



The pseudogap

André-Marie Tremblay



26 April 2016



UNIVERSITÉ DE
SHERBROOKE

Atomic structure

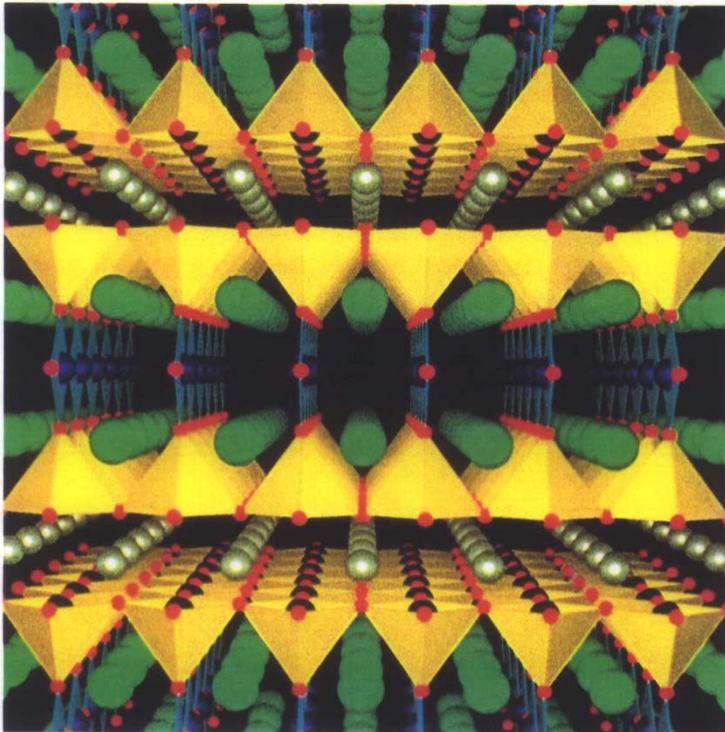
SCIENTIFIC AMERICAN

JUNE 1988
\$3.50

How nonsense is deleted from genetic messages.

R_x for economic growth: aggressive use of new technology.

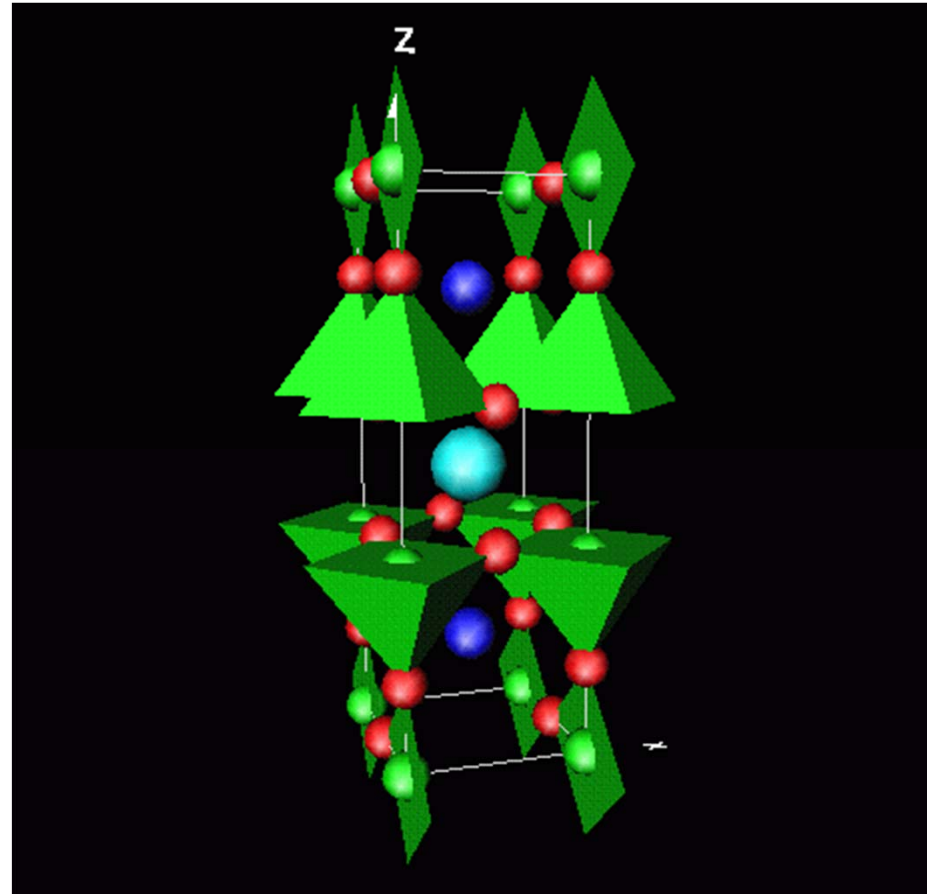
Can particle physics test cosmology?



High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.



92-37

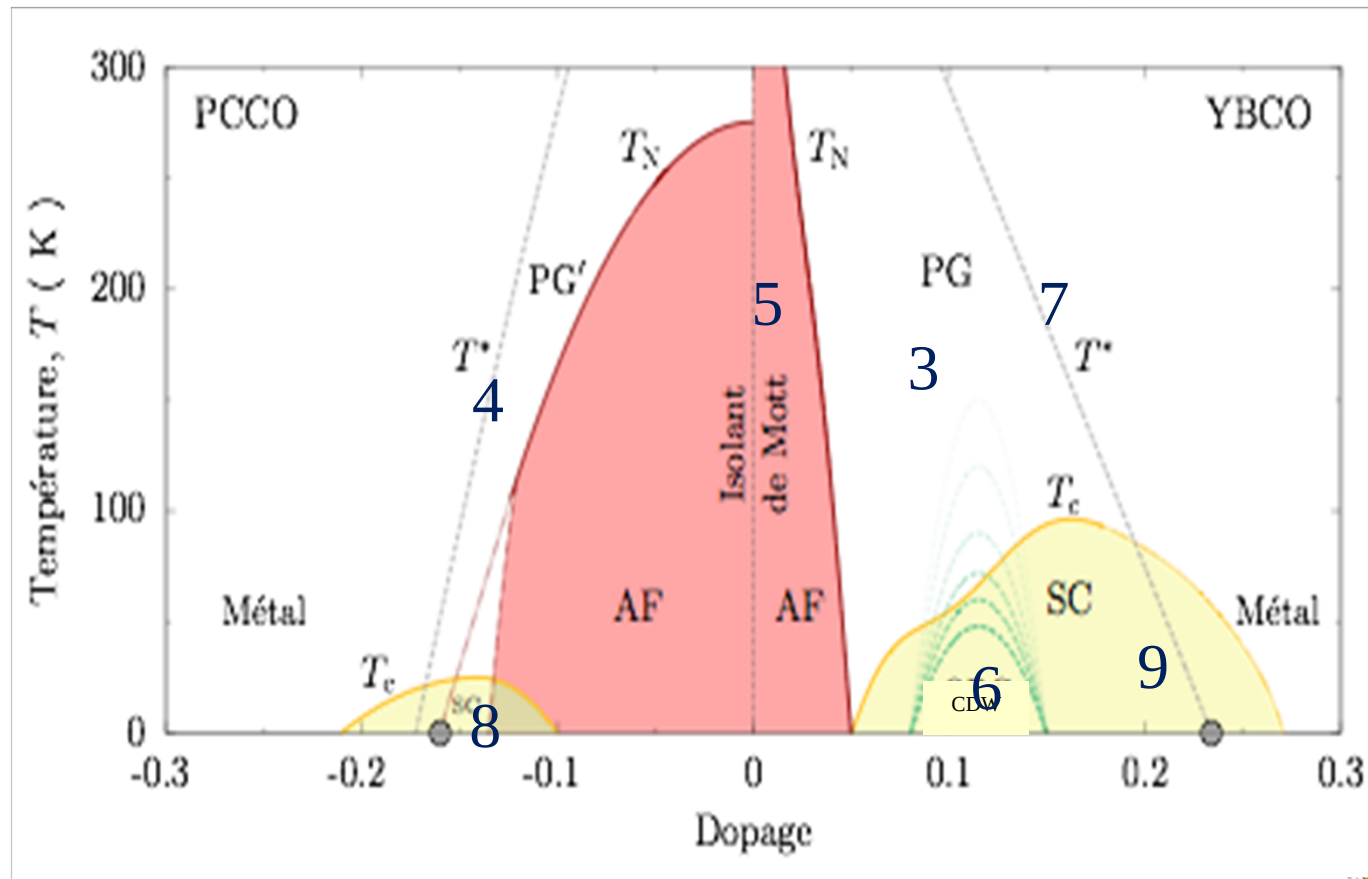


UNIVERSITÉ DE
SHERBROOKE

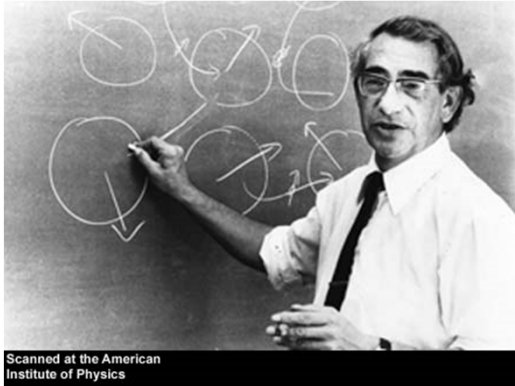
Outline

For references, September 2013 Julich summer school
Strongly Correlated Superconductivity

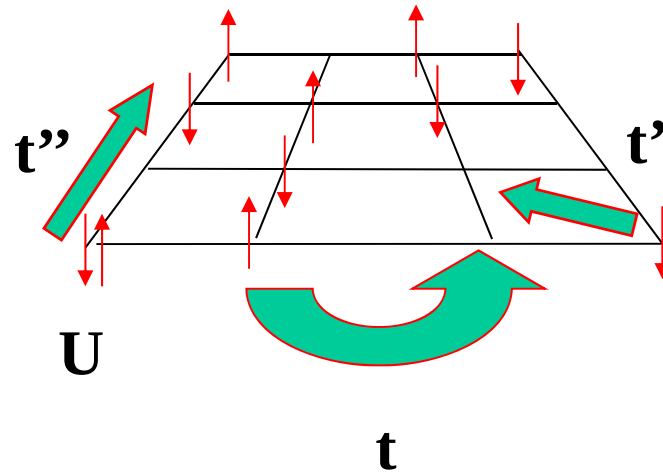
<http://www.cond-mat.de/events/correl13/manuscripts/tremblay.pdf>



Hubbard model



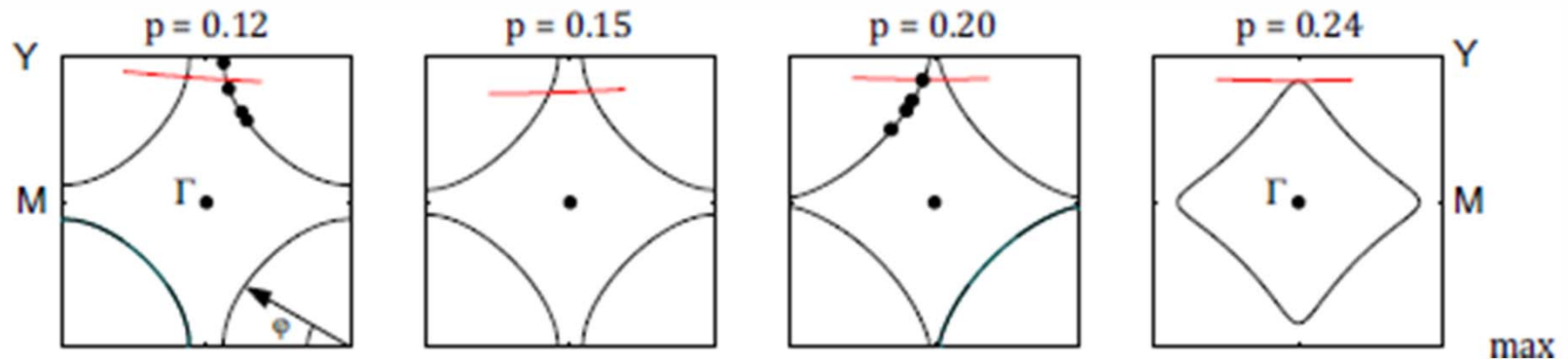
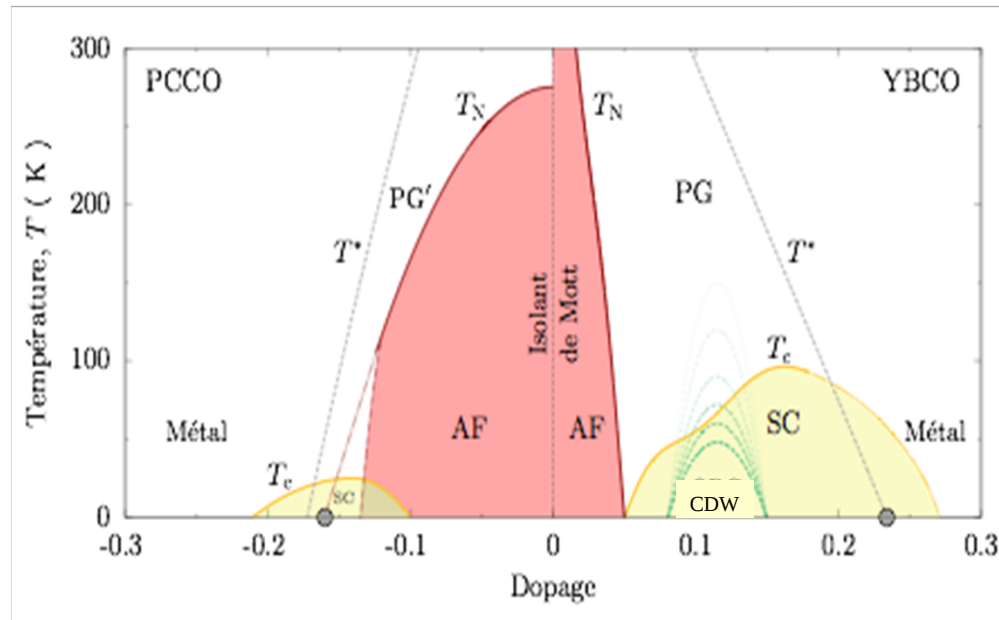
1931-1980



$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Attn: Charge transfer insulator

Non-interacting Fermi surface



Speakers

Friday 29 April, Forest Hill Ballroom

07:00-08:45 Breakfast, High Park Ballroom

09:00-09:45 Jeff TALLON (Wellington) – *Pseudogap in cuprate superconductors – what, when, why, where, how?*

09:45-10:30 Dan DESSAU (Colorado) – *Direct connection of ARPES with optics, transport, and thermodynamics experiments in cuprate superconductors: the key role of electron self-energies*

11:30-12:15 Louis TAILLEFER (Sherbrooke) – *Normal-state signatures of the pseudogap quantum critical point*



Outline

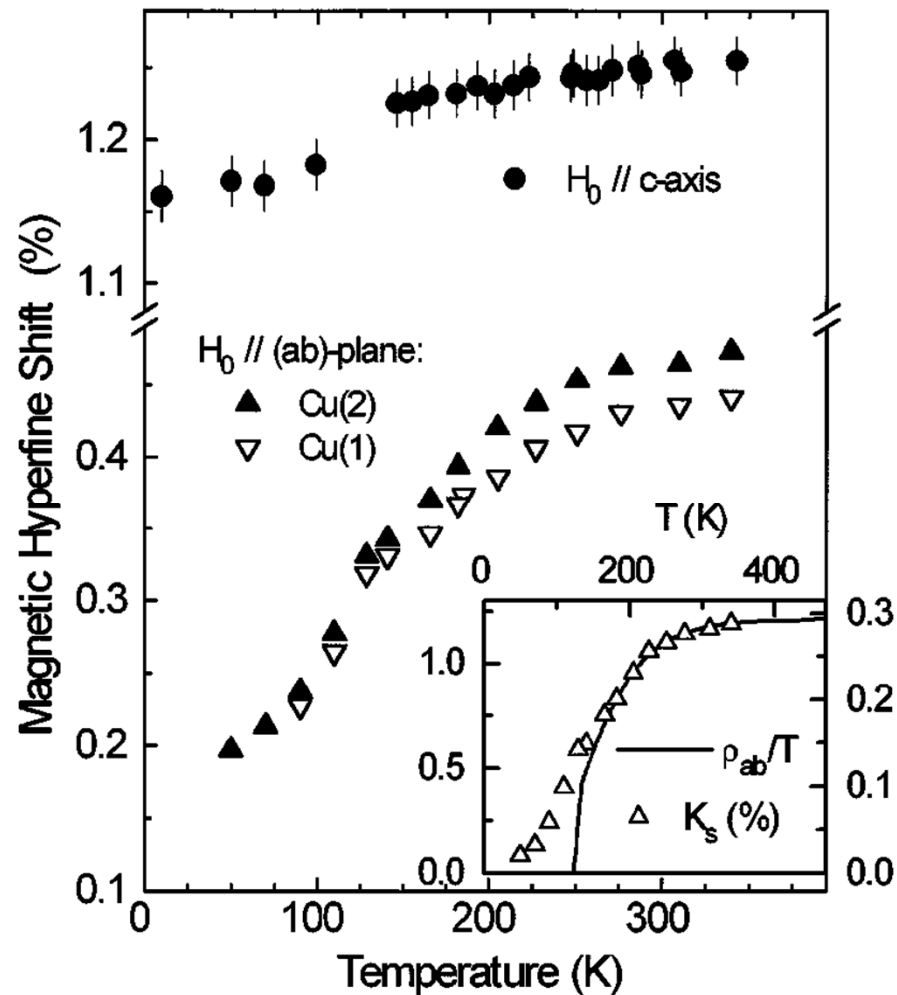
- Pseudogap: not an ordinary metal
 - Experimental evidence (Tallon)
- Link between various experiments
 - Self-energy etc (Dessau)
- Theories of the pseudogap
 - Broadened first-order transition
 - Precursor of LRO
 - Strong correlations: Mott physics



Pseudogap: not an ordinary metal



Spin susceptibility (Knight shift): Pseudogap



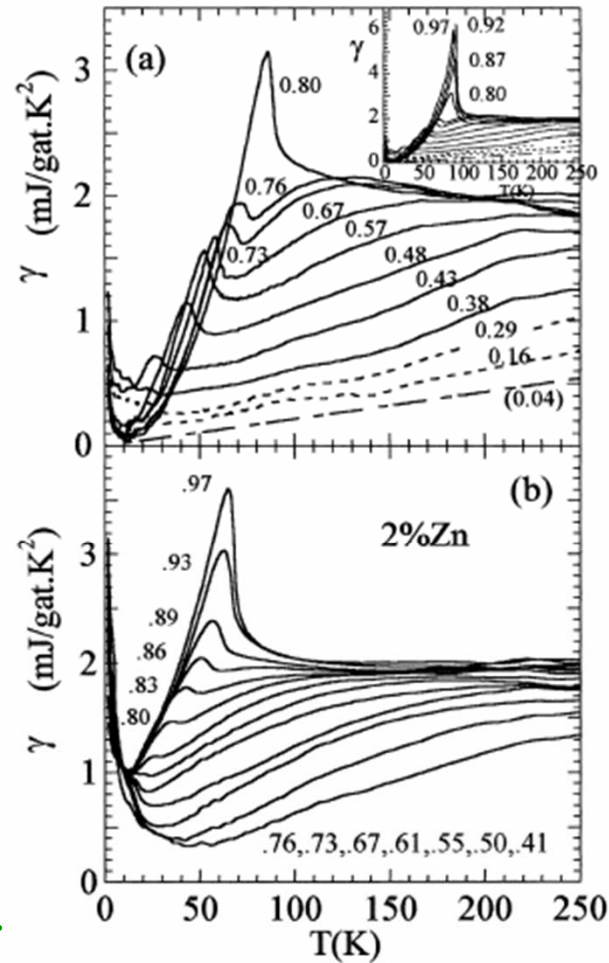
Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



UNIVERSITÉ DE
SHERBROOKE

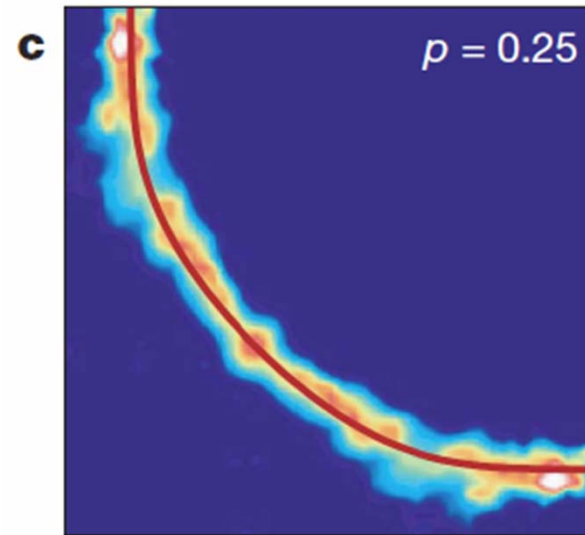
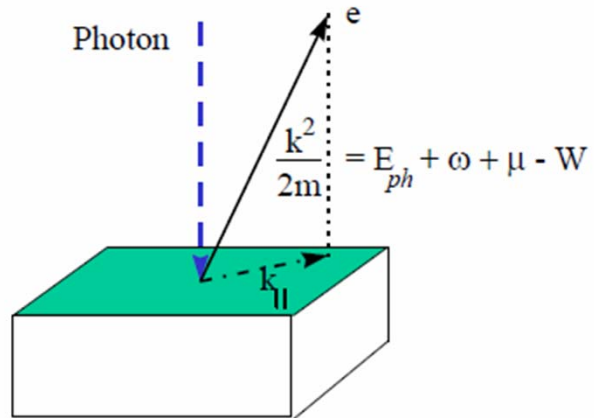
Specific Heat



Loram et al. J. Phys. Chem.
Solids, **62**, 59 (2001)

Fig. 5. γ vs T for: (a) YBa₂Cu₃O_{6+x}; and (b) YBa₂(Cu_{0.98}Zn_{0.02})₃O_{6+x}. Labels show x [7].

Fermi surface (ARPES)



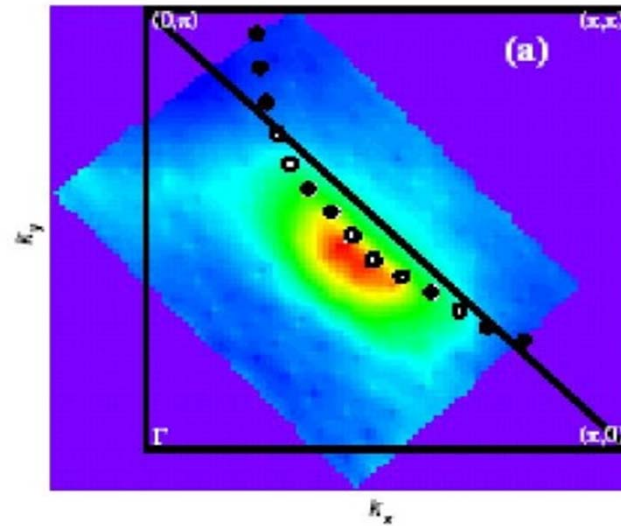
Tl 2201

Platé *et al.* PRL **95**, 077001 (2005)



Fermi surface?? ARPES: (Pseudogap)

Hole-doped, 10%

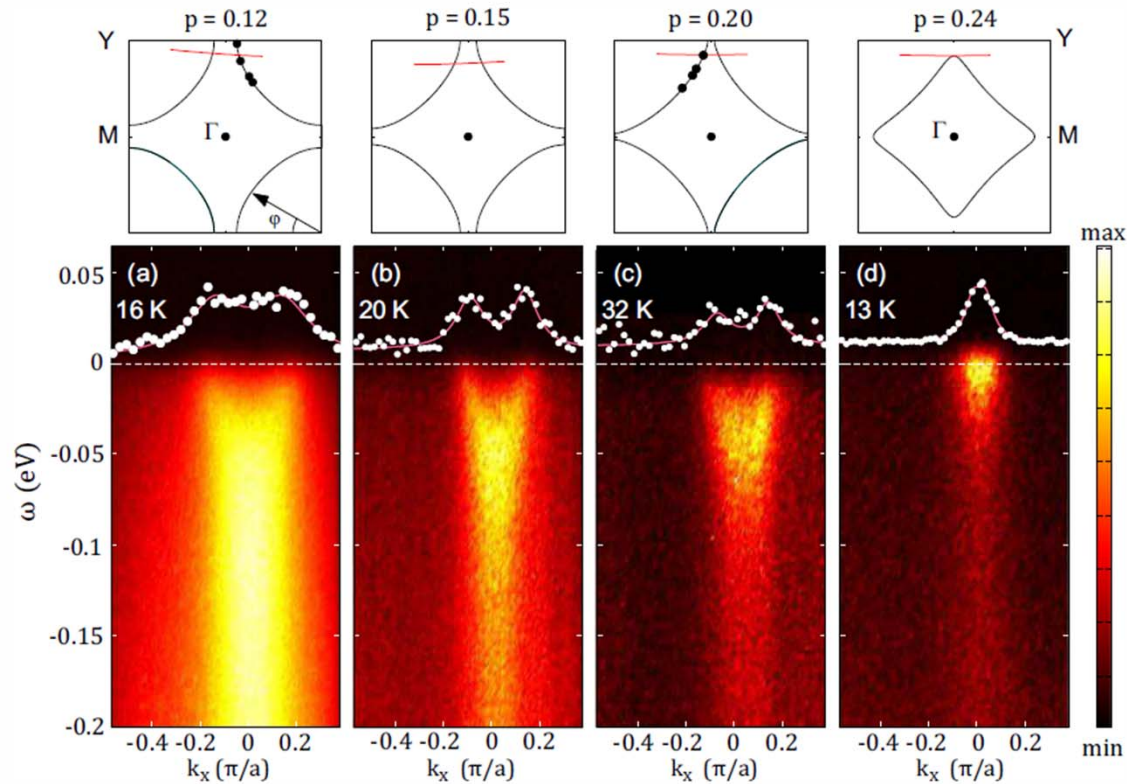


F. Ronning et al. Jan. 2002, $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$

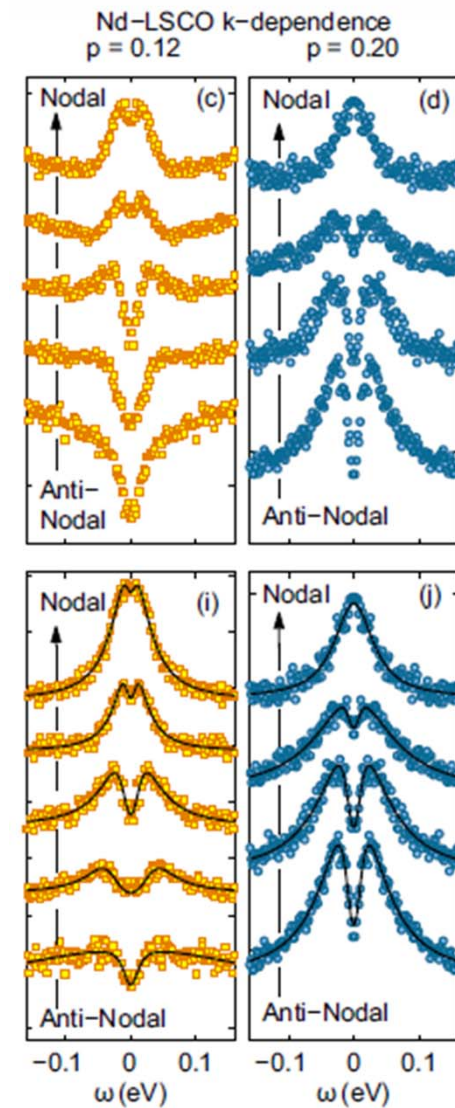
Ronning *et al.* (PRB 2003)



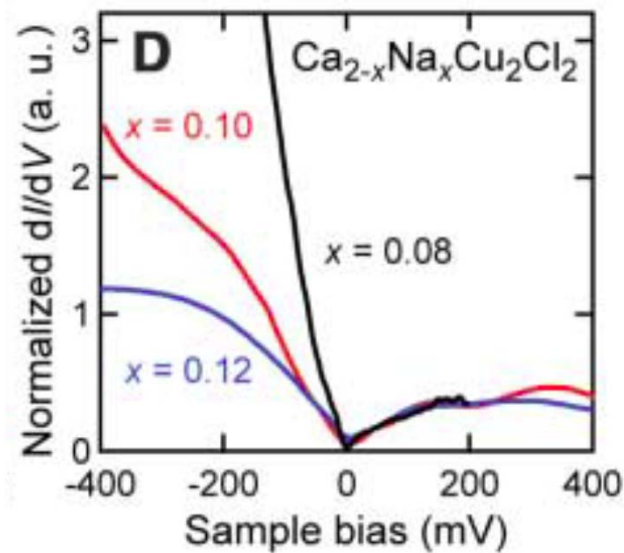
ARPES pseudogap Nd-LSCO



Matt *et al.* PRB 92, 134524 (2015)

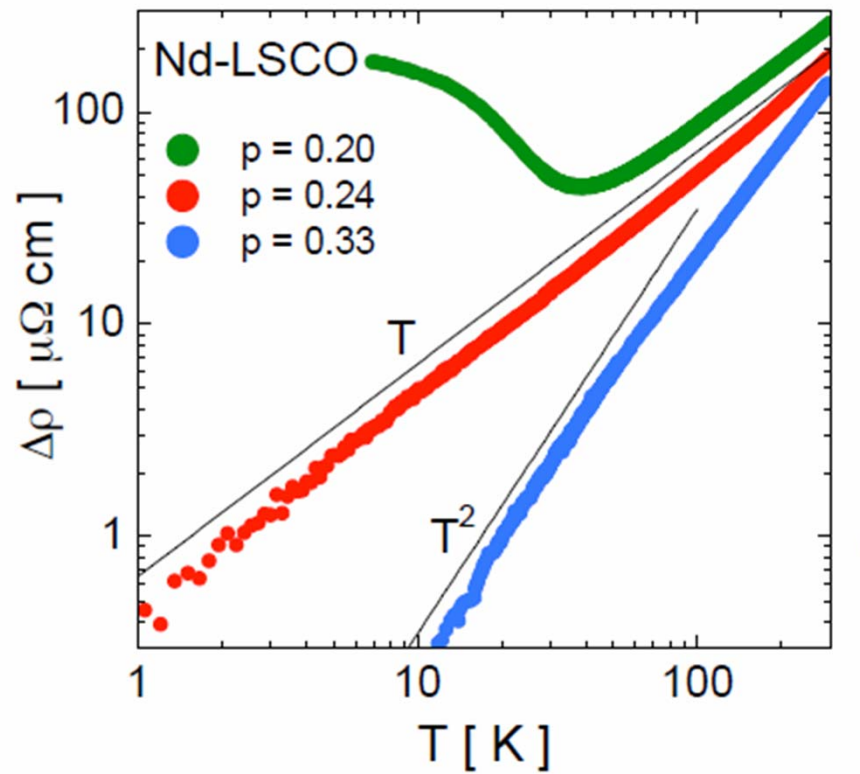


Local density of states (STM)



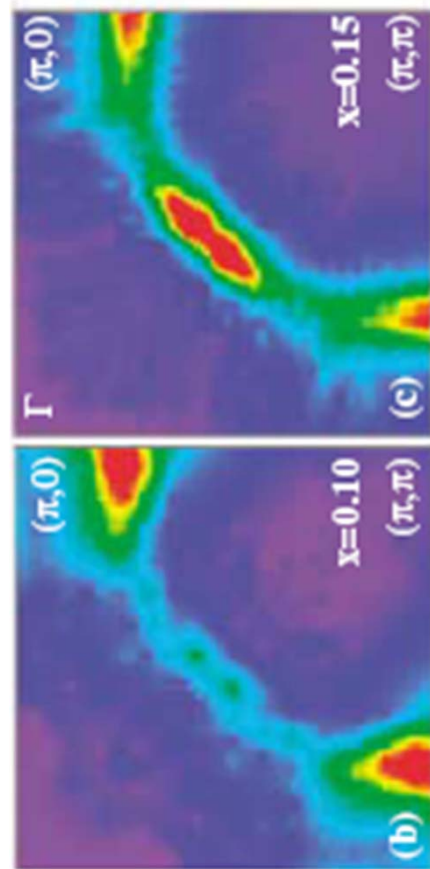
Khosaka et al. *Science* **315**, 1380 (2007);

Strange metal vs pseudogap



Daou R *et al.* Phys. Rev. B **79**,180505 (2009)

Electron-doped cuprates: Fermi surface



Armitage et al. PRL 2001

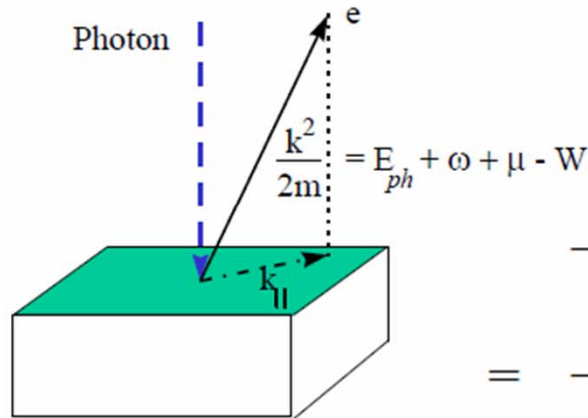


Outline

- Pseudogap: not an ordinary metal
 - Experimental evidence (Tallon)
- **Link between various experiments**
 - Self-energy etc (Dessau)
- Theories of the pseudogap
 - Broadened first-order transition
 - Precursor of LRO
 - Strong correlations: Mott physics



What does ARPES measure?



$$= - \sum_{\mathbf{k}'} \langle n | \otimes \langle \mathbf{k} | \otimes \langle 0 |_{em} \mathbf{j}_{\mathbf{k}'} \cdot \mathbf{A}_{-\mathbf{k}'} | m \rangle \otimes | 0 \rangle \otimes | 1_{\mathbf{q}} \rangle_{em} .$$

$$= - \sum_{\mathbf{k}'} \langle n | \otimes \langle \mathbf{k} | \mathbf{j}_{\mathbf{k}'} | m \rangle \otimes | 0 \rangle \cdot \langle 0 |_{em} \mathbf{A}_{-\mathbf{k}'} | 1_{\mathbf{q}} \rangle_{em}$$

$$\frac{\partial^2 \sigma}{\partial \Omega \partial \omega} \propto \sum_{mn} e^{-\beta K_m} |\langle n | c_{\mathbf{k}_{||}} | m \rangle|^2 \delta(\omega + \mu - (E_m - E_n))$$

$$\propto \sum_{mn} e^{-\beta K_m} |\langle n | c_{\mathbf{k}_{||}} | m \rangle|^2 \delta(\omega - (K_m - K_n))$$

$$\propto \int dt e^{i\omega t} \langle c_{\mathbf{k}_{||}}^\dagger c_{\mathbf{k}_{||}}(t) \rangle .$$

$$\frac{\partial^2 \sigma}{\partial \Omega \partial \omega} \propto f(\omega) A(\mathbf{k}_{||}, \omega)$$

$$c_{\mathbf{k}}(t) = e^{i\hat{H}t} c_{\mathbf{k}} e^{-i\hat{H}t} \quad ; \quad c_{\mathbf{k}}^\dagger(t) = e^{i\hat{H}t} c_{\mathbf{k}}^\dagger e^{-i\hat{H}t} .$$



Self-energy

$$G^R(\mathbf{k}; t) = -i \left\langle \left\{ c_{\mathbf{k}}(t), c_{\mathbf{k}}^\dagger \right\} \right\rangle \theta(t)$$

Relation to DOS
to kinetic energy...

Non-interacting case

$$c_{\mathbf{k}}(t) = e^{-i\epsilon_{\mathbf{k}}t} c_{\mathbf{k}}$$

$$G^R(\mathbf{k}; \omega) = \frac{1}{\omega + i\eta - \epsilon_{\mathbf{k}}} \quad A(\omega) = -2 \operatorname{Im} G^R(\omega)$$

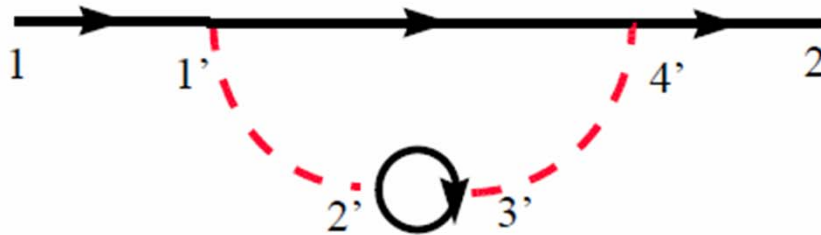
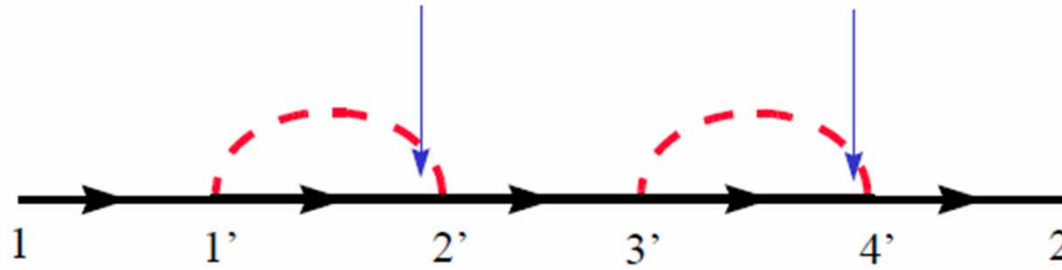
Effect of interactions

$$G^R(\mathbf{k}, \omega) = \frac{1}{\omega + i\eta - \zeta_{\mathbf{k}} - \Sigma^R(\mathbf{k}, \omega)}$$

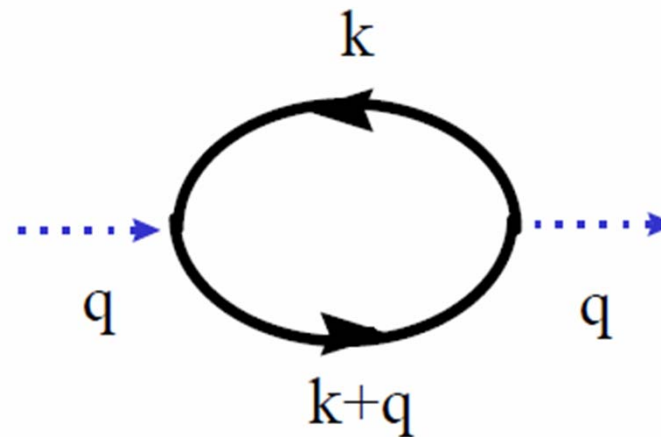
$$\begin{aligned} A(\mathbf{k}, \omega) &= -2 \operatorname{Im} G^R(\mathbf{k}, \omega) \\ &= \frac{-2 \operatorname{Im} \Sigma^R(\mathbf{k}, \omega)}{\left(\omega - \zeta_{\mathbf{k}} - \operatorname{Re} \Sigma^R(\mathbf{k}, \omega) \right)^2 + \left(\operatorname{Im} \Sigma^R(\mathbf{k}, \omega) \right)^2} \end{aligned}$$



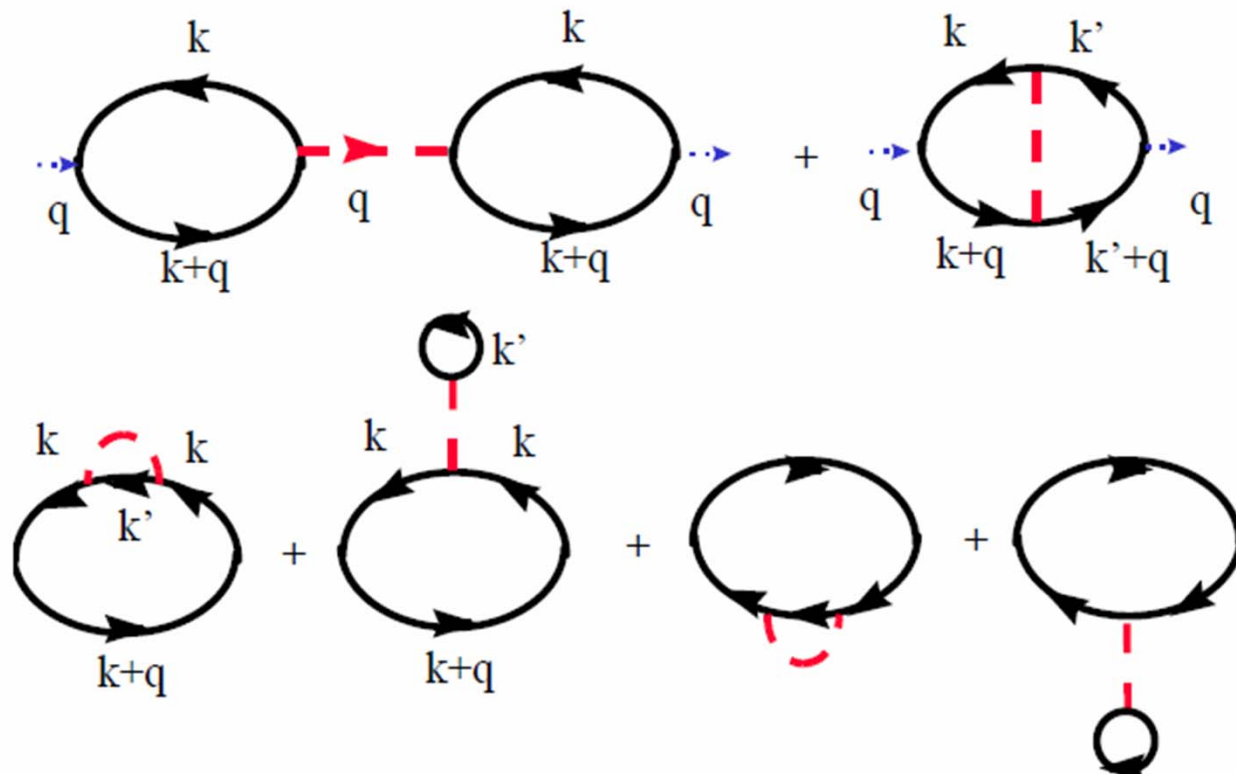
Relation to other observables



Conductivity



Vertex corrections



Outline

- Pseudogap: not an ordinary metal
 - Experimental evidence (Tallon)
- Link between various experiments
 - Self-energy etc (Dessau)
- Theories of the pseudogap
 - Broadened first-order transition
 - Precursor of LRO
 - Strong correlations: Mott physics

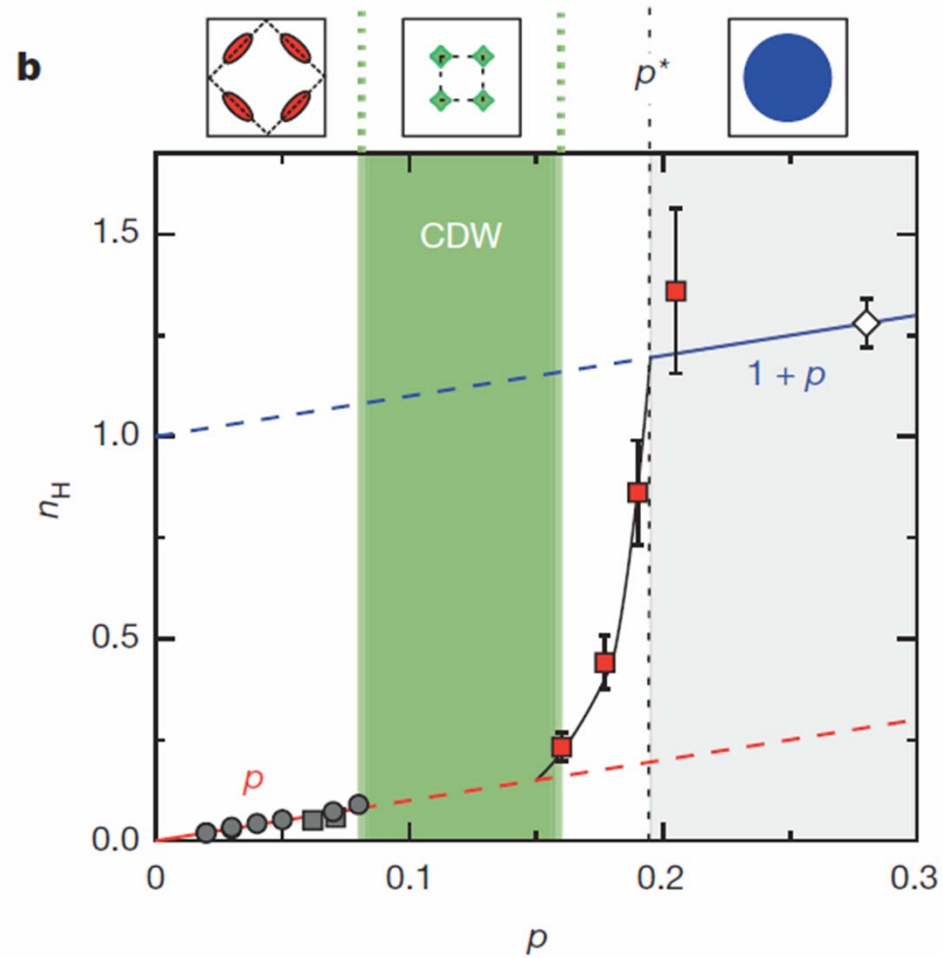


1. First-order transition broadened by disorder

- Loop current order ($\mathbf{q} = \mathbf{0}$) (Varma)
 - Nematic order ($\mathbf{q} = \mathbf{0}$) (Kivelson)
 - Charge order ?
-
- (See Simon Verret and Maxime Charlebois)



Not charge order



Badoux *et al.* Nature, March 2016



More generally

PHYSICAL REVIEW B 89, 201104(R) (2014)



Pseudogap in $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$ is not bounded by a line of phase transitions: Thermodynamic evidence

J. R. Cooper,¹ J. W. Loram,¹ I. Kokanović,^{1,2} J. G. Storey,³ and J. L. Tallon³



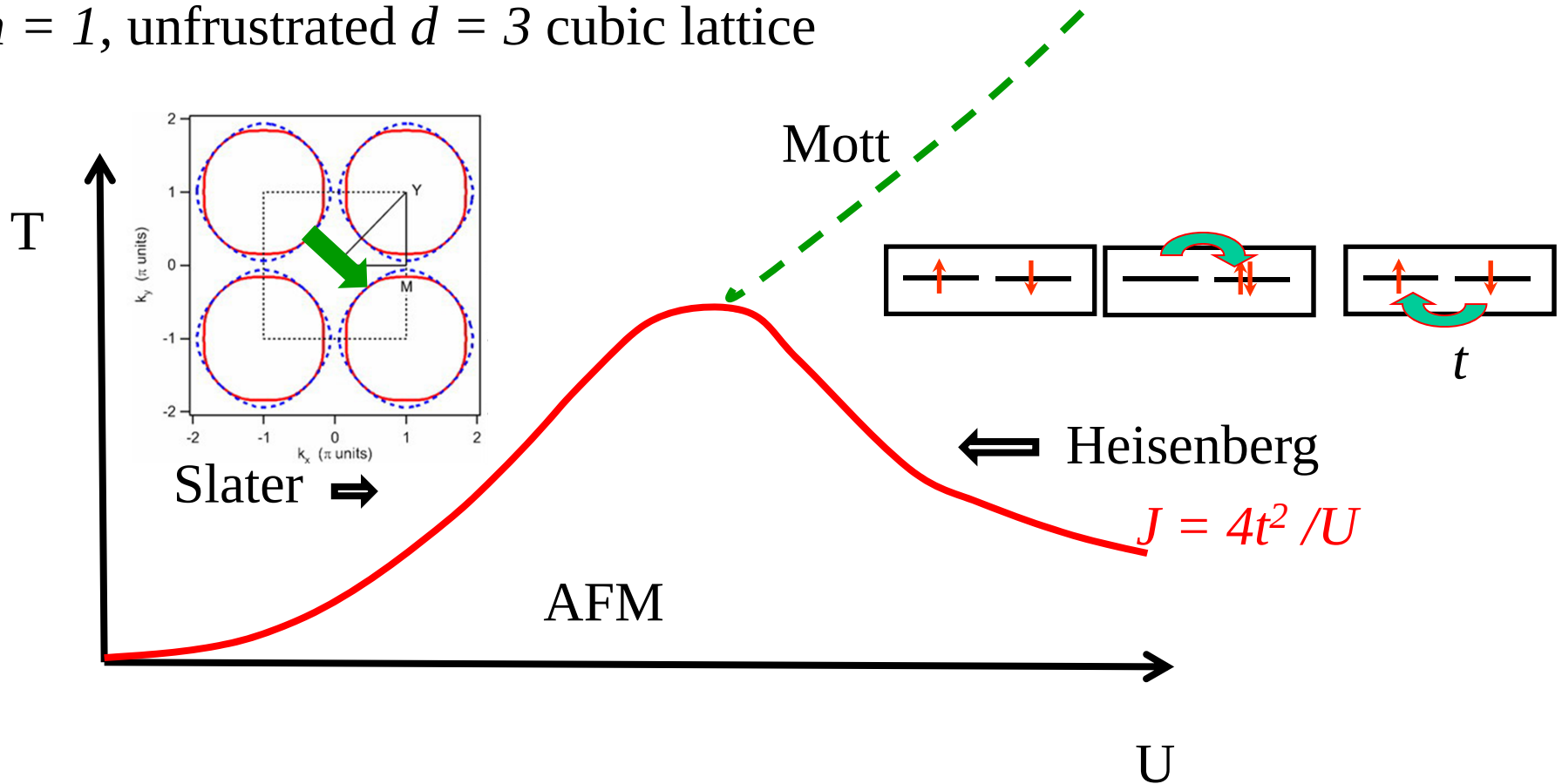
UNIVERSITÉ DE
SHERBROOKE

2. Precursor of $d=2$ long-range order



Weak vs Strong correlations

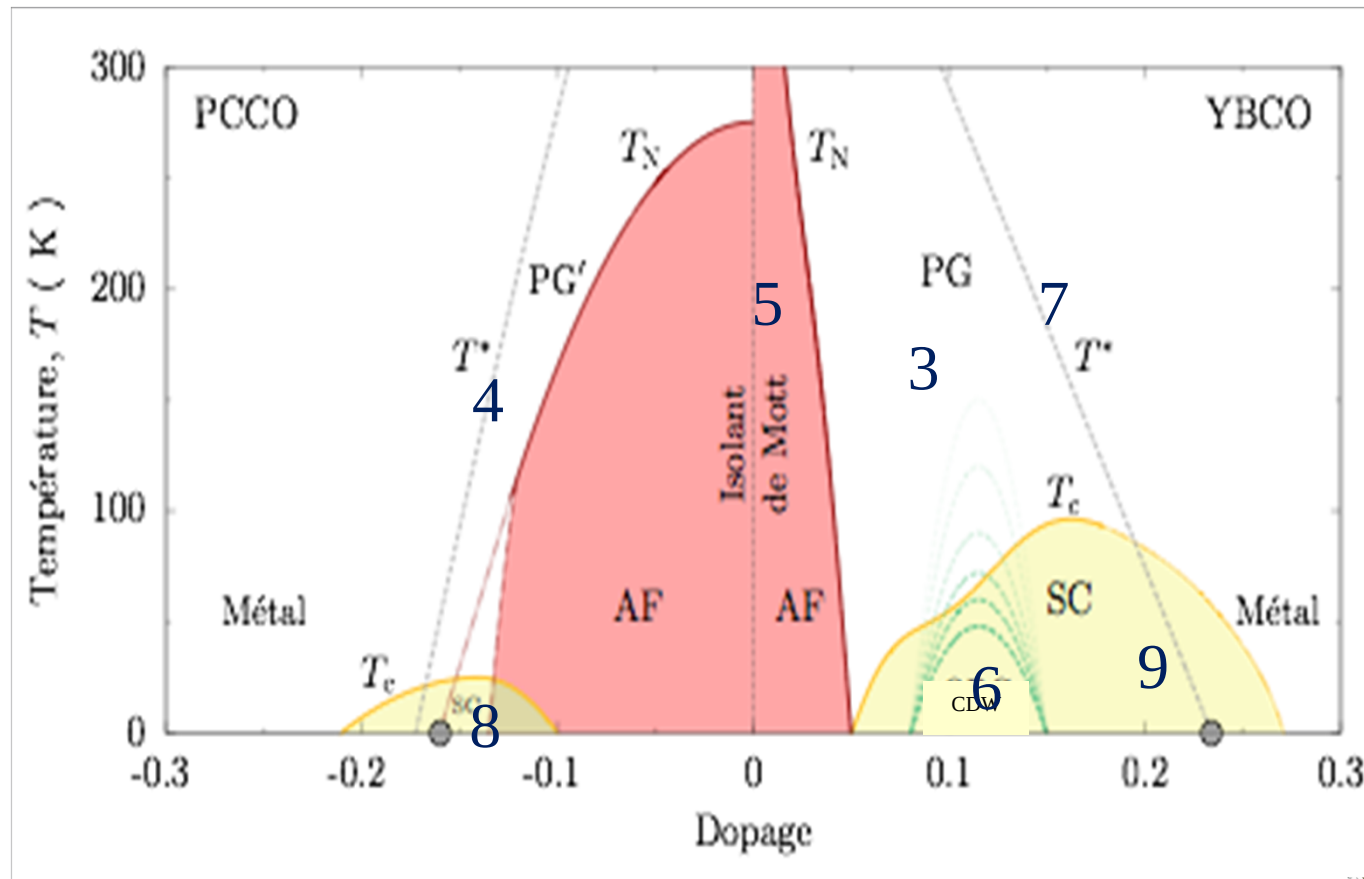
$n = 1$, unfrustrated $d = 3$ cubic lattice



Outline

For references, September 2013 Julich summer school
Strongly Correlated Superconductivity

<http://www.cond-mat.de/events/correl13/manuscripts/tremblay.pdf>



2. Doping an itinerant antiferromagnet

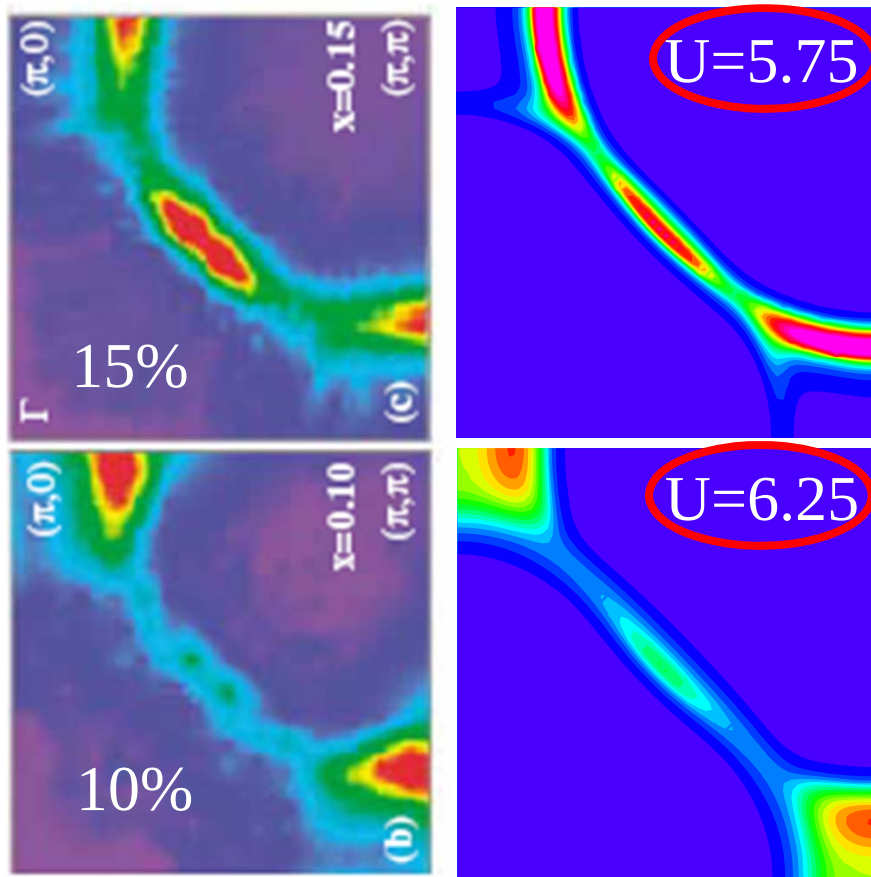
Quantum critical point

Electron-doped cuprates

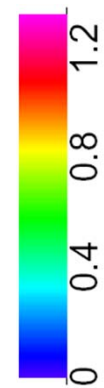


Fermi surface plots

Hubbard repulsion U has to...



be not too large



increase for
smaller doping

Hankevych, Kyung, A.-M.S.T., PRL, sept. 2004

B.Kyung *et al.*, PRB **68**, 174502 (2003)

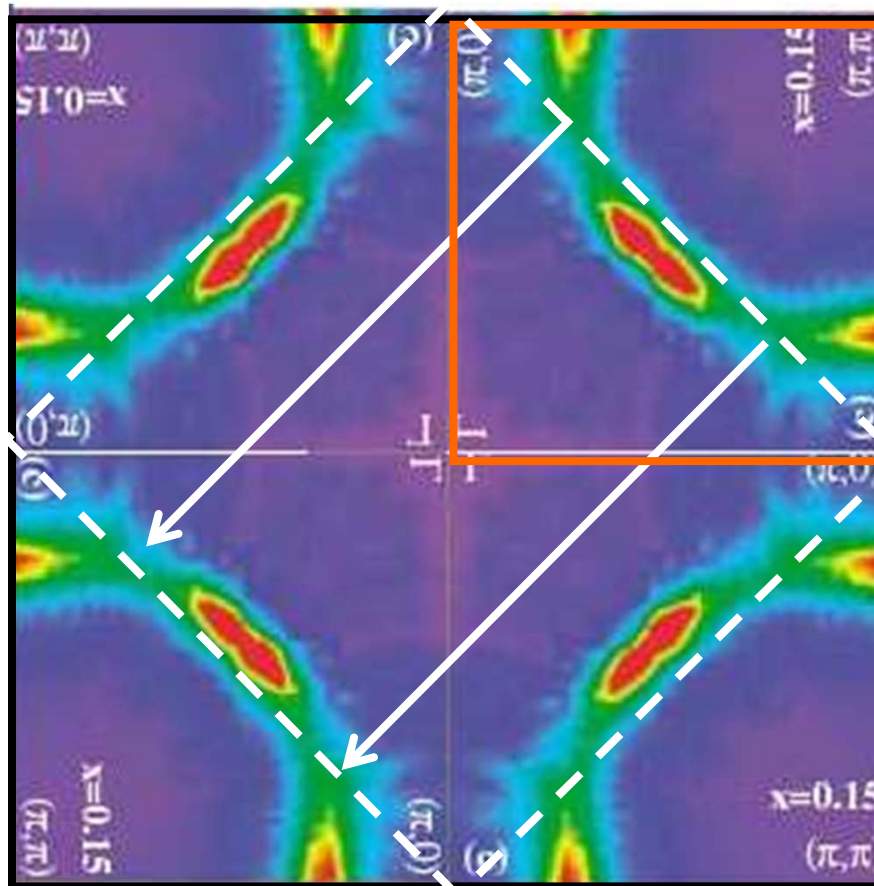


UNIVERSITÉ DE
SHERBROOKE

Hot spots from AFM quasi-static scattering

Mermin-Wagner

$d = 2$



Vilk, A.-M.S.T (1997)
Kyung, Hankevych,
A.-M.S.T., PRL, 2004

$$\xi^* = 2.6(2)\xi_{th}$$

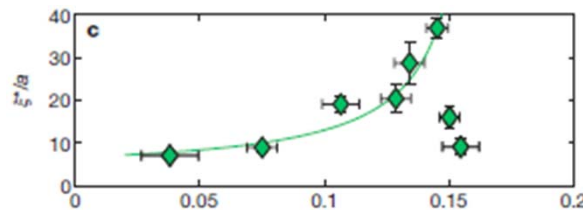
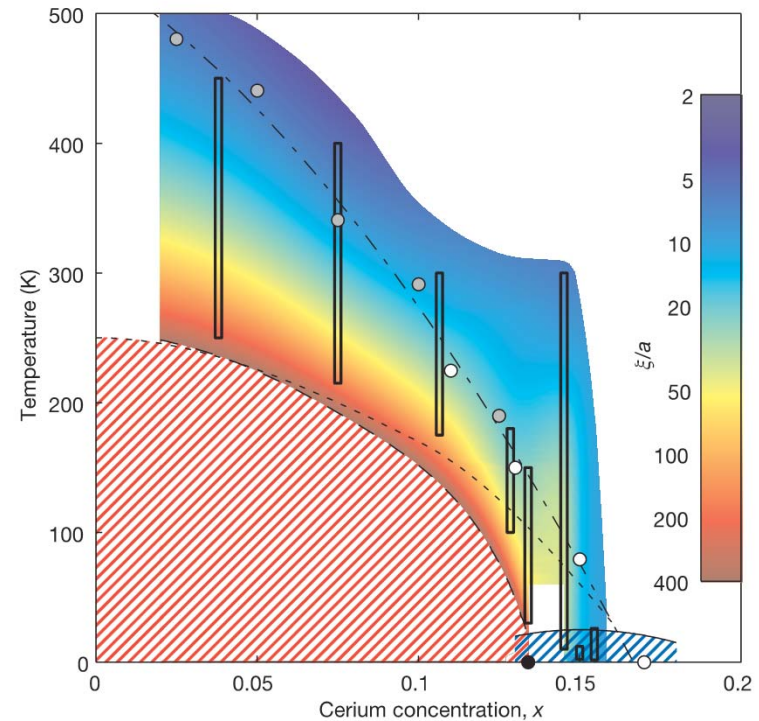
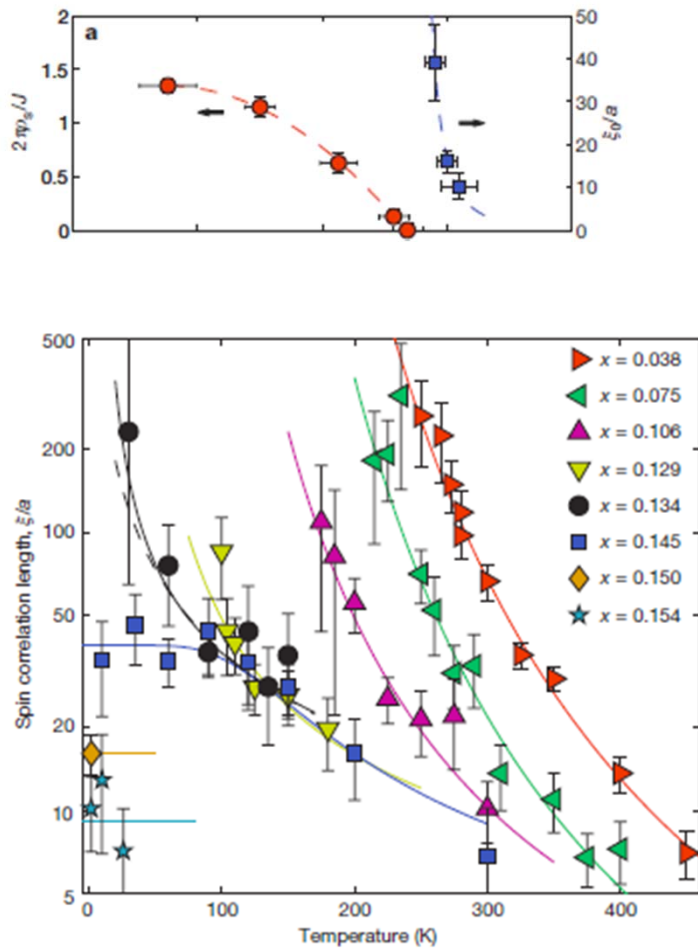
Motoyama, E. M. et al..
445, 186–189 (2007).

Armitage et al. PRL 2001

E-doped quantum critical

NCCCO

Motoyama, E. M. et al.. Nature 445, 186–189 (2007).



$$\xi^* = 2.6(2)\xi_{th}.$$



UNIVERSITÉ DE
SHERBROOKE

Precursor of d-wave superconducting fluctuations

- In a 15 K interval above T_c

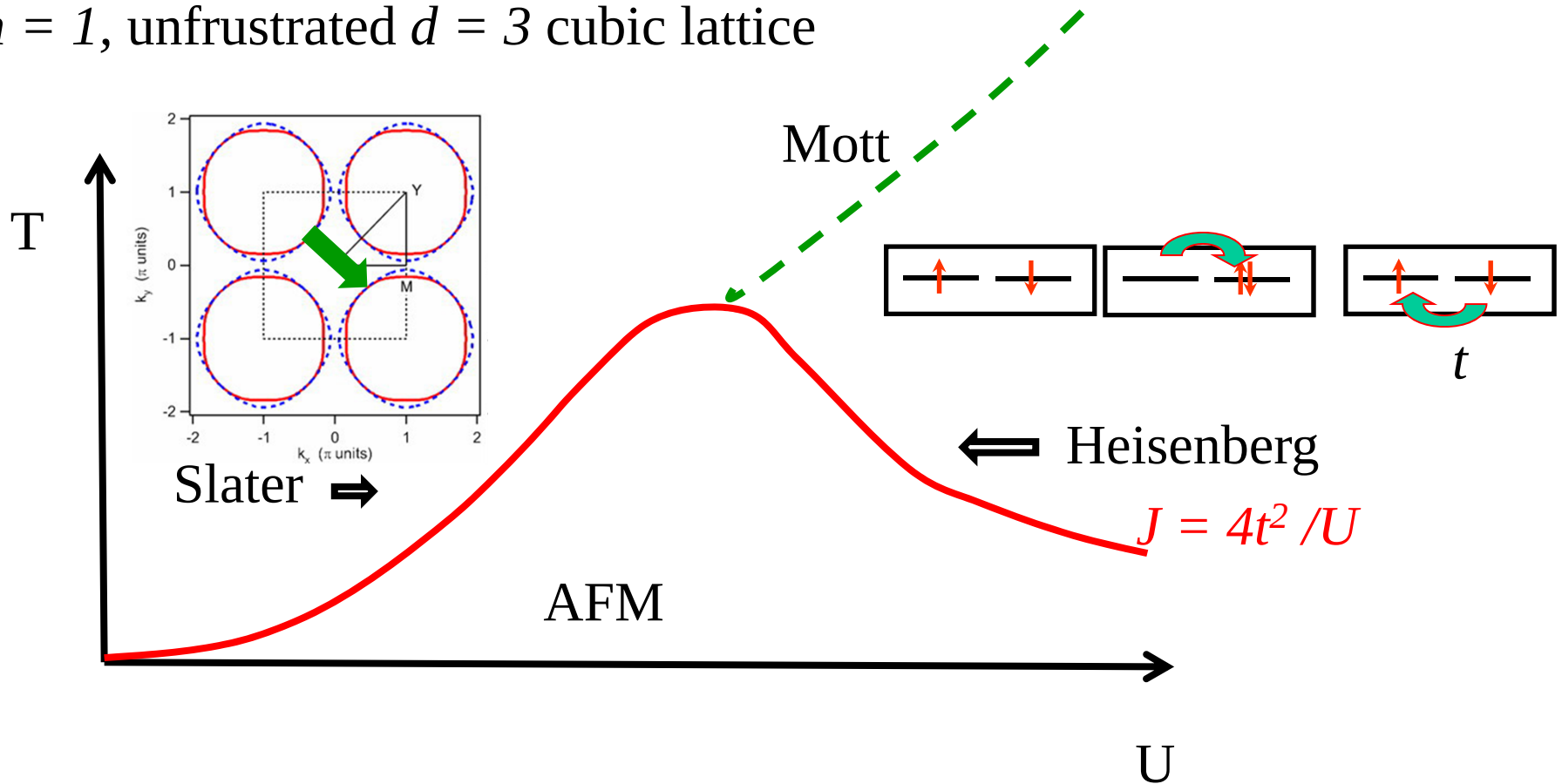


3. Strong correlation physics (Mott insulator)



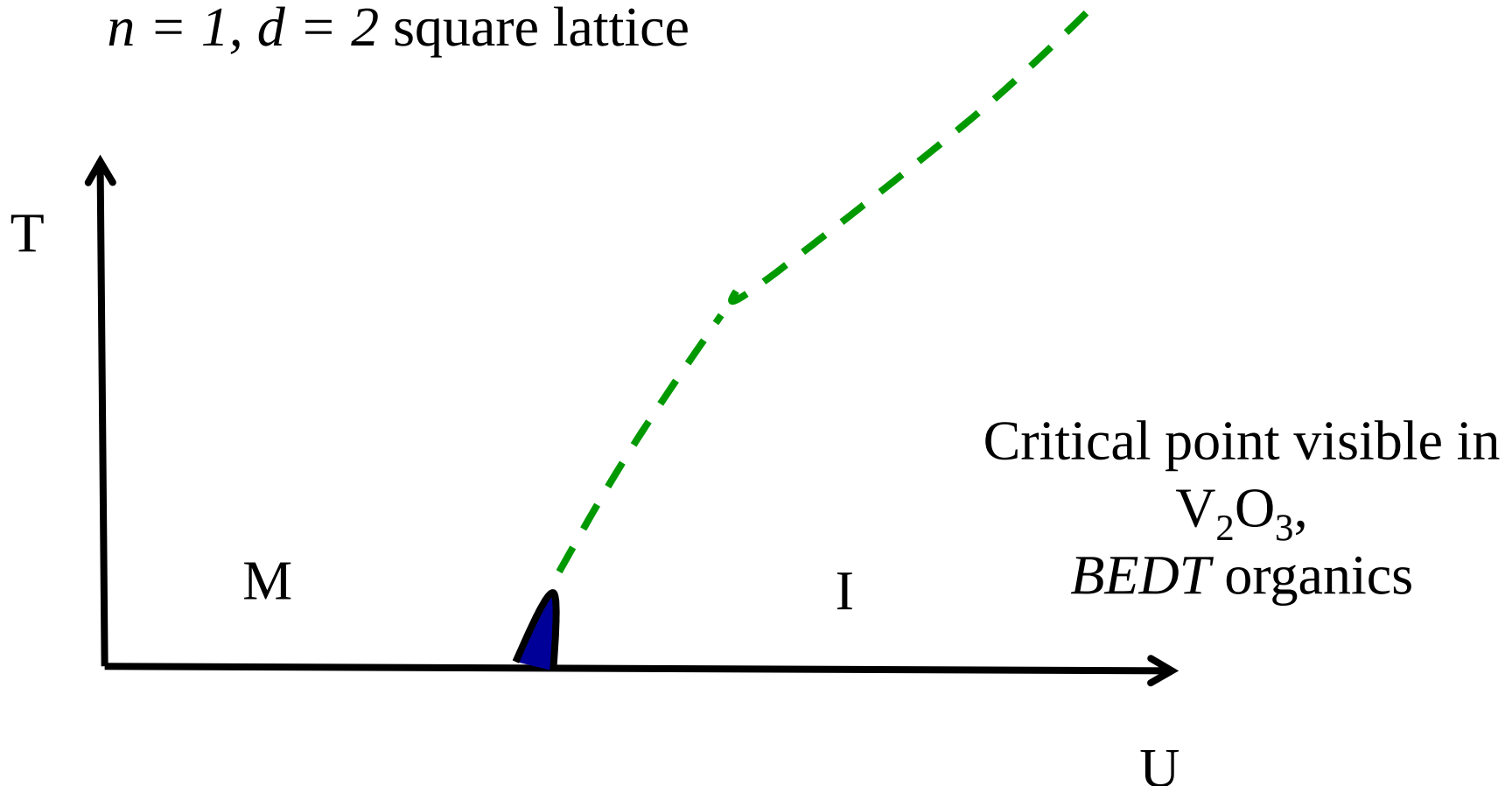
Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition

$n = 1, d = 2$ square lattice



Understanding finite temperature phase from a *mean-field theory* down to $T = 0$



YRZ

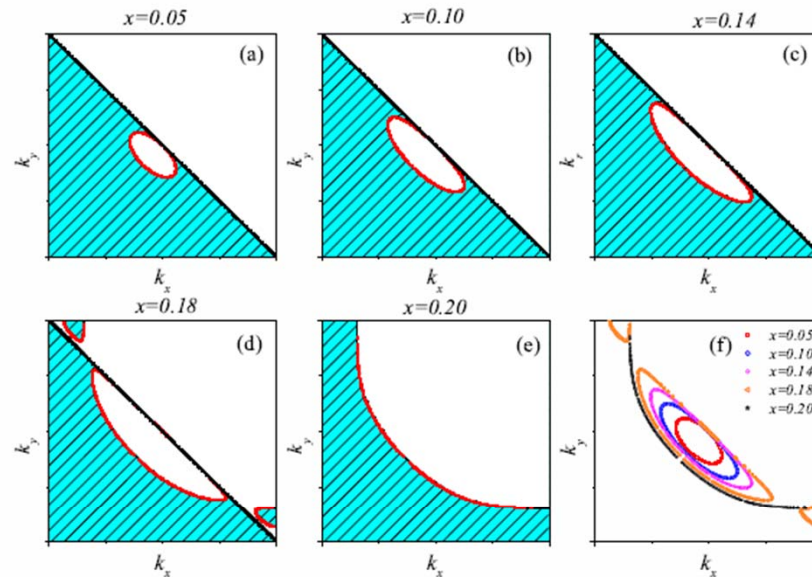


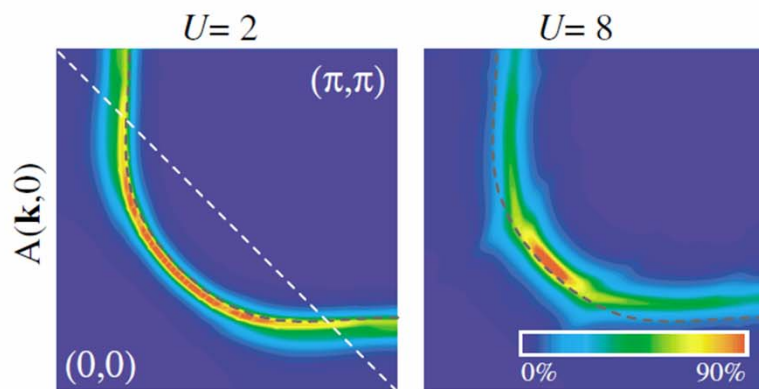
FIG. 1: Contours on which $G(\mathbf{k}, 0)$ changes sign at various hole concentrations x are shown in (a)-(e). In the shaded area $G(\mathbf{k}, 0) > 0$ satisfying the Luttinger Sum Rule. In the normal pseudogap phase, the Luttinger sum rule is violated. See also FL* Sachdev. The values of the parameters used here are given in Fig.2. The evolution of the contours of infinities in $G(\mathbf{k}, 0)$ is illustrated in (f).

Yang, Rice, Zhang PRB 73, 174501 (2006)

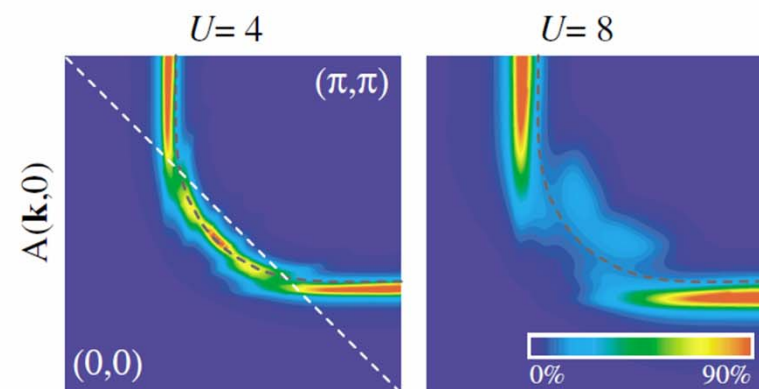


UNIVERSITÉ DE
SHERBROOKE

Quantum cluster approaches



17 % hole doped



17 % electron doped

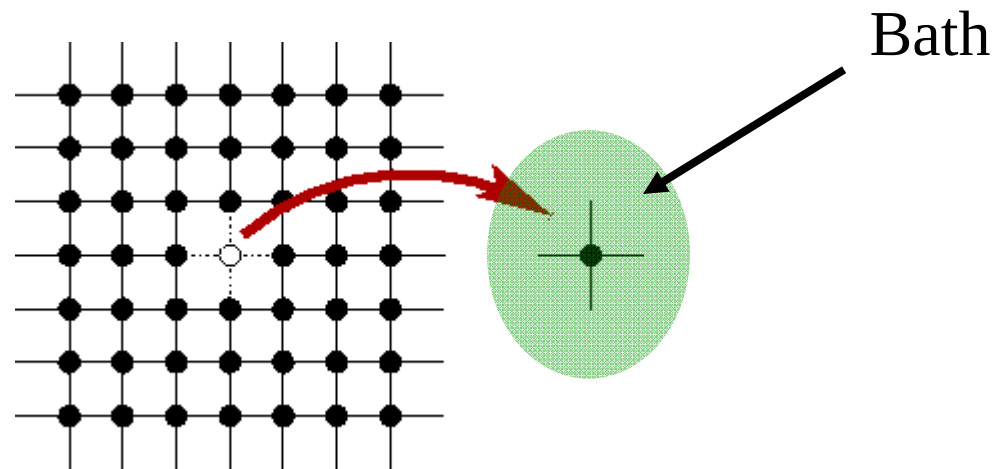
Sénéchal Tremblay, PRL 92, 126401 (2004)



Mott transition and Dynamical Mean-Field Theory.

The beginnings in $d = \text{infinity}$

- Compute scattering rate (self-energy) of impurity problem.
- Use that self-energy (ω dependent) for lattice.
- Project lattice on single-site and adjust bath so that single-site DOS obtained both ways be equal.



W. Metzner and D. Vollhardt, PRL (1989)

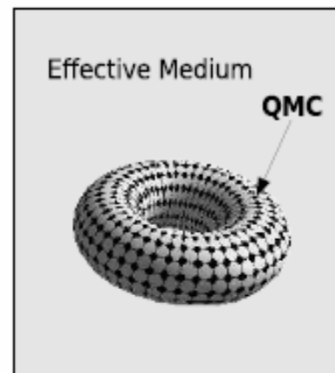
A. Georges and G. Kotliar, PRB (1992)

M. Jarrell PRB (1992)

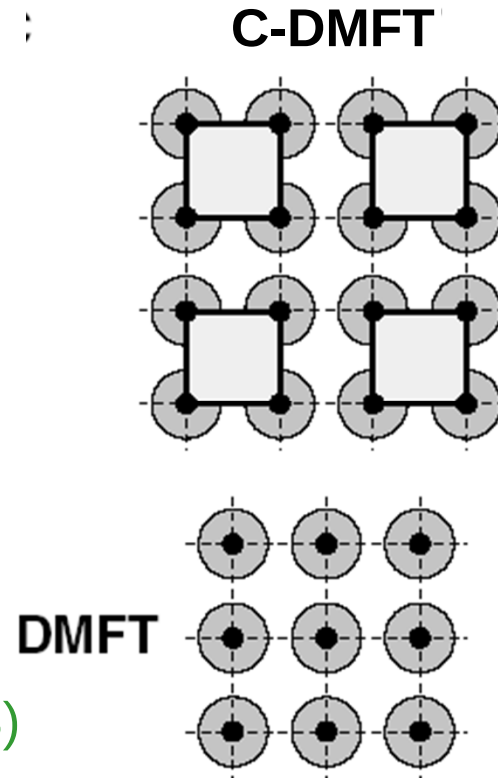
DMFT, ($d = 3$)



2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

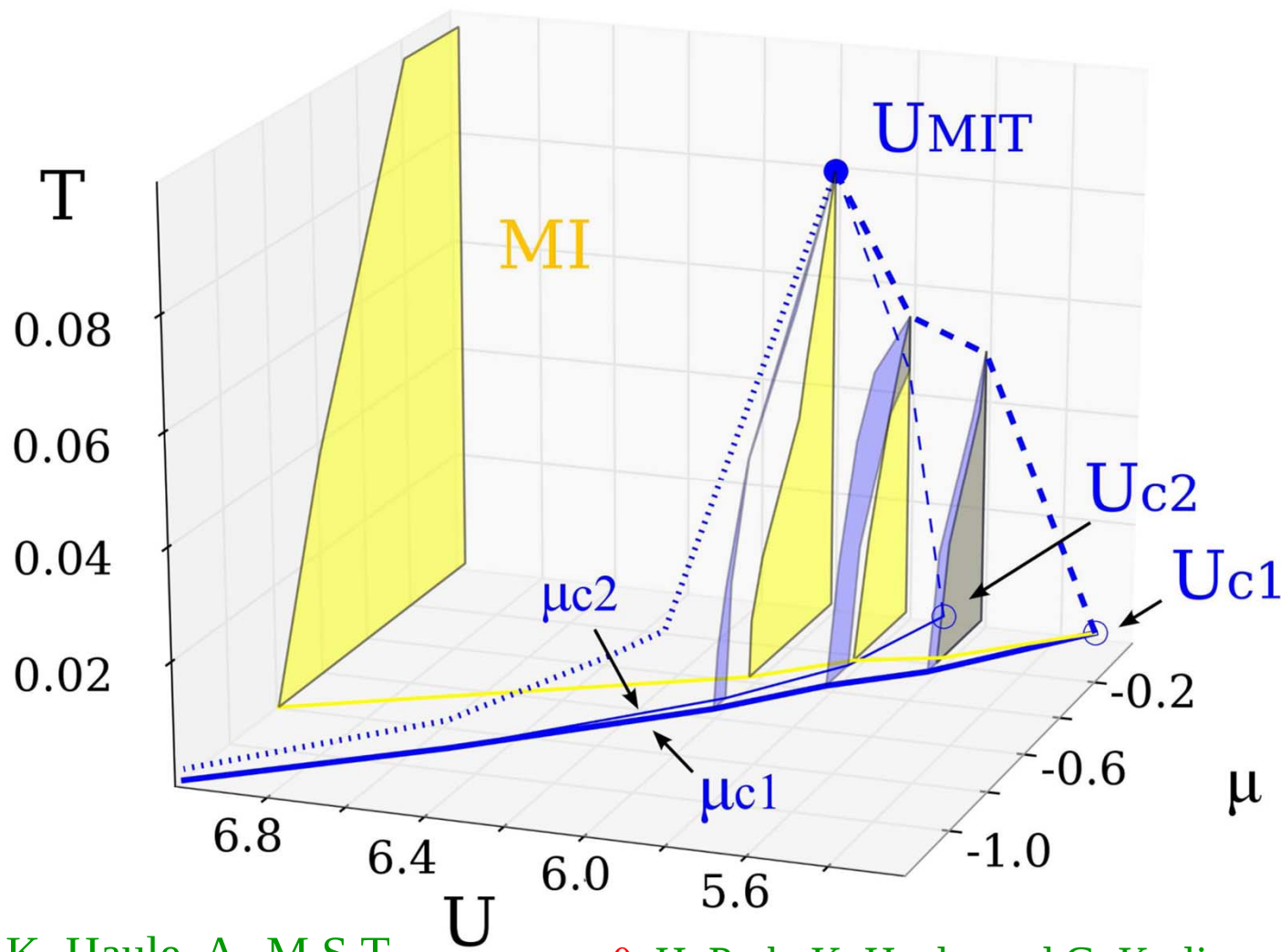
Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)



UNIVERSITÉ DE
SHERBROOKE

Normal state phase diagram



G. Sordi, K. Haule, A.-M.S.T
PRL, **104**, 226402 (2010)

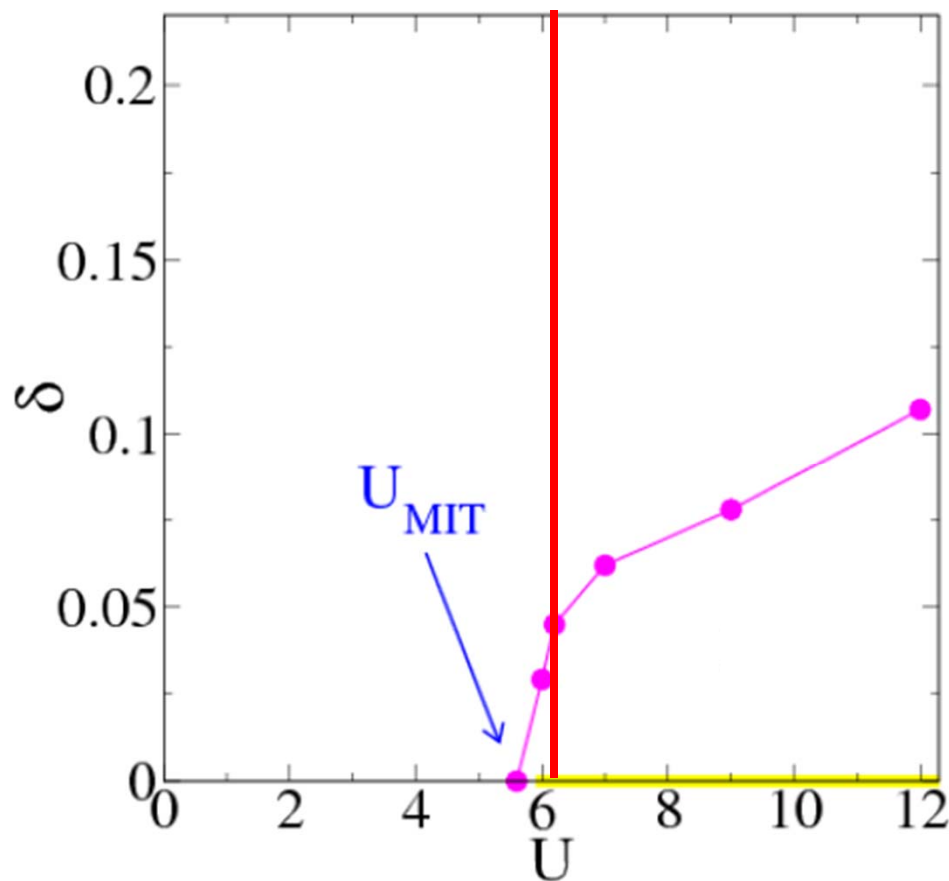
$\mu = 0$, H. Park, K. Haule, and G. Kotliar,
Phys. Rev. Lett. 101, 186403 (2008).



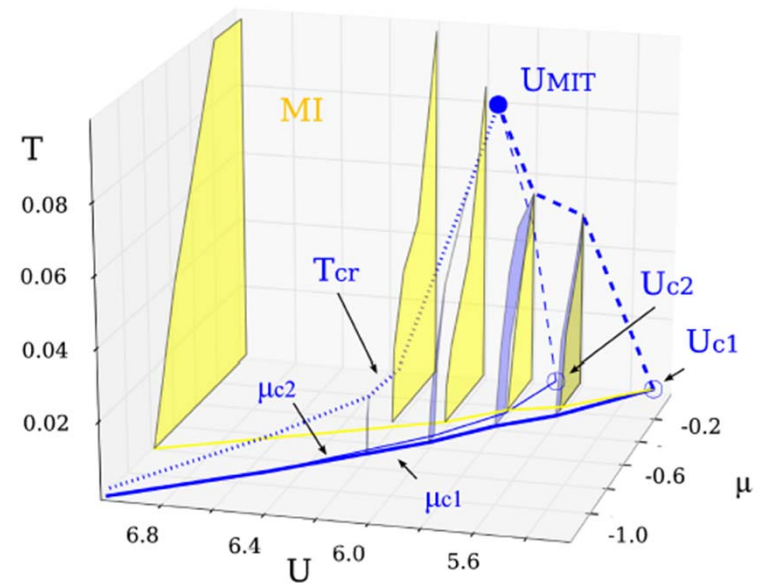
UNIVERSITÉ DE
SHERBROOKE

Link to Mott transition up to optimal doping

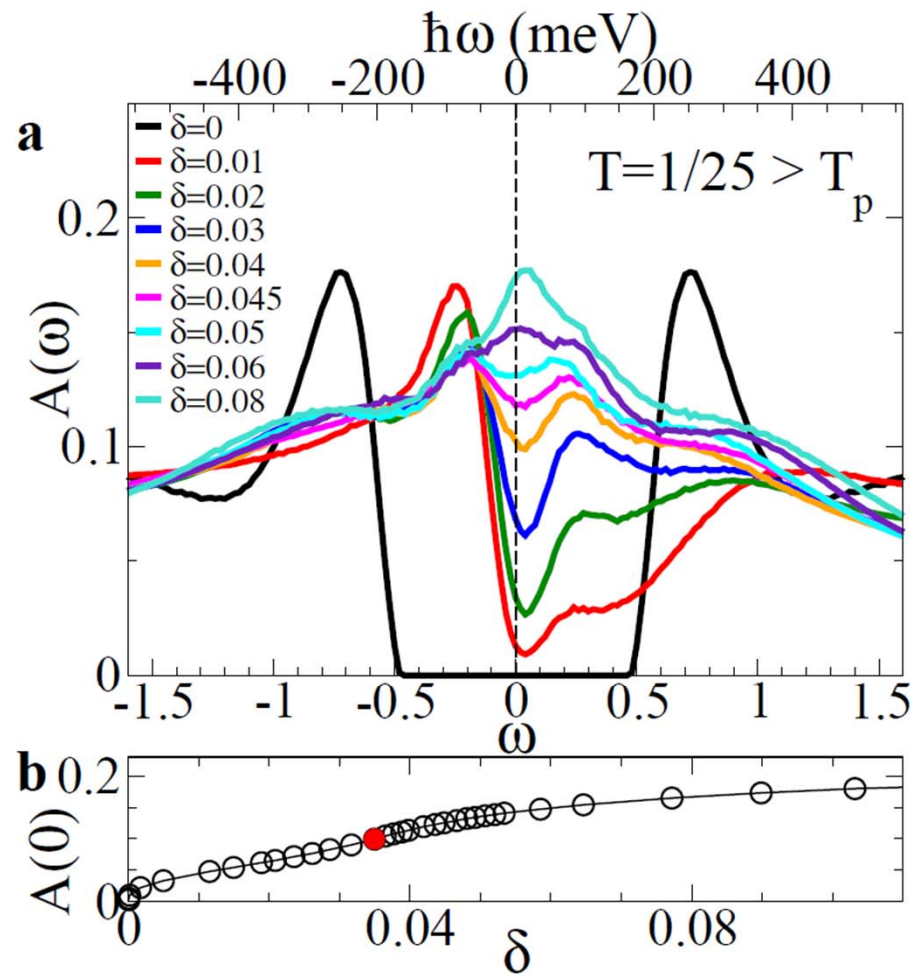
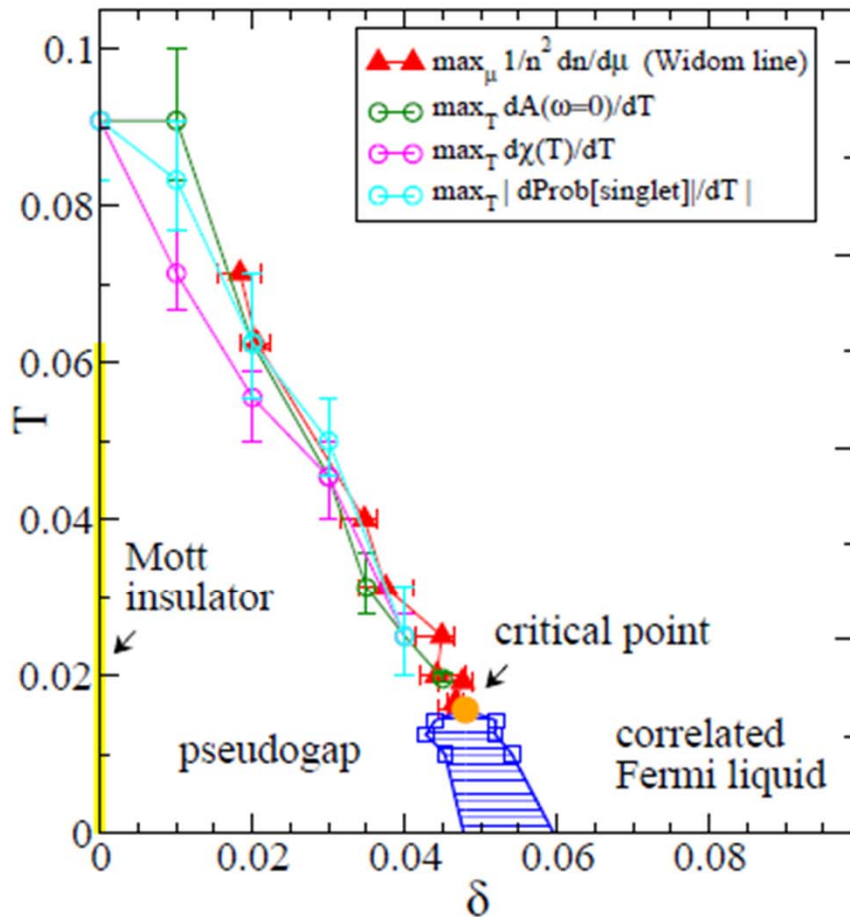
Doping dependence of critical point as a function of U



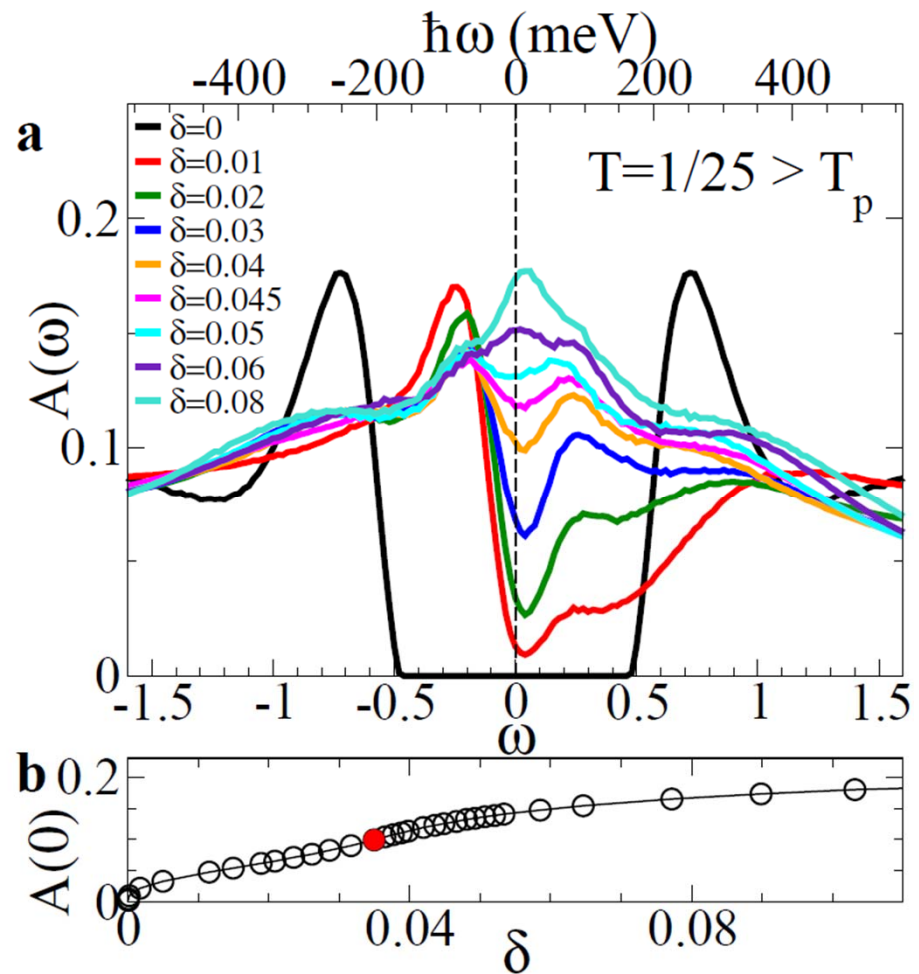
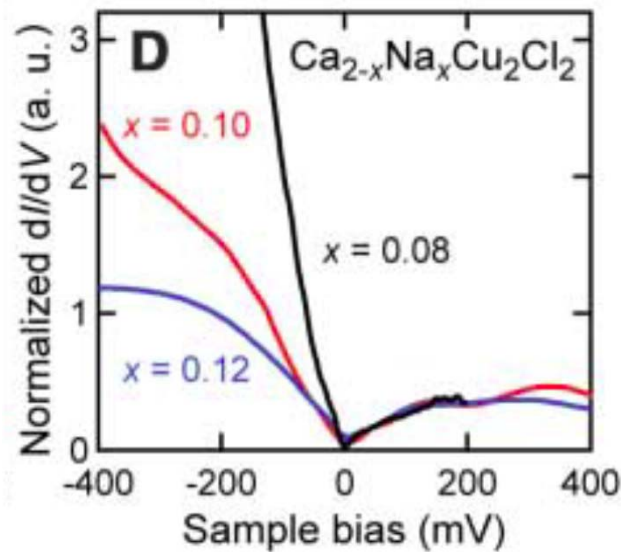
Smaller D and S



Density of states

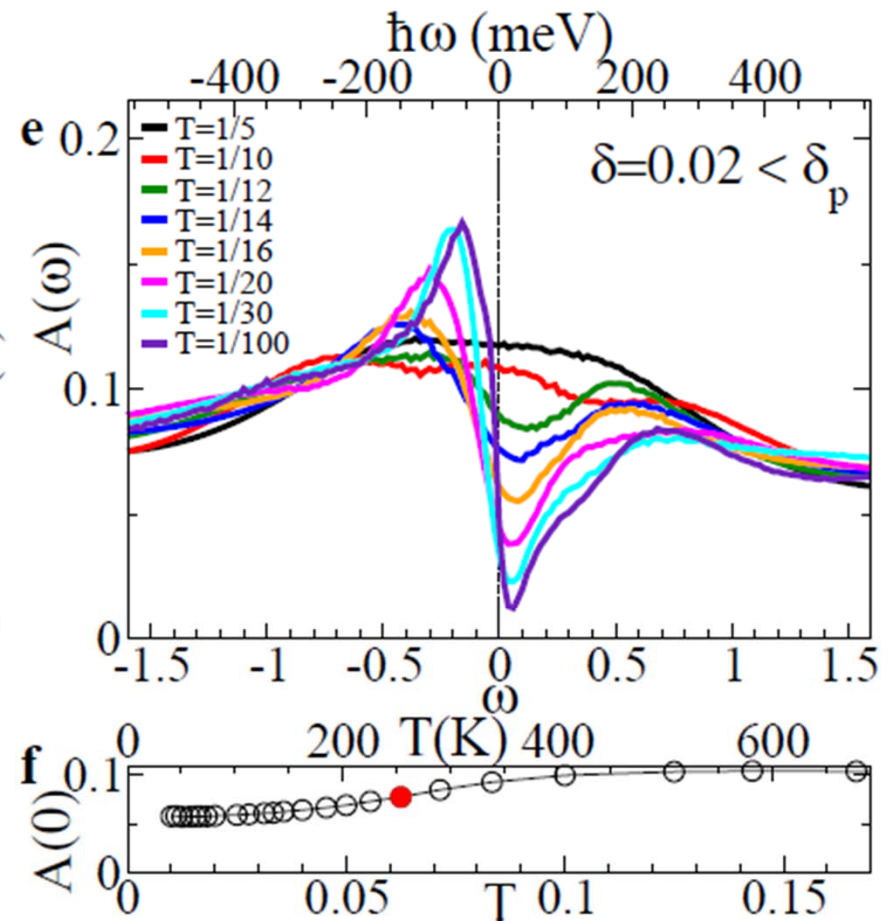
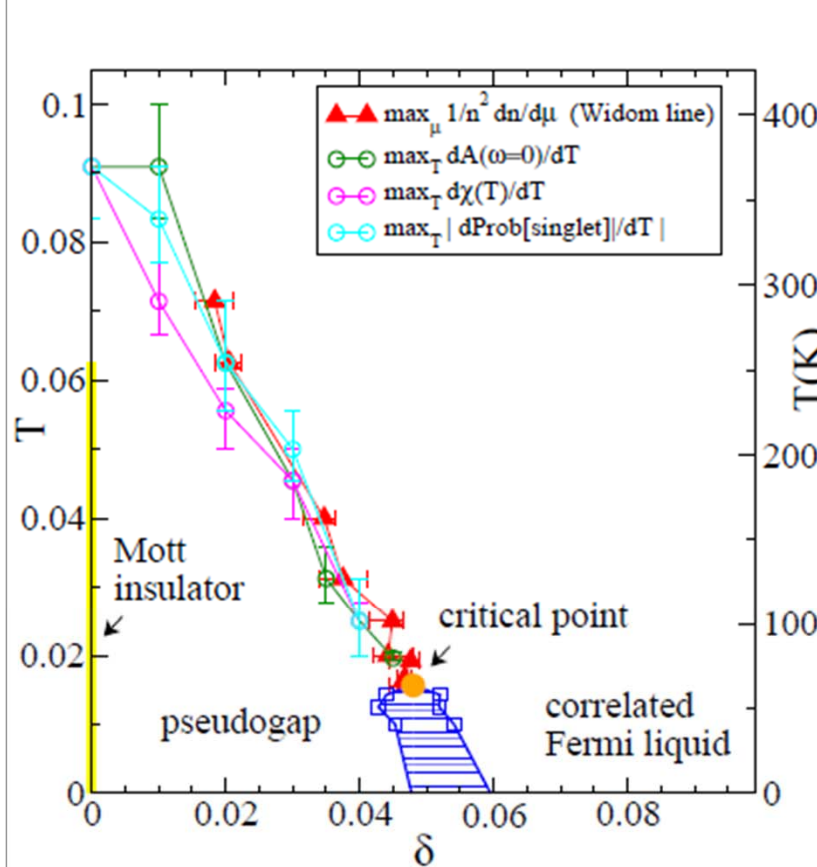


Density of states

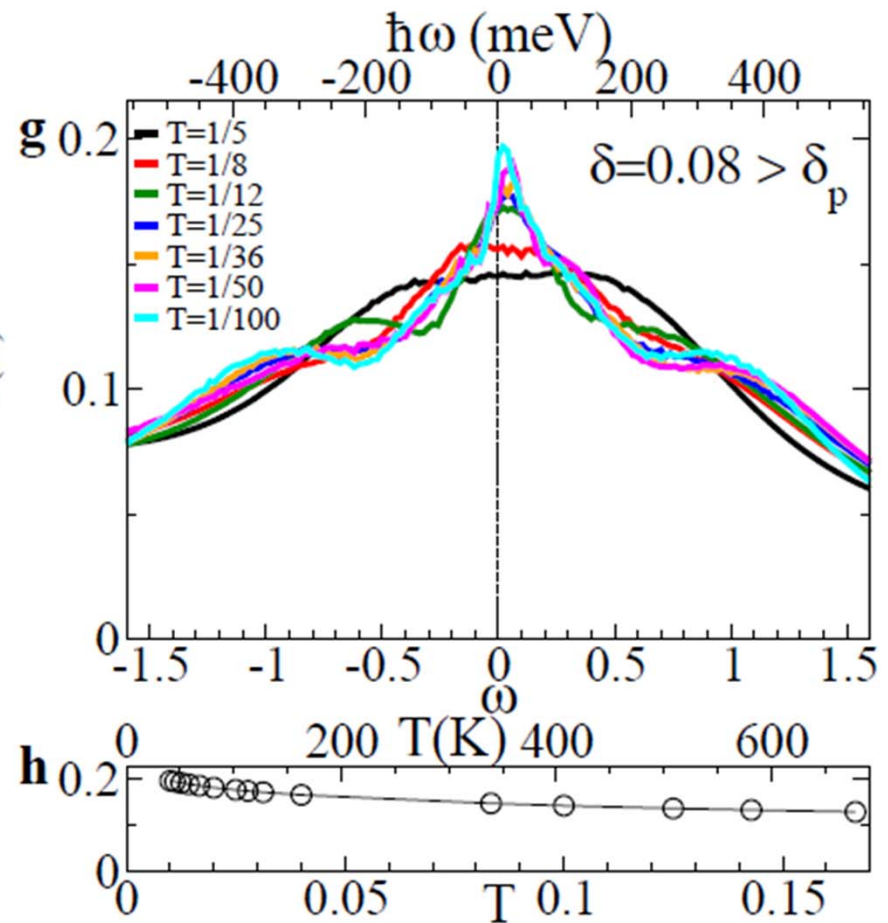
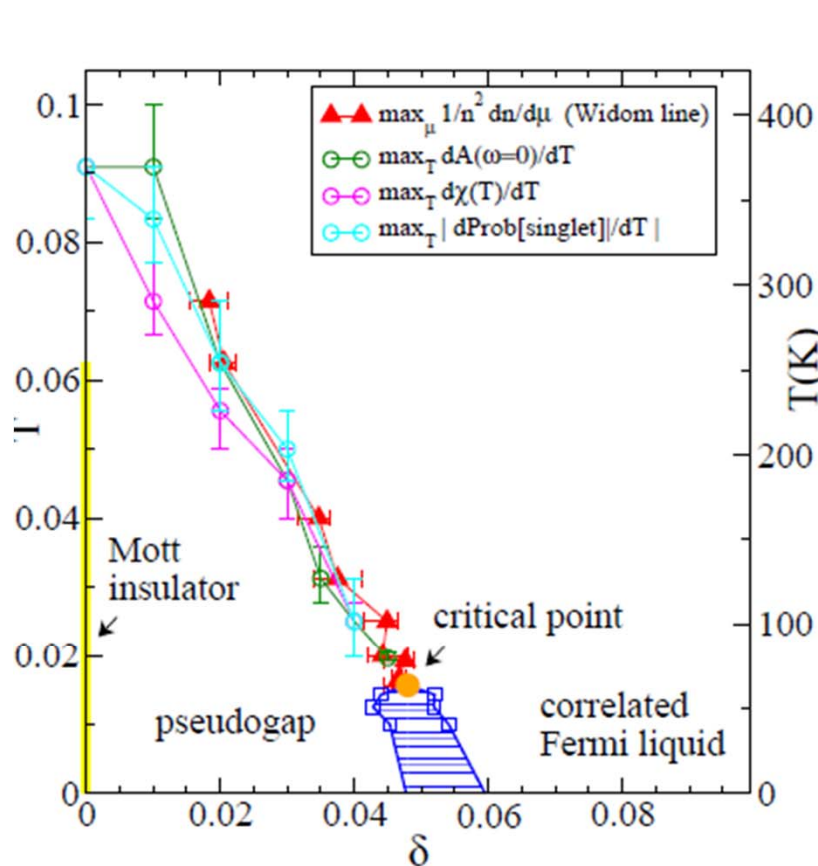


Khosaka et al. *Science* **315**, 1380 (2007);

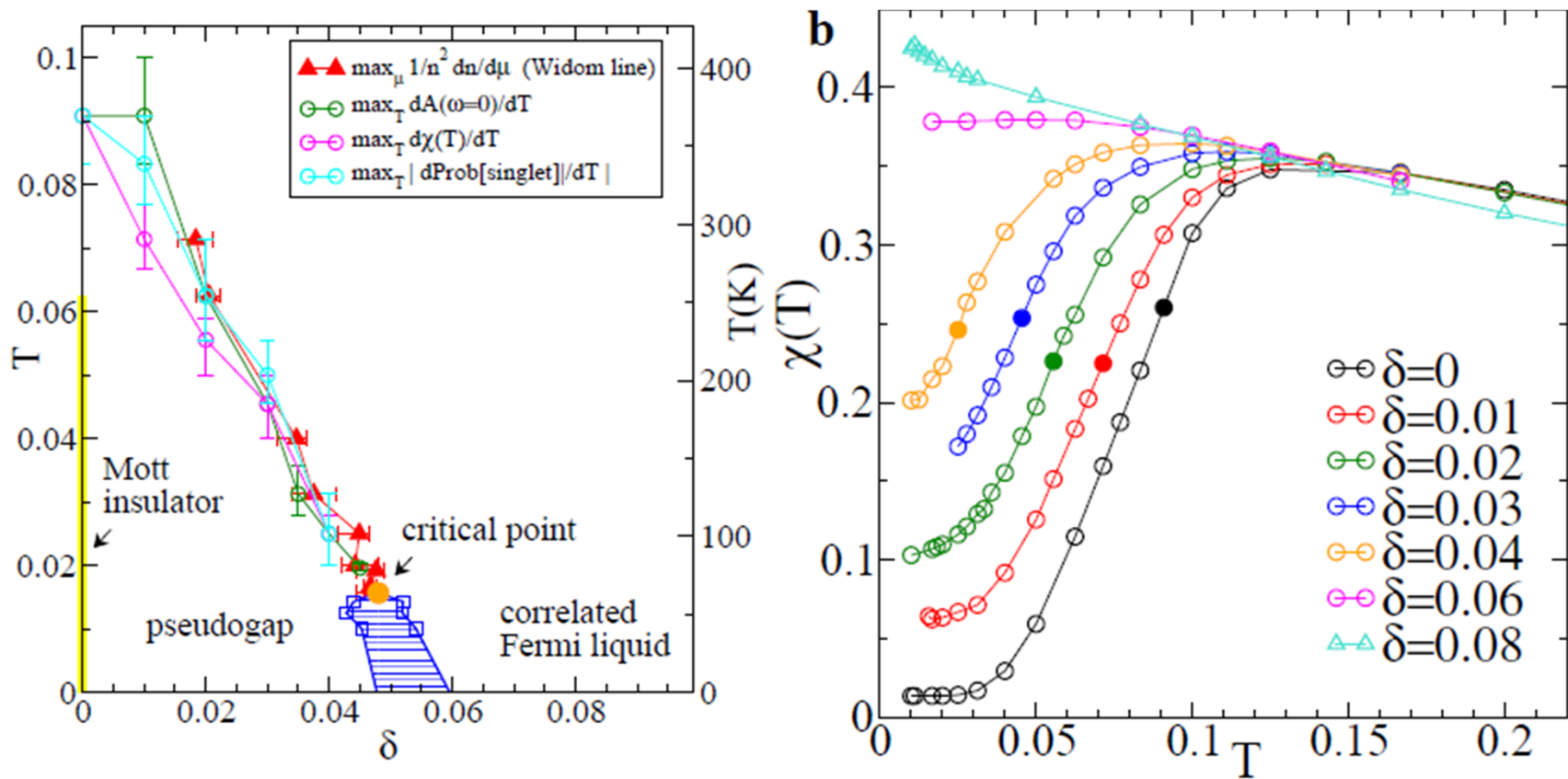
Density of states



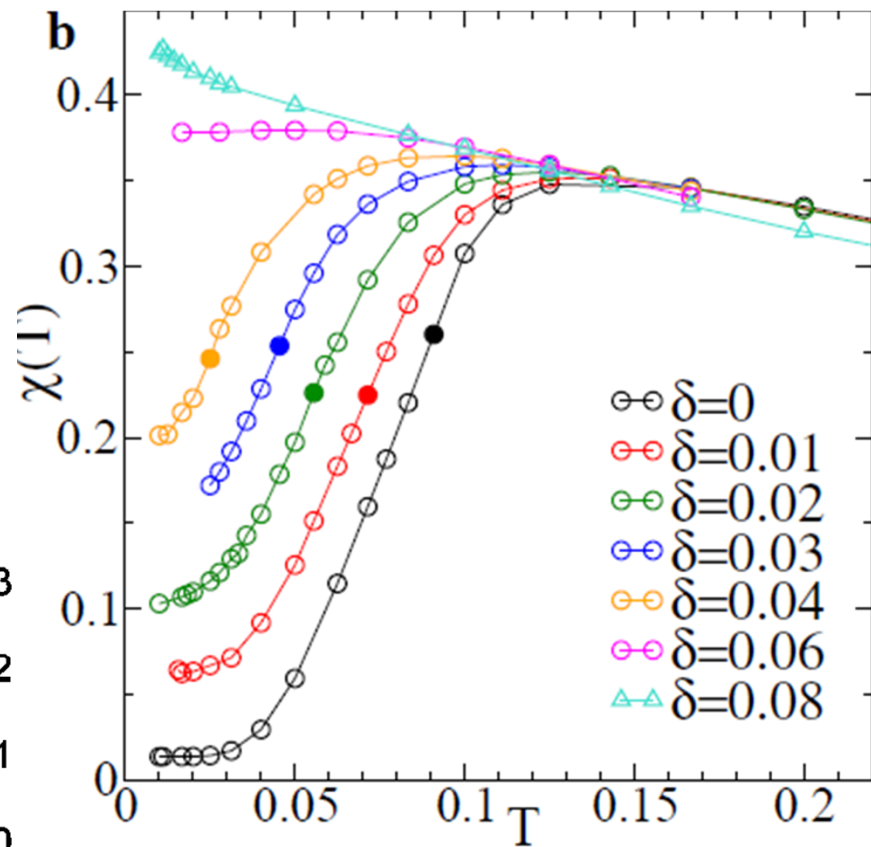
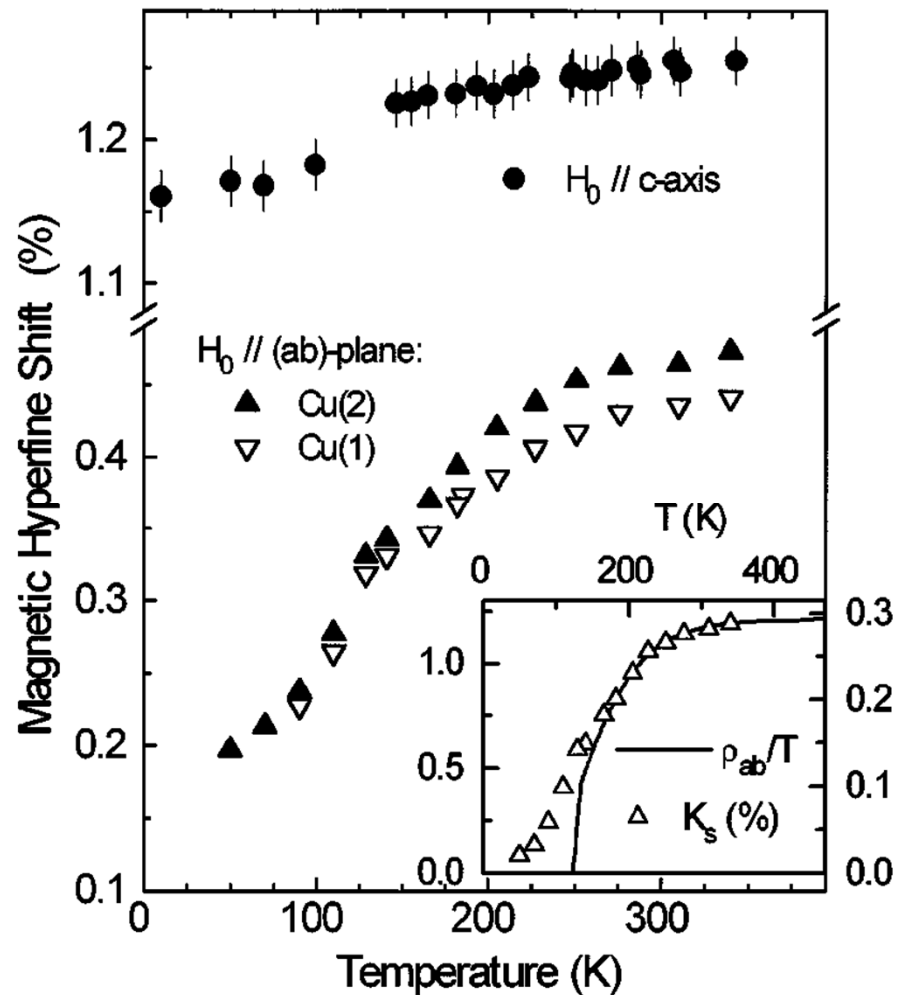
Density of states



Spin susceptibility



Spin susceptibility



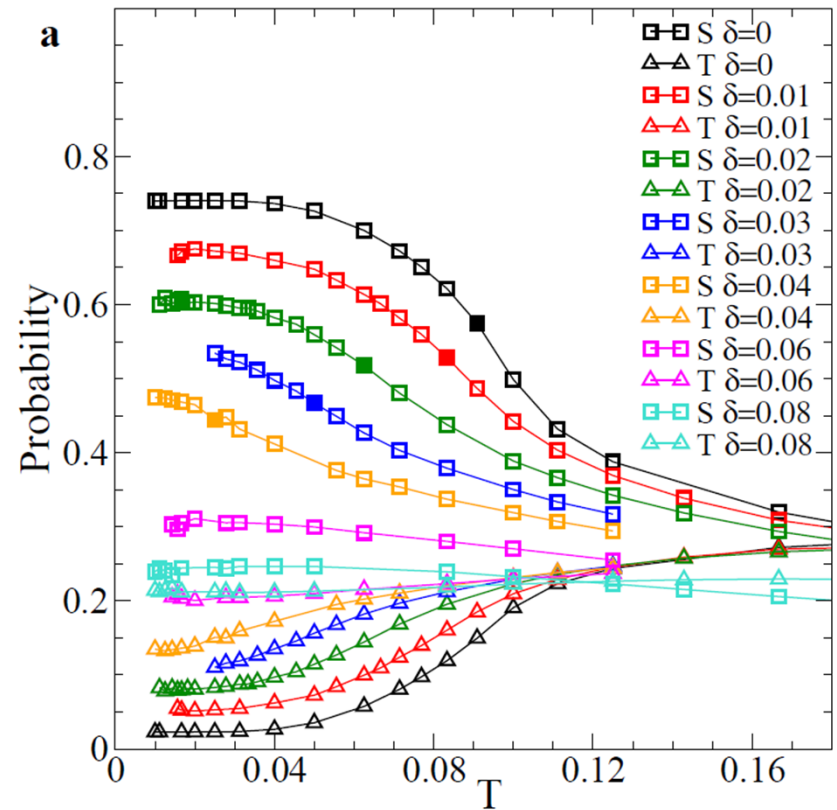
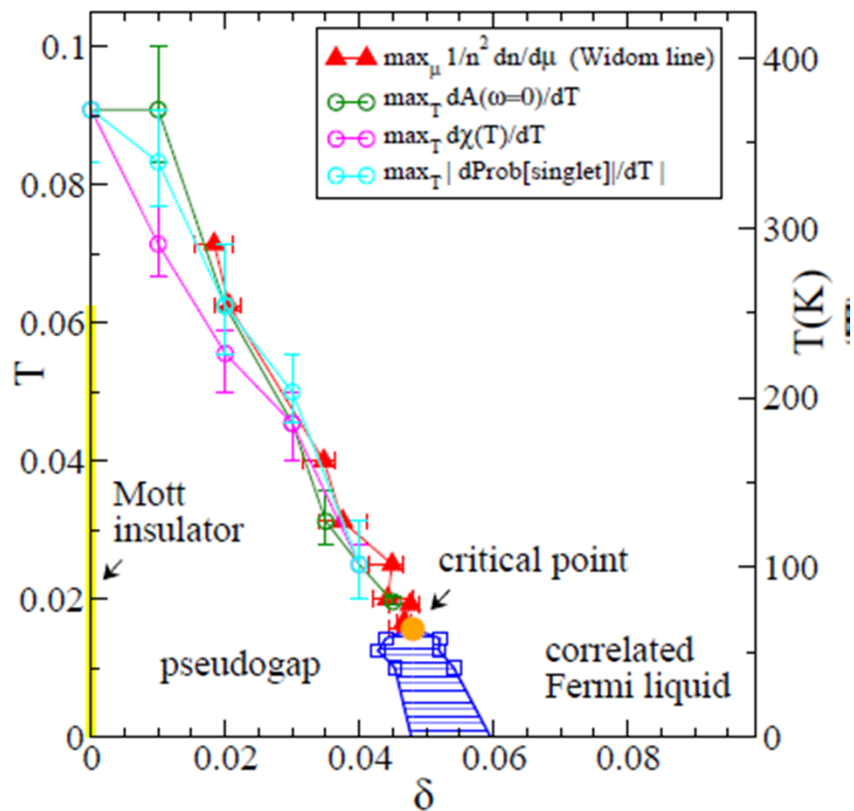
Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)

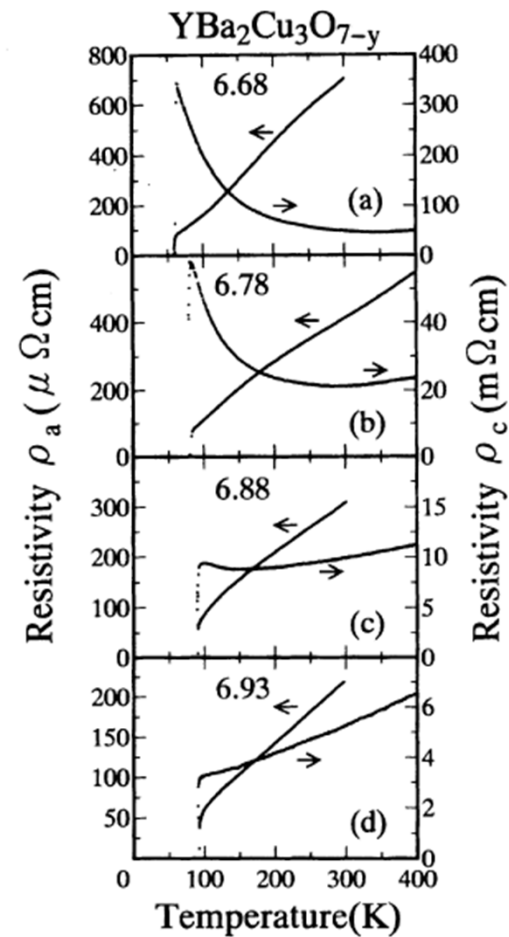
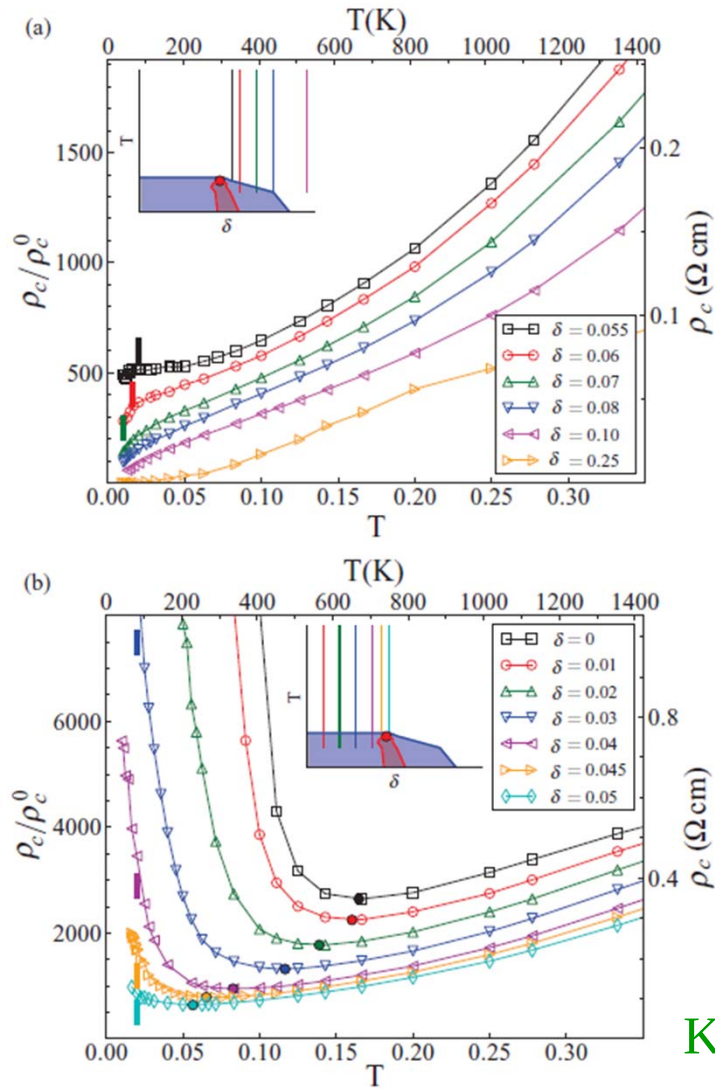


UNIVERSITÉ DE
SHERBROOKE

Plaquette eigenstates



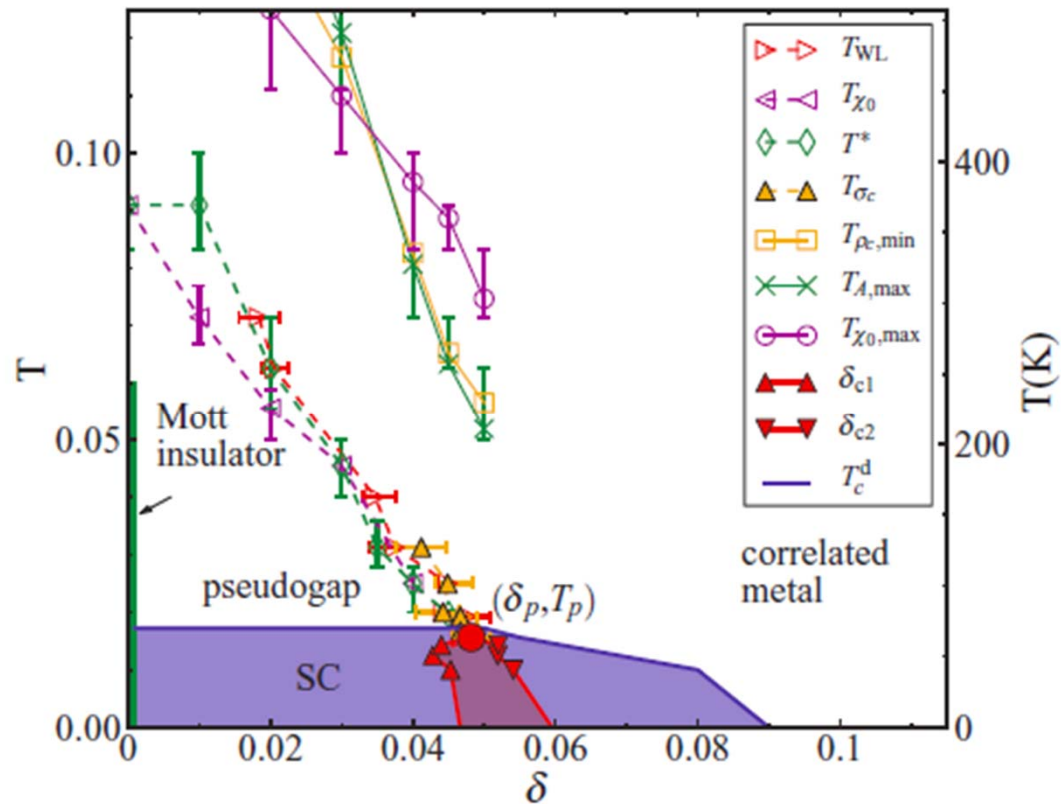
C-axis resistivity



K. Takenaka, K. Mizuhashi, H. Takagi, and S. Uchida,
Phys. Rev.B 50, 6534 (1994).

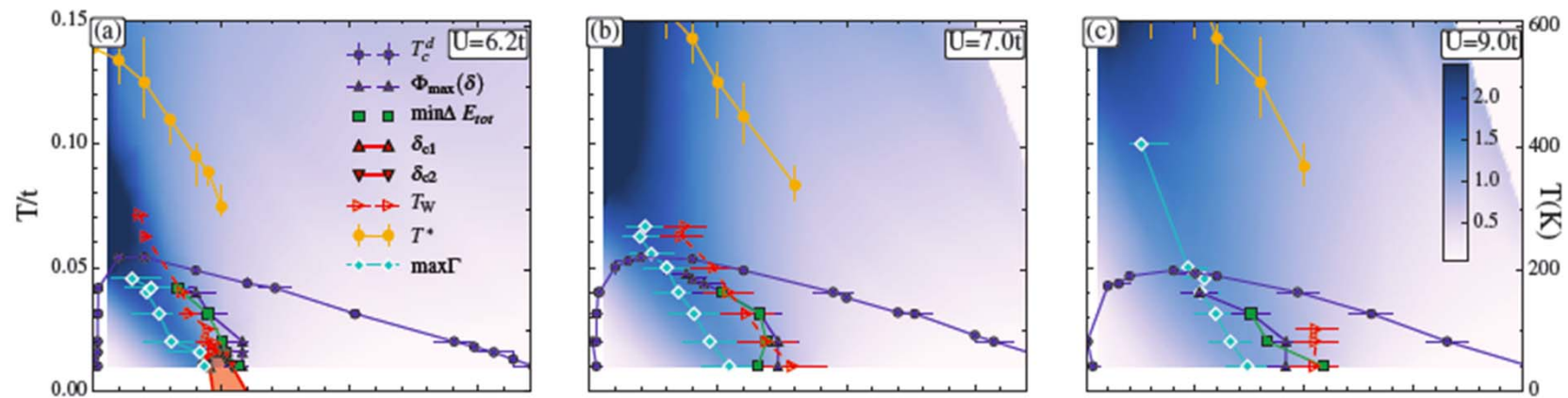


Two crossover lines



Sordi et al. PRL 108, 216401 (2012)
PRB **87**, 041101(R) (2013)

Two crossover lines



Fratino, Sémon, Sordi, A-MS T Sci. Rep. **6**, 22715 (2016)

What is the minimal model?

H. Alloul arXiv:1302.3473

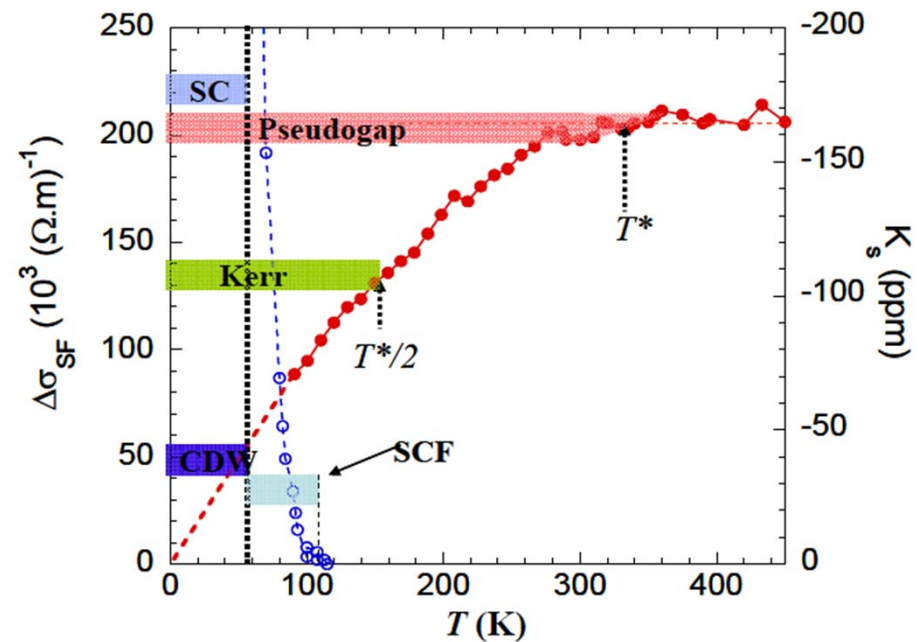
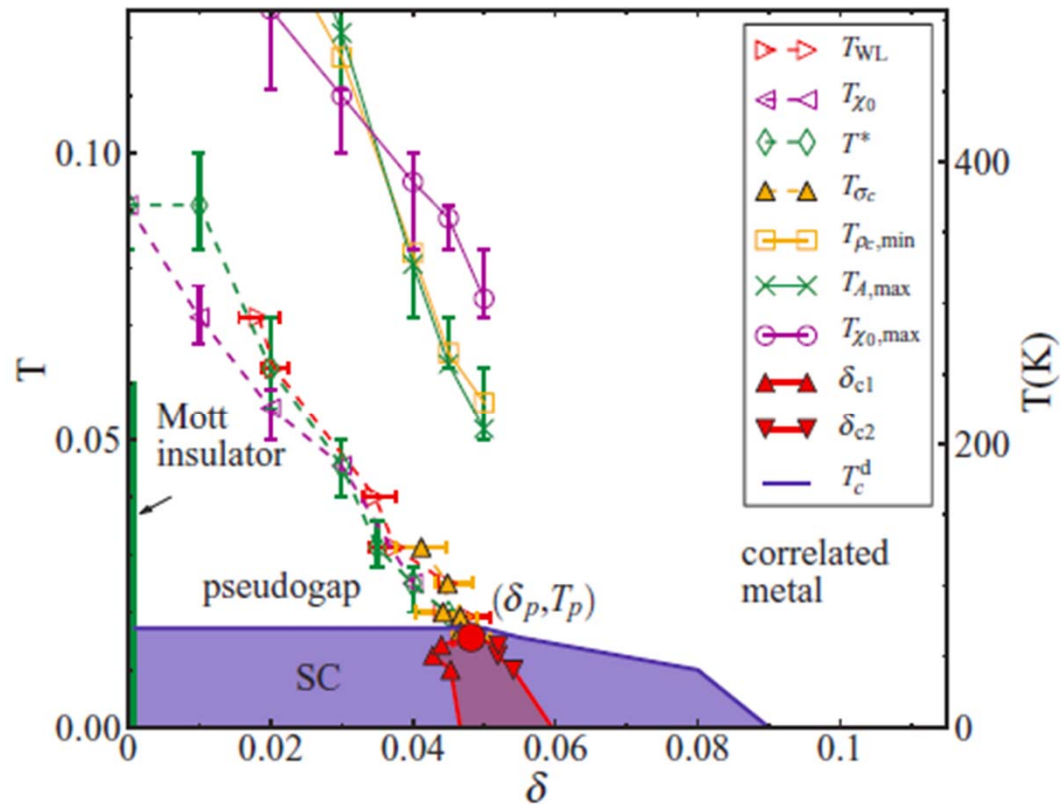


Fig 1 Spin contribution K_s to the ^{89}Y NMR Knight shift [11] for $\text{YBCO}_{6.6}$ permit to define the PG onset T^* . Here K_s is reduced by a factor two at $T \sim T^*/2$. The sharp drop of the SC fluctuation conductivity (SCF) is illustrated (left scale) [23]. We report as well the range over which a Kerr signal is detected [28], and that for which a CDW is evidenced in high fields from NMR quadrupole effects [33] and ultrasound velocity data [30]. (See text).

Crossover along the Widom line



Sordi et al. PRL 108, 216401 (2012)
PRB 87, 041101(R) (2013)



Giovanni Sordi



Patrick Sémon



Kristjan Haule

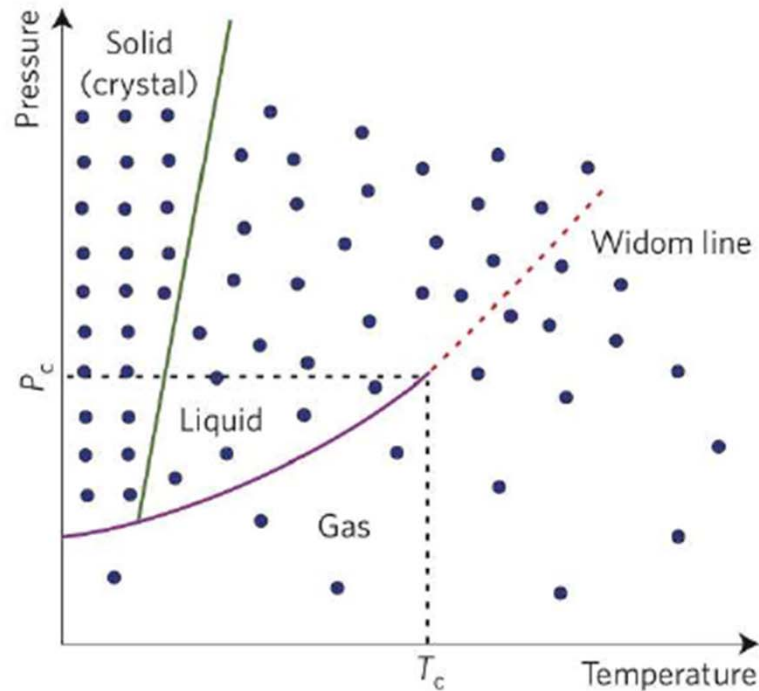
The Wisdom line

G. Sordi, *et al.* Scientific Reports 2, 547 (2012)



UNIVERSITÉ DE
SHERBROOKE

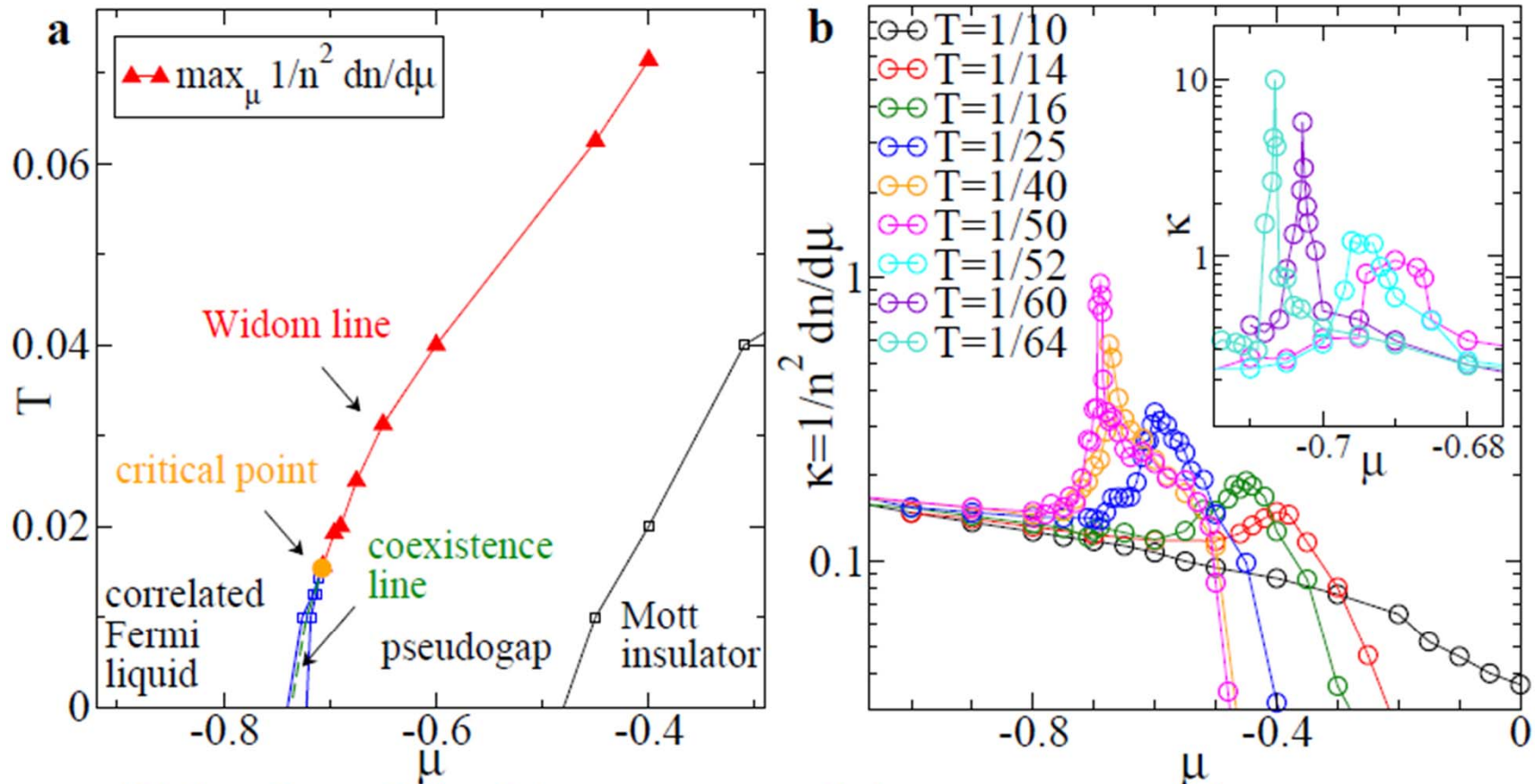
What is the Widom line?



McMillan and Stanley, Nat Phys 2010

- ▶ it is the continuation of the coexistence line in the supercritical region
- ▶ line where the **maxima of different response functions** touch each other asymptotically as $T \rightarrow T_p$
- ▶ liquid-gas transition in water: max in isobaric heat capacity C_p , isothermal compressibility, isobaric heat expansion, etc
- ▶ **DYNAMIC crossover arises from crossing the Widom line!**
water: Xu et al, PNAS 2005, Simeoni et al Nat Phys 2010

Pseudogap T^* along the Widom line

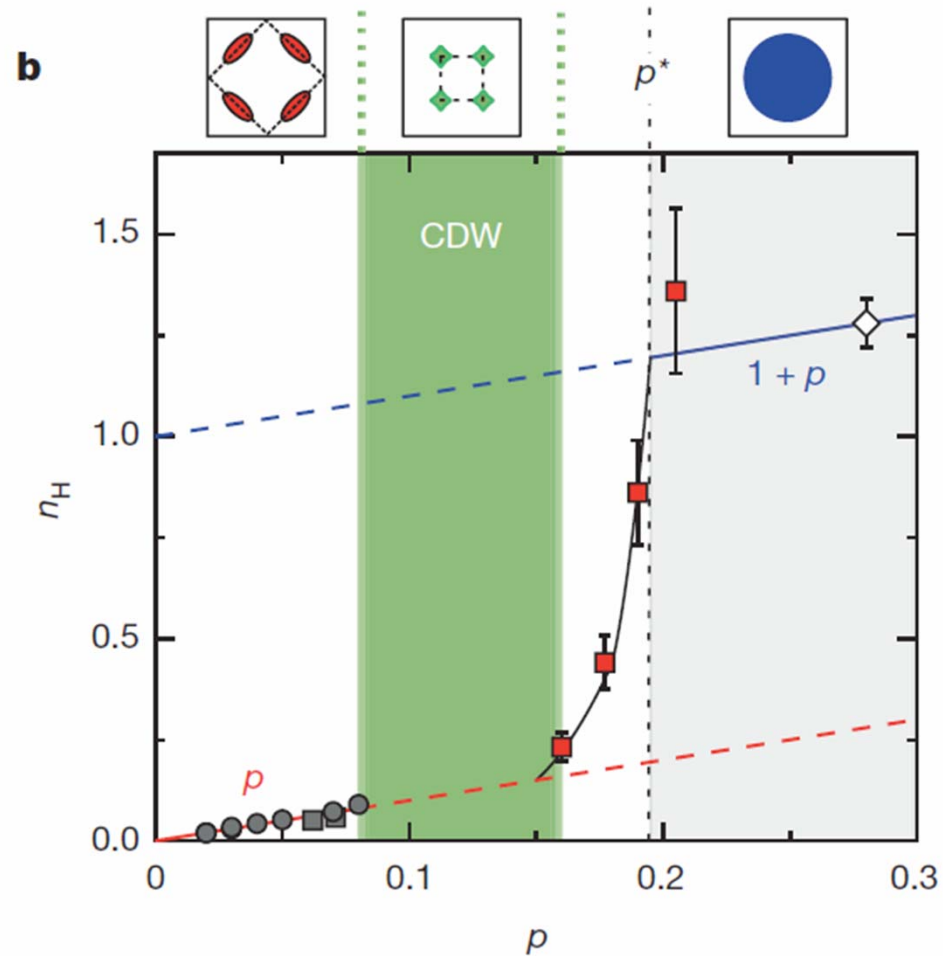


Widom line: defined from maxima of charge compressibility

$$\kappa = 1/n^2 (dn/d\mu)_T$$

divergence of κ at the (classical) critical point!

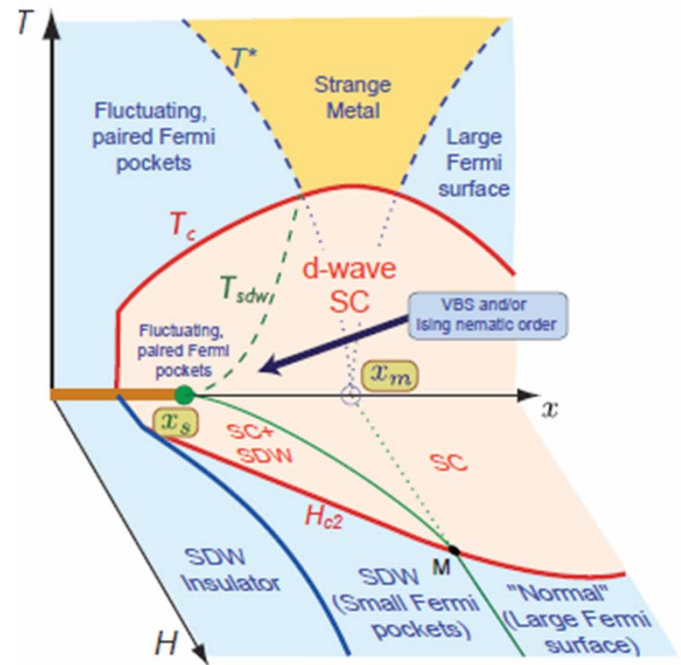
Coming back to: Not charge order



Badoux *et al.* Nature, March 2016



Applying a B field to remove superconductivity



S. Sachdev, Physica C **470**, S4 (2010)



Antiferromagnetic phase diagram (TPSC)

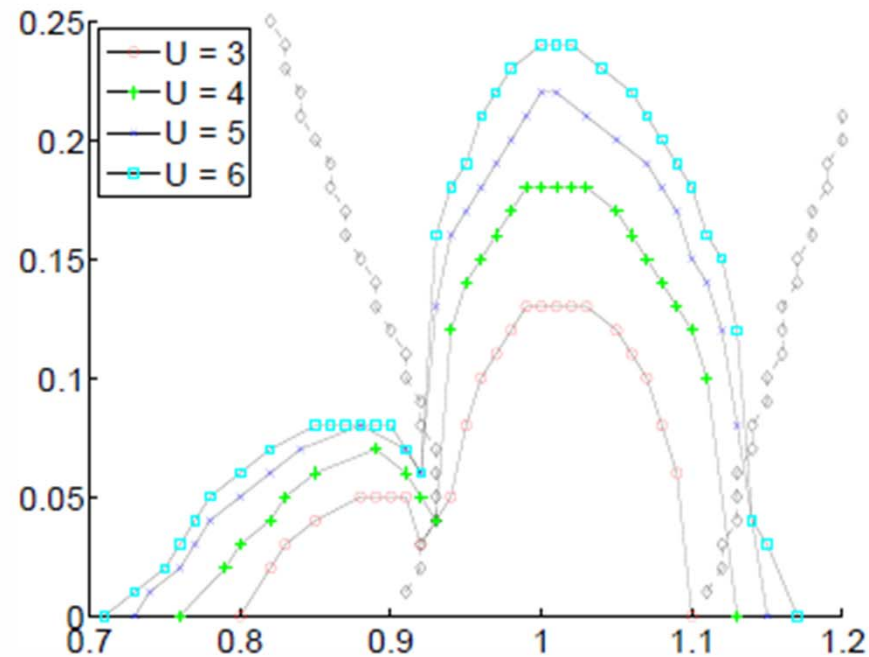


FIGURE 5.5 – Diagramme de phase température-dopage pour $t' = -0.1t$ pour quelques valeurs de U en fonction de n et T . Les diamants ($\diamond$) indique la frontière commensurable-incommensurable.

Sébastien Roy, PhD thesis 2007

2 possible scenarios



A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS

EPL, 113 (2016) 27003
doi: 10.1209/0295-5075/113/27003

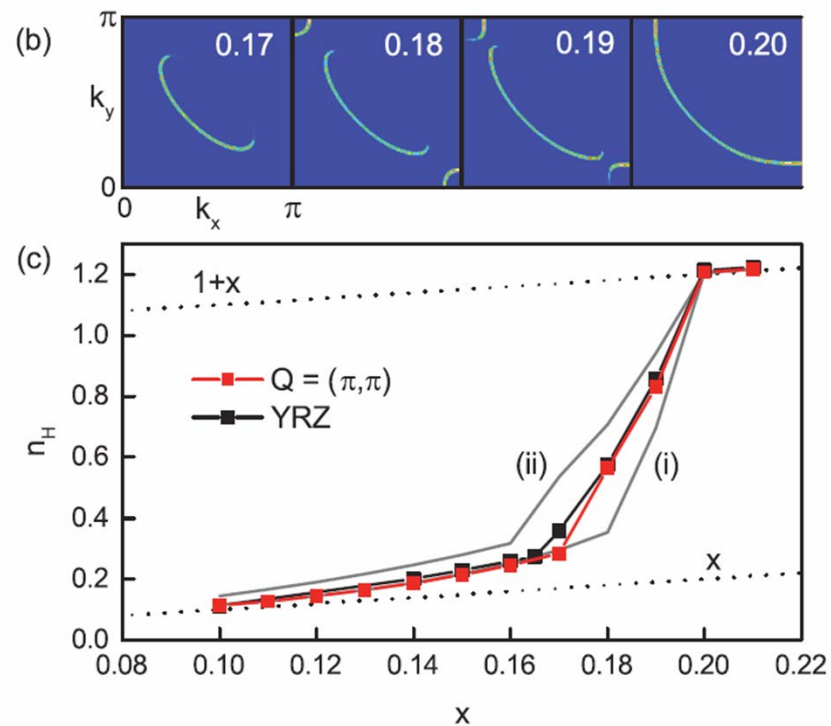
January 2016

www.epljournal.org

Hall effect and Fermi surface reconstruction via electron pockets in the high- T_c cuprates

J. G. STOREY

Robinson Research Institute, Victoria University - P.O. Box 600, Wellington, New Zealand



UNIVERSITÉ DE
SHERBROOKE

