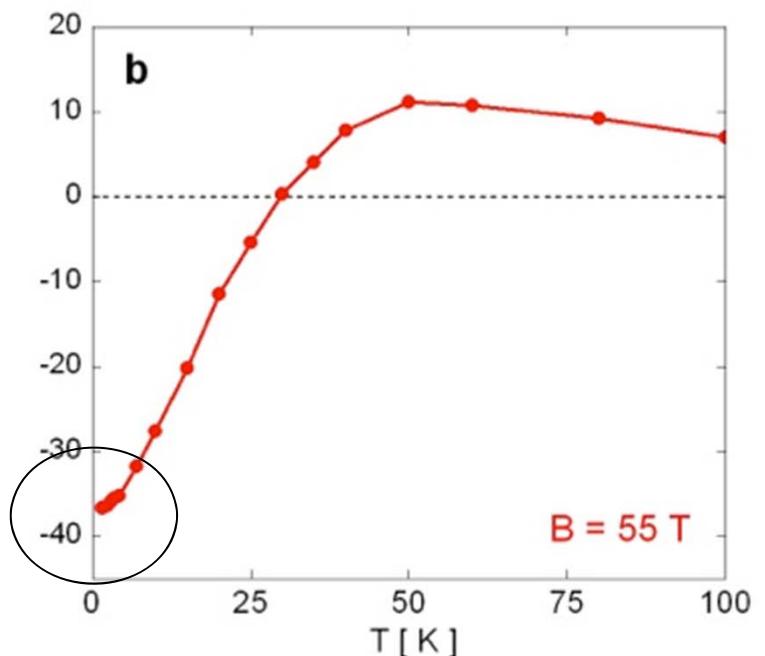
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Quantum oscillations in cuprates: 2007

N. Doiron-Leyraud et al., Nature 2007



D. LeBoeuf et al., Nature 2007



Quantum oscillations

Fermi surface includes a small *electron pocket* !

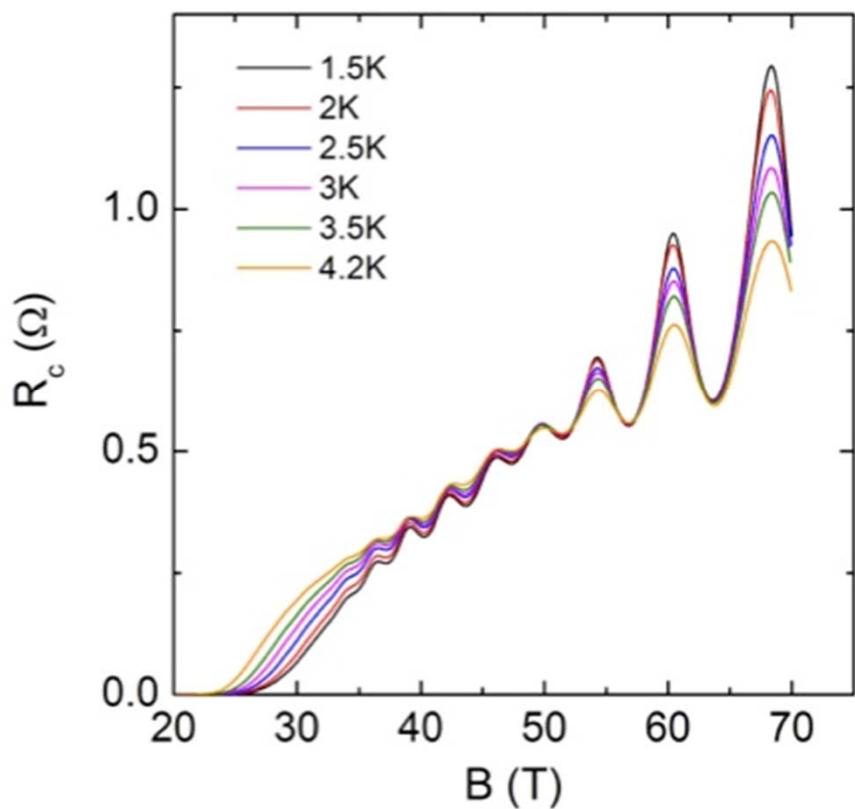
$R_H < 0$



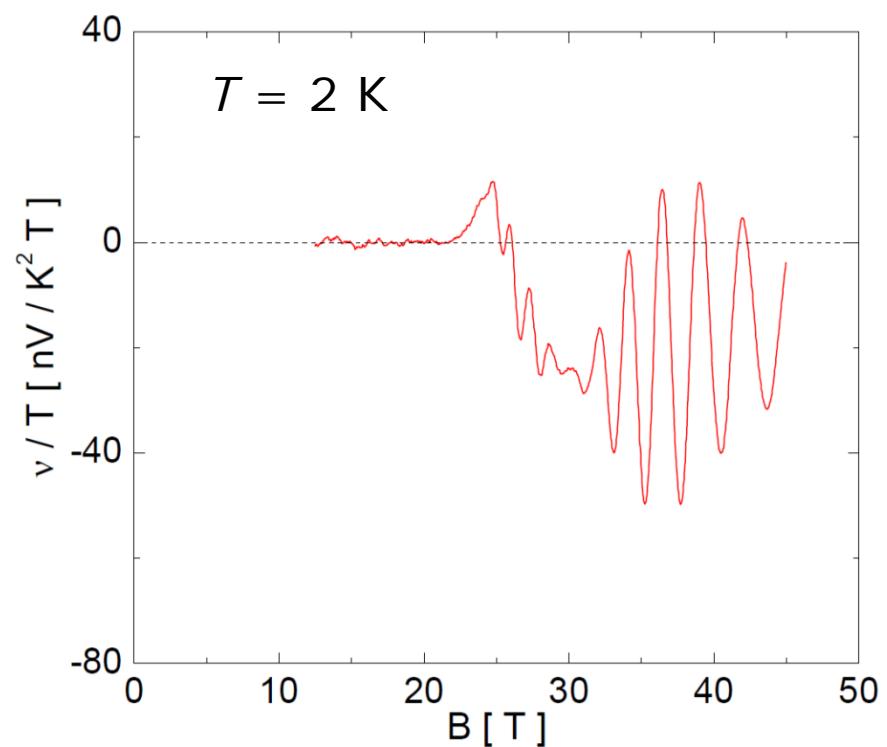
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Quantum oscillations in cuprates: 2013

Resistance



Nernst



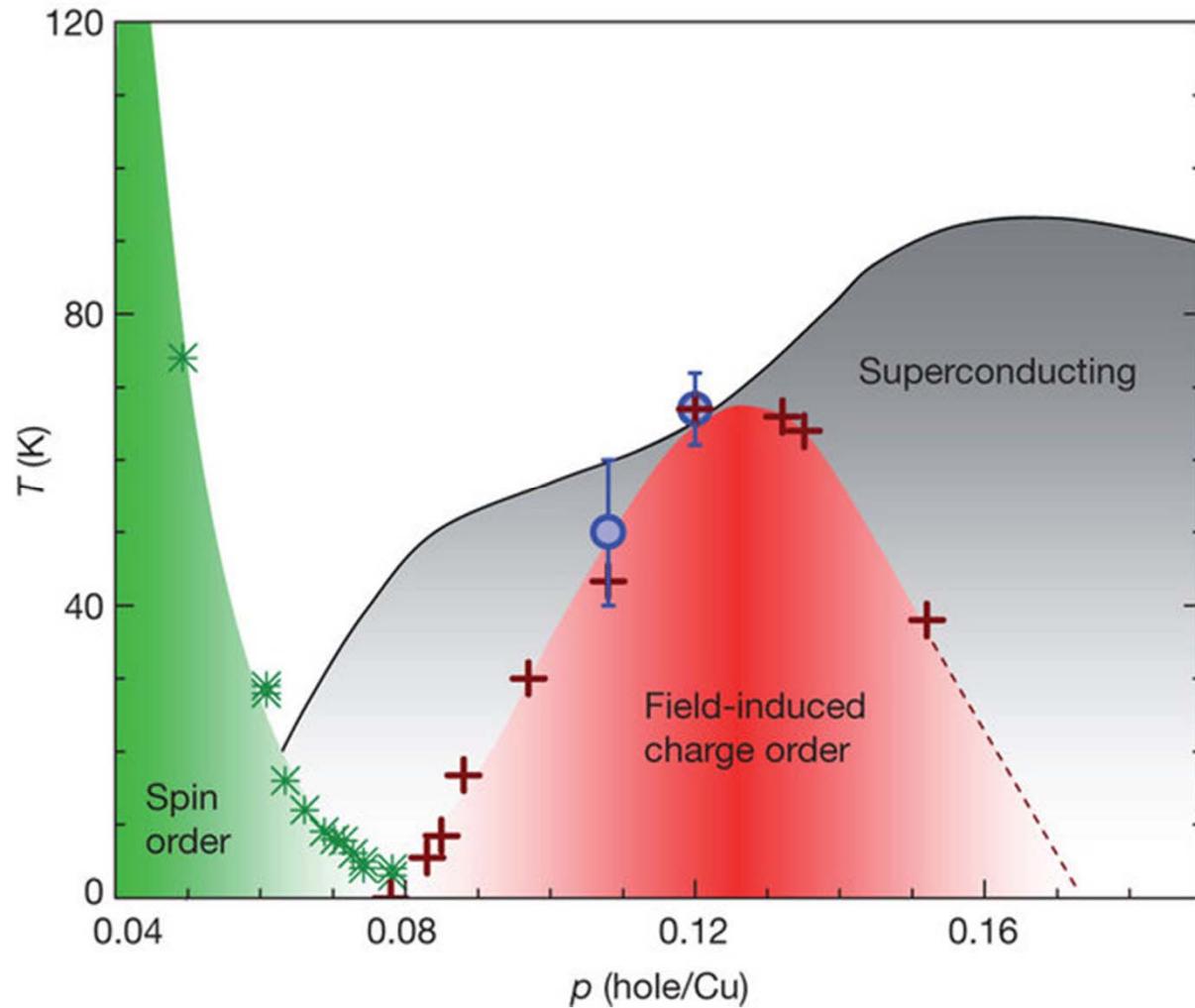
LNCMI, Toulouse

NHMFL, Tallahassee



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Stripes and reconstructed Fermi surface



Wu et al. Julien, Nature 477, 191–194 (2011)

Direct observation of competition between superconductivity and charge density wave order in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

J. Chang^{1,2*}, E. Blackburn³, A. T. Holmes³, N. B. Christensen⁴, J. Larsen^{4,5}, J. Mesot^{1,2},
Ruixing Liang^{6,7}, D. A. Bonn^{6,7}, W. N. Hardy^{6,7}, A. Watenphul⁸, M. v. Zimmermann⁸, E. M. Forgan³
and S. M. Hayden⁹

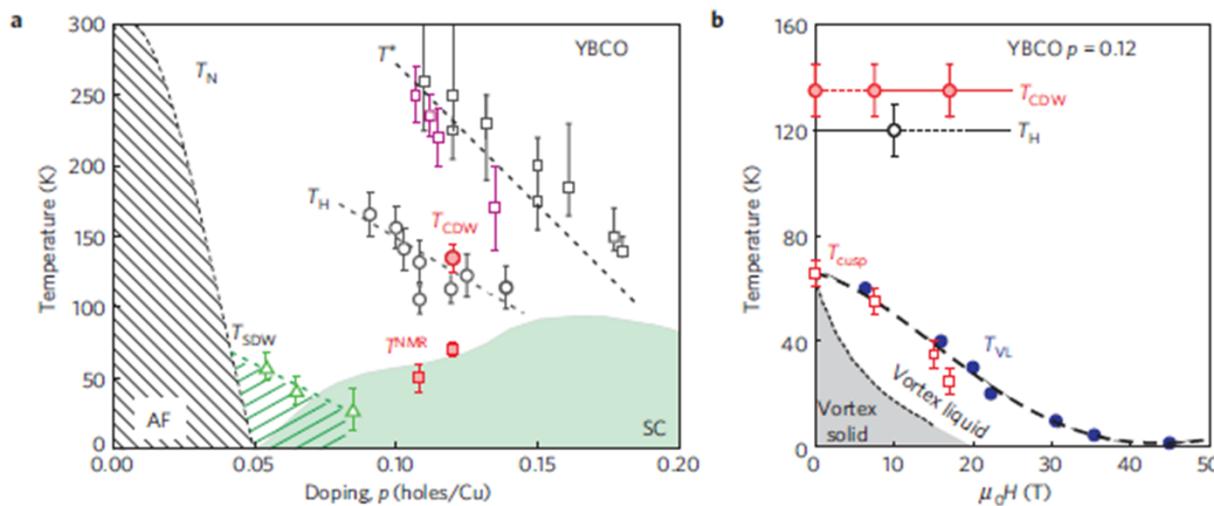
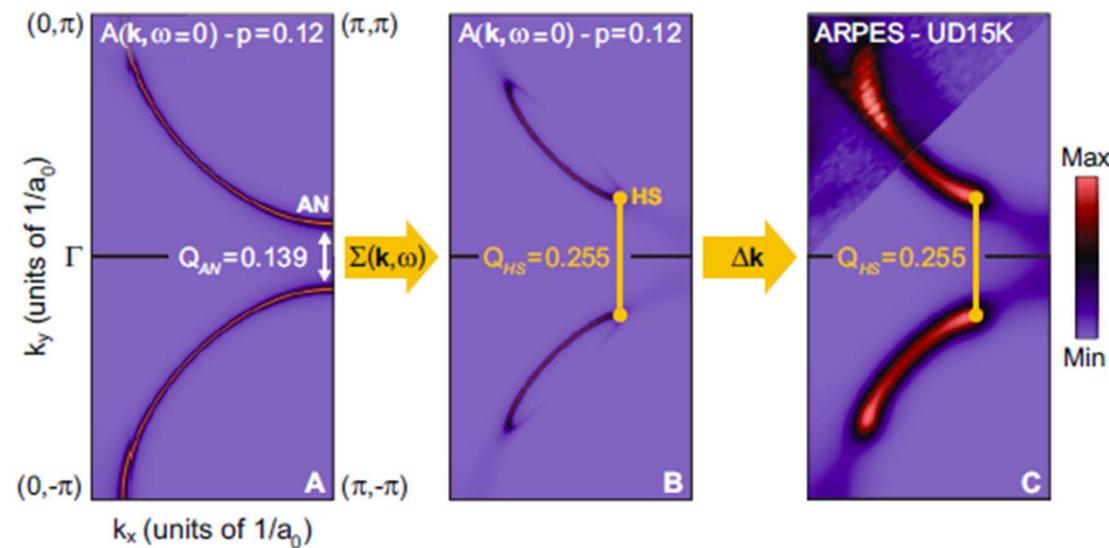


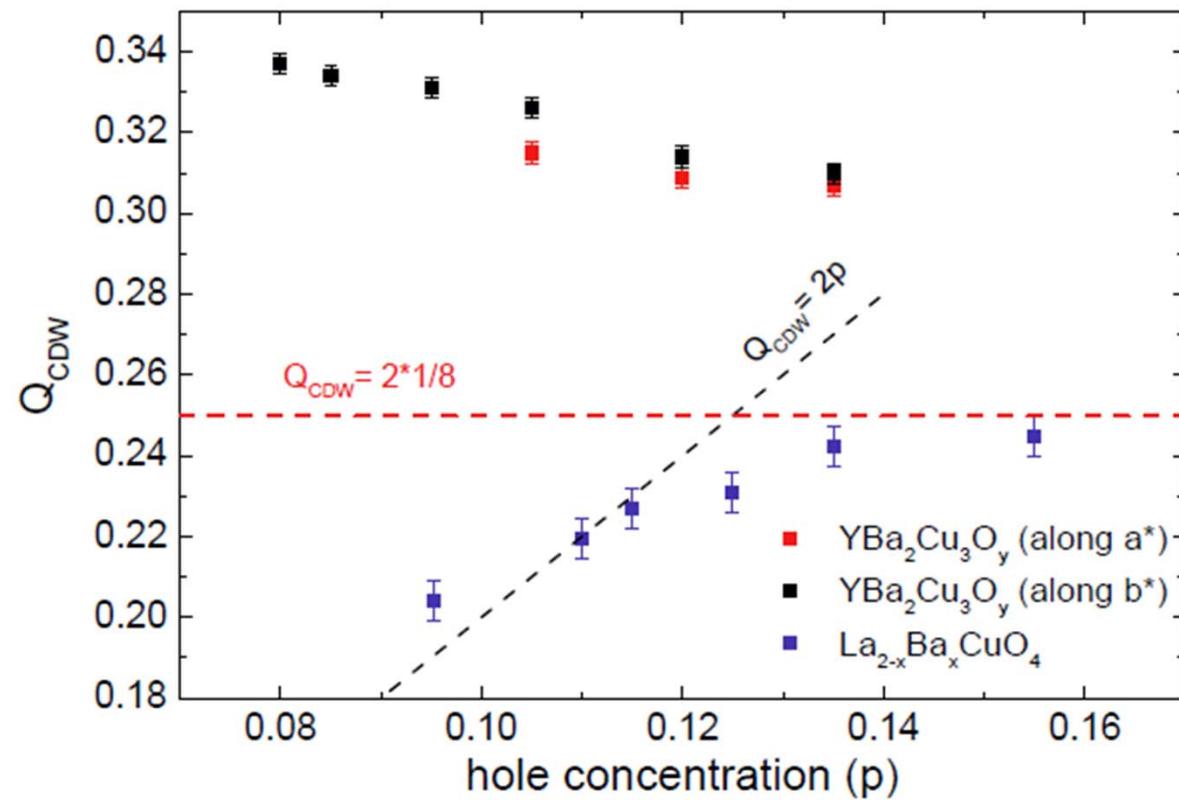
Figure 4 | Phase diagram of $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$. a, Doping dependence of the antiferromagnetic ordering temperature T_N , the incommensurate spin-density wave order T_{SDW} (green triangles; ref. 21), the superconducting temperature T_c and the pseudogap temperature T^* as determined from the Nernst effect³⁰ (black squares) and neutron diffraction²⁹ (purple squares). Notice that the Nernst effect³⁰ indicates a broken rotational symmetry inside the pseudogap region, whereas a translational symmetry preserving magnetic order is found by neutron scattering²⁹. Below temperature scale T_H (black circles), a larger and negative Hall coefficient was observed²⁶ and interpreted in terms of a Fermi surface reconstruction. Our X-ray diffraction experiments show that in $\text{YBCO } p = 0.12$ incommensurate CDW order spontaneously breaks the crystal translational symmetry at a temperature T_{CDW} that is twice as large as T_c . T_{CDW} is also much larger than T_{NMR} (red squares), the temperature scale below which NMR observes field-induced charge order¹³. b, Field dependence of T_{CDW} (filled red circles) and T_{cusp} (open squares), the temperature below which the CDW is suppressed by superconductivity, compared with T_H (open black circle) and T_{VL} (filled blue circles), the temperature where the vortex liquid state forms²⁶. Error bars on T_{SDW} , T_H , T_{NMR} , and T^* are explained in refs 21,26,30,33. The error bars on T_{CDW} and T_{cusp} reflect the uncertainty in determining the onset and suppression temperature of CDW order from Fig. 2. KE

Fermi surface vs wave vector of instability



Comin et al. arXiv:1312.1343
Da Silva Neto, arXiv:1312.1347 (also in BSCCO)

Wave vector



Keimer, Julich summer school 2013

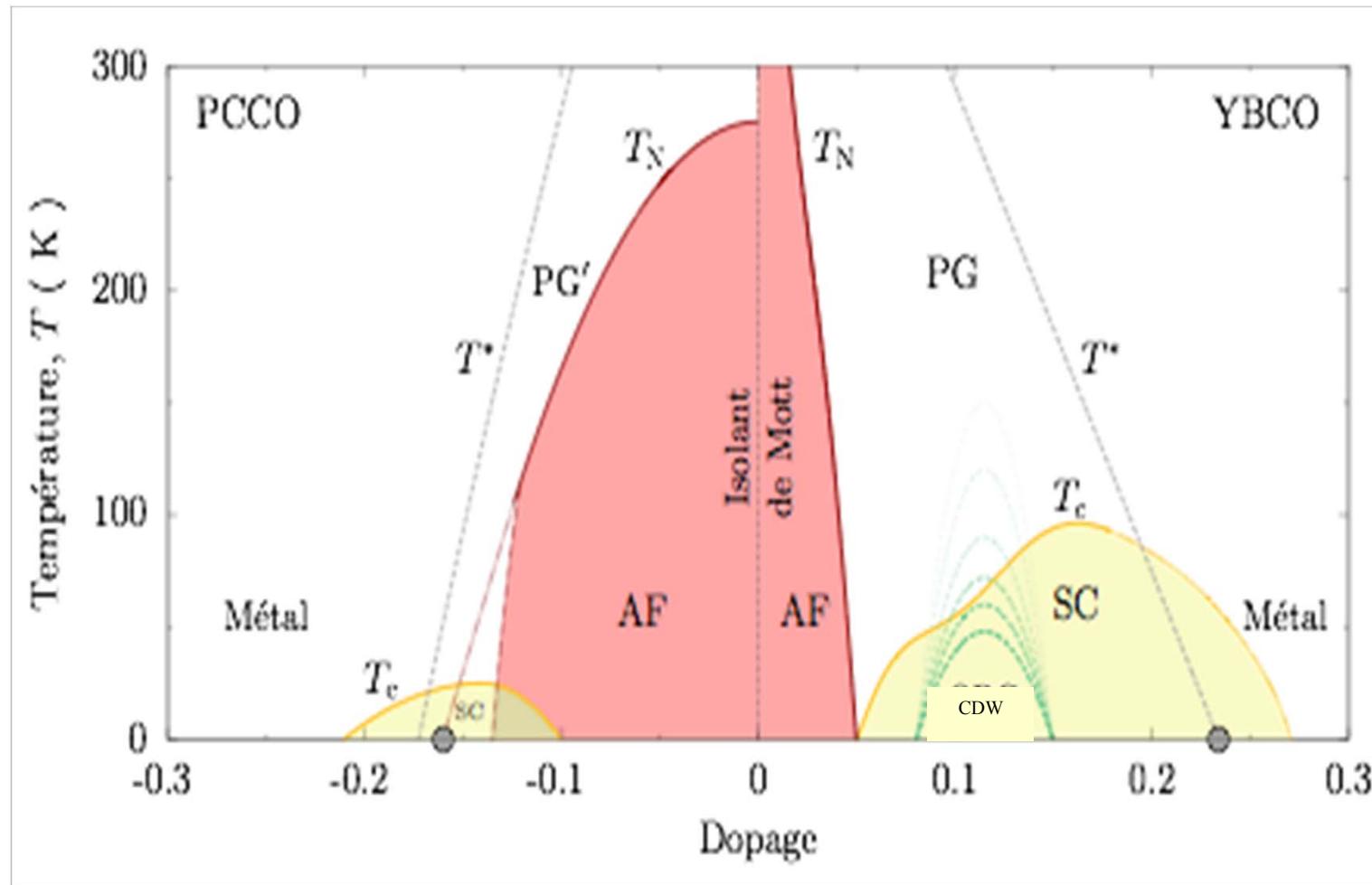
7. Pseudogap

h-doped



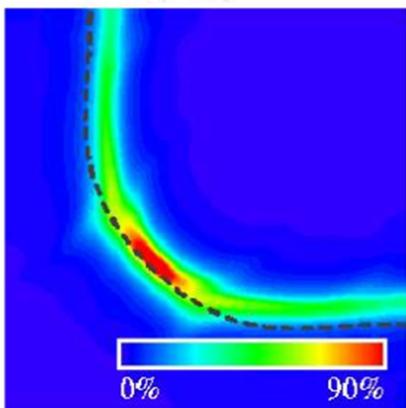
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Our road map



Hole-doped 17%, U=8t (CPT)

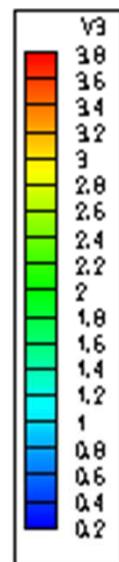
$U=8$



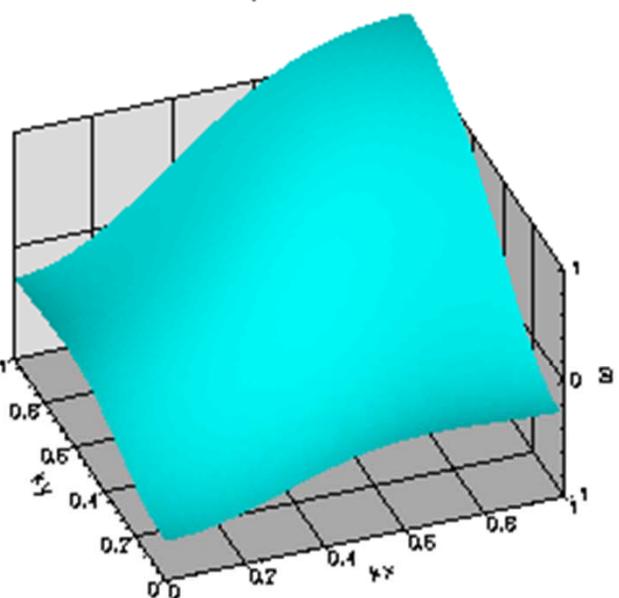
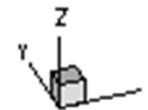
Sénéchal, AMT,
PRL (2004)

Frequency $\omega=-1$

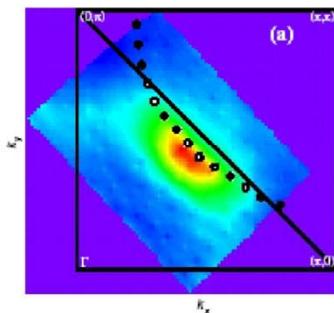
$U=2, n=10, t=0.25$
 $t_d=-0.075, t_2=0.05$



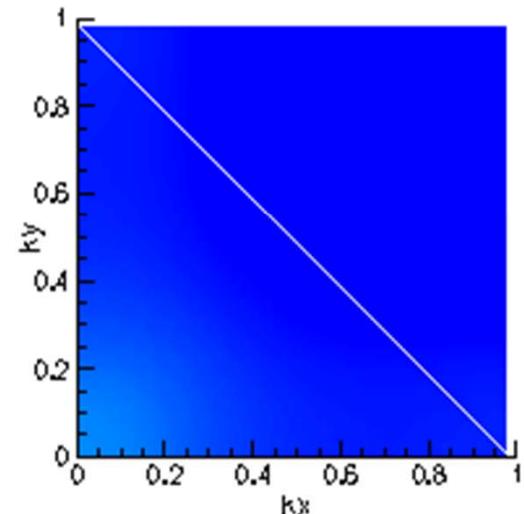
Dispersion relation
 $U=0, n=10, t=0.25$
 $t_d=-0.075, t_2=0.05$



Hole-doped, 10%

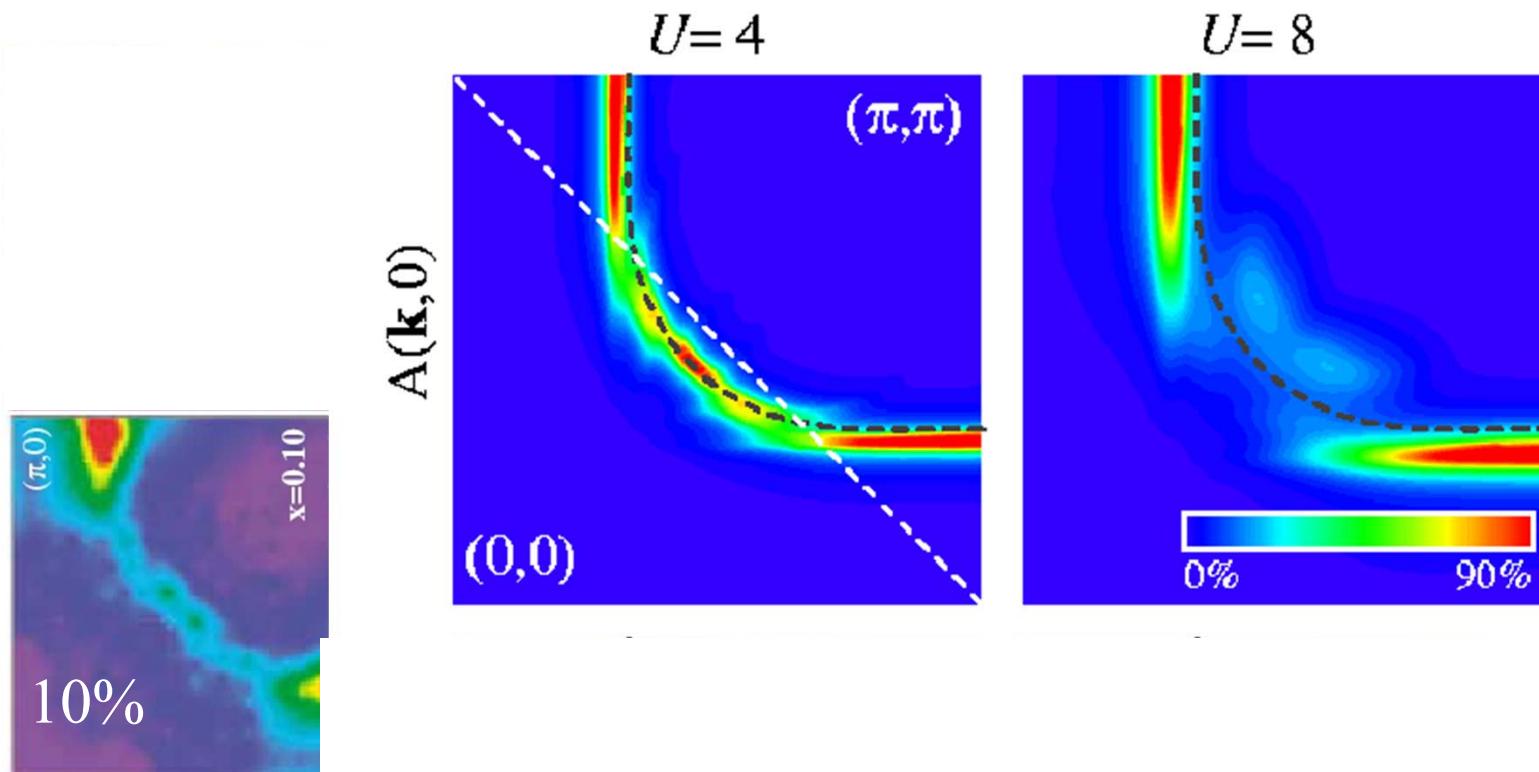


F. Ronning et al. Jan. 2002, $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$



Ronning *et*
al. (PRB
2003)

Electron-doped (17%) (CPT)



$$t' = -0.3t$$
$$t'' = 0.2t$$

$$\eta = 0.12t$$
$$\eta = 0.4t$$

Armitage *et al.*
PRL 2003

Sénéchal, AMT,
PRL (2004)

How does the pseudogap develops as a function of T ?



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NMR Knight shift?

H. Alloul arXiv:1302.3473

Noninteracting case:

$$\chi_{nn}^{0R}(\mathbf{q}, \omega) = -2 \int \frac{d^3 k}{(2\pi)^3} \frac{f(\zeta_k) - f(\zeta_{k+q})}{\omega + i\eta + \zeta_k - \zeta_{k+q}}$$

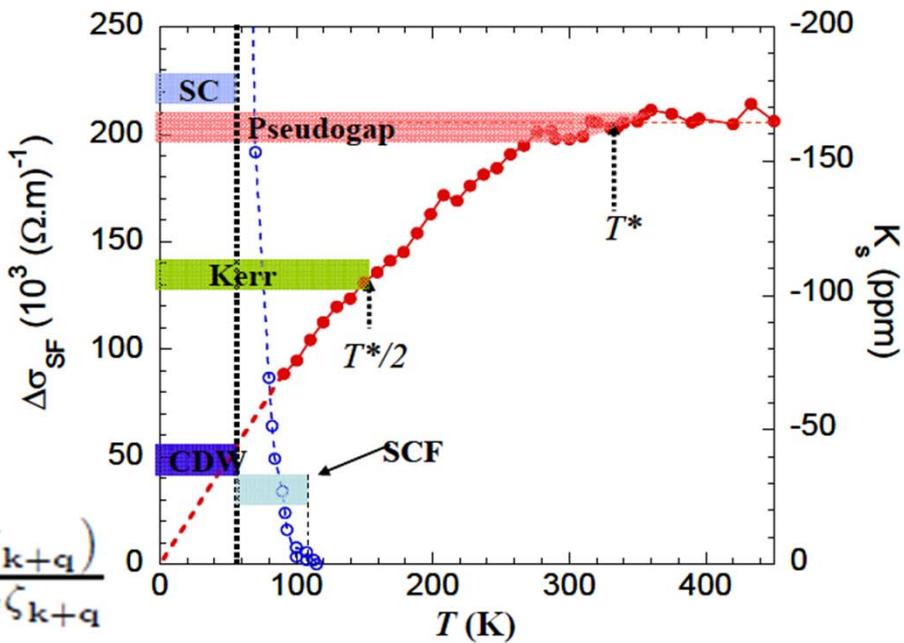
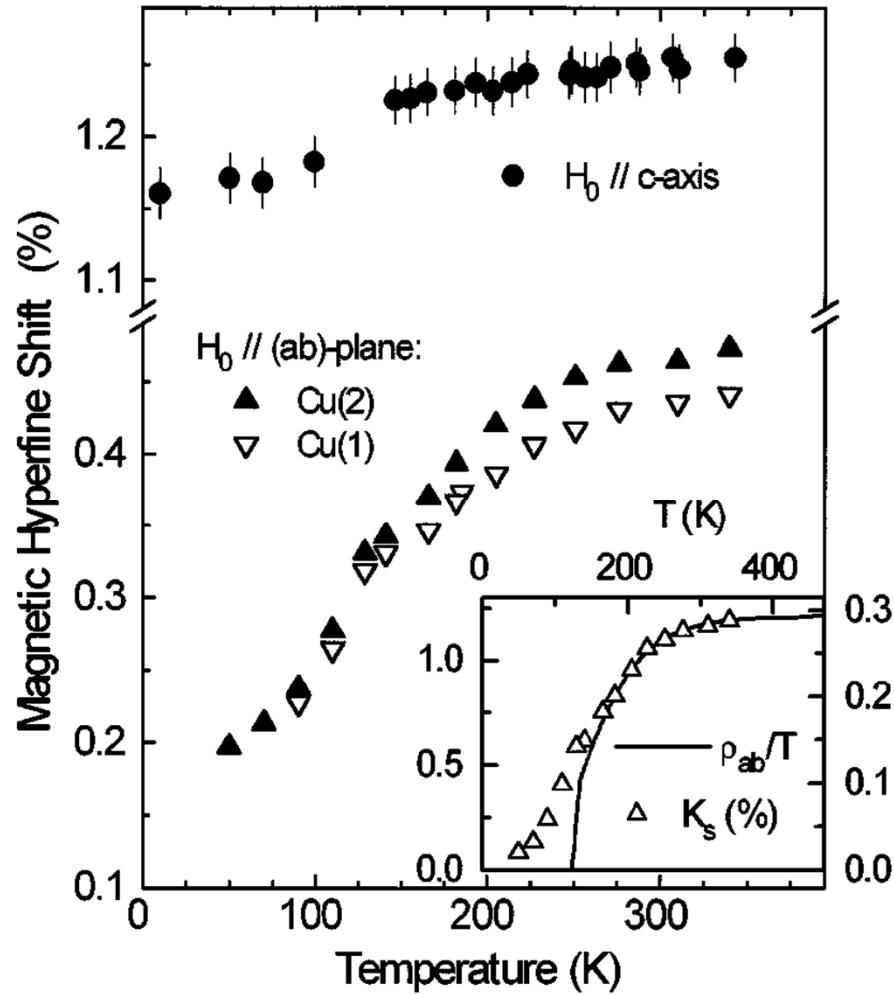


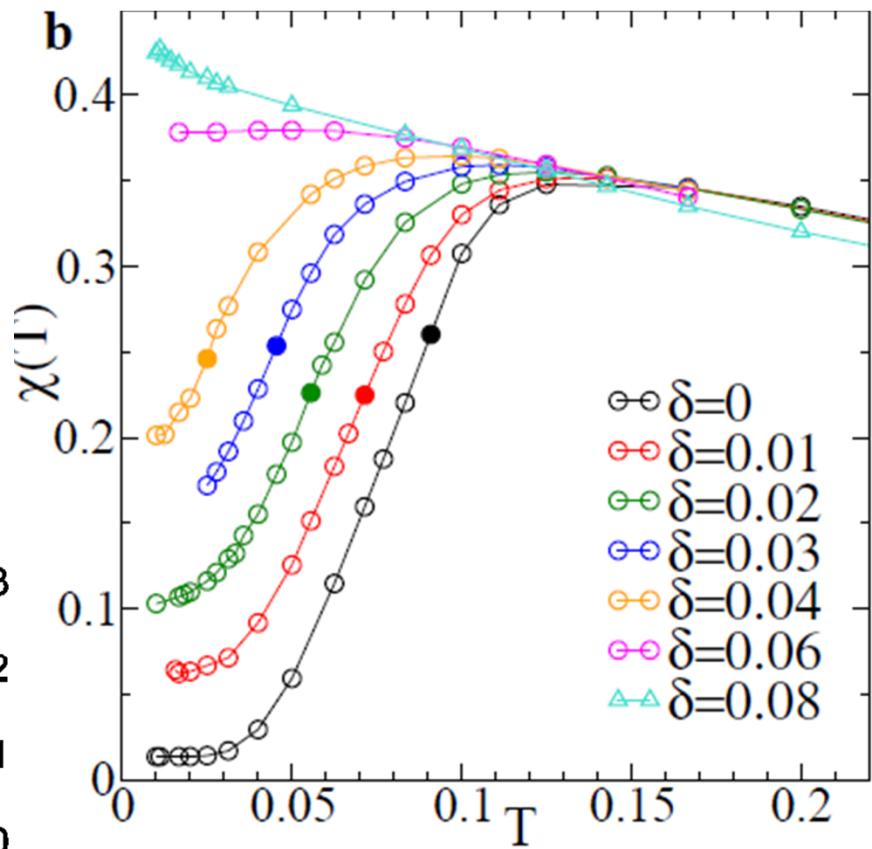
Fig 1 Spin contribution K_s to the ^{89}Y NMR Knight shift [11] for YBCO_{6.6} permit to define the PG onset T^* . Here K_s is reduced by a factor two at $T \sim T^*/2$. The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).

Spin susceptibility



Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)

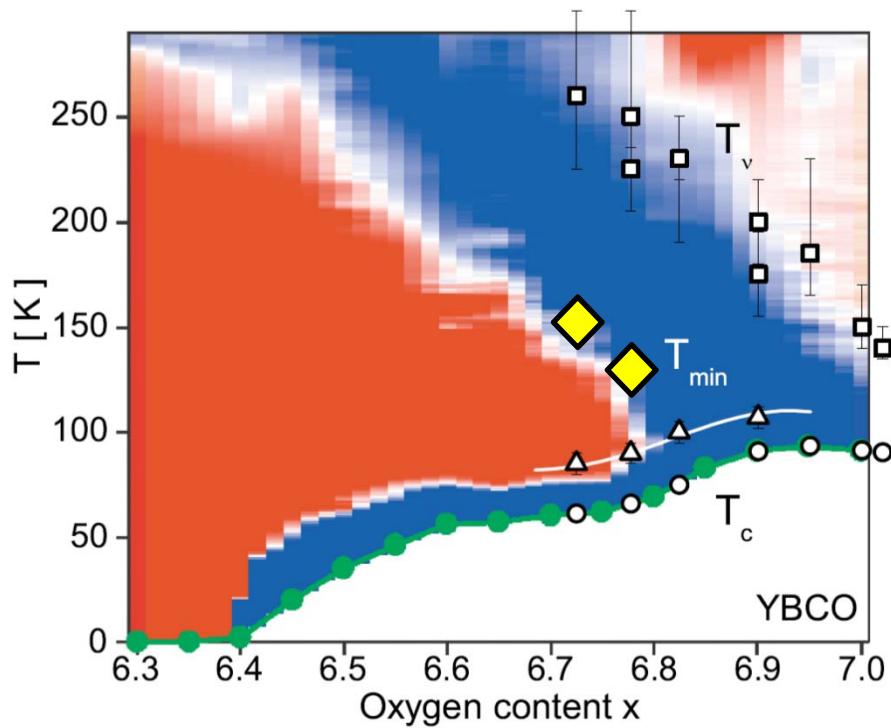


Sordi et al. Scientific Repts. 2012



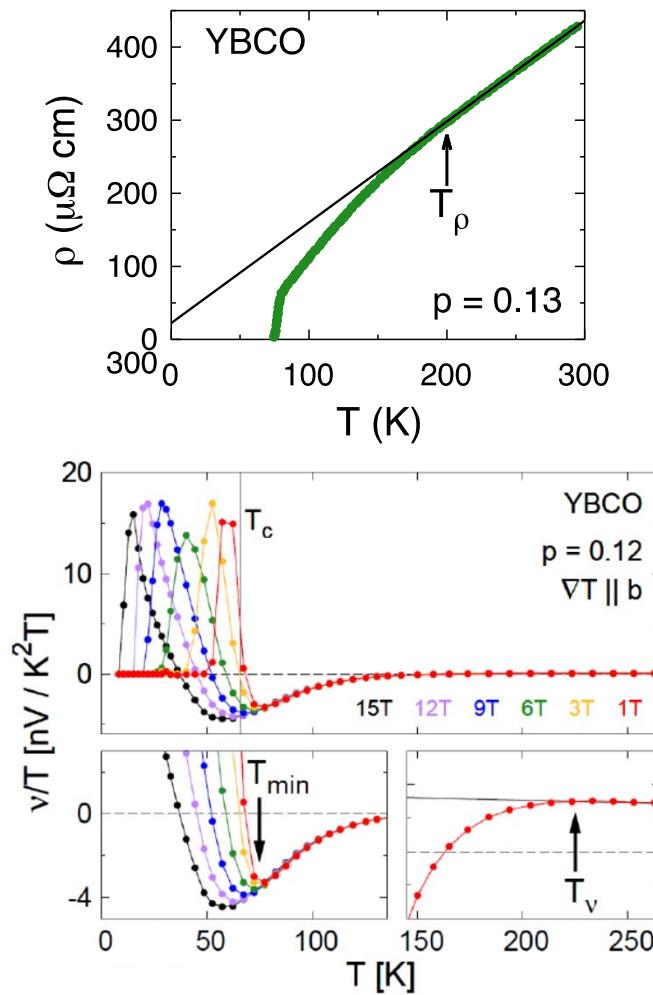
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Pseudogap from transport



CDW onset :

Color map : Ando *et al.*, PRL 2004



Daou *et al.*, Nature 463, 519 (2010)

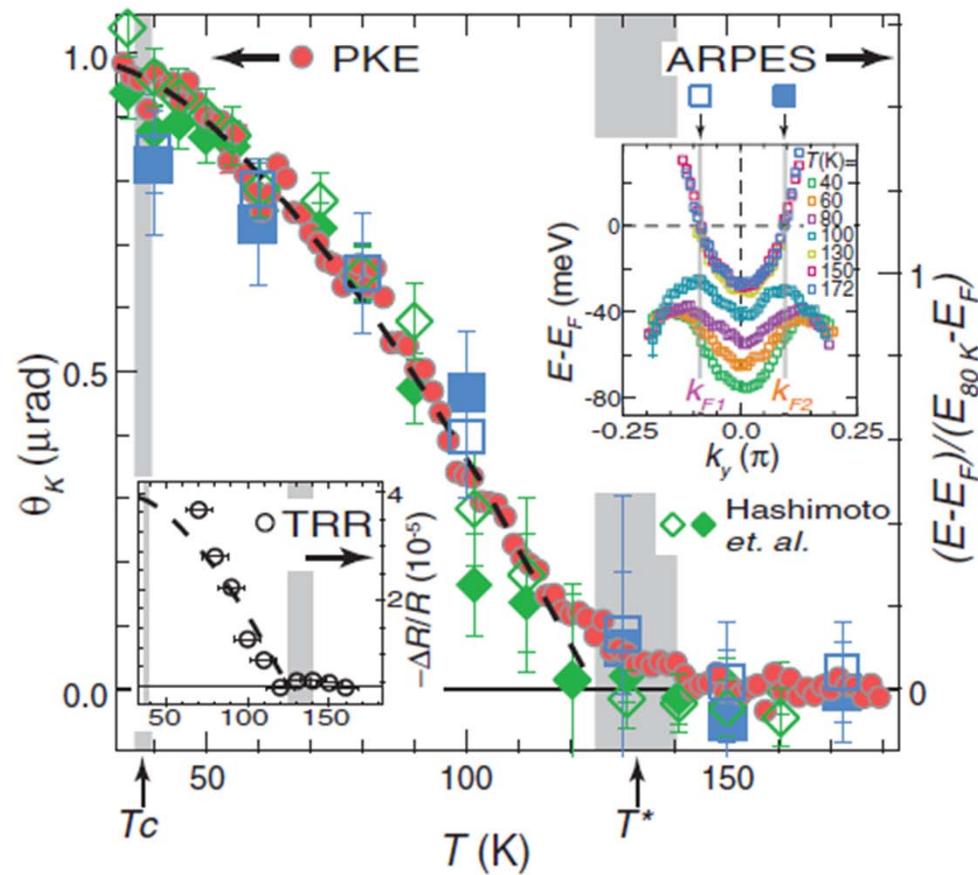


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3 measurements: Kerr, ARPES, TRR

Fig. 3. Temperature dependence of Kerr rotation (θ_K) measured by PKE, in comparison with that of the binding energy position of the EDC maximum at k_F given by ARPES [reproduced from fig. S1F and (29)]. ARPES results are normalized to the 80 K values (free from the interference of fluctuating superconductivity). The dashed black curve is a guide to the eye for the PKE data, showing a mean-field-like critical behavior close to T^* [see additional discussion in (27)]. **(Left inset)** Temperature dependence of the transient reflectivity change measured by TRR (right axis). The dashed

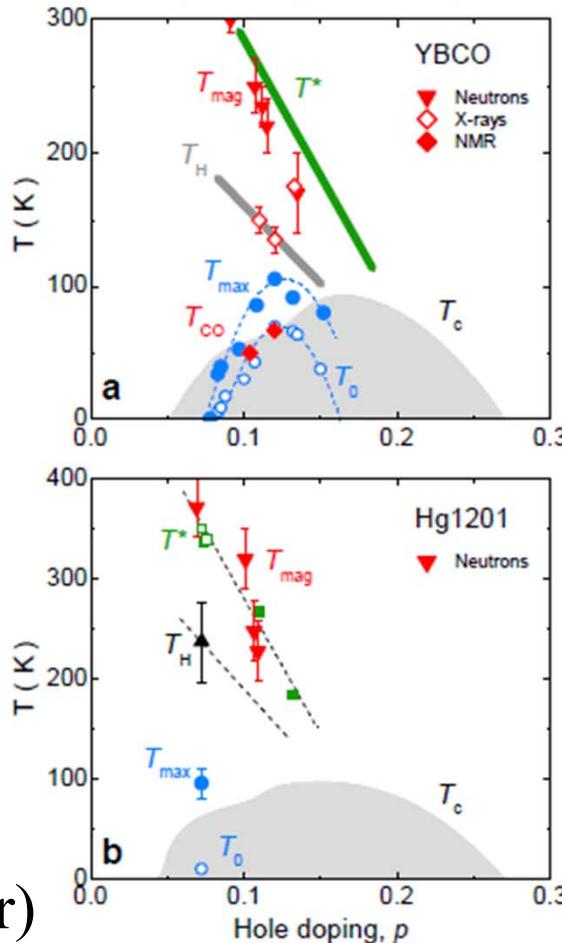
black curve (left axis) is reproduced from the main panel. Error bars (if not visible) are smaller than the symbol size. **(Right inset)** Dispersion of the EDC maximum at various temperatures above T_c , summarizing the results of Figs. 2A and 4A and fig. S1, A to E. All data were taken on samples from the same growth and annealing batch, except those reproduced from (29) on differently annealed samples.



From transport measurements same signatures in single layer material HgBaCuO

Hall and Nernst Coefficients of Underdoped $\text{HgBa}_2\text{CuO}_{4+\delta}$:
Fermi-Surface Reconstruction in an Archetypal Cuprate Superconductor

Nicolas Doiron-Leyraud,^{1,*} S. Lepault,² O. Cyr-Choinière,¹ B. Vignolle,² F. Laliberté,¹ J. Chang,¹ N. Barišić,³ M. K. Chan,³ L. Ji,³ X. Zhao,^{3,4} Y. Li,⁵ M. Greven,³ C. Proust,^{2,6} and Louis Taillefer^{1,6,†}



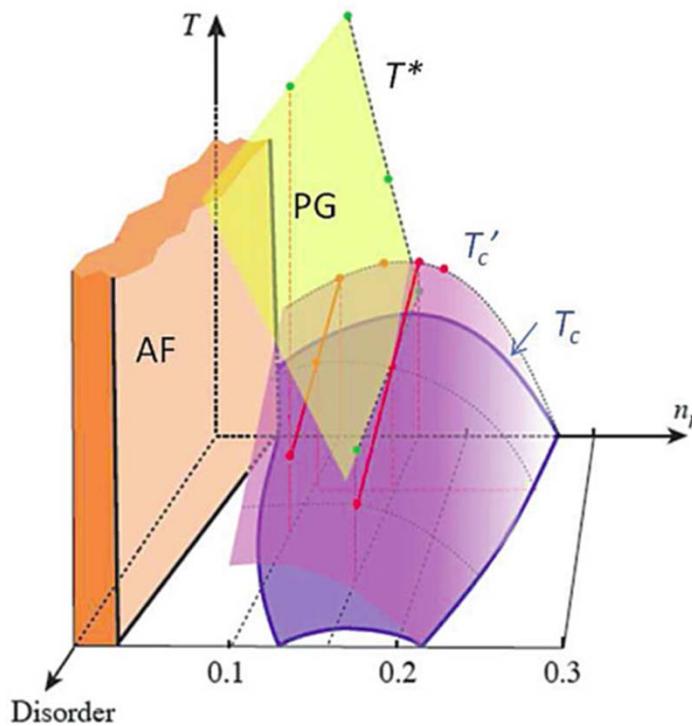
31 oct 2012

Also, Nernst very
anisotropic at T^* (Taillefer)



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Pseudogap not from SC fluctuations Effect of disorder



F. Rullier-Albenque, H. Alloul, and G.Rikken,
Phys. Rev. B **84**, 014522 (2011).

Pseudogap not from SC fluctuations

Torque magnetometry,
Hg1201
Bi2201
LSCO

Yu et al. cond-mat/1210.6942

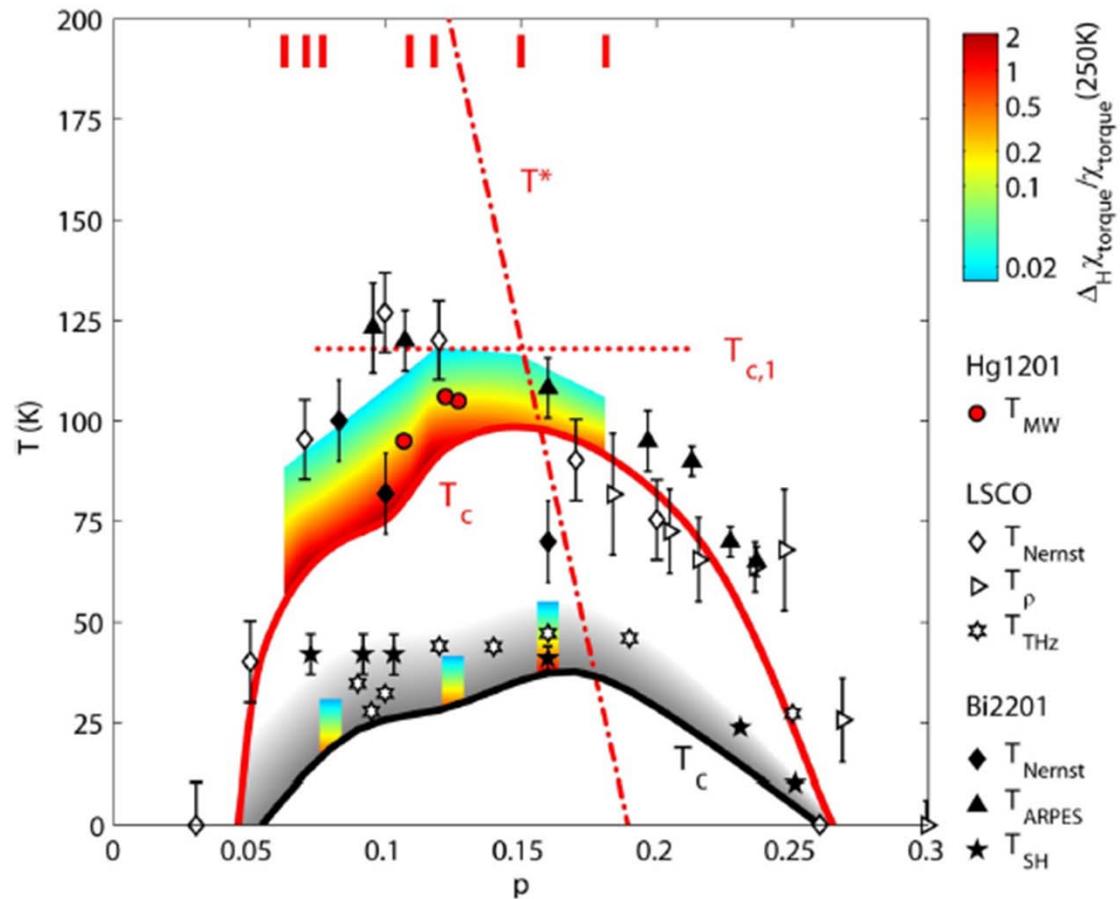
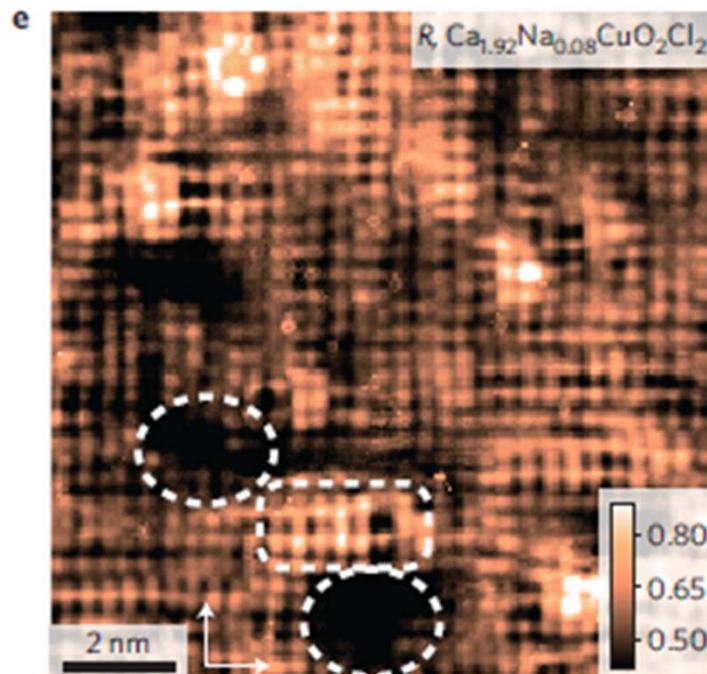


Figure 4 | Phase diagram. The color contour shows $\log_{10}(\Delta_H \chi_{\text{torque}} / \chi_{\text{torque}}^{250})$ for Hg1201 (same data as in Fig. 3d, but for wider temperature range), obtained from an interpolation of measurements of seven samples (indicated by vertical red bars), and for optimally-doped Bi2201 and underdoped LSCO ($p = x = 0.08$ and 0.125). The grey shaded area indicates schematically the extent of SC fluctuations in LSCO and

Intra-unit cell nematic order: STM

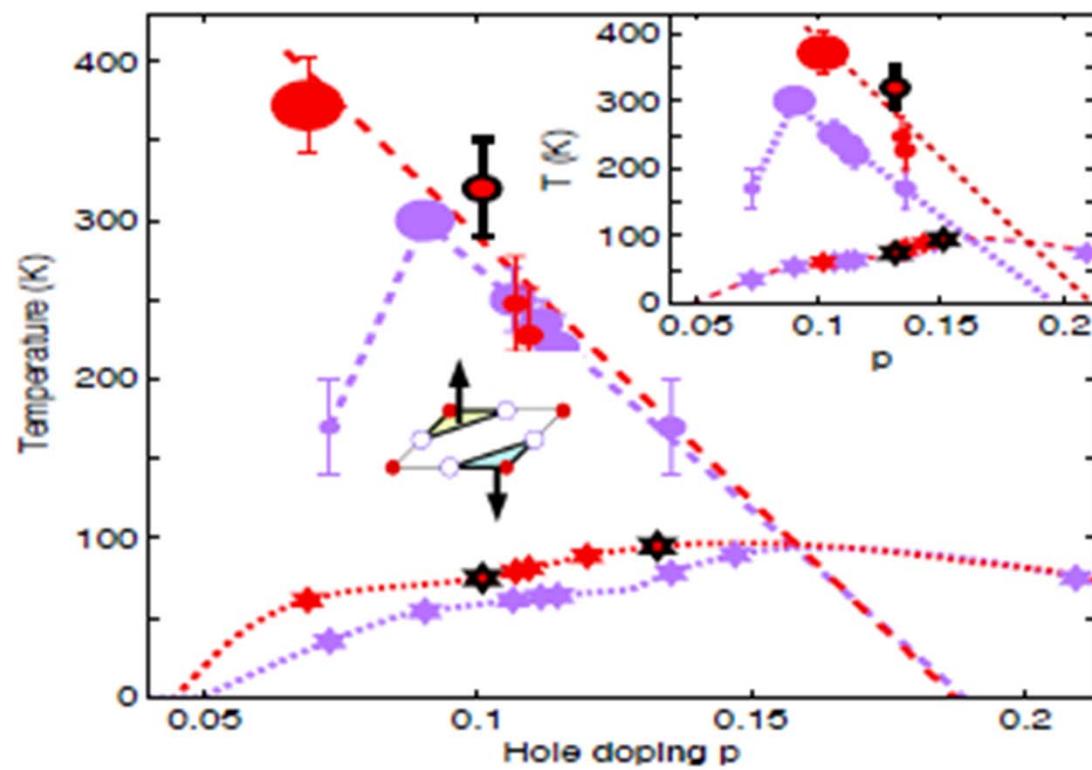


Kohsaka et al. Nature Physics 2012

Loop current order (Varma)

YBaCuO : Bourges et al.

HgCaCuO: Greven et al.



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8. Superconductivity in general

Analog to weakly and strongly
correlated antiferromagnets



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Superconducting phase: identical properties

- Emergent:
 - Same broken symmetry $U(1)$ for s-wave,
 - $U(1)$ and C_{4v} for d-wave
 - Single-Particle gap, point or line node.
 - T dependence of C_p and κ at low T
 - Goldstone modes (+Higgs)



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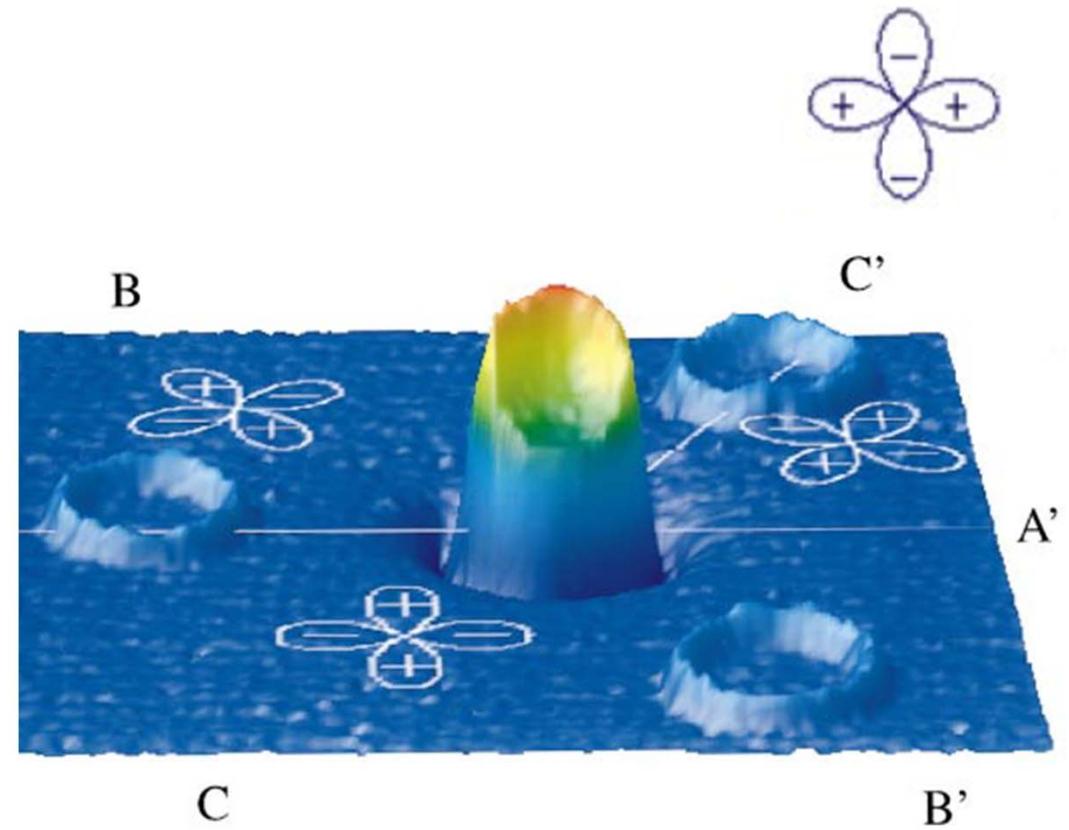
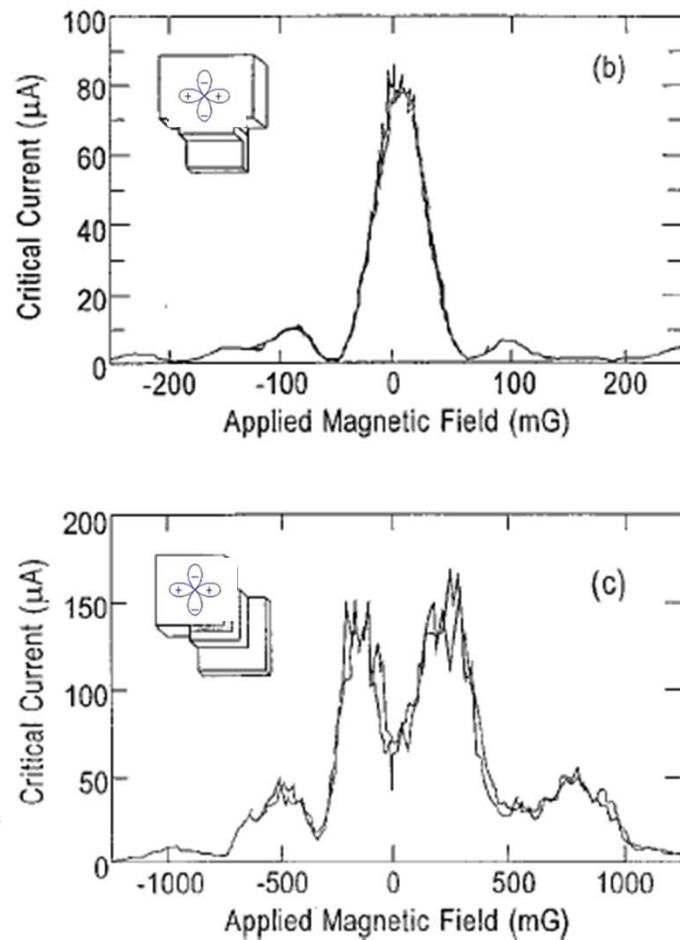
Superconductivity not universal even with phonons: weak or strong coupling

- In BCS universal ratios: e.g. $\Delta/k_B T_c$
 - Would never know the mechanism for sure if only BCS!
 - N.B. Strong coupling in a different sense



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High T_c are d-wave (interference)



Tsuei Kirtley, Rev. Mod. Phys. 2000

Wollman et al. PRL 1993

Two emergent universal laws ?

Basov

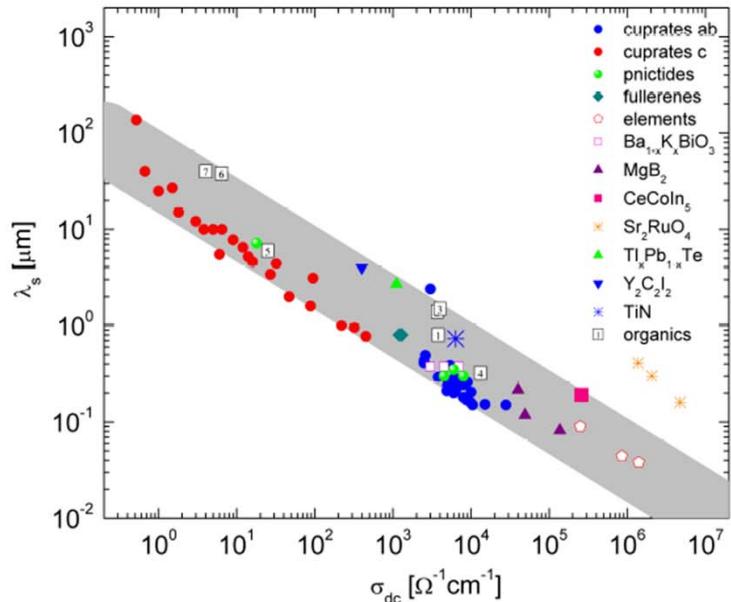


FIG. 1. **Basov scaling plot**, Eq. (1). The gray stripe corresponds to $\lambda_s = (45 \pm 25) \sigma_{dc}^{-0.5}$. Only data from optical spectroscopies (IR and MW SI) are included in the plot. The data points are from: cuprates *ab*-cuprates *c*-axis [2], pnictides [13, 14], elements [2], TiN [15], $\text{Ba}_{1-x}\text{K}_x\text{BiO}_3$ [16], MgB_2 [17, 18], organic SC [19–21], fullerenes [22], heavy fermion CeCoIn_5 [23], negative-U induced SC $\text{Tl}_x\text{Pb}_{1-x}\text{Te}$ [24] and $\text{Y}_2\text{C}_2\text{I}_2$ [25].

Homes

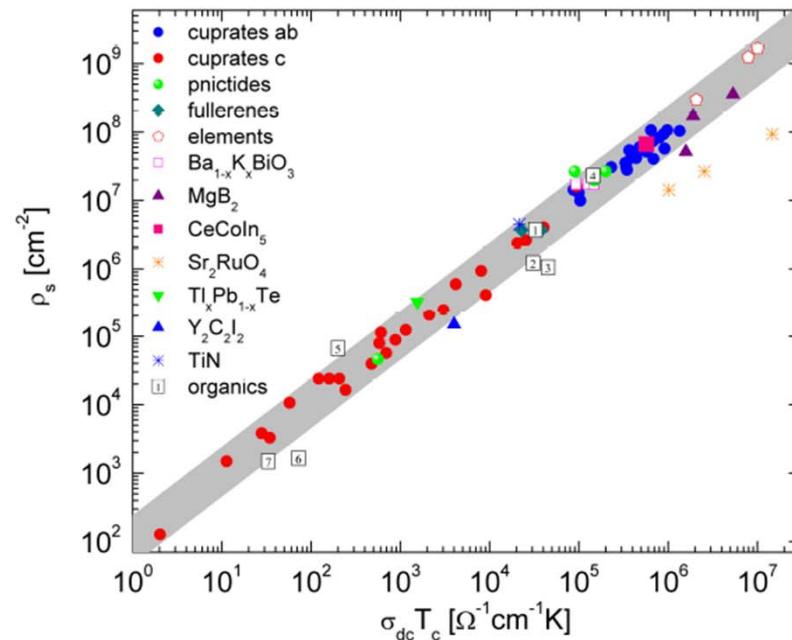
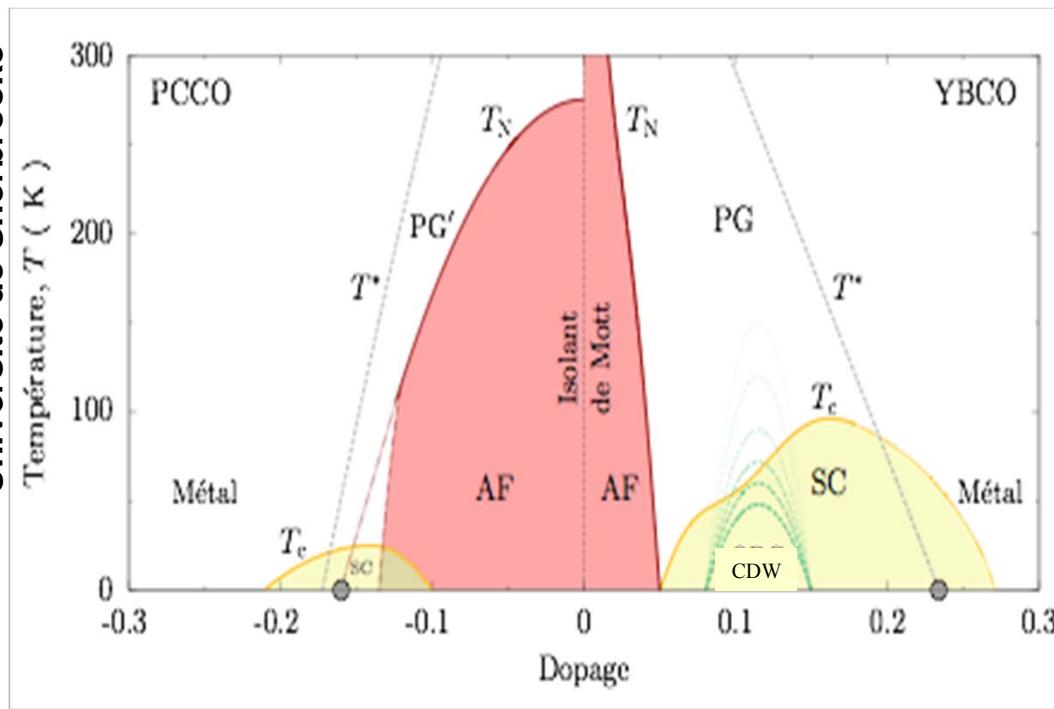


FIG. 2. **Homes' scaling plot**, Eq. (2). The gray stripe corresponds to $\rho_s = (110 \pm 60) T_c \sigma_{dc}$. The data points are the same as in Fig. 1.

Dordevich et al. arXiv:1305.0019v1

High-temperature superconductors



- Competing order
 - Current loops: Varma, PRB **81**, 064515 (2010)
 - Stripes or nematic: Kivelson et al. RMP **75** 1201(2003); J.C.Davis
 - d-density wave : Chakravarty, Nayak, Phys. Rev. B **63**, 094503 (2001); Affleck et al. flux phase
 - SDW: Sachdev PRB **80**, 155129 (2009) ...

What is under the dome?
Mott Physics away from $n = 1$

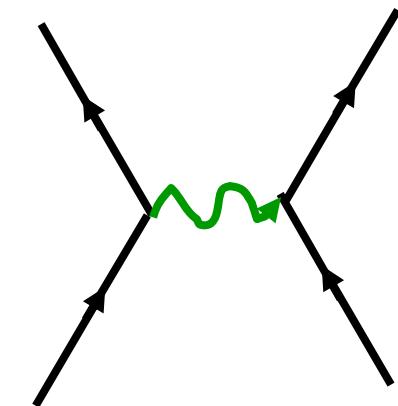
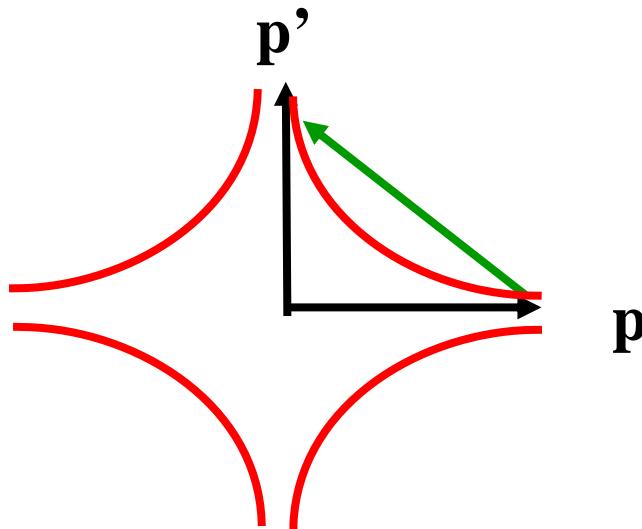
- Or Mott Physics?
 - RVB: P.A. Lee Rep. Prog. Phys. **71**, 012501 (2008)

8. Superconductivity

a. Weakly correlated case
(e-doped ?)

Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?
Kohn-Luttinger
D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch

P.R. B **34**, 8190-8192 (1986).

T_c with pressure
Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)

Weak coupling methods

- Functional renormalization group

$$(a) \partial_\ell \text{ (loop diagram)} = \text{ (loop diagram with one dashed line)} + \text{ (loop diagram with one dashed line)}$$

$$+ \text{ (loop diagram with one dashed line)} + \dots$$

$$\partial_\ell \text{ (loop diagram)} = \text{ (loop diagram with one dashed line)} + \dots$$

$$(b) \partial_\ell \rightarrow (\Sigma_+) = \text{ (dashed line with cross)} + \text{ (dashed line with cross)} + \dots$$

Zanchi, Schultz 2000

Honerkamp, Salmhofer 2000 +
Bourbonnais Sedeki PRB 2012

- Weak coupling perturbation theory

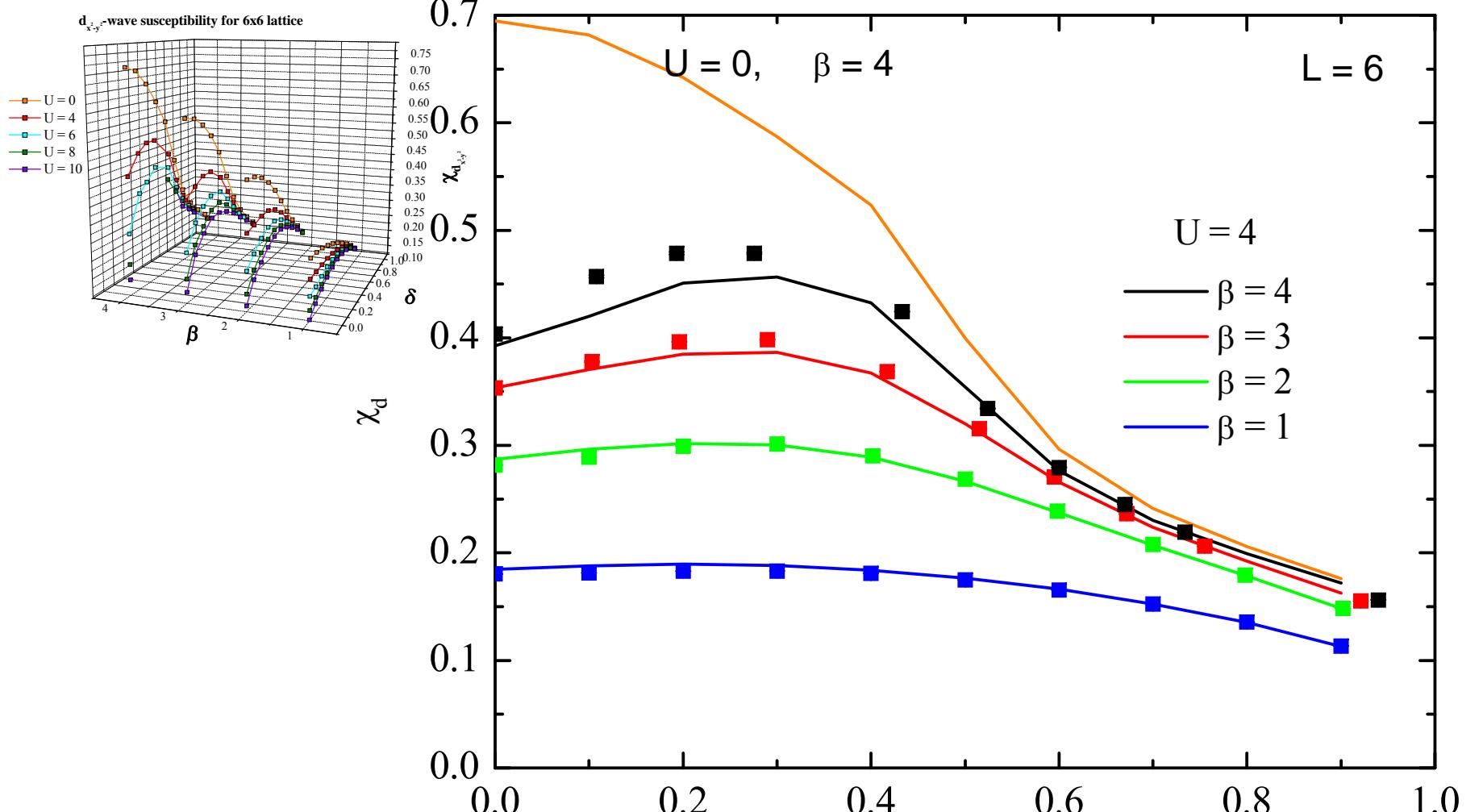
S. Maiti and A.V. Chubukov: arXiv:1305.4609

Results from TPSC

Satisfies Mermin-Wagner



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QMC: symbols.

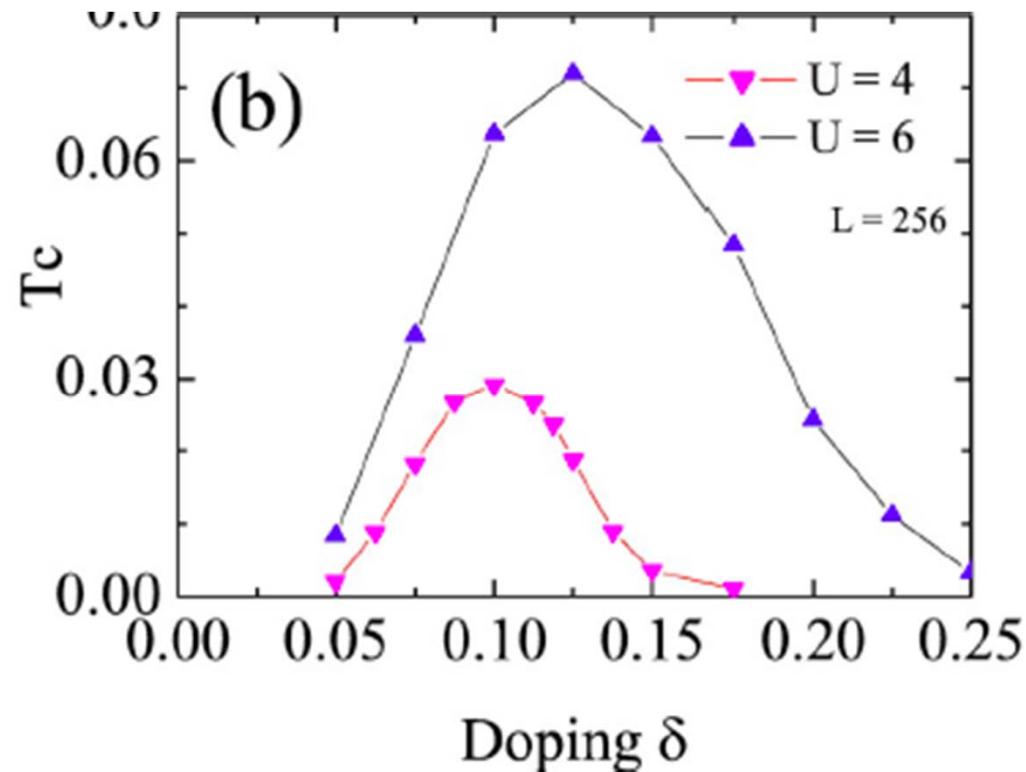
Solid lines analytical. Kyung, Landry, A.-M.S.T., PRB 2003

Doping



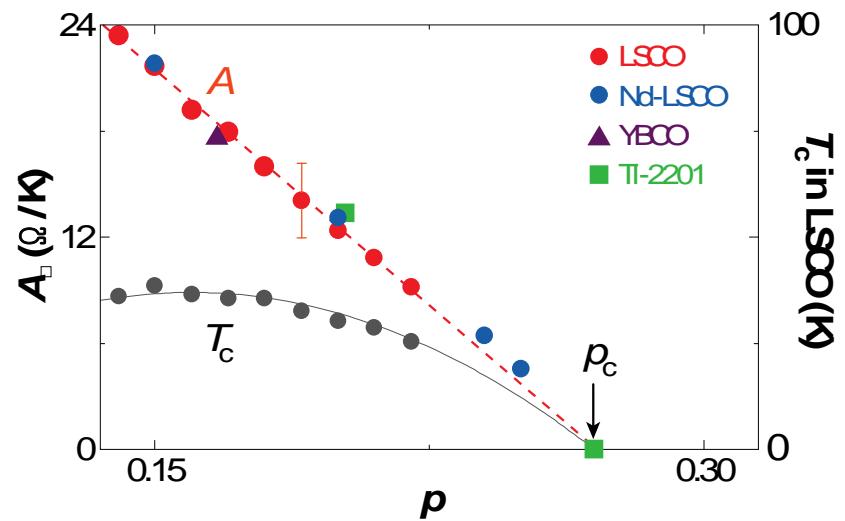
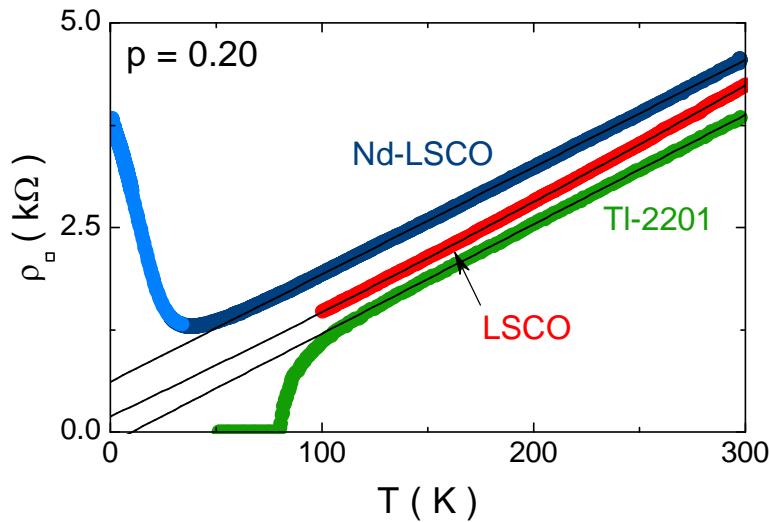
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T_c from TPSC



Kyung et al. PRB **68** (2003)

Correlation between T_c and coefficient of linear T dependence



Linear-T resistivity is universal in hole-doped cuprates

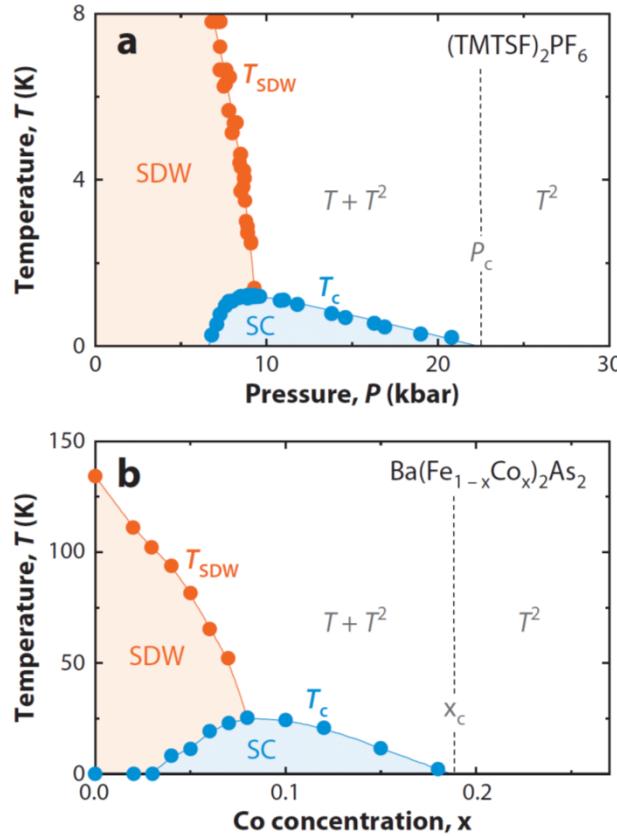
Correlation between linear-T resistivity and T_c

Doiron-Leyraud *et al.*, arXiv:0905.0964

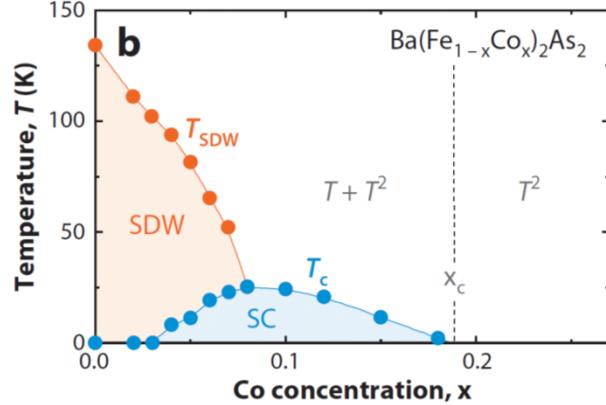
Taillefer, Annual Review of CMP 1, 51 (2010)

Linear T coefficient of R and T_c in other systems

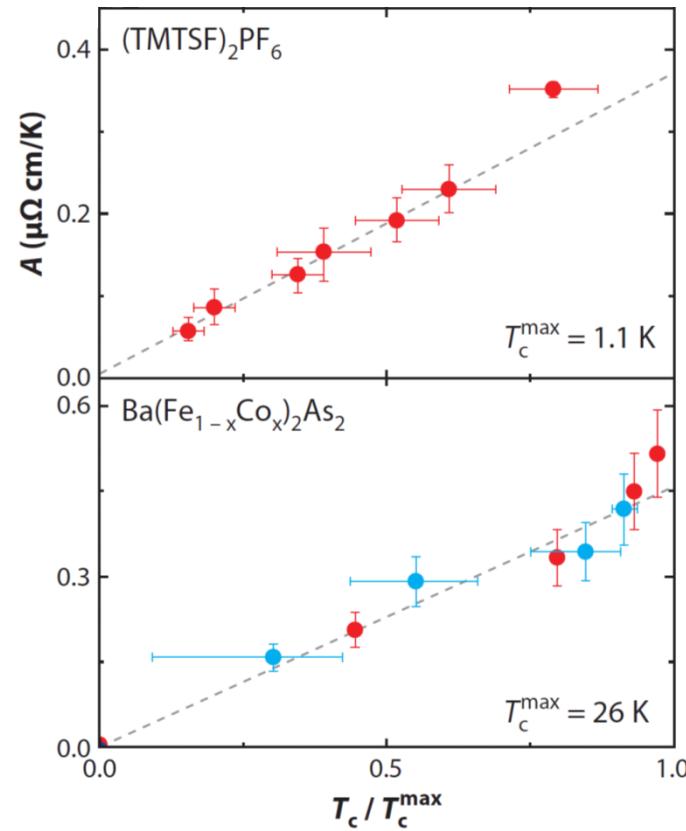
Organic



Pnictide



Bourbonnais, Sedeki, 2012

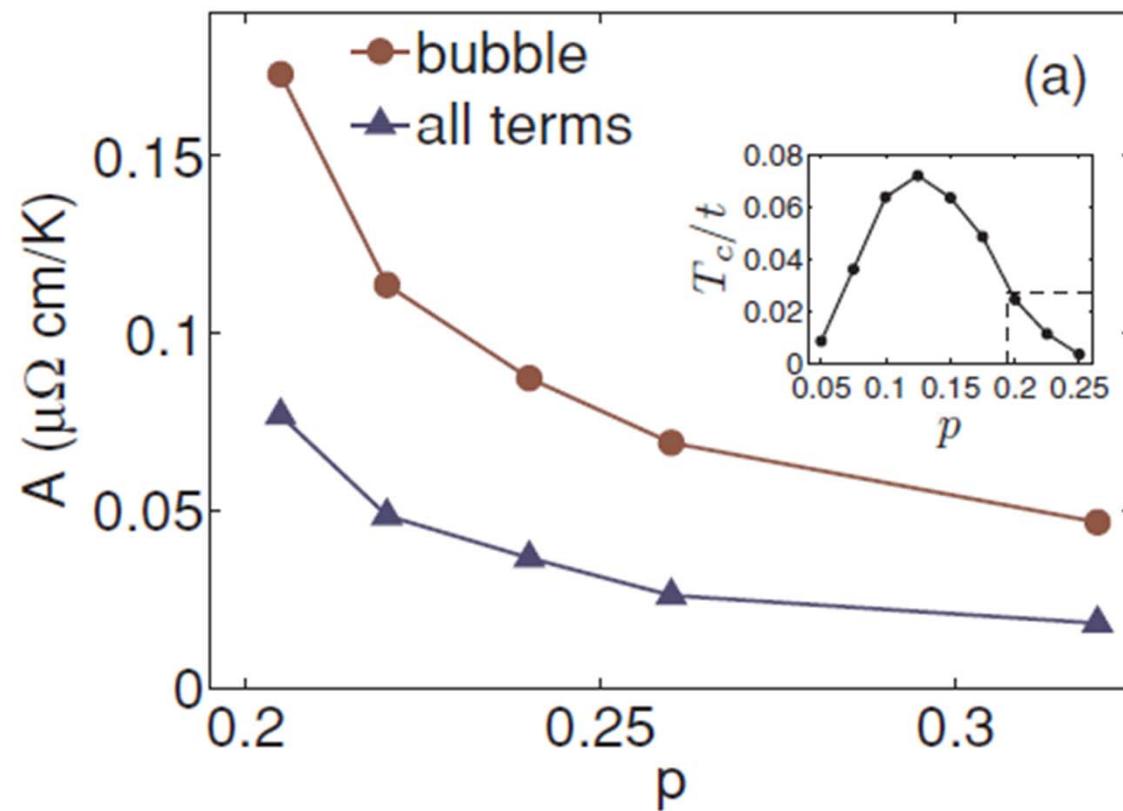


Doiron-Leyraud et al., PRB **80**, 214531 (2009)



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Correlation resistivity vs T_c



Dominic Bergeron et al. TPSC
PRB **84**, 085128 (2011)

8. Superconductivity

b. Strong coupling point of view



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A cartoon strong coupling picture

P.W. Anderson Science 317, 1705 (2007)

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow}^\dagger c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

Miyake, Schmitt–Rink, and Varma
P.R. B 34, 6554-6556 (1986)

Strongly correlated superconductors

- T_c does not scale like order parameter
- Superfluid stiffness scales like doping
- Superconductivity can be largest close to the metal-insulator transition
- Resilience to near-neighbor repulsion



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The view from dynamical mean-field
theory (I am not supposed to do this)

International summer school on
Computational Methods for Quantum
Materials

26 May - 6 June 2014



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