What is special about strongly correlated superconductivity

A.-M. Tremblay

Giovanni Sordi, Patrick Sémon, Kristjan Haule, David Sénéchal, Alexandre Day,

Vincent Bouliane, Simon Verret, Jyotirmoy Roy

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VERITATEM IN CHARITATE

ISS22, ENSE^3-2, Exotic Superconductivity, SCES2014 Grenoble, 9 July 2014

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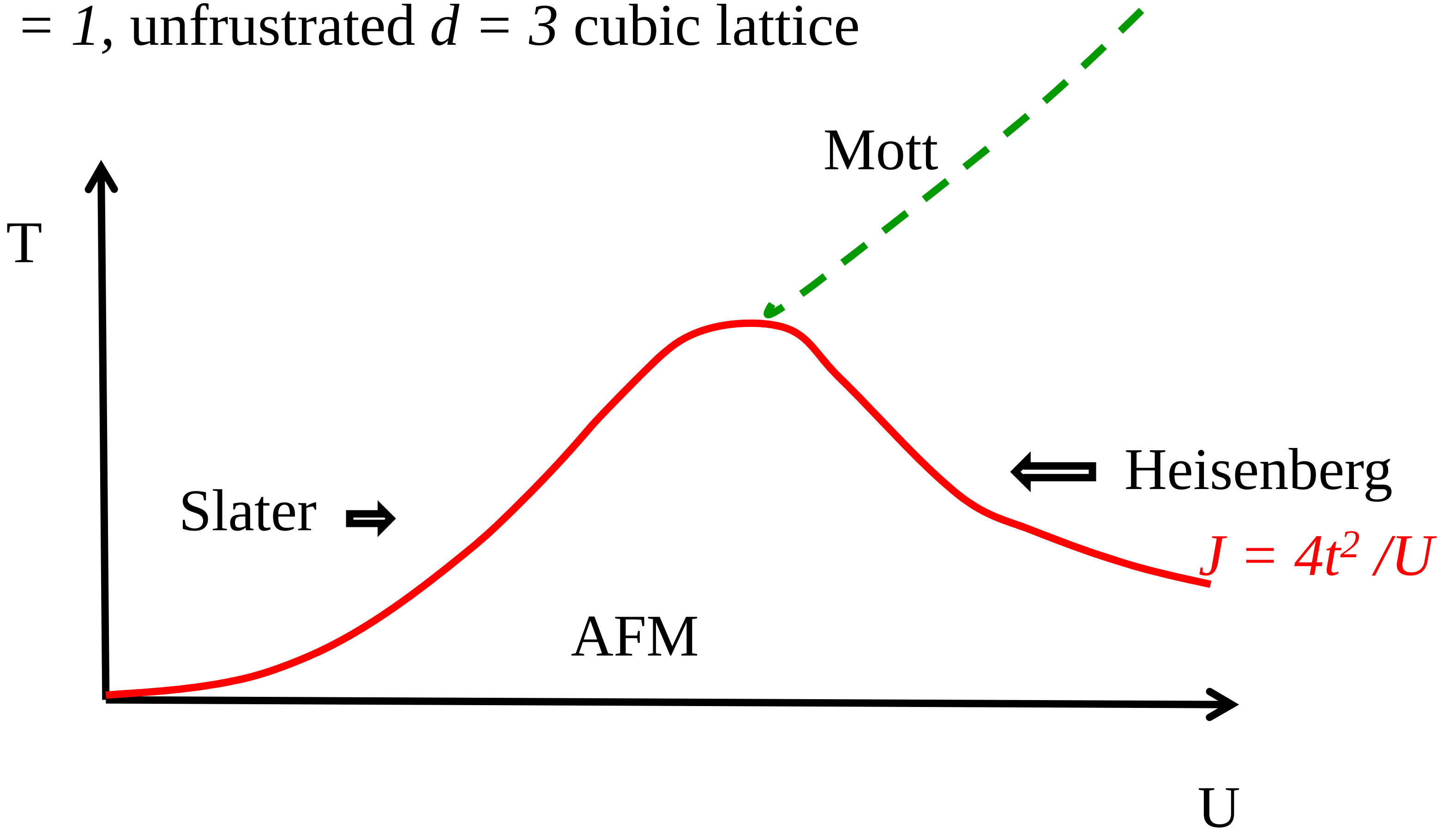
Weakly and strongly correlated antiferromagnets

What is a phase?

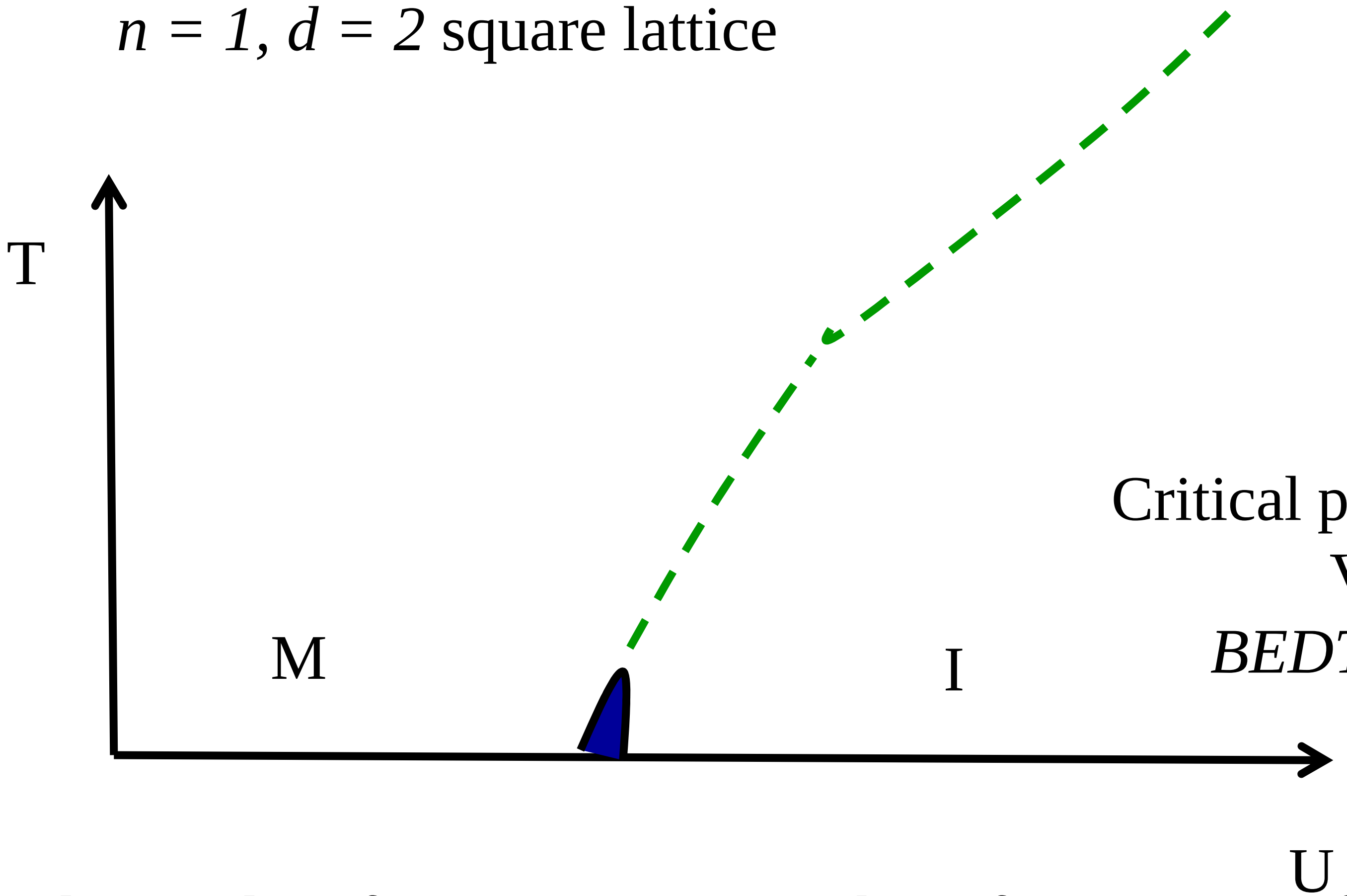
Weak vs Strong correlations

n = 1, unfrustrated *d = 3* cubic lattice



Local moment and Mott transition

n = 1, d = 2 square lattice



Critical point visible in V_2O_3, *BEDT* organics

Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

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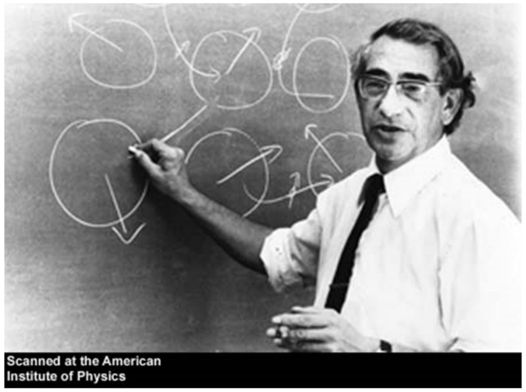
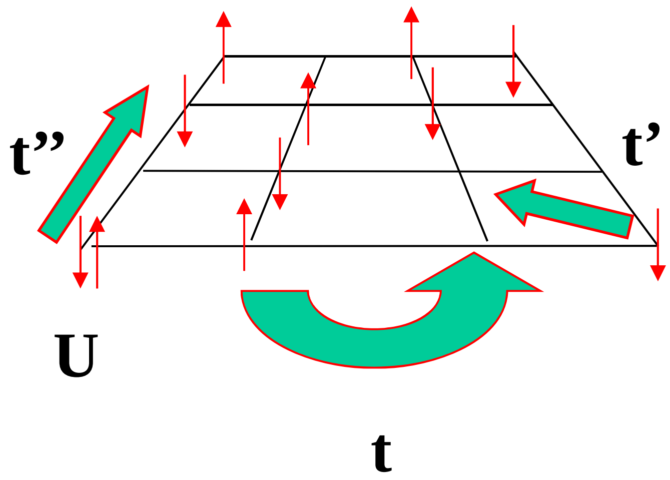
Hubbard model

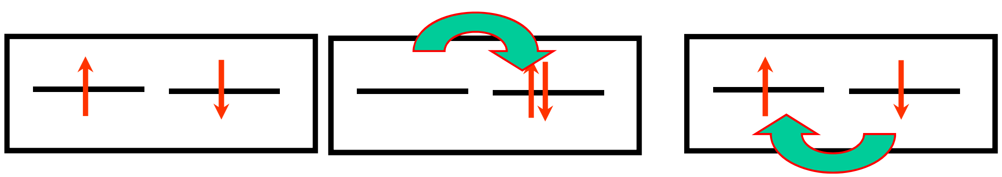


1931-1980

μ



$$H = -\sum_{<ij>\sigma} t_{i,j}\left(c^{\dagger}_{i\sigma}c_{j\sigma} + c^{\dagger}_{j\sigma}c_{i\sigma}\right) + U\sum_i n_{i\uparrow}n_{i\downarrow}$$



$t = 1$

Effective model, Heisenberg: $J = 4t^2/U$

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Overview

- **An apetizer:**
  - Two gaps
  - Superfluid density
- A cartoon picture of pairing at weak and strong correlations
- The view from dynamical mean-field theory
  - The glue
  - A paradox: The goo remover (V) ?

Two gaps in underdoped regime of cuprates

Le Tacon *et al.* Nature Physics 2, 537 - 543 (2006)

….

Sakai et al. PRL 111, 107001 (2013)

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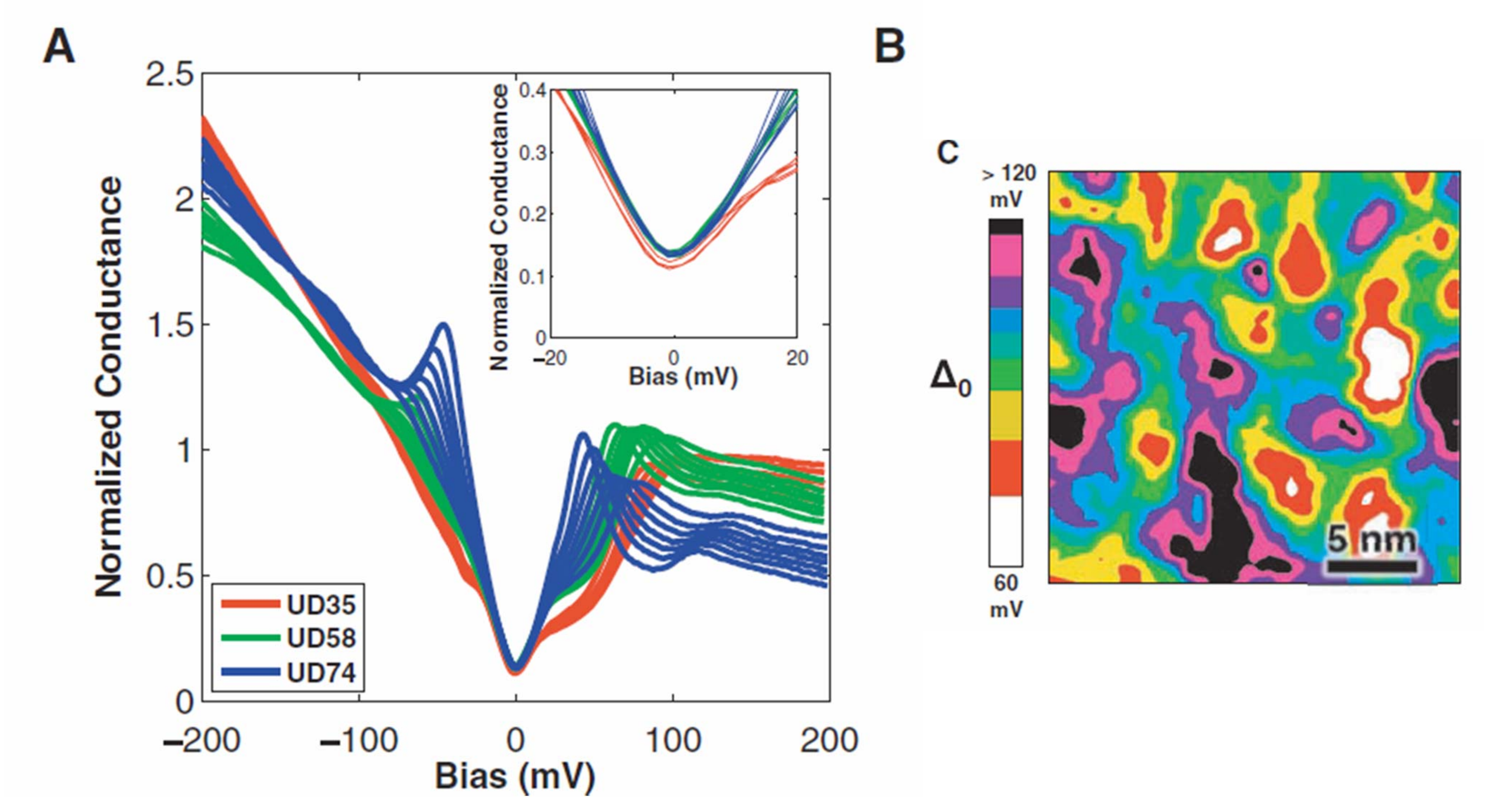
Superconducting gap in STM



A. Pushp, Parker, … A. Yazdani,
Science **364**, 1689 (2009)

Experiment vs Theory, STM

Simon Verret

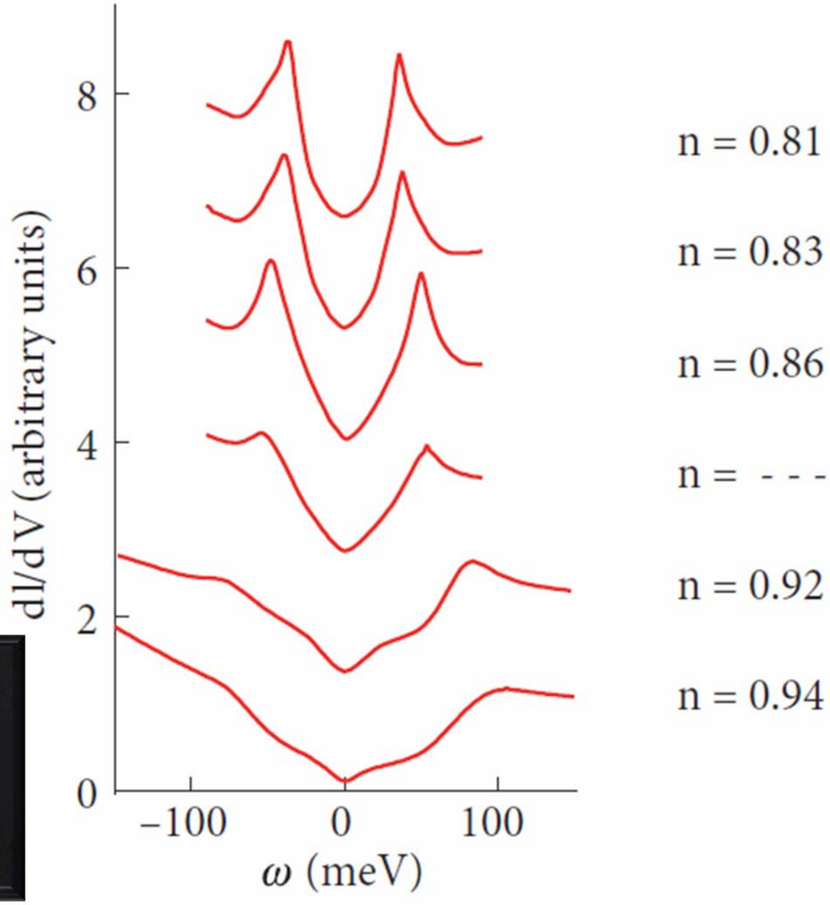
Jyotirmoy Roy

STM data

Kohsaka *et al.*, Nature **454** 1072 (2008)

CDMFT

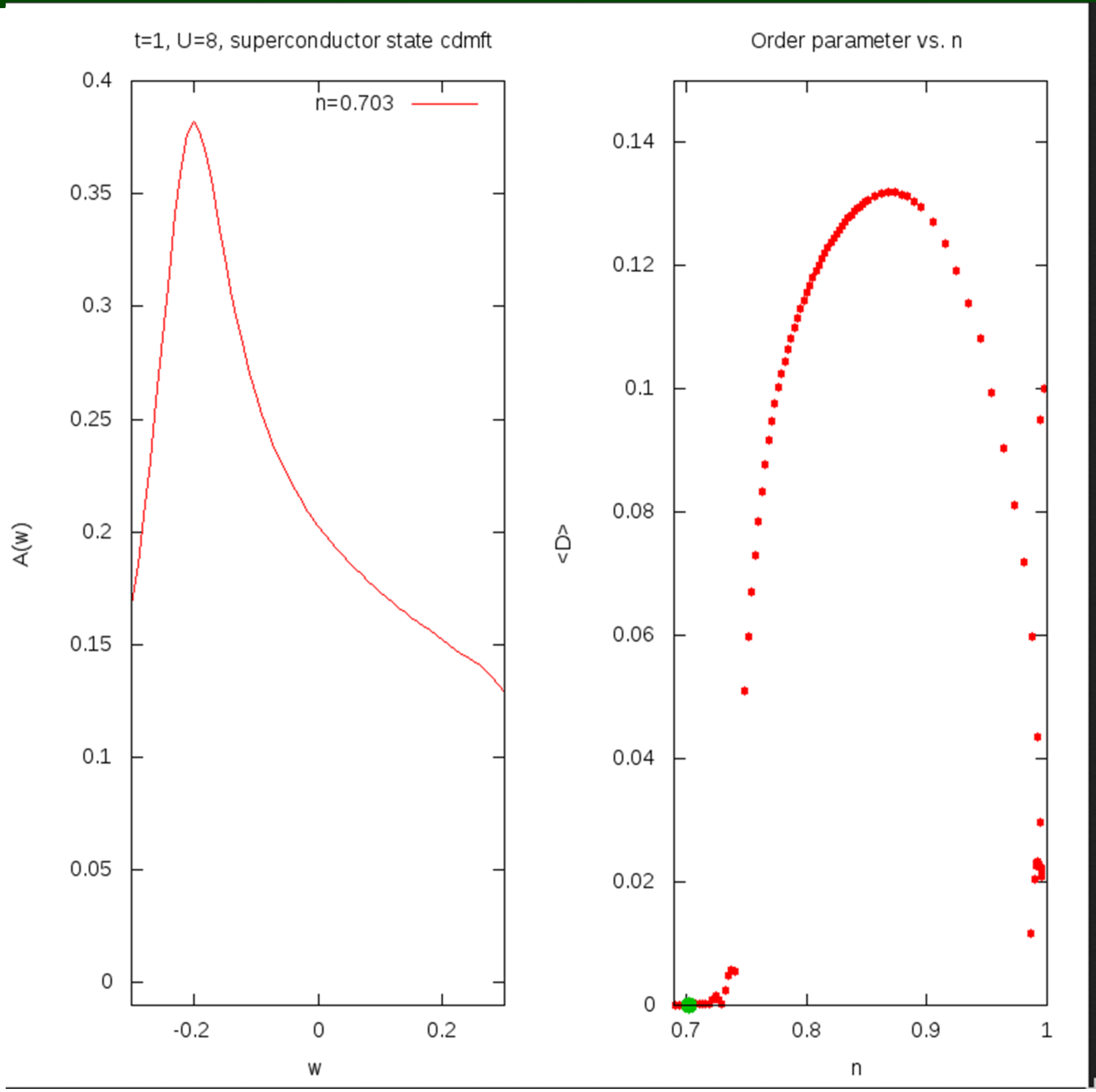
Unpublished



David Sénéchal

Evolution of SC gap and pseudogap with *n*

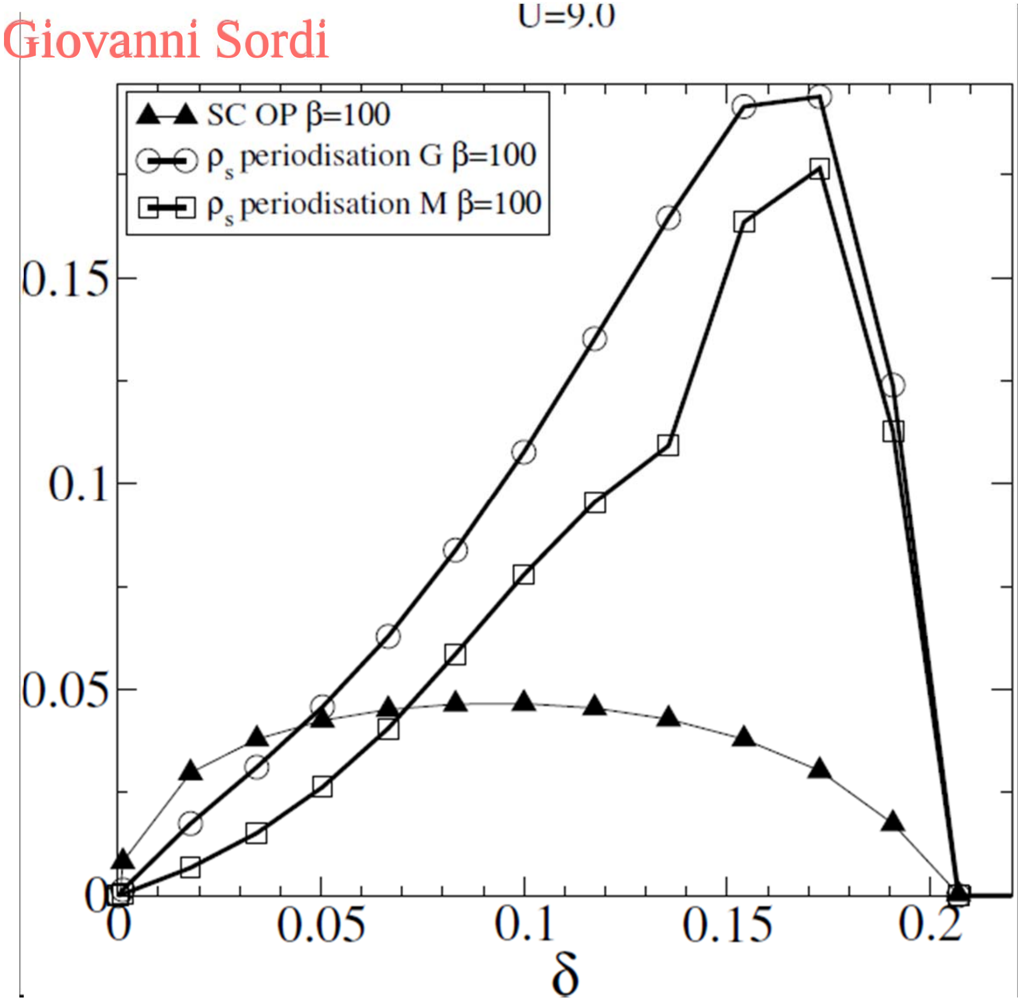
$t' = -0.3\,t$
$t'' = 0.2\,t$
$U=8t$



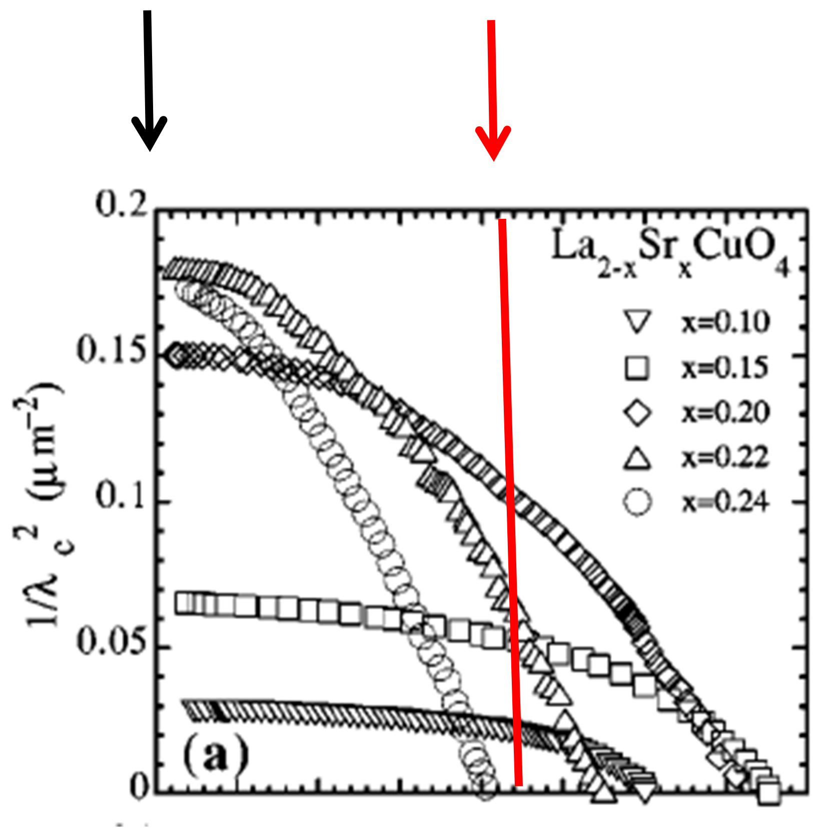
c-axis Superfluid density

U = 9t, T=1/100t

Giovanni Sordi



See also, Gull Millis, PRB, 2013



T

Panagopoulos et al. PRB 2000

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Weakly and strongly correlated superconductivity

Analog to weakly and strongly correlated antiferromagnets

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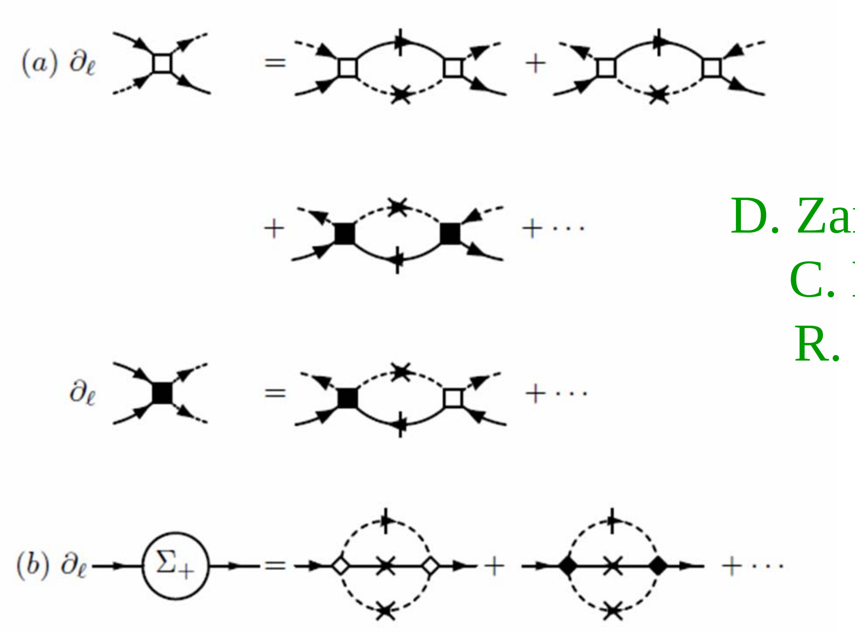
Weakly and strongly correlated superconductivity

Weakly correlated case

Weak coupling methods

- Functional renormalization group

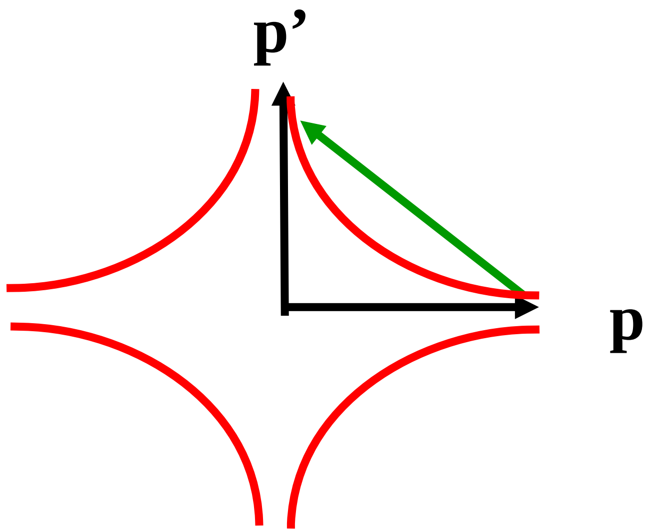


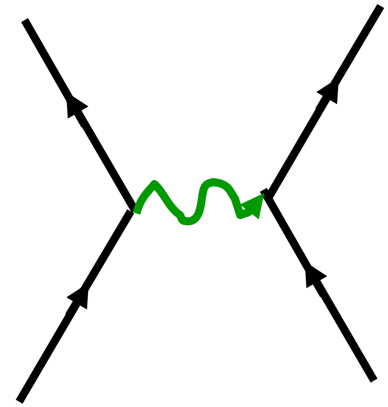
D. Zanchi and H.J. Schulz, PRB 61, 13609 (2000)
C. Honerkamp, et al.PRB 63, 035109 (2001)
R. Shankar, Rev. Mod. Phys. 66, 129 (1994)
C. Bourbonnais Sedeki PRB 2012

- Other weak coupling methods
  - N.E. Bickers, et al. Phys. Rev. Lett. 62, 961 (1989) FLEX
  - B. Kyung, J.-S. Landry, and A.-M.S.T.,PRB 68, 174502 (2003)

Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V}\sum_{\mathbf{p}'} U(\mathbf{p}-\mathbf{p}')\frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}}\left(1-2n\left(E_{\mathbf{p}'}\right)\right)$$





Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?
Kohn-Luttinger
T_c with pressure

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).
Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)

Weakly and strongly correlated superconductivity

Strongly correlated point of view

Many methods

A cartoon strong coupling picture

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson Science 317, 1705 (2007)

Miyake, Schmitt–Rink, and Varma P.R. B **34**, 6554-6556 (1986)

More sophisticated Slave Boson: Kotliar Liu PRB 1988

A paradox (Scalapino)

Resilience to near-neighbor repulsion V

$$\hat{\mathcal{H}}_{Hubbard} = -\sum_{\langle i,j\rangle_{1,2,3}\,\sigma} \left(t_{ij}\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + c.h \right) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_{\langle i,j\rangle} \hat{n}_i \hat{n}_j - \mu \sum_{i\,\sigma} \hat{n}_{i\sigma}$$

$YBa_2Cu_3O_7$: $t = 1$ $t' = -0.3$ $t'' = 0.2$

We expect superconductivity to disappear when:

$V > \frac{U^2}{W}$ **In weakly correlated case U/W < 1**

$V > J$ **In mean-field strongly correlated case**

In cuprates:

$V = 400\,\text{meV}$

$J = 130\,\text{meV}$

$$U_c = V_c / [1 + N(0) V_c \ln(E_F/\omega_c)]$$

S. Onari, R. Arita, K. Kuroki et H. Aoki, PRB **70**, 094523 (2004)
S. Raghu, E. Berg, A. V. Chubukov et S. A. Kivelson, PRB **85**, 024516 (2012)
S. Sorella, et al. Phys. Rev. Lett. 88, 117002 (2002)

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Strongly correlated superconductivity: the view from dynamical mean-field theory

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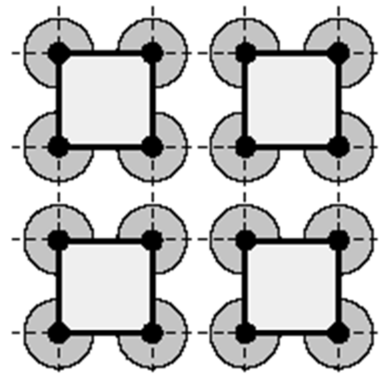
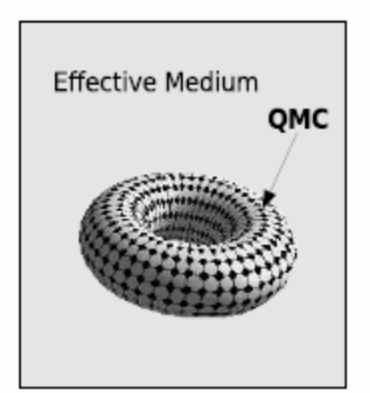
2d Hubbard: Quantum cluster method

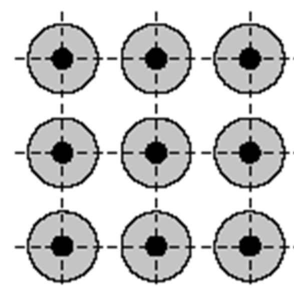
C-DMFT





DCA

DMFT



Hettler …Jarrell…Krishnamurty PRB **58** (1998)
Kotliar et al. PRL **87** (2001)
M. Potthoff *et al.* PRL **91,** 206402 (2003).

REVIEWS
Maier, Jarrell et al., RMP. (2005)
Kotliar *et al.* RMP (2006)
AMST *et al.* LTP (2006)

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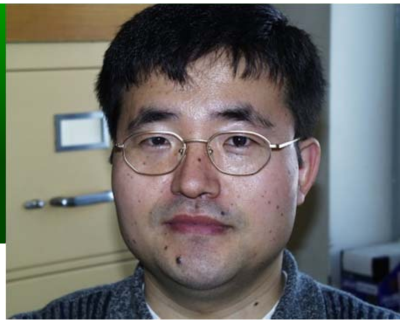
+ and -

- Long range order:
  - Allow symmetry breaking in the bath (mean-field)
- Included:
  - Short-range dynamical and spatial correlations
- Missing:
  - Long wavelength p-h and p-p fluctuations

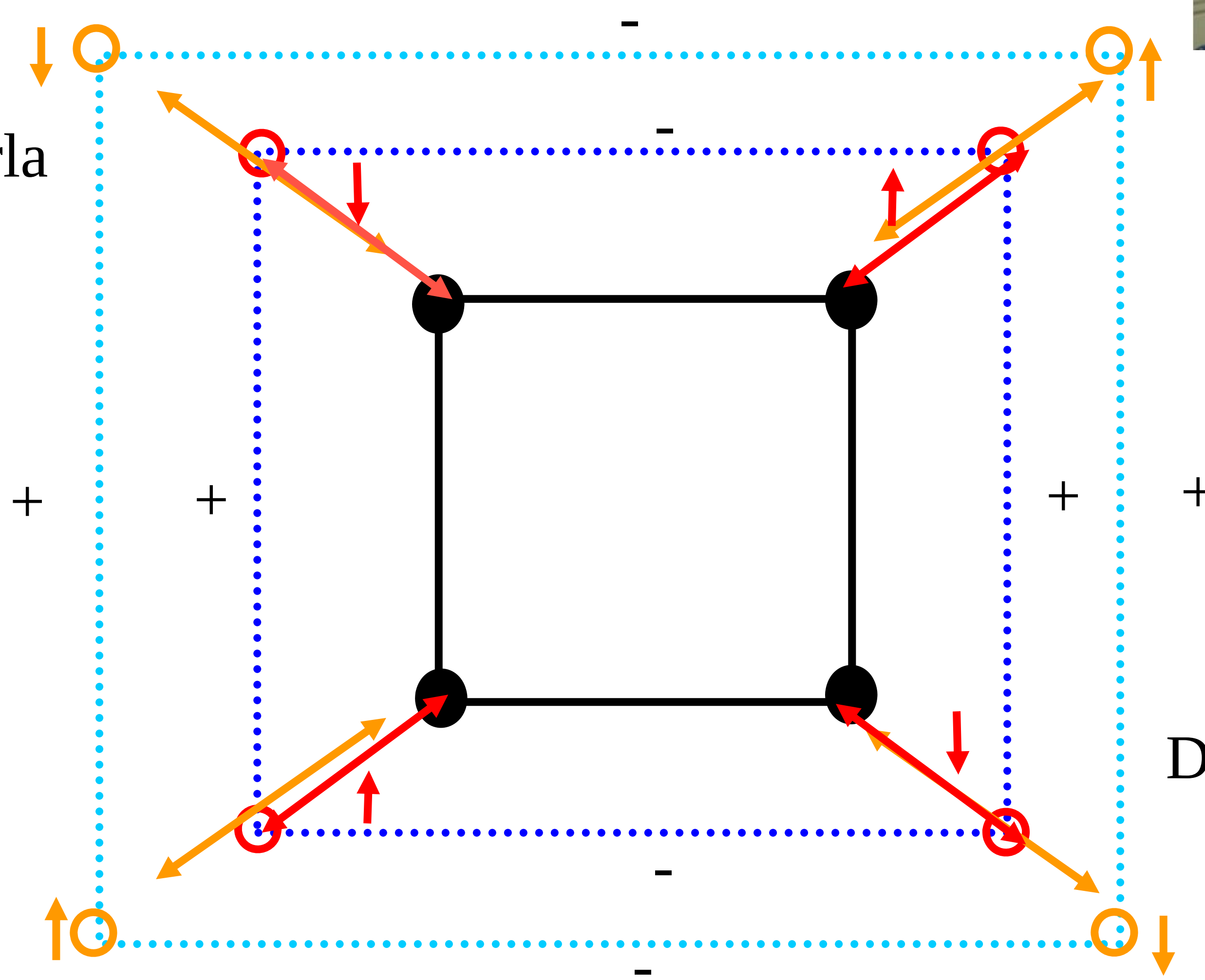
$T = 0$ results

Competition AFM-dSC

S. Kancharla



B. Kyung



David Sénéchal

See also, Capone and Kotliar, Phys. Rev. B 74, 054513 (2006),
Macridin, Maier, Jarrell, Sawatzky, Phys. Rev. B 71, 134527 (2005)

Pairing, glue and glue remover

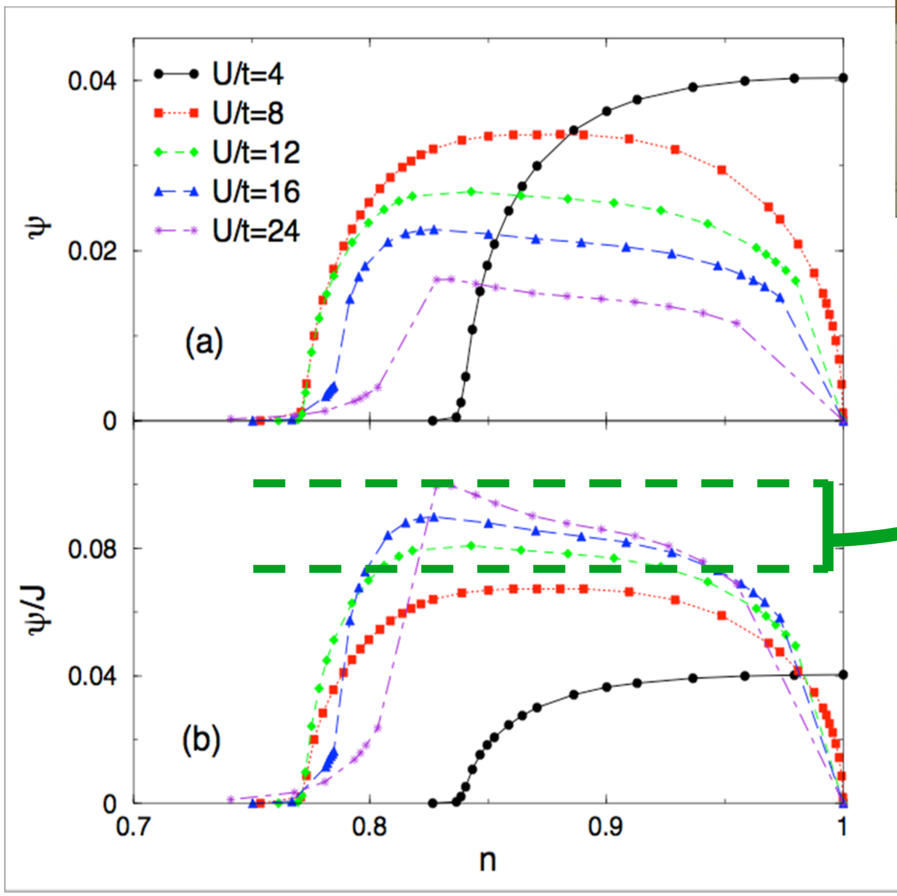
Strength of pairing

Kancharla, Kyung, Civelli,
Sénéchal, Kotliar AMST PRB (2008)

In cuprates:

$$V = 400\,\mathrm{meV}$$

$$J = 130\,\mathrm{meV}$$



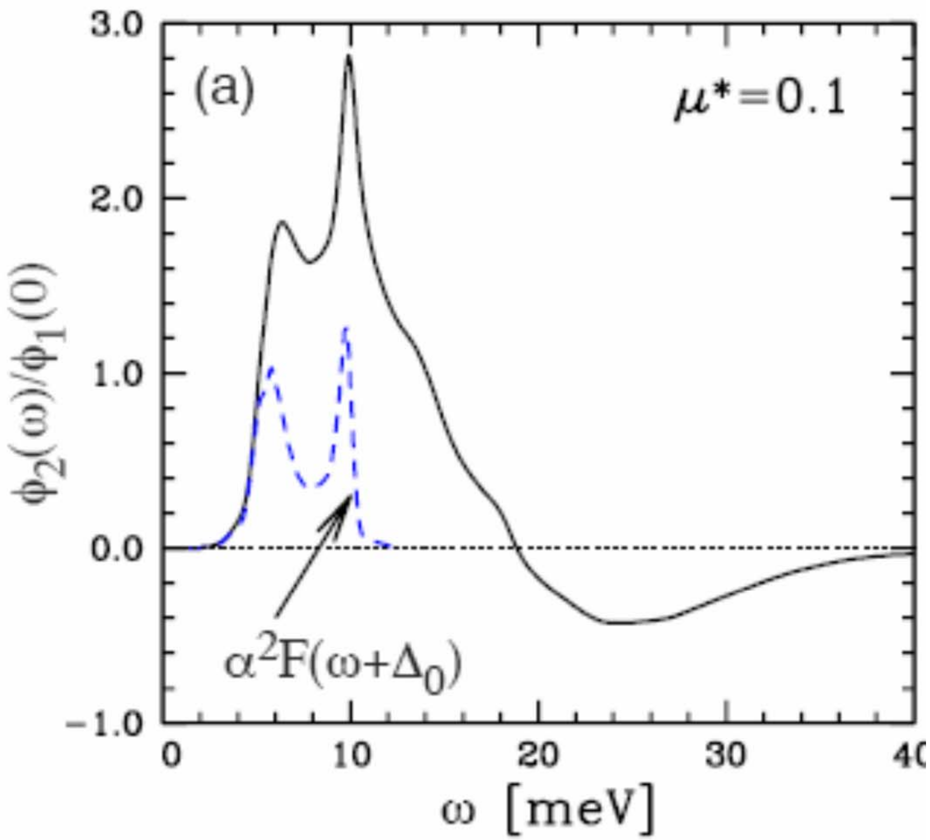
$$J = \frac{4t^2}{U}$$

The superconducting order parameter scales like *J*

The glue

Im Σ_{an} and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

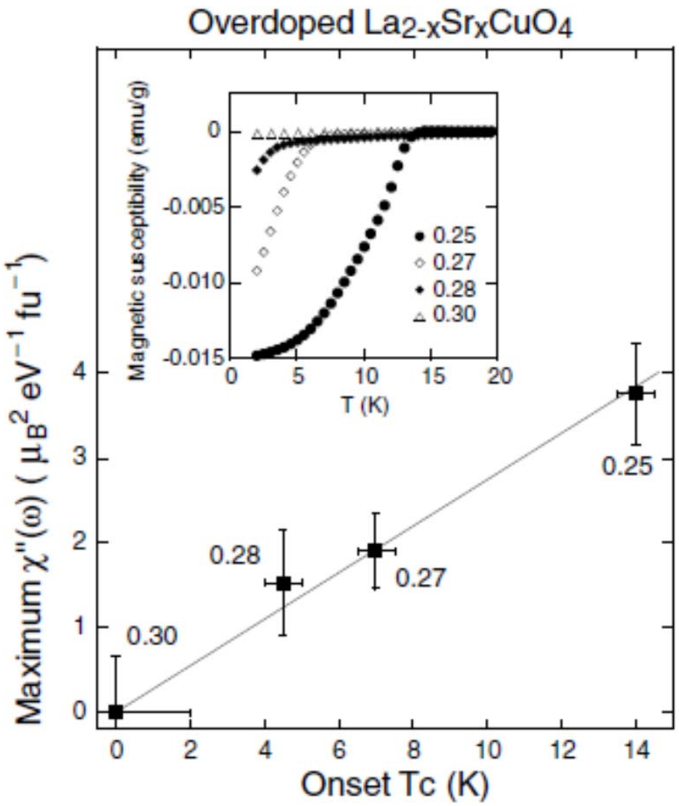


Retardation

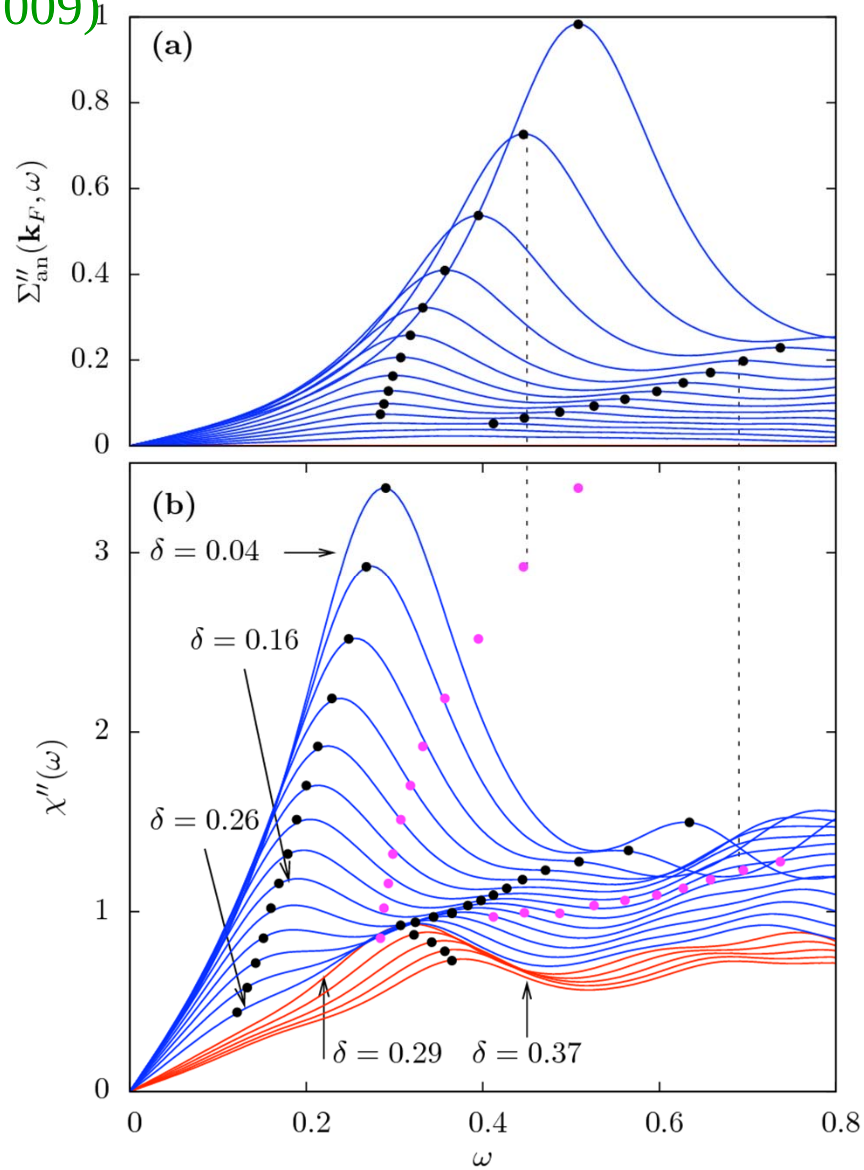
$$V^{eff}_{él-ph}(\vec{q},\omega) = \frac{e^2}{4\pi\varepsilon_0(q^2+k_{TF}^2)}\left[1+\frac{\omega_{ph}^2(\vec{q})}{\omega^2-\omega_{ph}^2(\vec{q})}\right]$$

The glue for strong corrections

Kyung, Sénéchal, AMST PRB **80,** 205109 (2009)
Civelli, PRL + PRB 79 195113 (2009)
Gull and Millis arXiv:1407.0704



Wakimoto … Birgeneau
PRL (2004)



The glue and neutrons

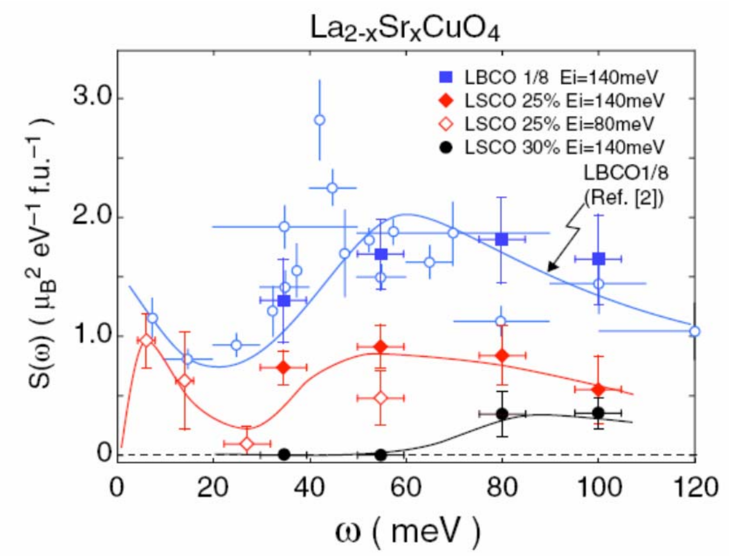
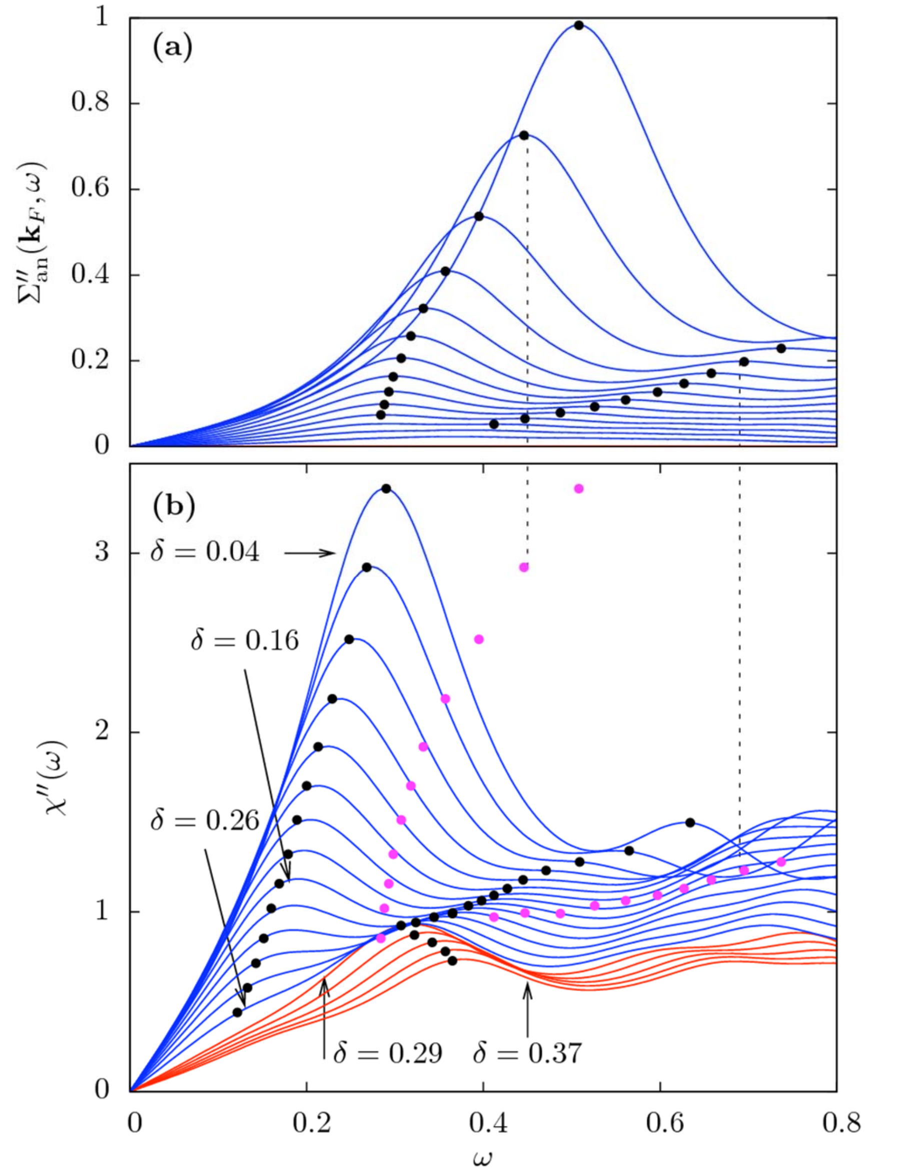


FIG. 3 (color online). **Q**-integrated dynamic structure factor $S(\omega)$ which is derived from the wide-H integrated profiles for LBCO 1/8 (squares), LSCO $x = 0.25$ (diamonds; filled for E_i = 140 meV, open for E_i = 80 meV), and $x = 0.30$ (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO $x = 0.25$ and 0.30 are guides to the eyes.



Wakimoto … Birgeneau PRL (2007);
PRL (2004)

Kyung, Sénéchal, AMST PRB **80,** 205109 (2009); Haule Kotliar (2007)

Frequencies important for pairing

Bumsoo Kyung

David Sénéchal

Anomalous Green function

$$[\mathcal{F}_{an}(t)]_{lm} = -i\theta(t)\,\langle\{\hat{c}_{l\uparrow}(t), \hat{c}_{m\downarrow}(0)\}\rangle_{\mathcal{H}_{AIM}}$$

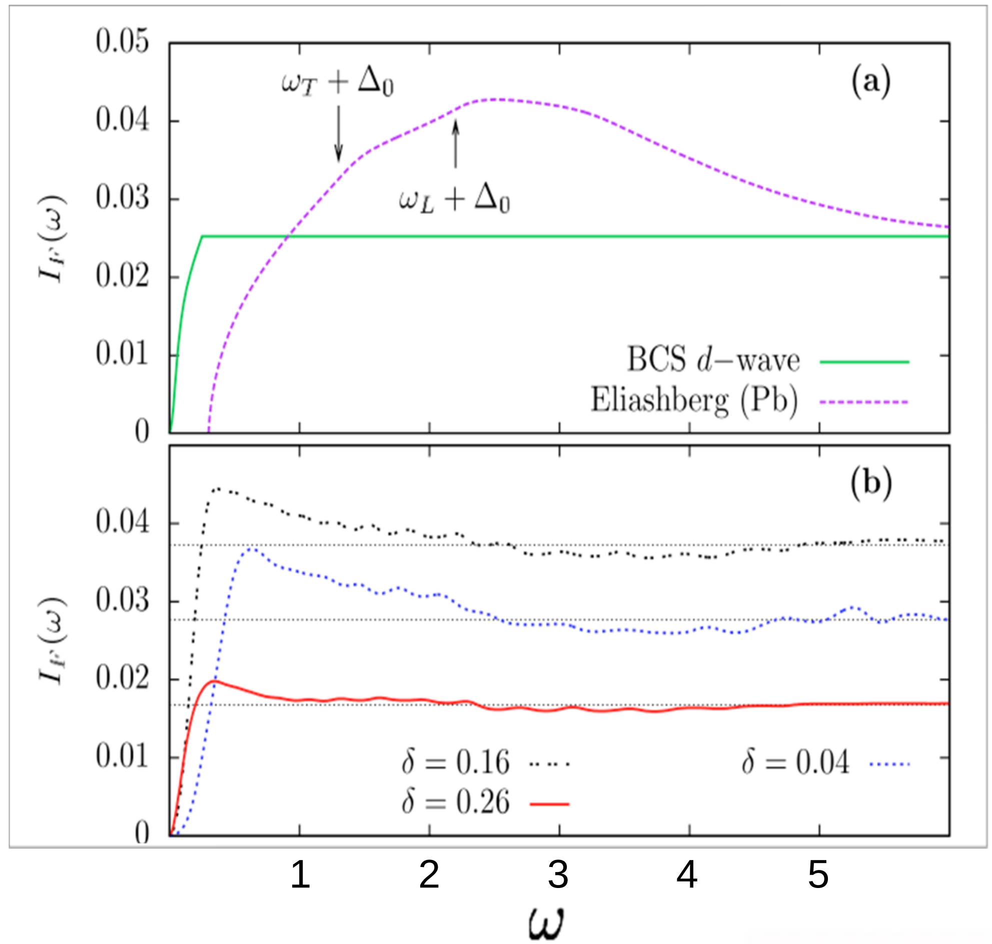
Anomalous spectral function

$$[\mathcal{A}_{an}(\omega)]_{lm} = -\frac{1}{\pi}\,\mathrm{Im}\,[\mathcal{F}_{an}(\omega)]_{lm}$$

Cumulative order parameter:

$$I_{\mathcal{F}}(\omega) = -\int_0^{\omega} \frac{d\omega'}{\pi}\,\mathrm{Im}\,[\mathcal{F}_{an}(\omega')]_{lm}$$

$$I_{\mathcal{F}}(\omega) \xrightarrow[\omega\to+\infty]{} \langle\hat{c}_{l\uparrow}\hat{c}_{m\downarrow}\rangle_{\mathcal{H}_{AIM}}$$



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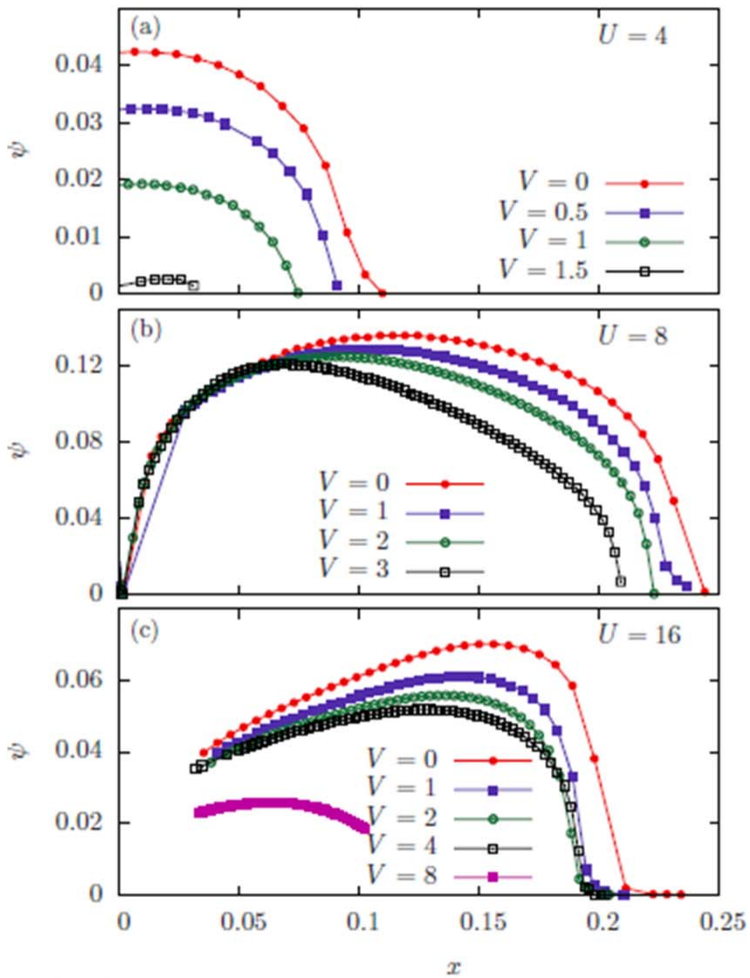
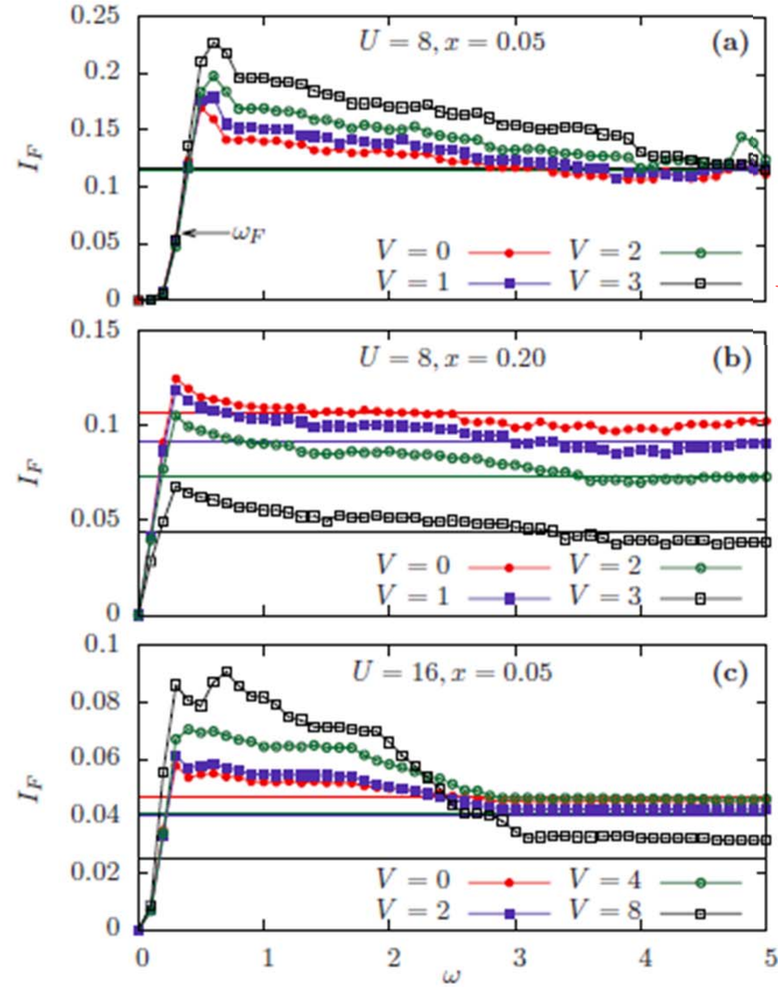
Resilience to near-neighbor repulsion

David Sénéchal

Alexandre Day

Vincent Bouliane





Sénéchal, Day, Bouliane, AMST PRB **87**, 075123 (2013)

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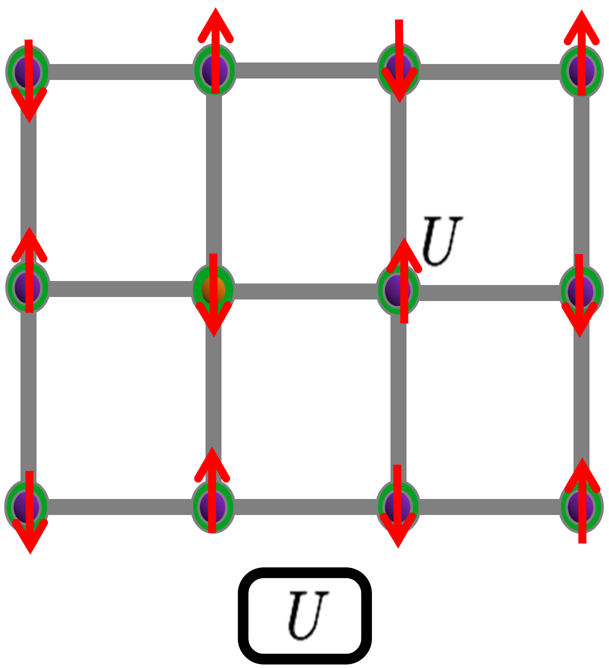
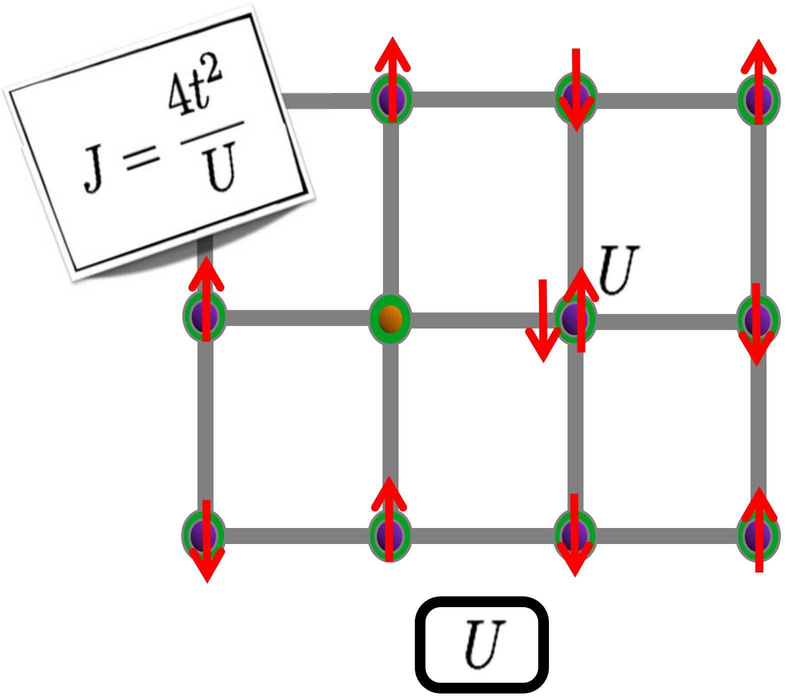
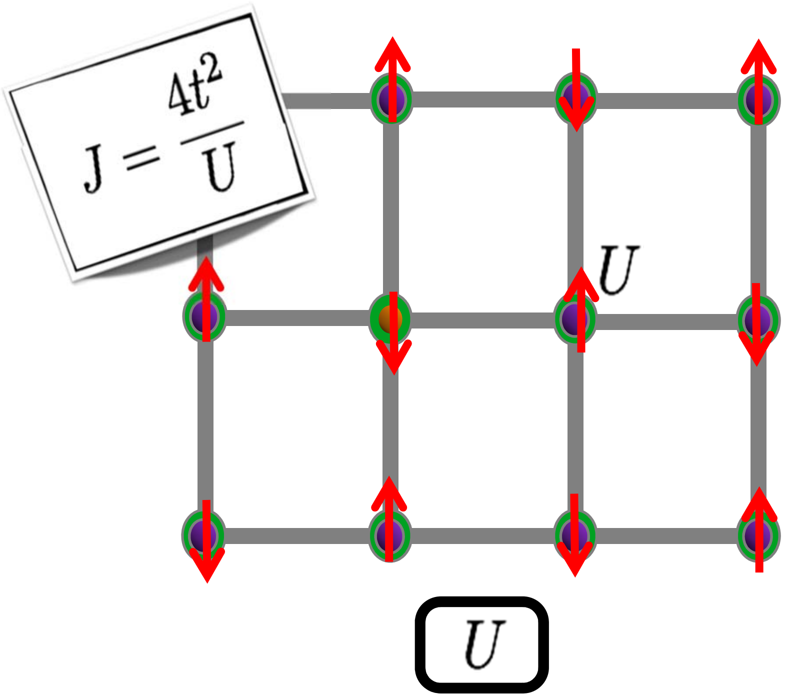
V also increases J



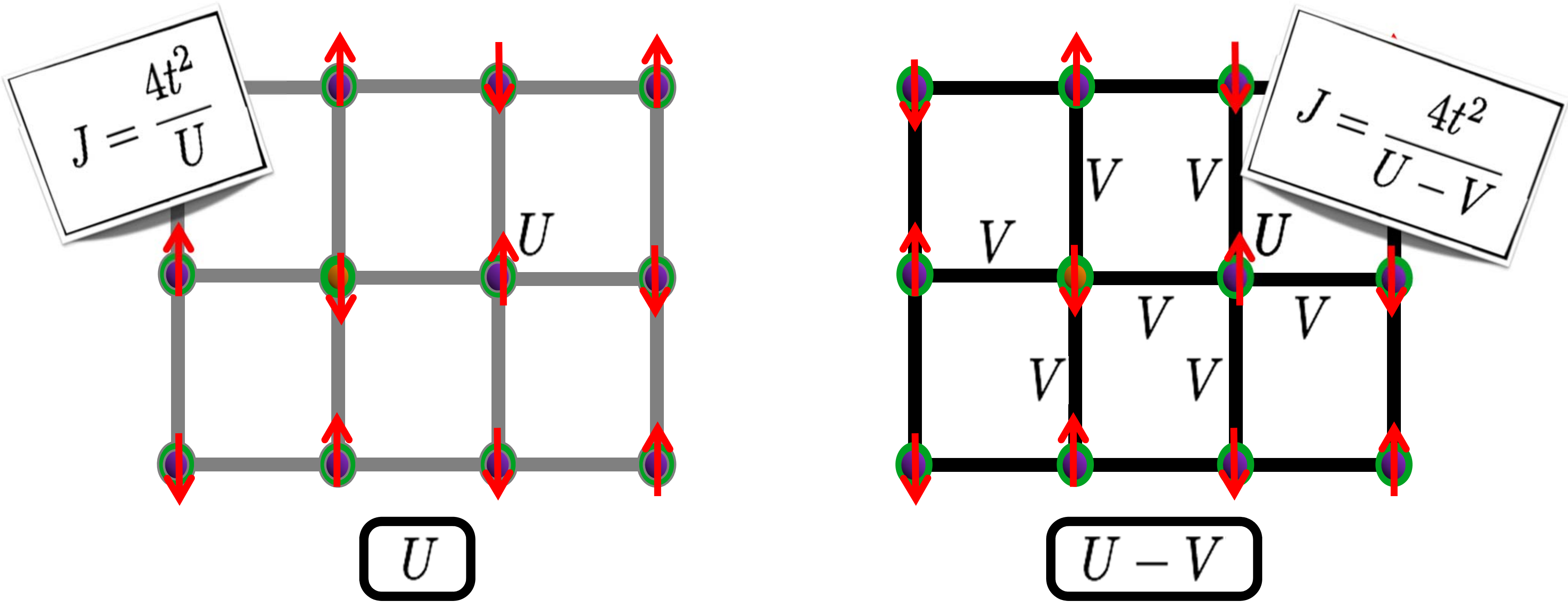
V also increases *J*



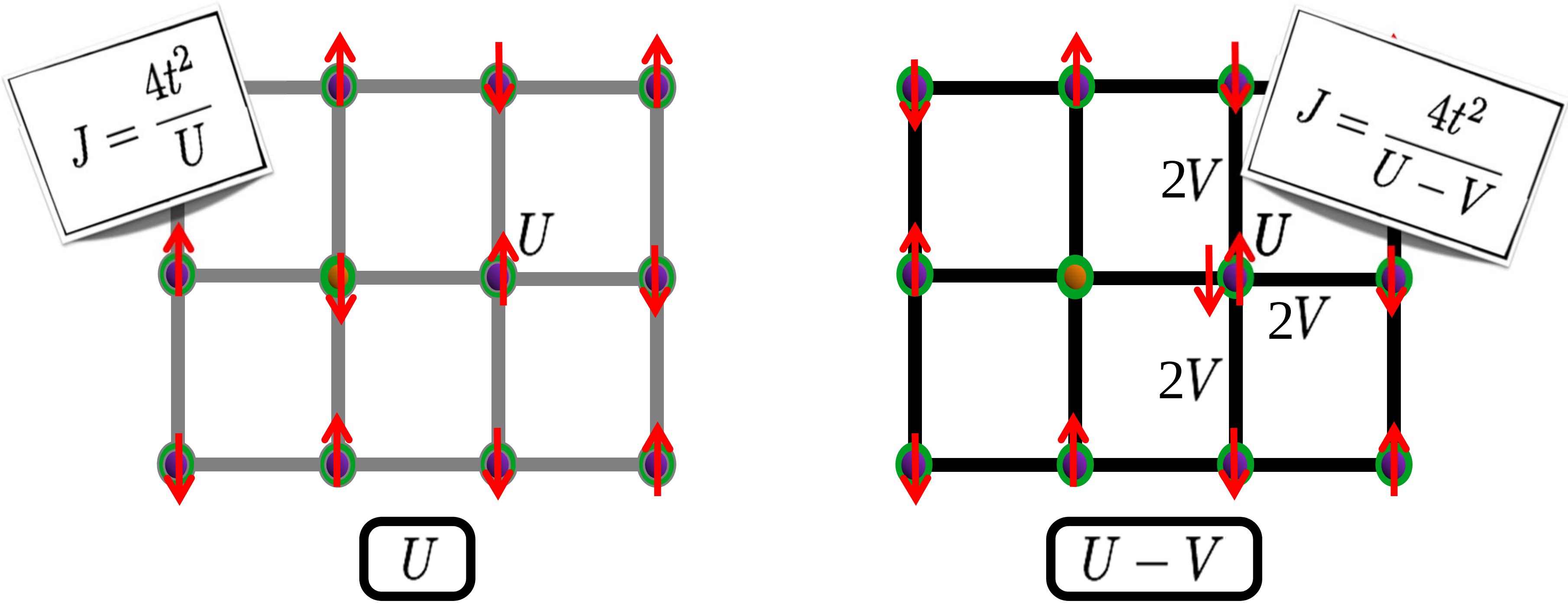
V also increases *J*



V also increases J



V also increases J

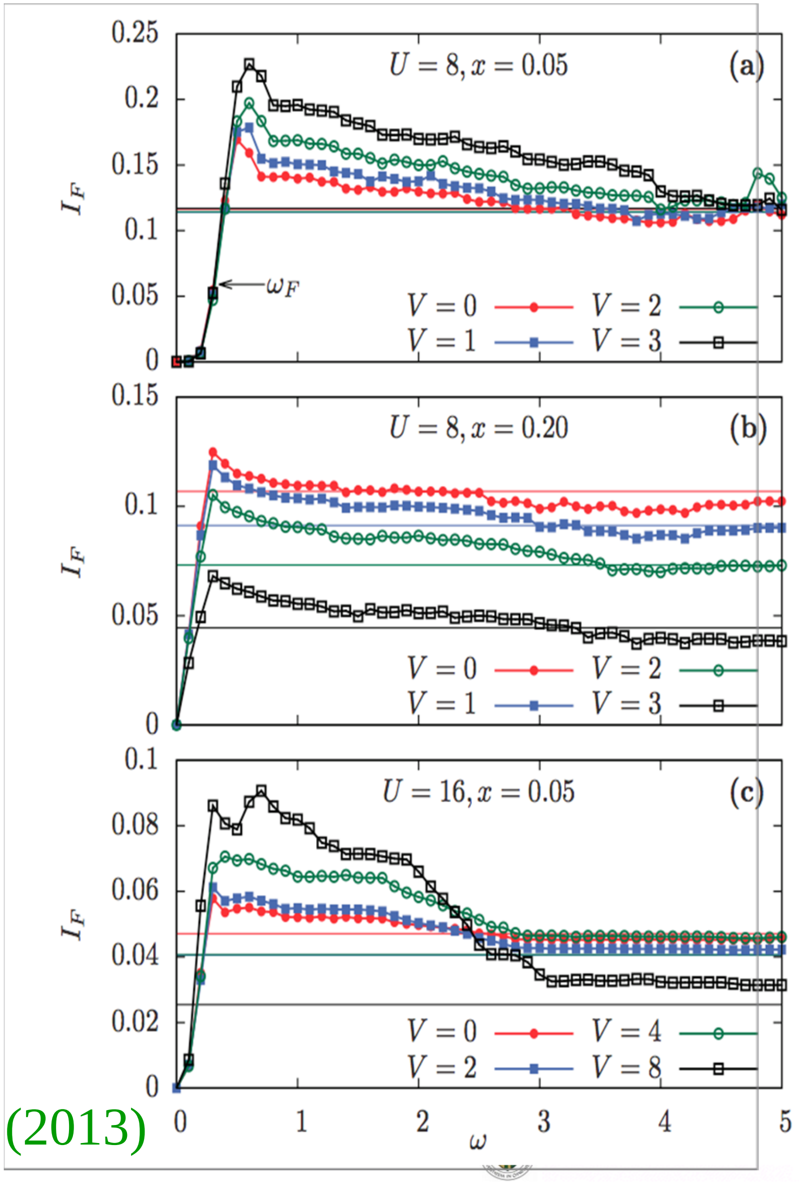


Binding aspects of V

$$J = \frac{4t^2}{U - V}$$

***J* increases with *V* explaining better pairing at low frequency**

But V also induces more repulsion at high frequency, explaining the negative impact at high frequency on binding



Sénéchal, Day, Bouliane, AMST PRB **87**, 075123 (2013)

Finite *T* and more experiments

H. Alloul C.R. Physique, **15** (2014)

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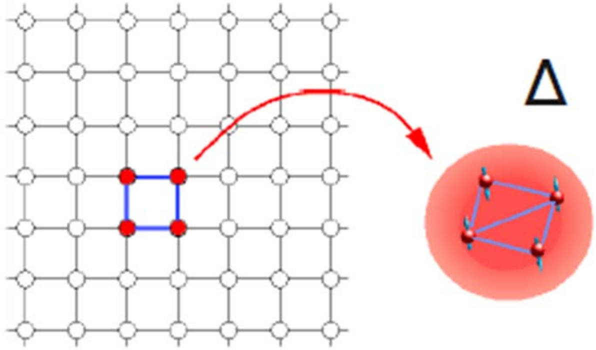
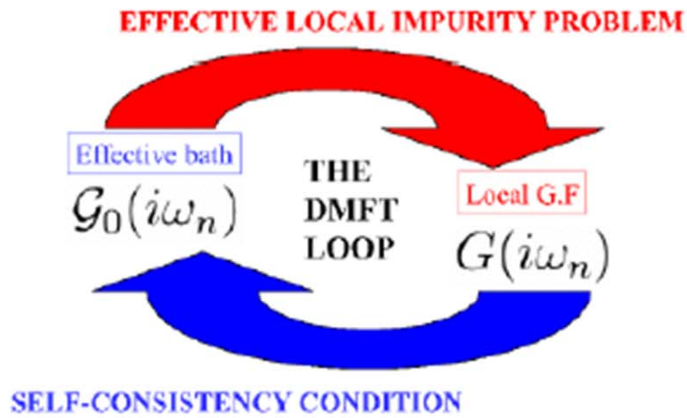
C-DMFT

$$Z = \int \mathcal{D}[\psi^\dagger, \psi]\, \mathrm{e}^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{K}} \psi^\dagger_{\mathbf{K}}(\tau) \Delta_{\mathbf{K}}(\tau,\tau') \psi_{\mathbf{K}}(\tau')}$$





Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006
P. Werner, PRB 2007
K. Haule, PRB 2007

$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n) - \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

Giovanni Sordi

Patrick Sémon

Standard 2 operator updates in CTQMC are not ergodic in SC state (fixed here)

Please ask in question period…

P. Sémon, G. Sordi, A.-M.S. Tremblay, Phys. Rev. B **89**, 165113/1-6 (2014)

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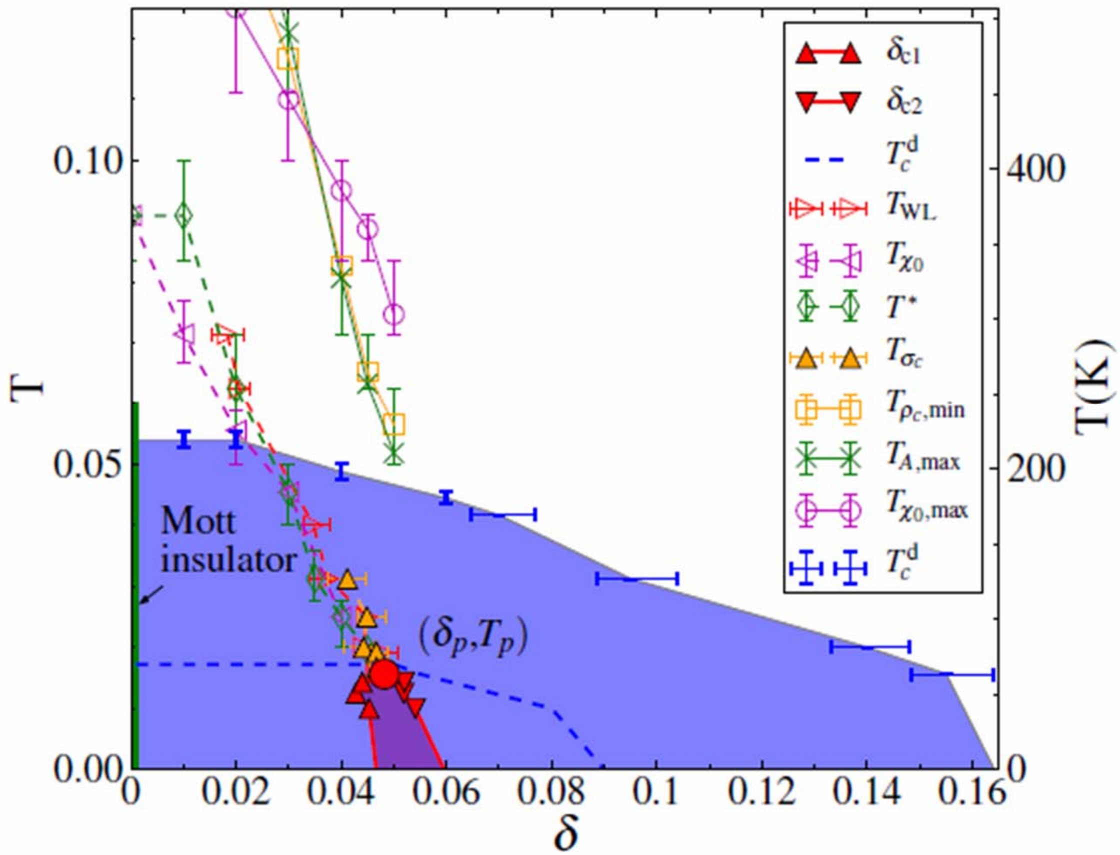
Superconductivity at $U > U_{MIT}$

Phase diagram for *U* =6.2 *t*

Giovanni Sordi

Patrick Sémon

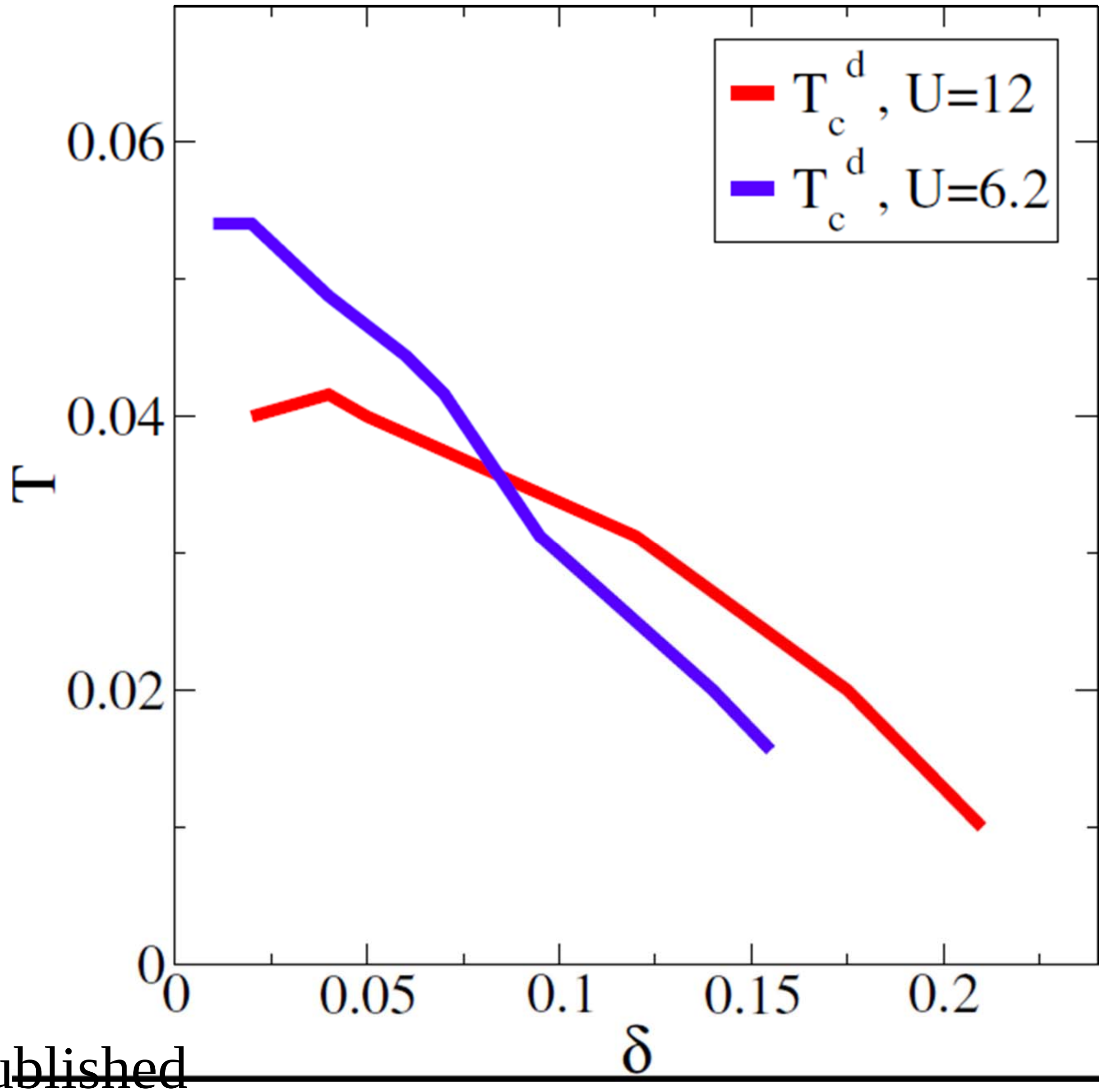


G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)

T_c^d

Giovanni Sordi

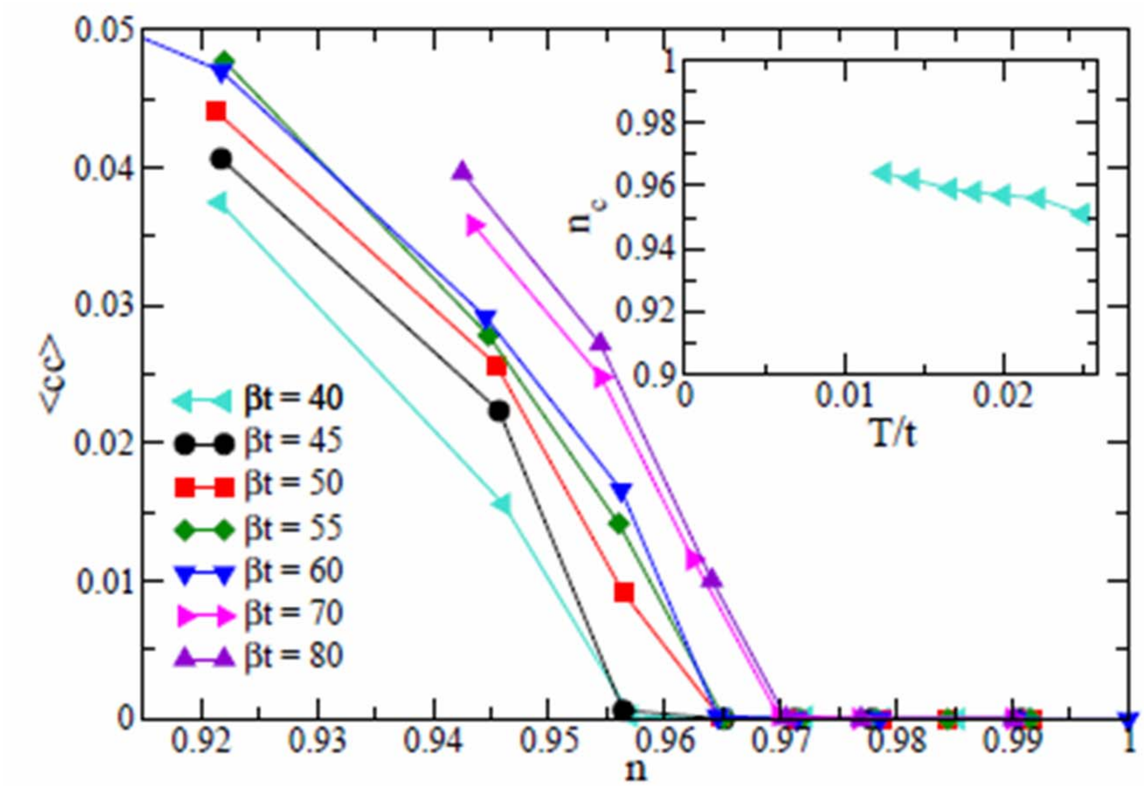


G. Sordi Unpublished

8 site cluster

Ends at finite doping

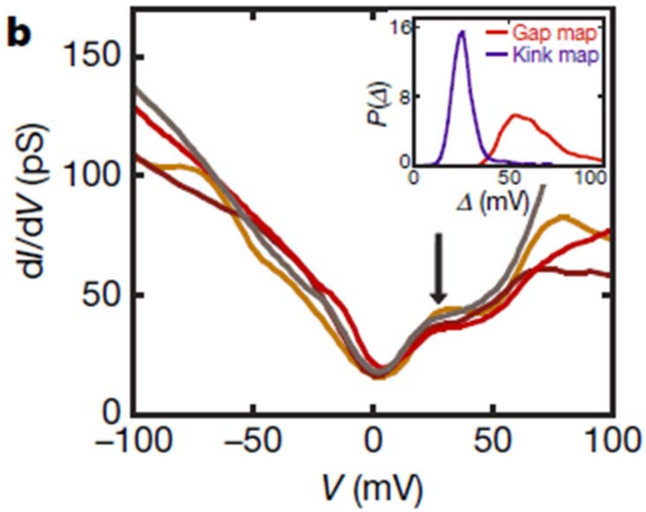
Gull, Parcollet, Millis, PRL 110, 216405 (2013)



Gull, Parcollet, Millis PRL 110, 216405 (2013)

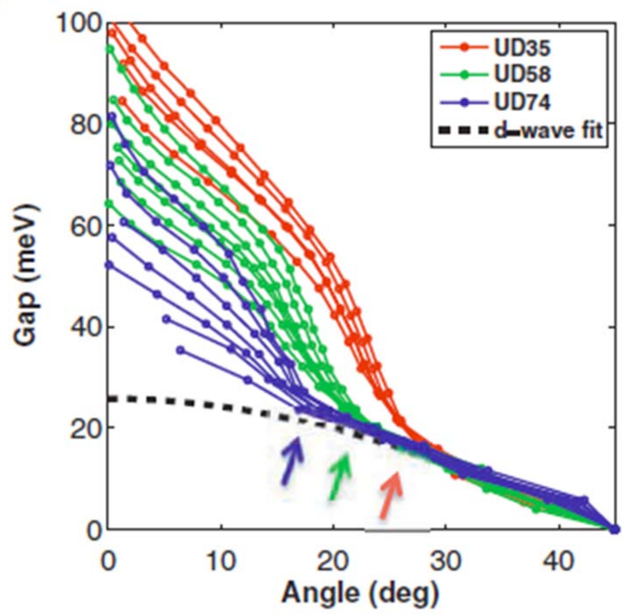
Meaning of T_c^d

- Local pair formation



K. K. Gomes, A. N. Pasupathy, A. Pushp,
S. Ono, Y. Ando, and A. Yazdani,
Nature **447**, 569 (2007)

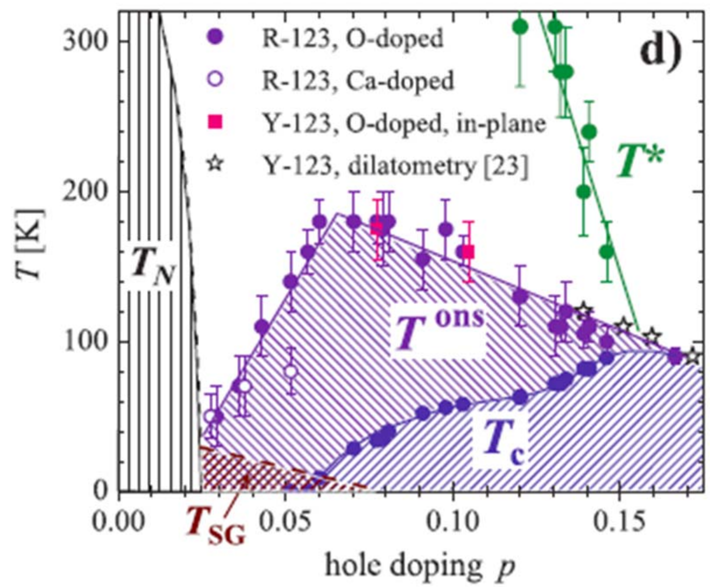
Meaning of T_c^d : Local pair formation



A. Pushp, Parker, … A. Yazdani, Science **364**, 1689 (2009)

However, our measurements demonstrate that the nodal gap does not change with reduced doping. The pairing strength does not get weaker or stronger as the Mott insulator is approached; rather, it saturates.

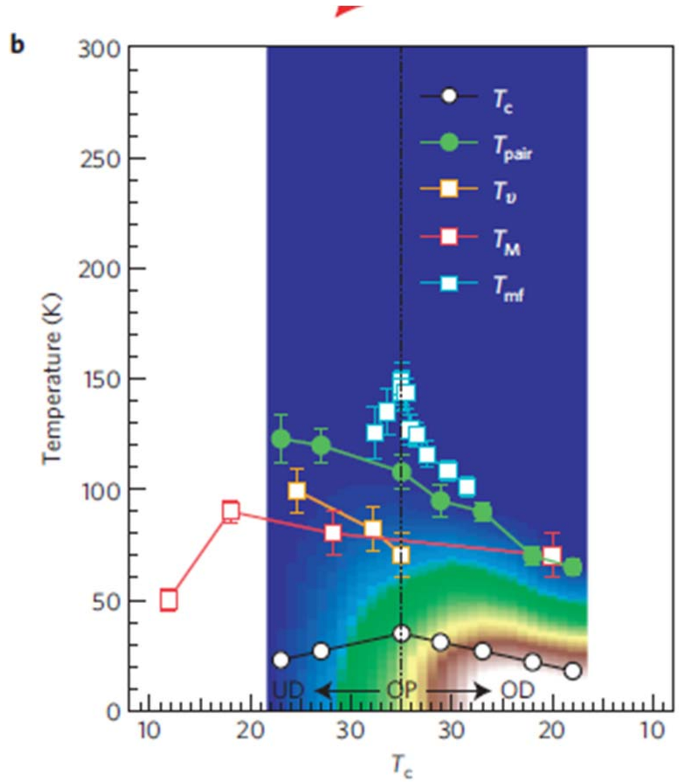
Fluctuating region



Infrared response

Dubroka et al. 106, 047006 (2011)

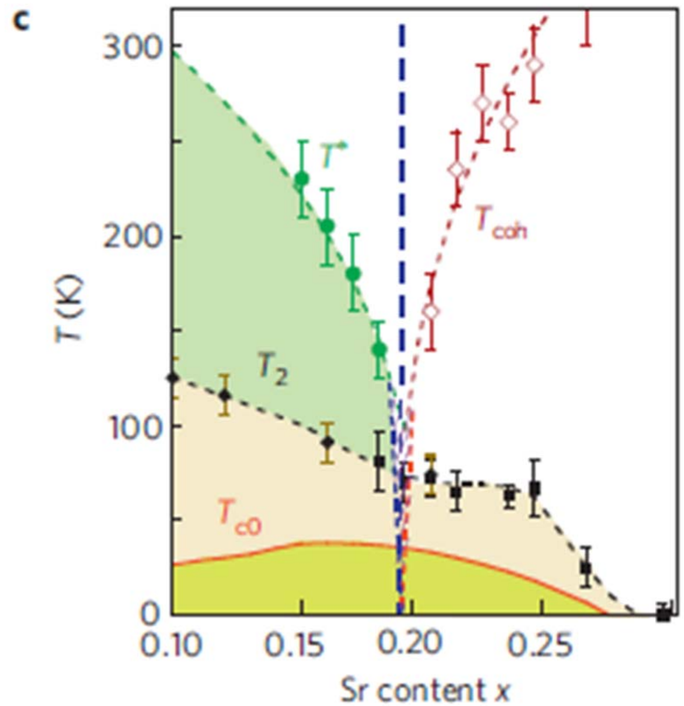
T_{pair}



ARPES
Bi2212

Kondo, Takeshi, et al. Kaminski Nature Physics **2011**, *7*, 21-25

T_2



Magnetoresistance, LSCO
Fluctuating vortices

Patrick M. Rourke, et al. Hussey Nature Physics 7, 455–458 (2011)

Giant proximity effect

$T_c = 32\ K$
$T_c < 5K$

Morenzoni et al.,
Nature Comms. **2** (2011)

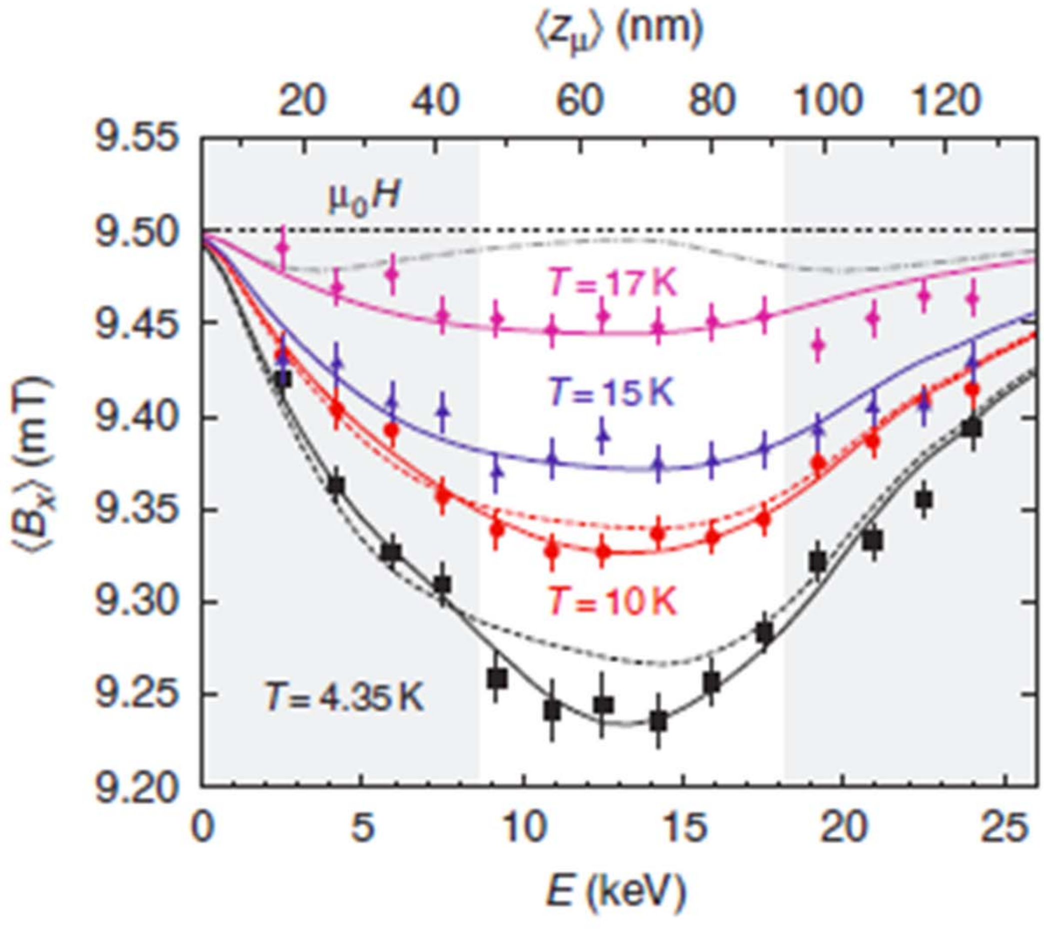
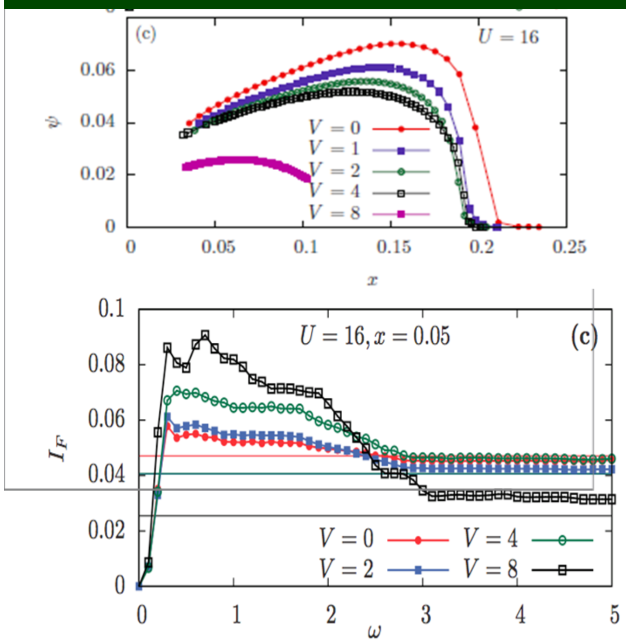
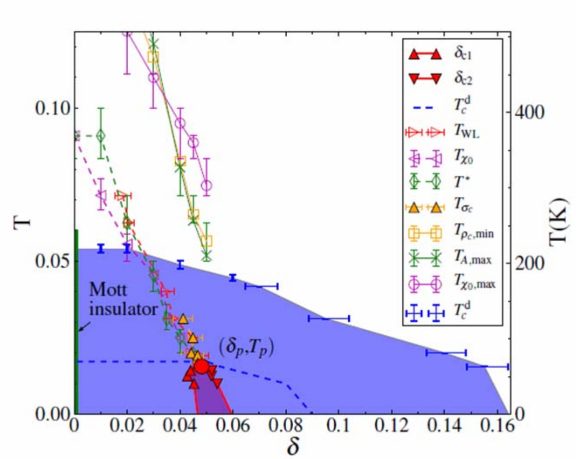
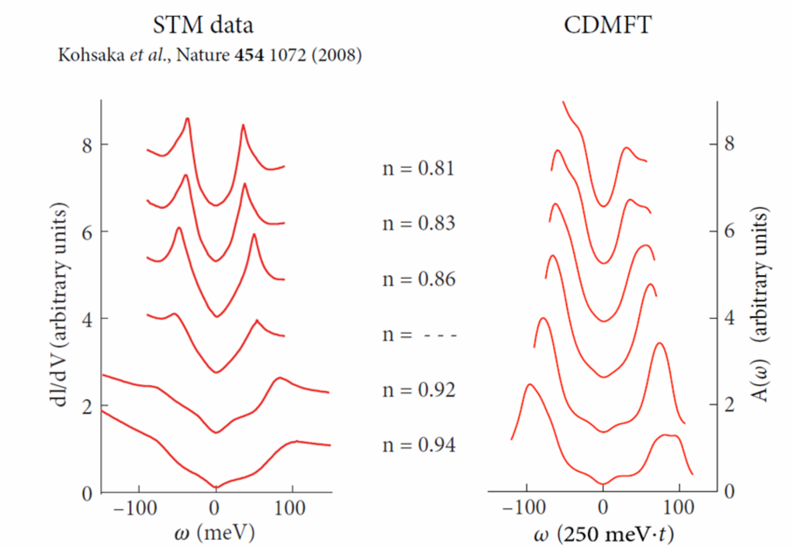


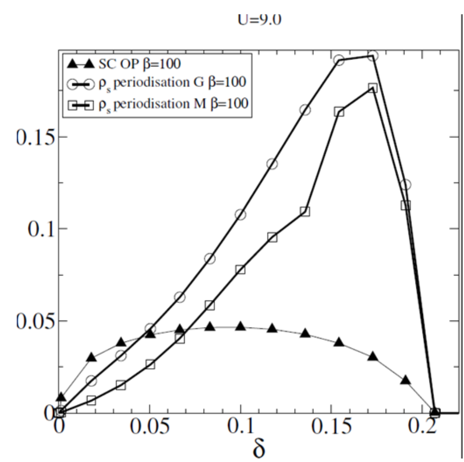
Figure 6 | Depth profile of the local field at different temperatures. The

Summary









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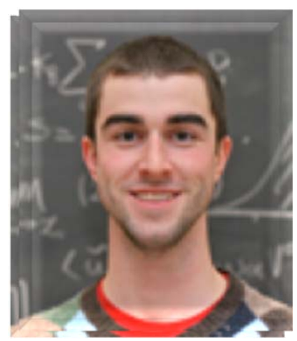

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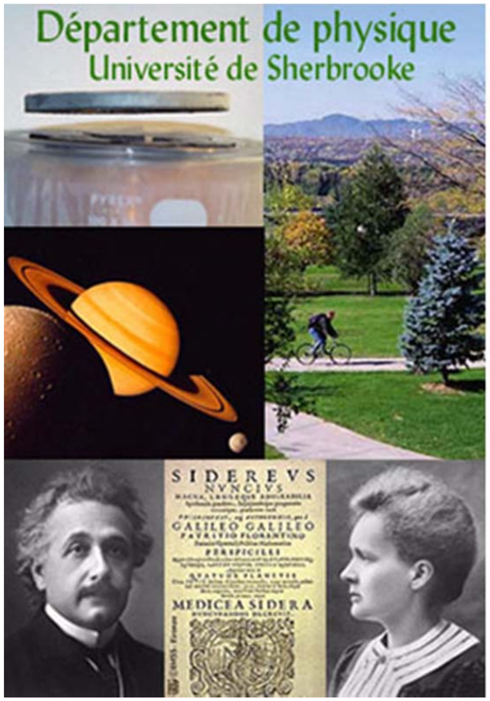

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Merci

Thank you

A.-M.S. Tremblay

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