

Unconventional superconductivity with and without an antiferromagnetic quantum critical point

A.-M. Tremblay

**Quantum criticality and topology in itinerant electron systems
15 to 19 August 2016**



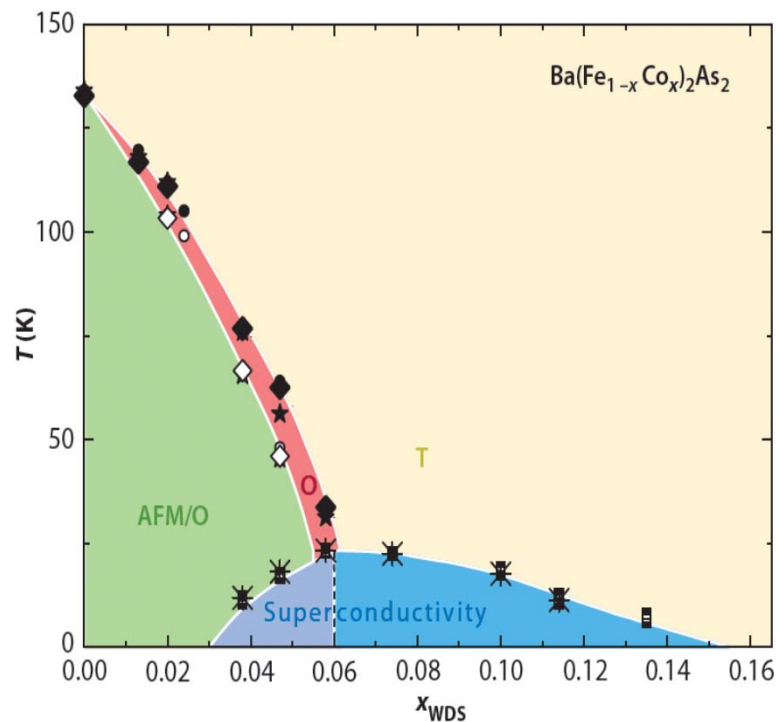
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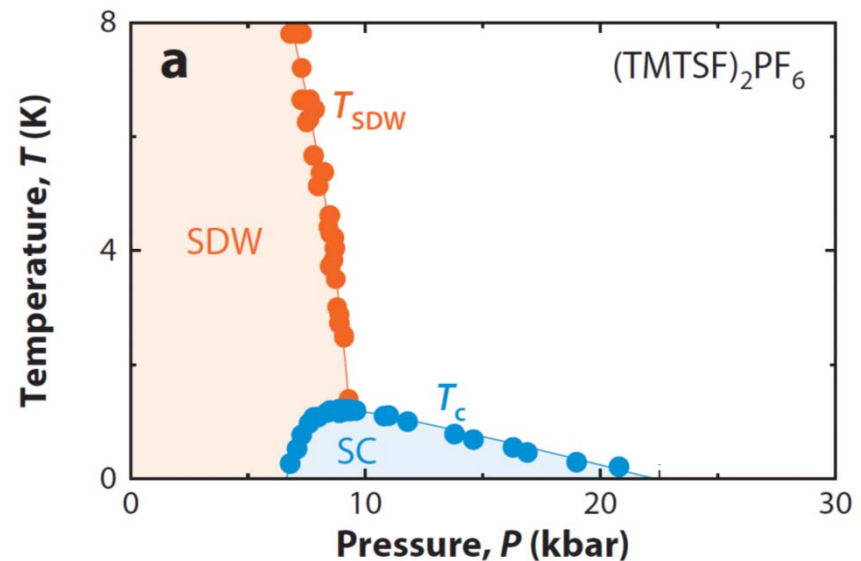
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Pnictides and organics

Pnictides



Bechgaard salts



The Archetype

Magnetic superconductivity

Nicolas Doiron-Leyraud, Bourbonnais, Taillefer 2010

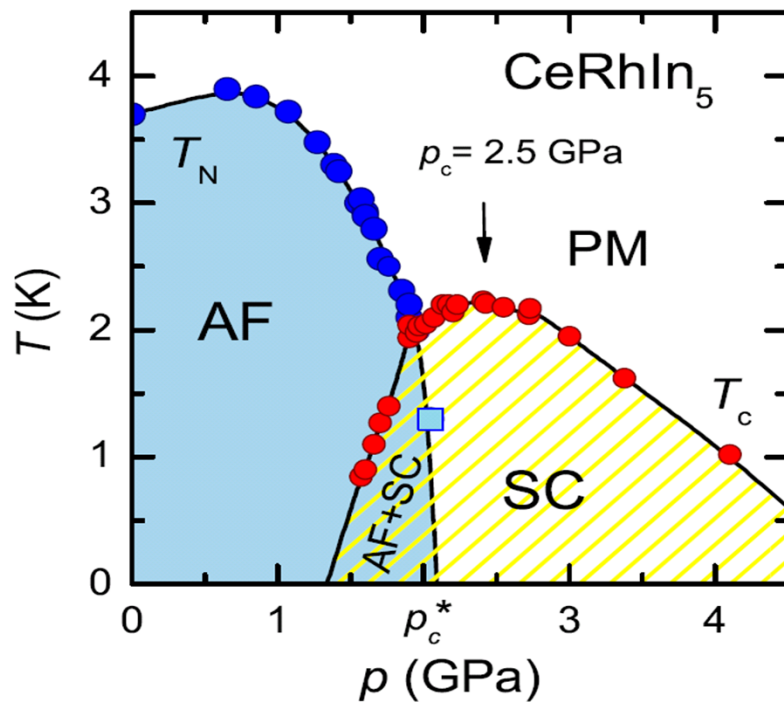
Canfield et al. (2010)



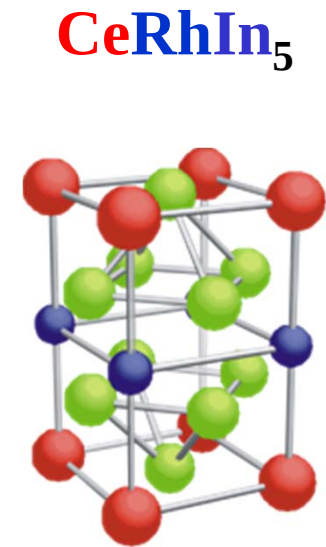
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Heavy fermions

Heavy fermions 3D metals tuned by pressure, field or concentration



Knebel et al. (2009)



**Magnetic
superconductivity**

Quantum criticality

Mathur et al., Nature 1998



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Method for strongly correlated matter

Dynamical Mean Field Theory (+ clusters)

Concept: atomic localized correlations
consistent with delocalized aspect



Dynamical “variational” principle

$$\Omega_t[G] = \Phi[G] - \text{Tr}[(G_{0t}^{-1} - G^{-1})G] + \text{Tr} \ln(-G)$$

$$\Phi[G] = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$

The diagrams represent terms in the expansion of $\Phi[G]$. The first diagram is a circle with a dashed red line connecting two points on its circumference. The second diagram is a circle with a dashed red line connecting two points on its circumference, with an arrow pointing from the left point to the right point. The third diagram is a rectangle with two vertical ovals inside, and dashed red lines connecting the top and bottom corners of the rectangle.

$$\frac{\delta \Phi[G]}{\delta G} = \Sigma[G]$$

$$\frac{\delta \Omega_t[G]}{\delta G} = \Sigma[G] - G_{0t}^{-1} + G^{-1} = 0$$

DMFT

$$\Phi[G] = \sum_i \Phi[G_{ii}(i\omega_n)]$$

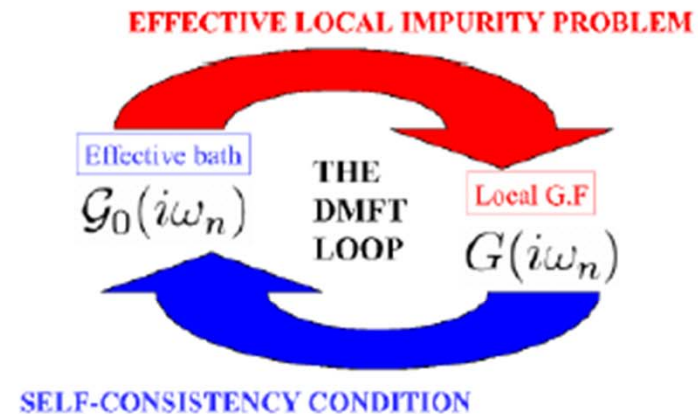
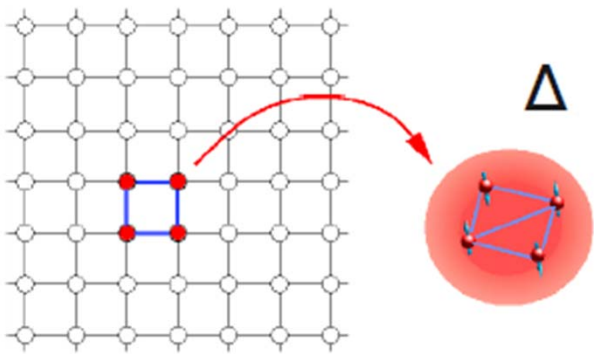
Luttinger and Ward 1960, Baym and Kadanoff (1961)



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Impurity solver

$$Z = \int D[d^\dagger, d] \exp \left[-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_i [d_i^\dagger(\tau) \Delta_{i'i}(\tau, \tau') d_{i'}(\tau')] \right]$$



Mean-field is not a trivial problem! Many impurity solvers.

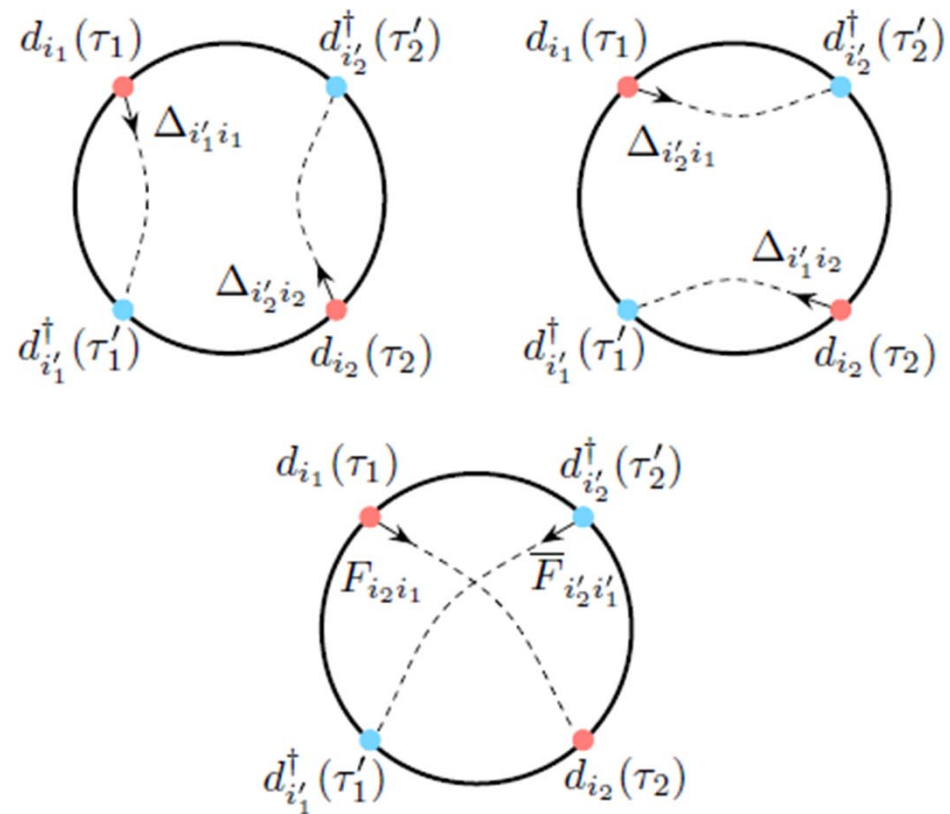
Here: continuous time QMC

P. Werner, PRL 2006
P. Werner, PRB 2007
K. Haule, PRB 2007

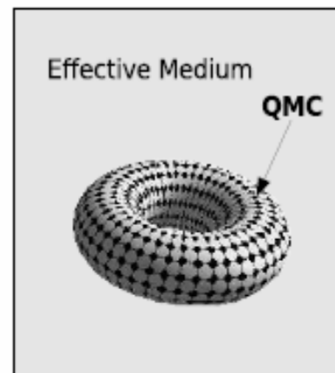
$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$= \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

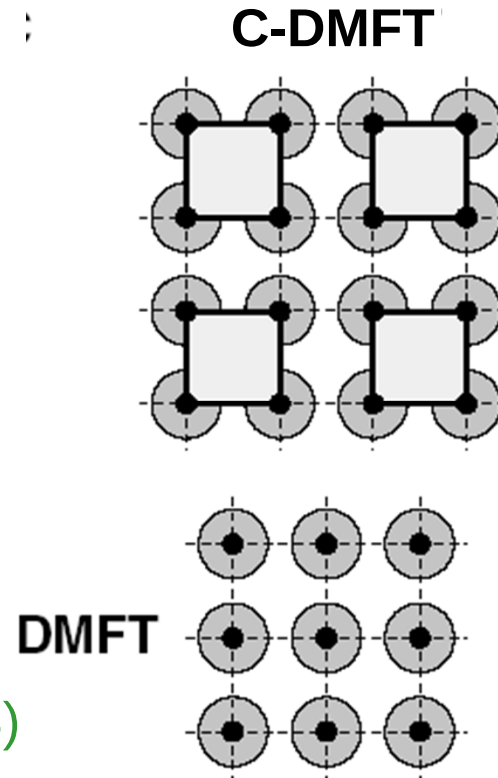
Some not Feynman diagrams



2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)



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+ and -

- Long range order:
 - Allow symmetry breaking in the bath (mean-field)
- Included:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations



Groups using these methods for cuprates

- Europe:
 - Georges, Parcollet, Ferrero, Civelli, (Paris)
 - de Medici (Grenoble) Capone (Italy)
- USA:
 - Gull (Michigan) Millis (Columbia)
 - Kotliar, Haule (Rutgers)
 - Jarrell (Louisiana)
 - Maier, Okamoto (Oakridge)
- Japan
 - Imada (Tokyo) Sakai





Wei Wu

Superconductivity around an AFM quantum critical point

A heavy fermion example

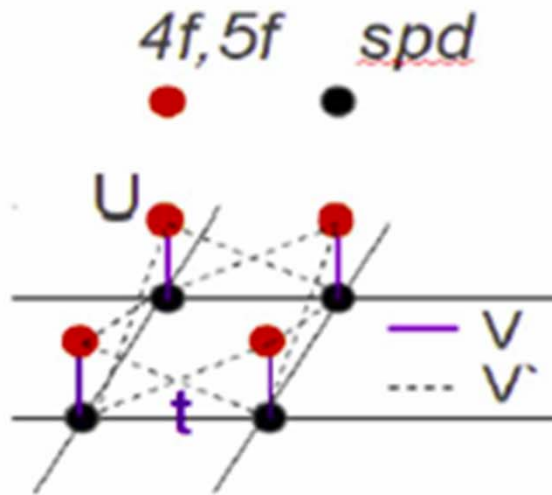
W. Wu A.-M.S.T. Phys. Rev. X, 2015



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Heavy fermions

$$H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{k,\sigma} \epsilon_f^f f_{k,\sigma}^\dagger f_{k,\sigma} \\ + \sum_{k,\sigma} V_k (f_{k,\sigma}^\dagger c_{k,\sigma} + \text{H.c.}) + \sum_i U \left(n_f^\uparrow - \frac{1}{2} \right) \left(n_f^\downarrow - \frac{1}{2} \right)$$



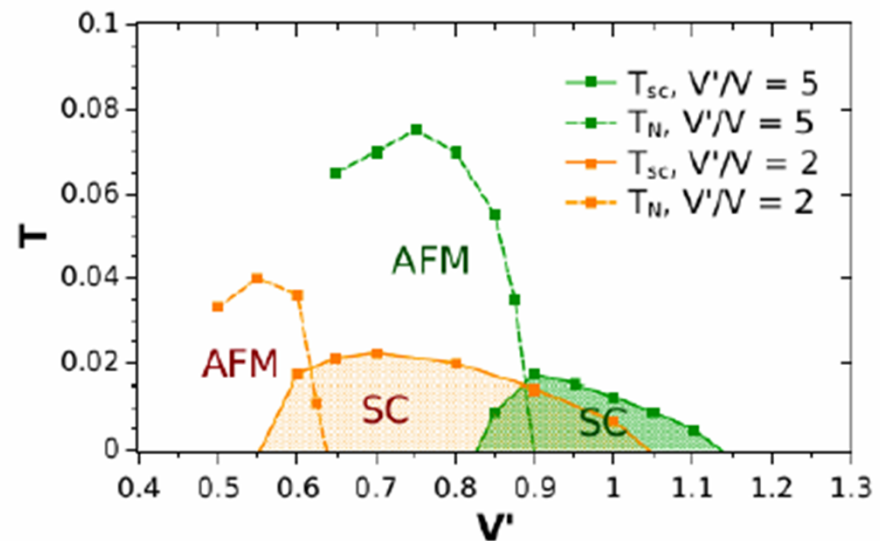
W. Wu A.-M.S.T. Phys. Rev. X, 2015

Phase diagram

$$U=4$$

AFM: antiferro-magnetism
SC: superconducting

$V'/V = 2$: more frustrated case
 $V'/V = 5$: less frustrated case



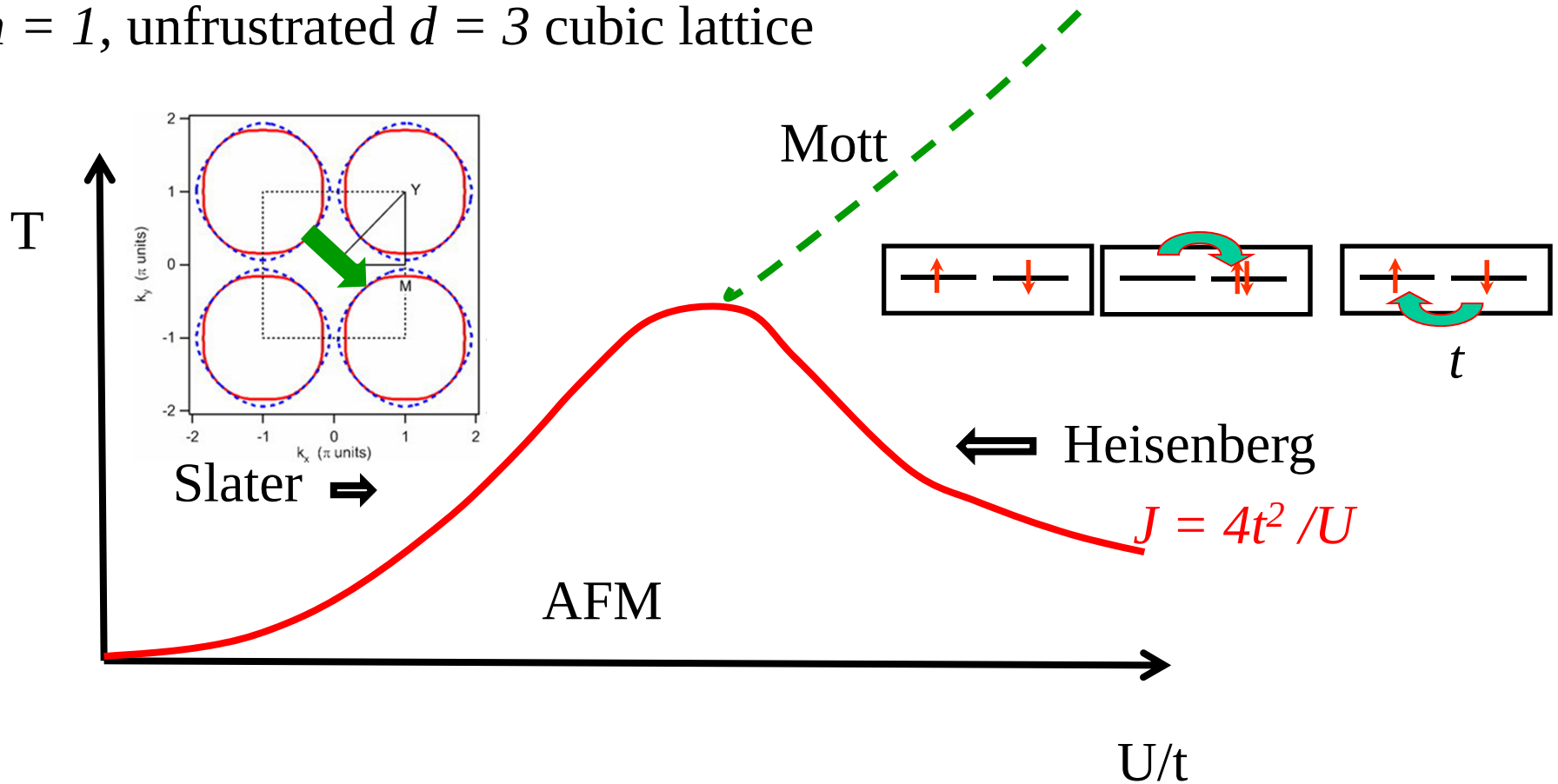
Weakly vs strongly correlated superconductivity

Analog to weakly and strongly
correlated antiferromagnets



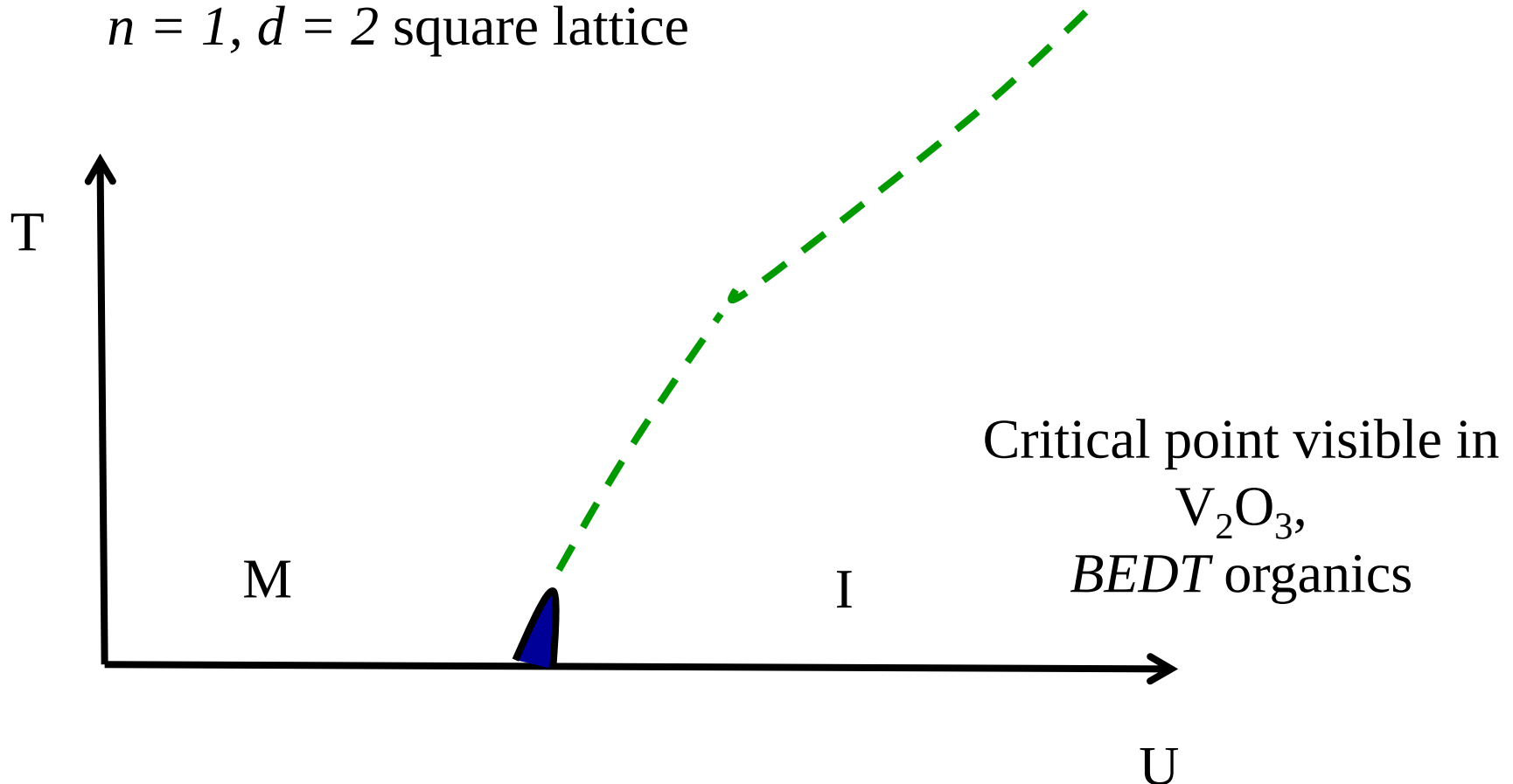
Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition

$n = 1, d = 2$ square lattice

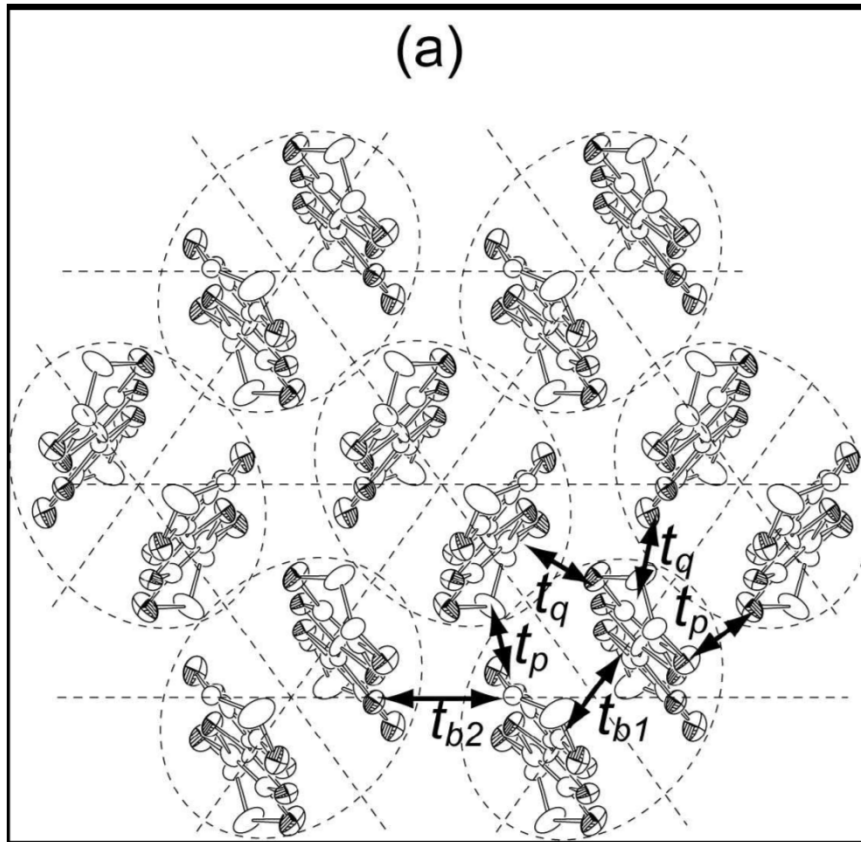


Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

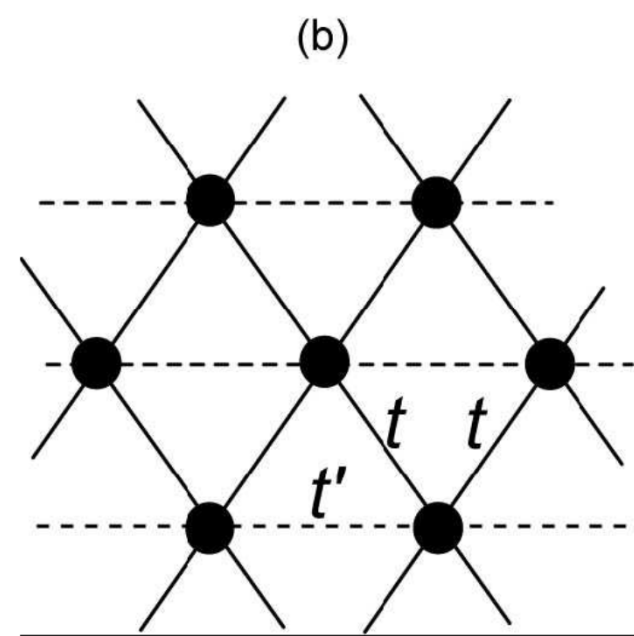


One-band Hubbard model of BEDT organics

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),
R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)



Y. Shimizu, et al. Phys. Rev. Lett. **91**,
107001(2003)



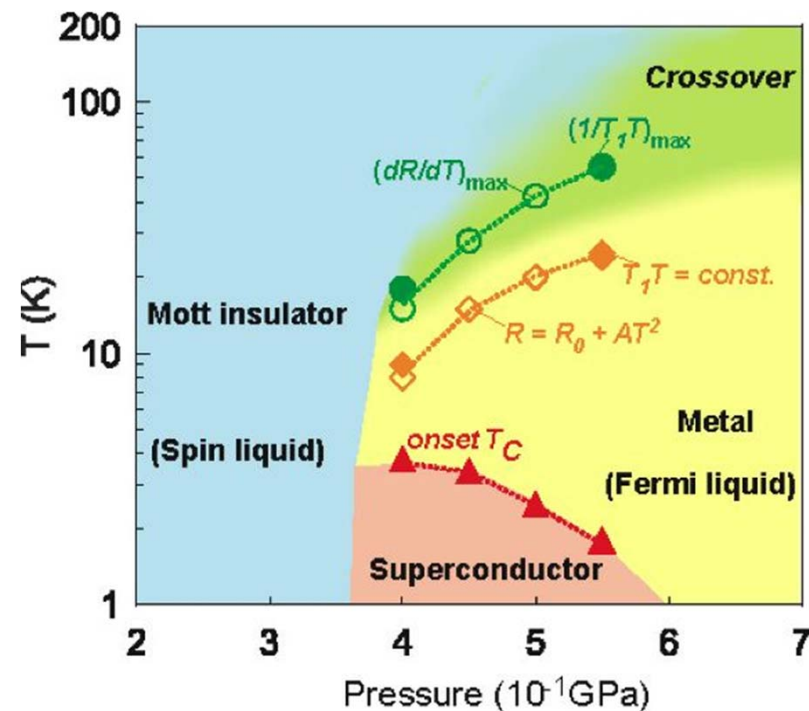
$$t \approx 50 \text{ meV}$$

$$\Rightarrow U \approx 400 \text{ meV}$$

$$t'/t \sim 0.6 - 1.1$$



Phase diagram at $n = 1$



Y. Kurisaki, et al.

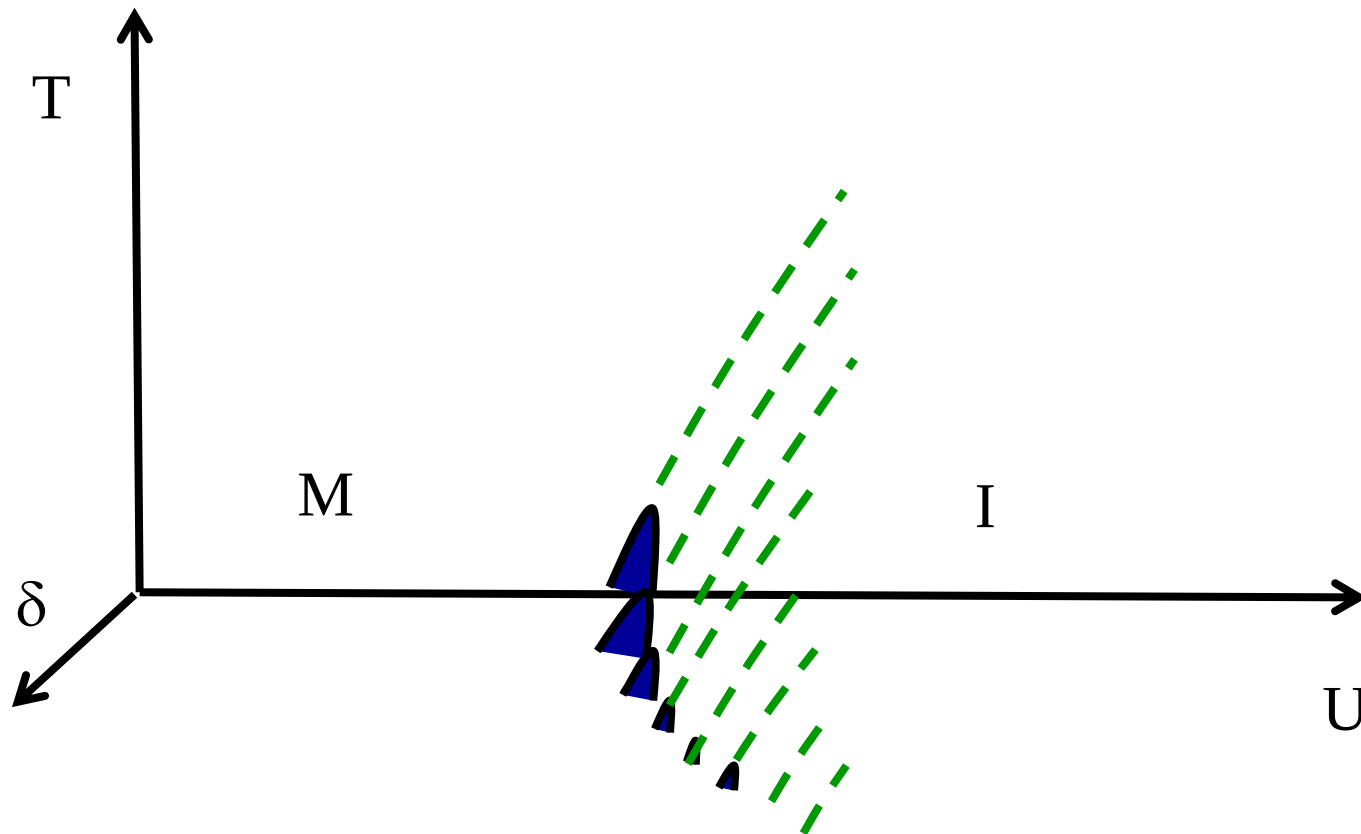
Phys. Rev. Lett. **95**, 177001(2005)

Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)



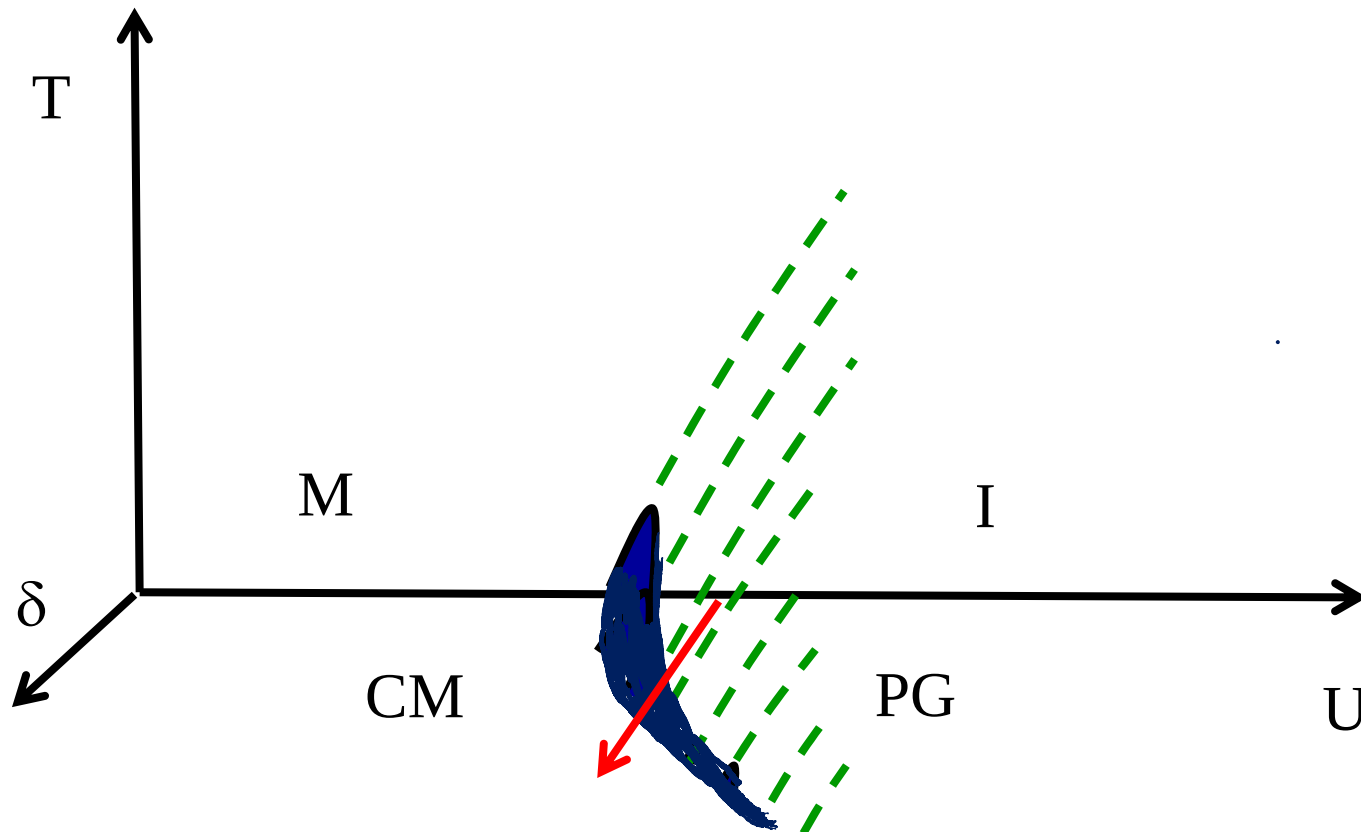
Influence of Mott transition away from half-filling

$n = 1, d = 2$ square lattice

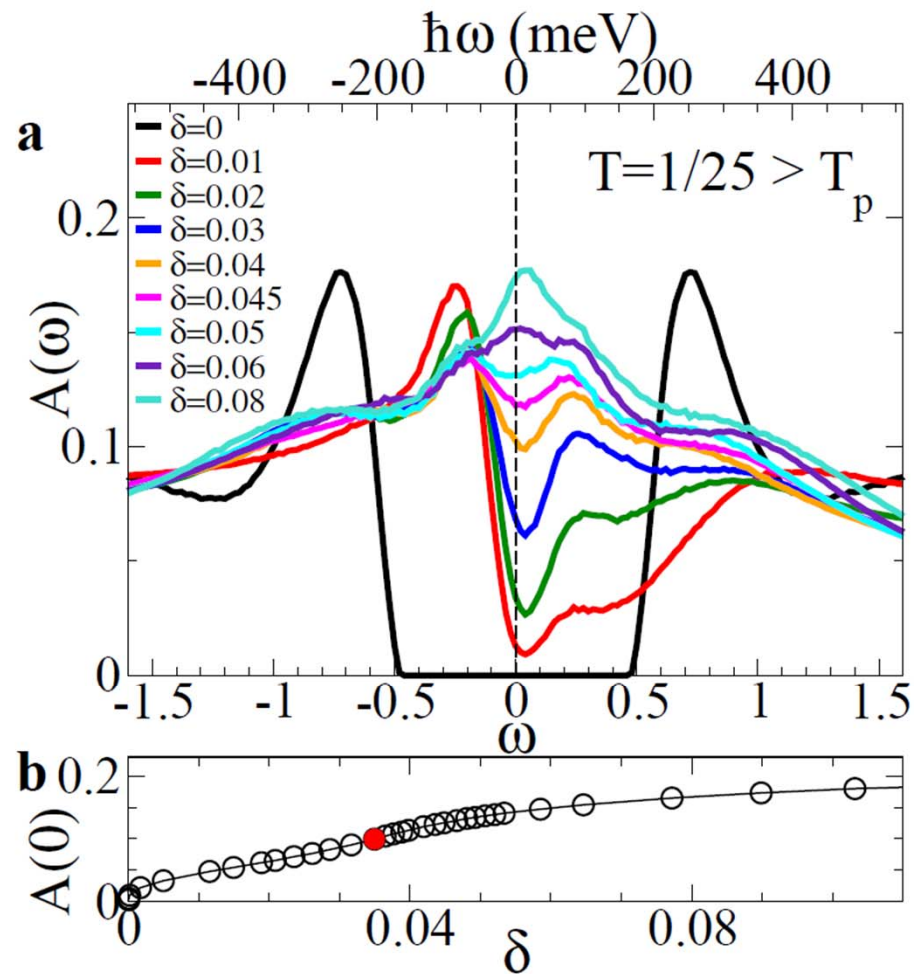
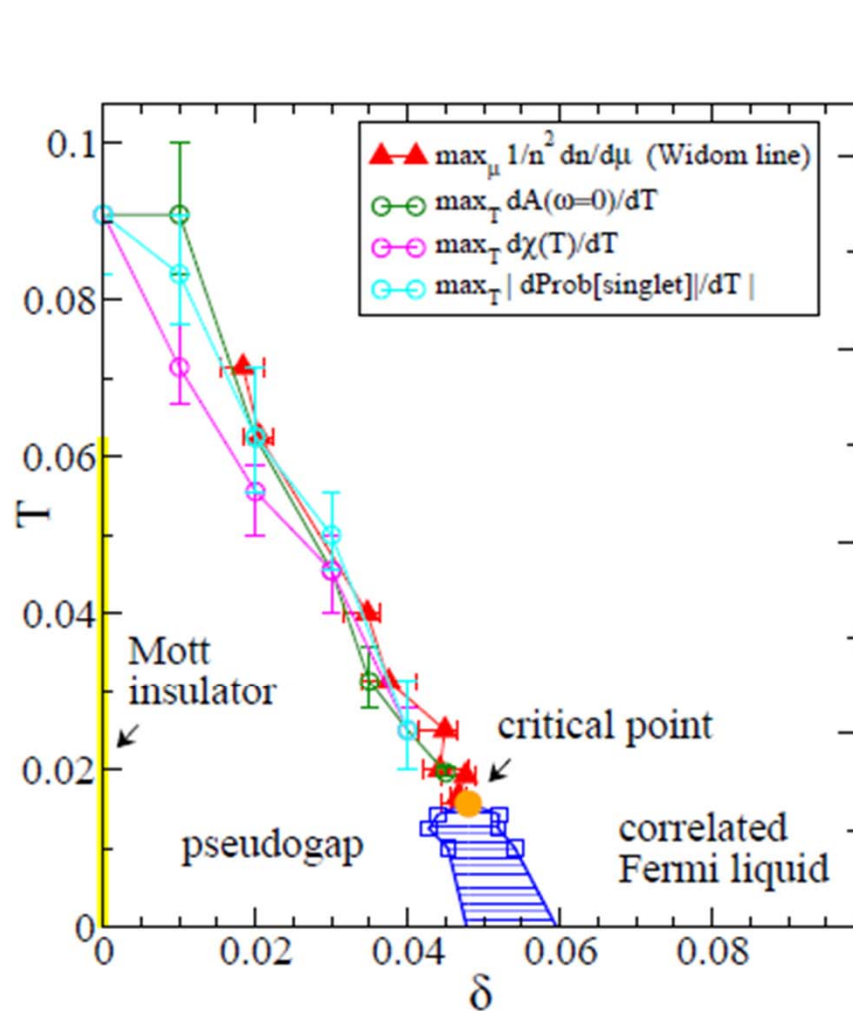


Influence of Mott transition away from half-filling

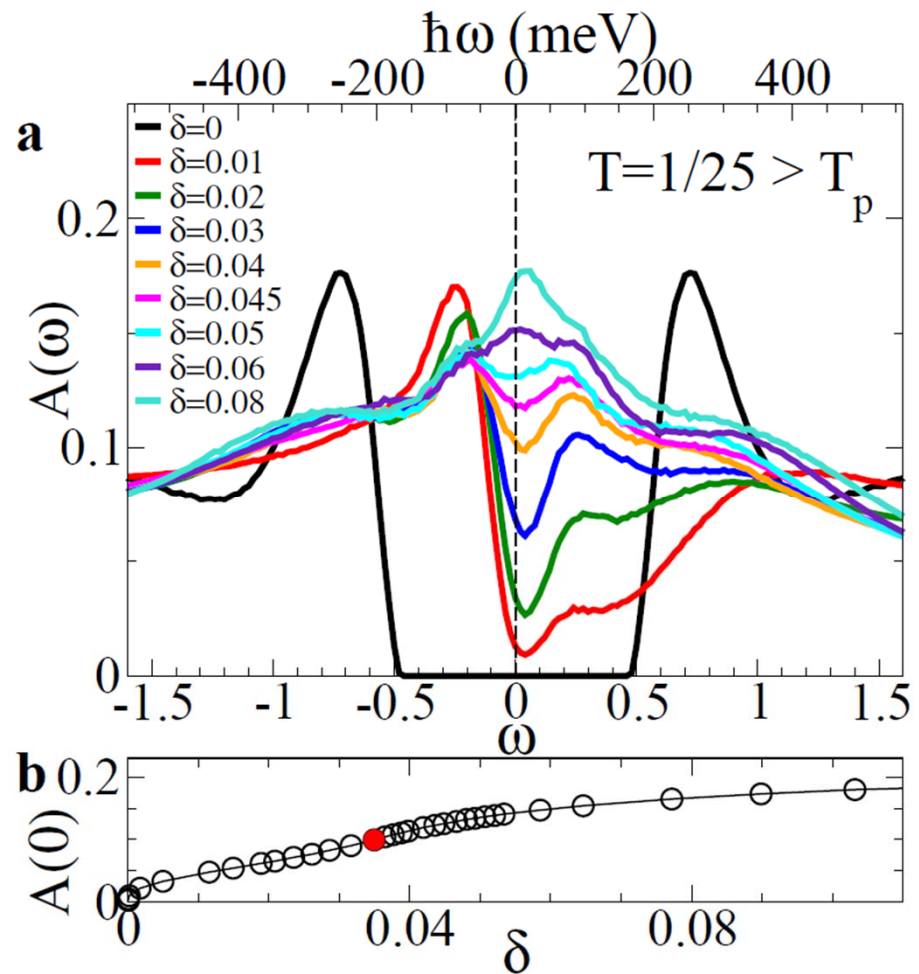
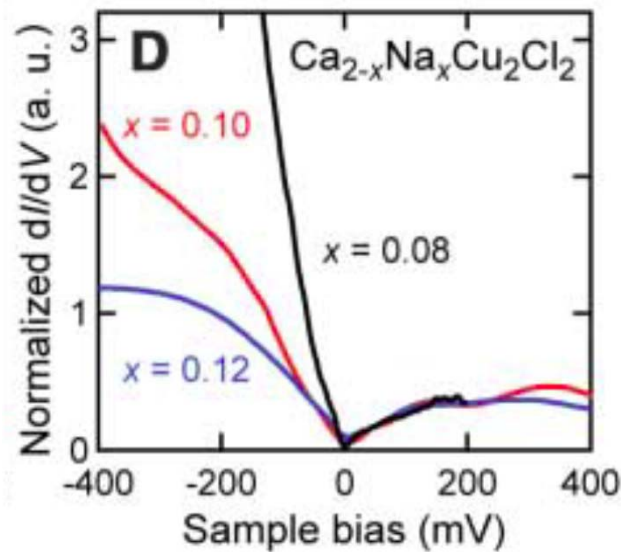
$n = 1, d = 2$ square lattice



$U = 6.2 t$ Normal state. Density of states

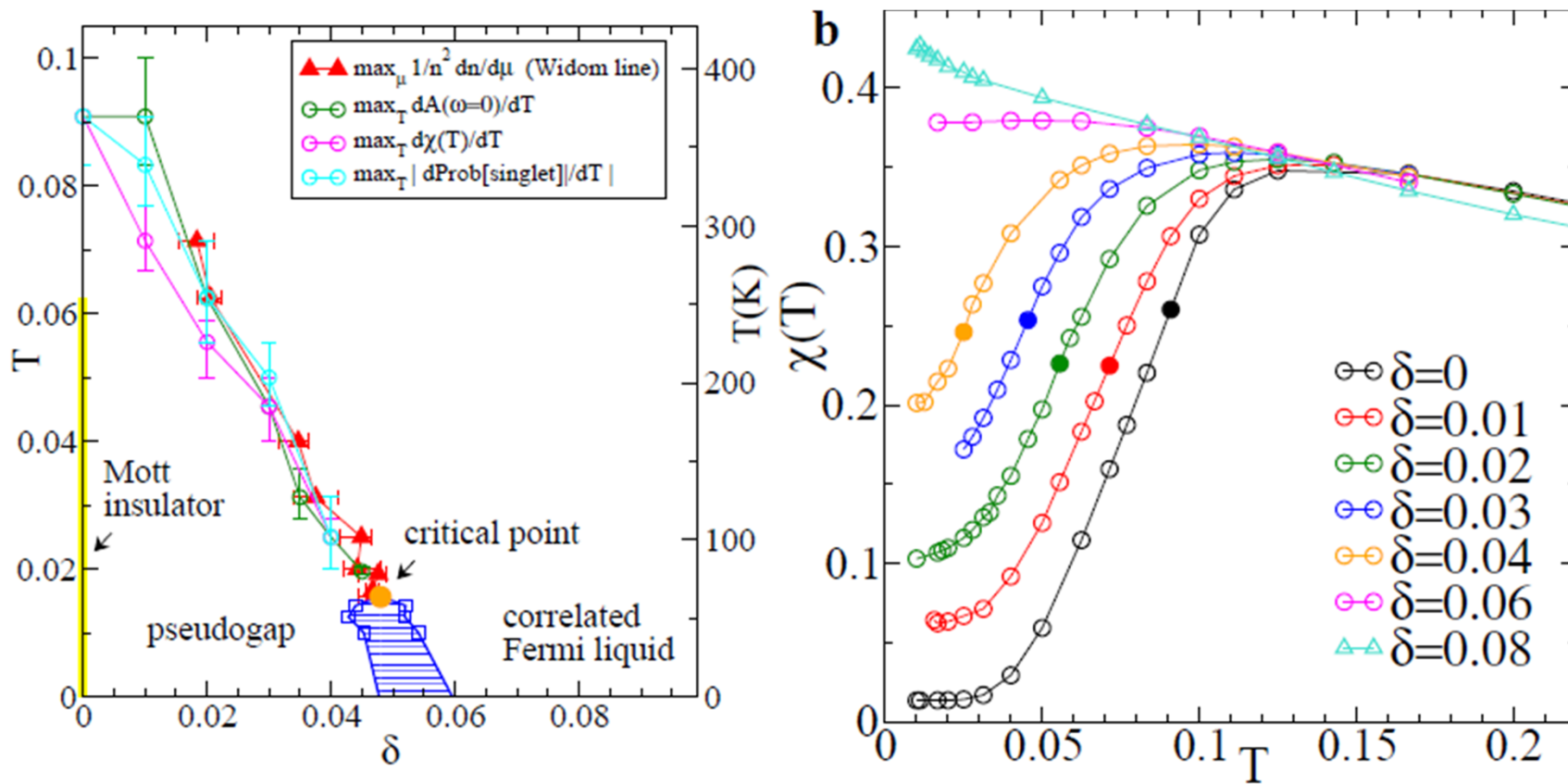


Density of states

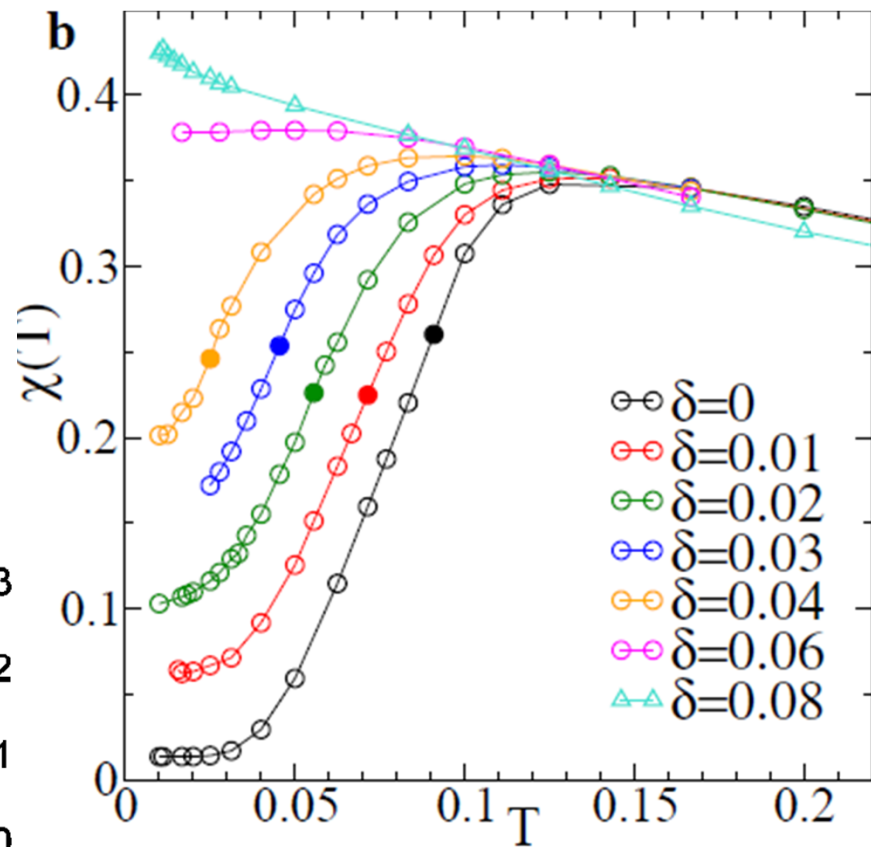
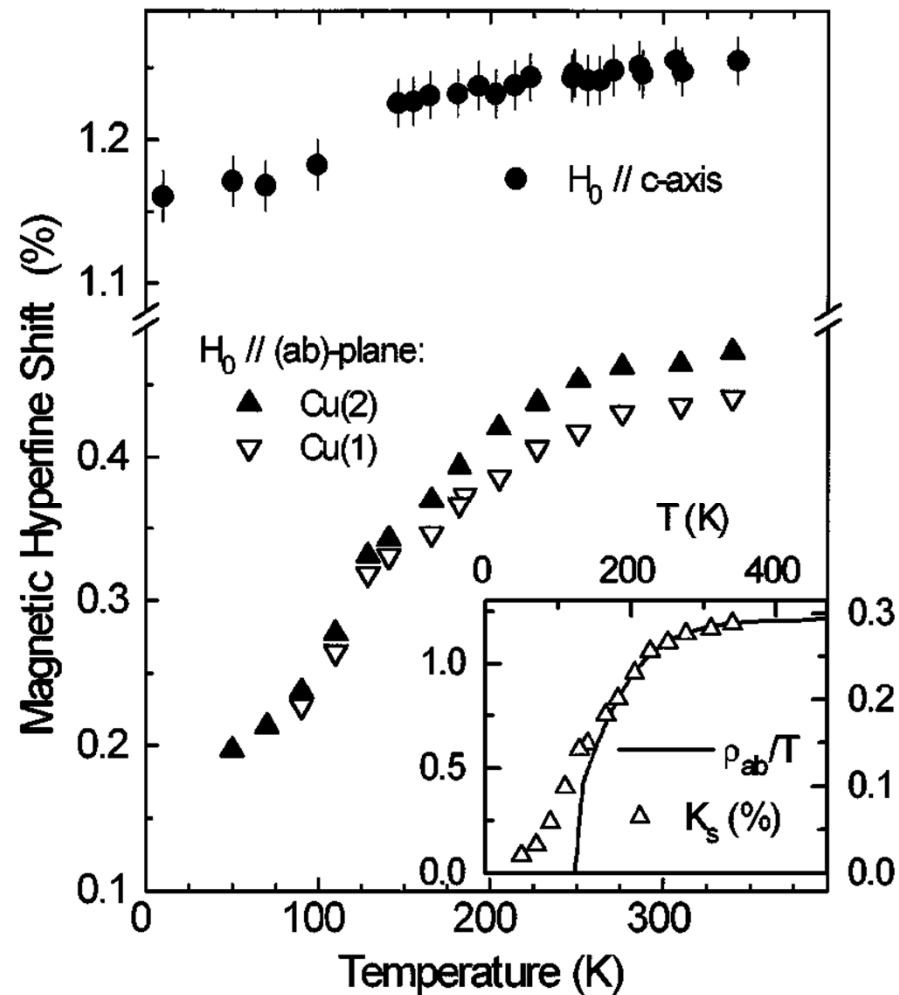


Khosaka et al. *Science* **315**, 1380 (2007);

Spin susceptibility



Spin susceptibility



Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



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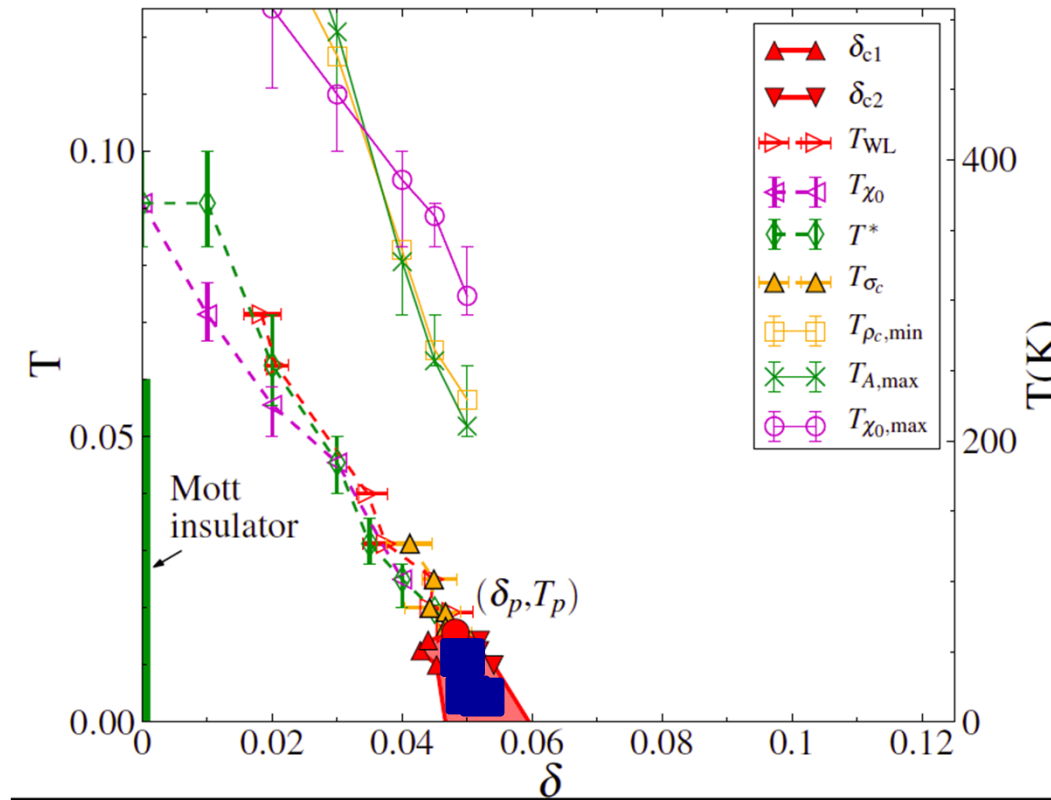


Giovanni Sordi

Two crossover lines



Patrick Sémon



G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

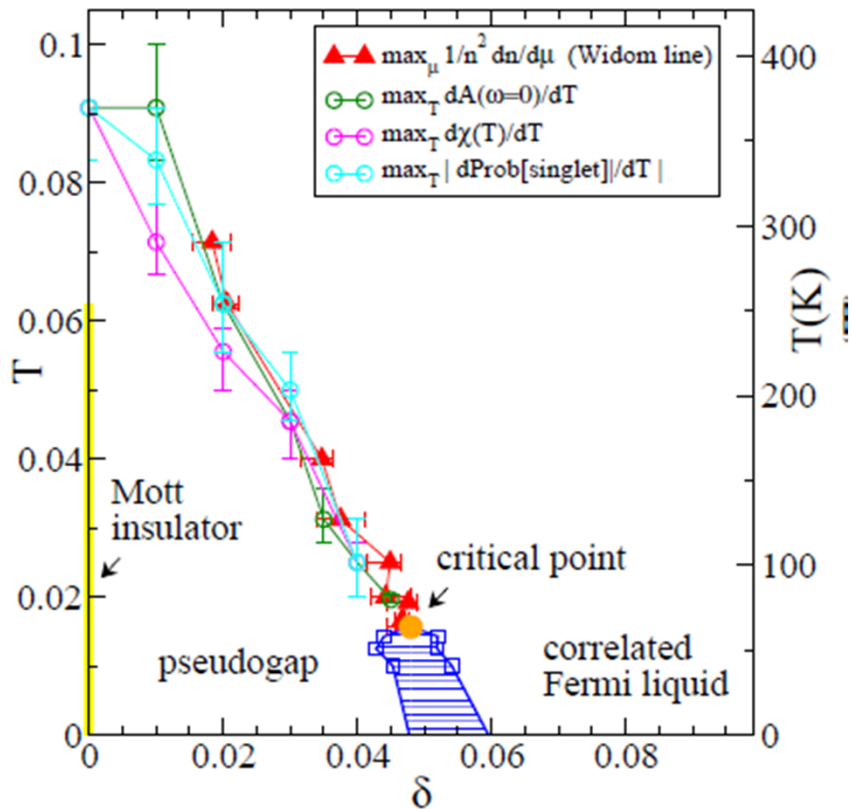
P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)



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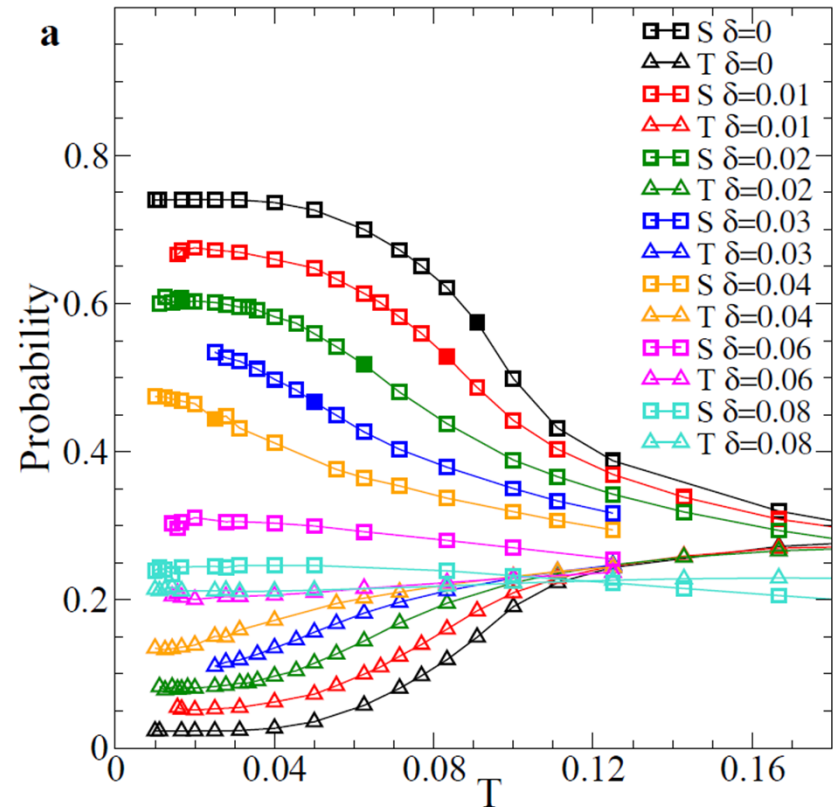
Physics

Plaquette eigenstates



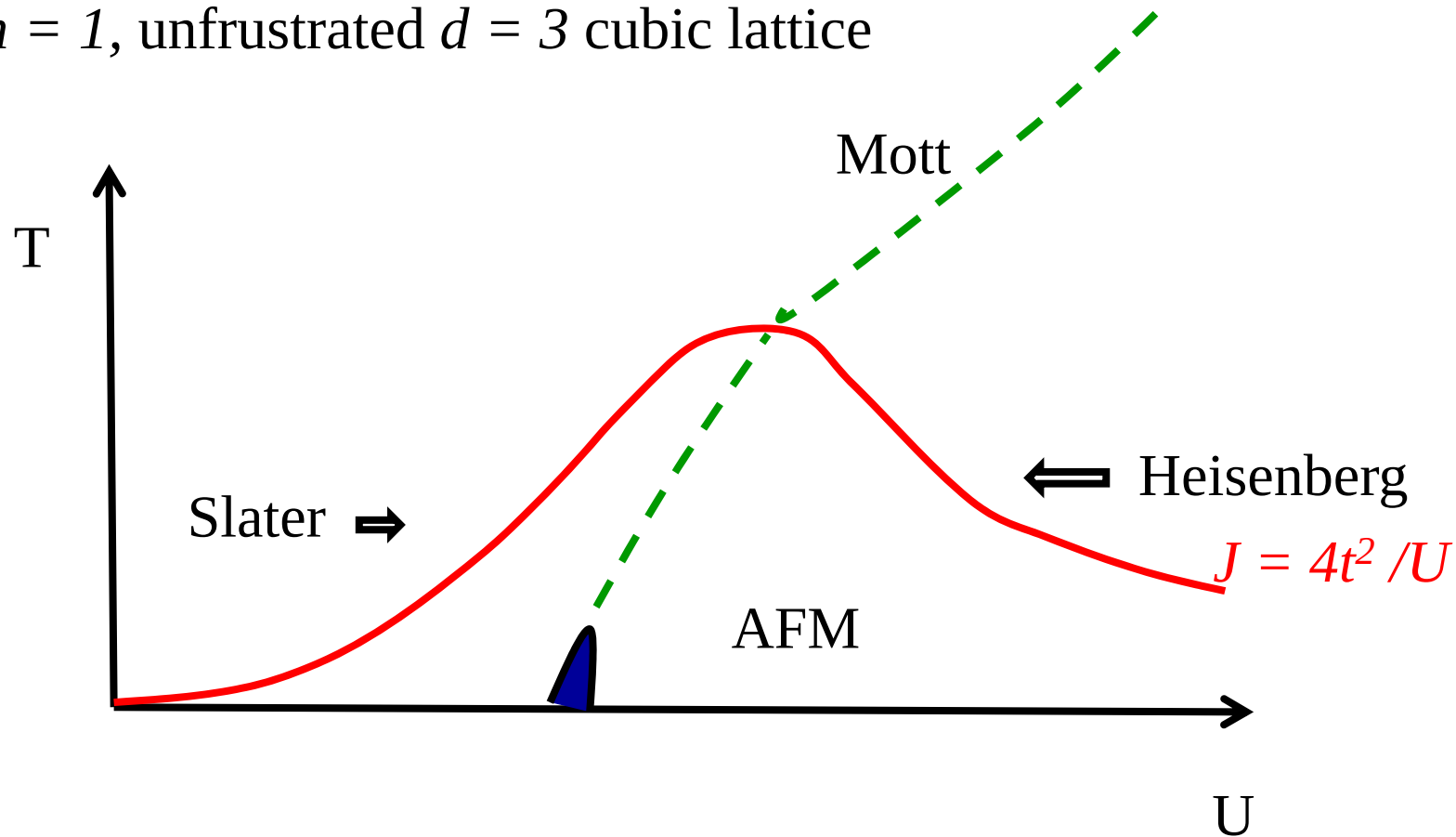
See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges
PRB **80**, 064501 (2009)



Crossovers inside the AFM phase

$n = 1$, unfrustrated $d = 3$ cubic lattice





Giovanni Sordi



Patrick Sémon



Lorenzo Fratino

Finite T phase diagram Superconductivity

Sordi et al. PRL **108**, 216401 (2012)

Fratino et al.

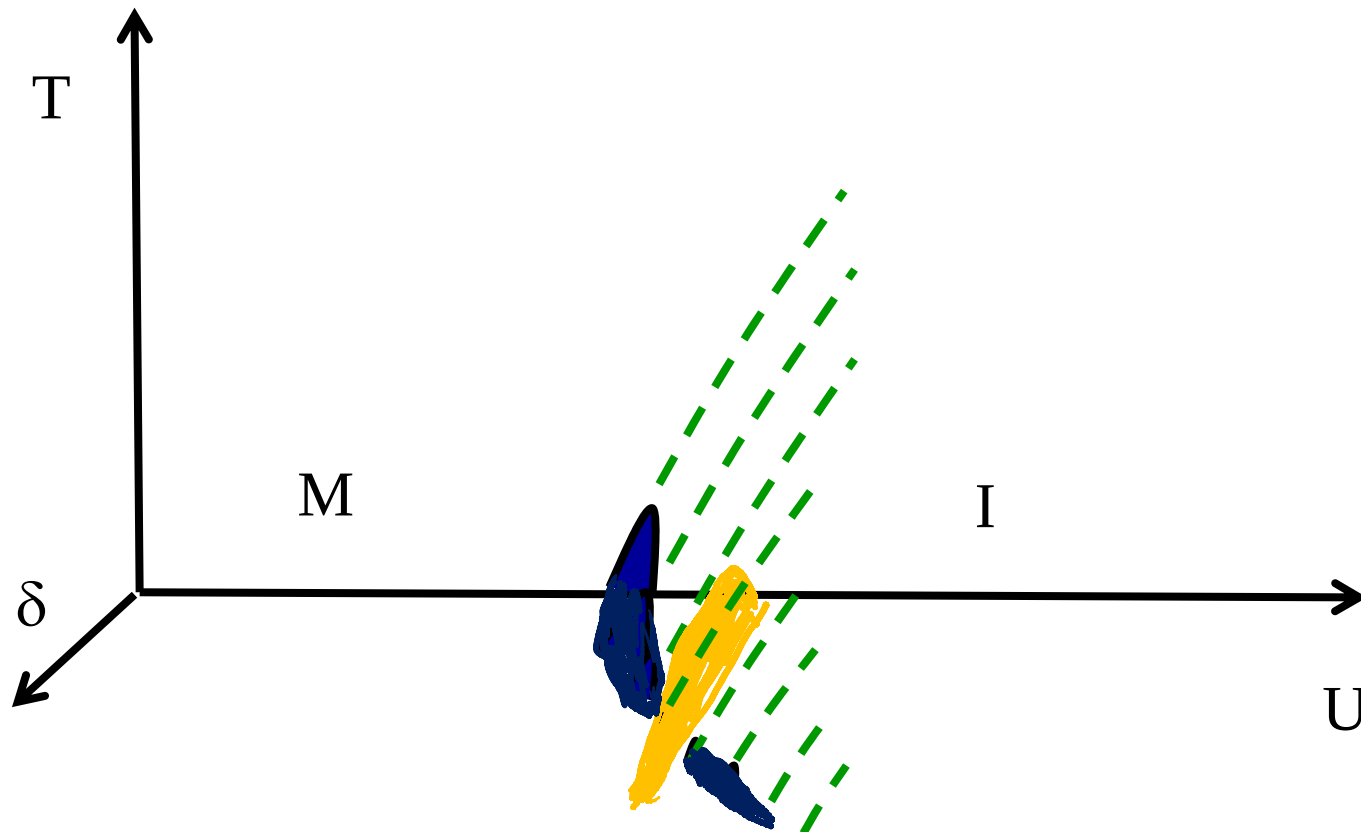
Sci. Rep. **6**, 22715 (2016)



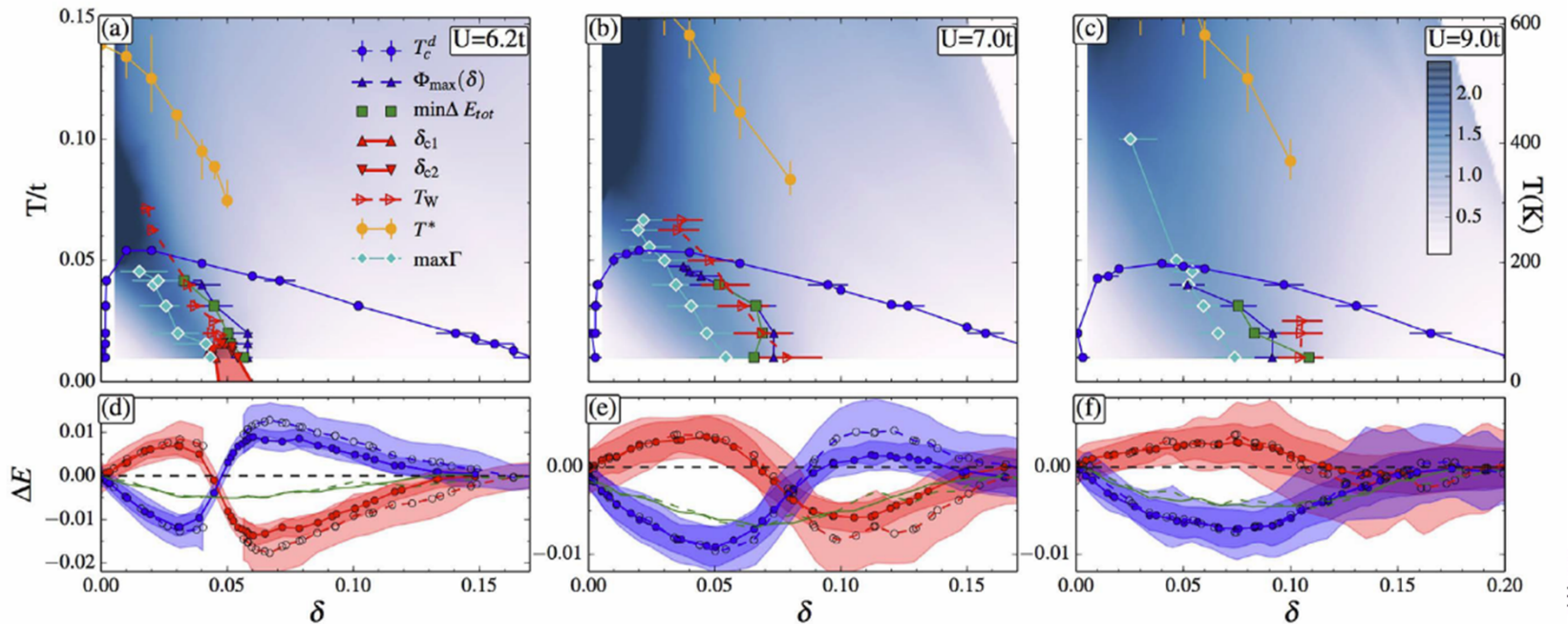
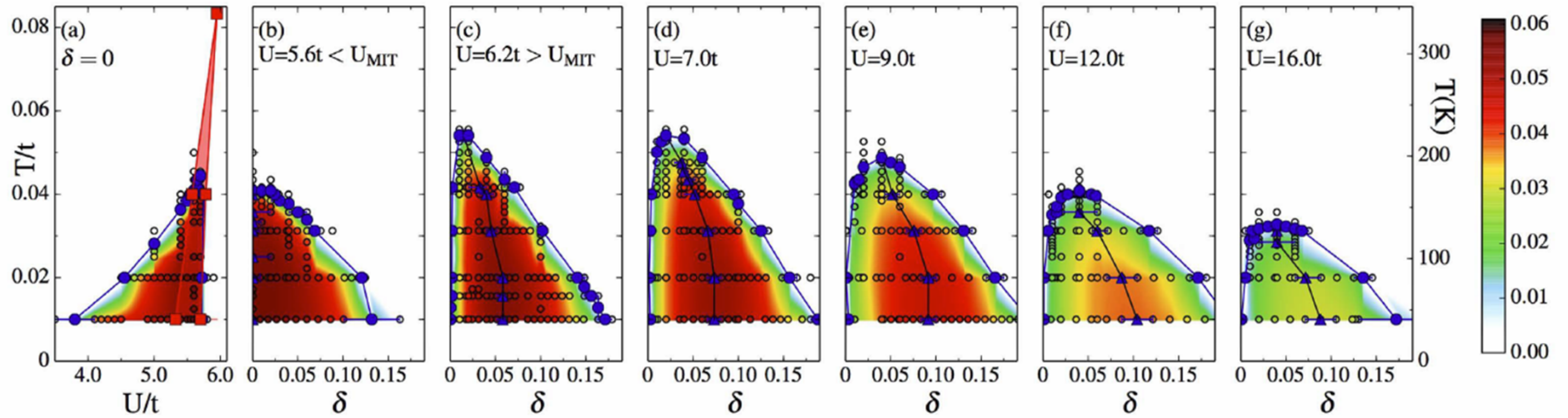
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Superconductivity in Doped Mott insulator

$n = 1, d = 2$ square lattice



An organizing principle



Fratino et al.
Sci. Rep. 6, 22715

3 bands, charge transfer insulator

Fratino et al. PRB **93**, 245147 (2016)

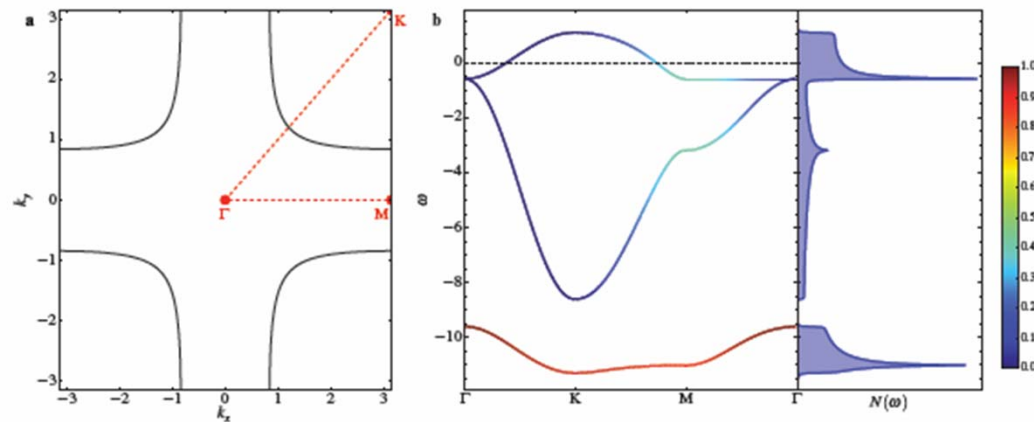
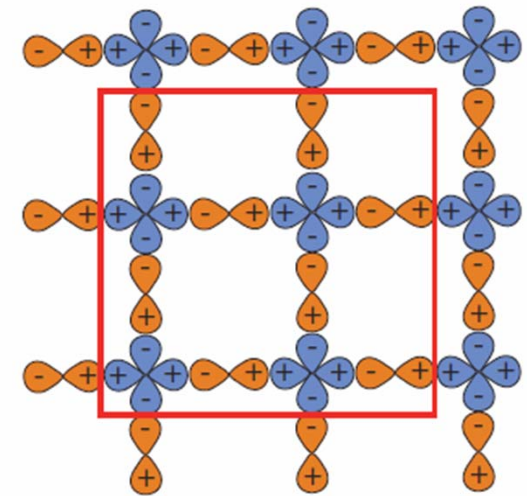


FIG. 2. (a) Noninteracting Fermi surface for the model parameter investigated in Fig. 1a of main text, namely $\epsilon_p = 9$, $t_{pp} = 1$, $t_{pd} = 1.5$, which gives a total occupation n_{tot} equal to five. (b) Non-interacting band structure for the same model parameter along with the resulting total density of states. Color corresponds to the d-character of the hybridised bands. The band crossing the Fermi level has mostly oxygen character.



Giovanni Sordi

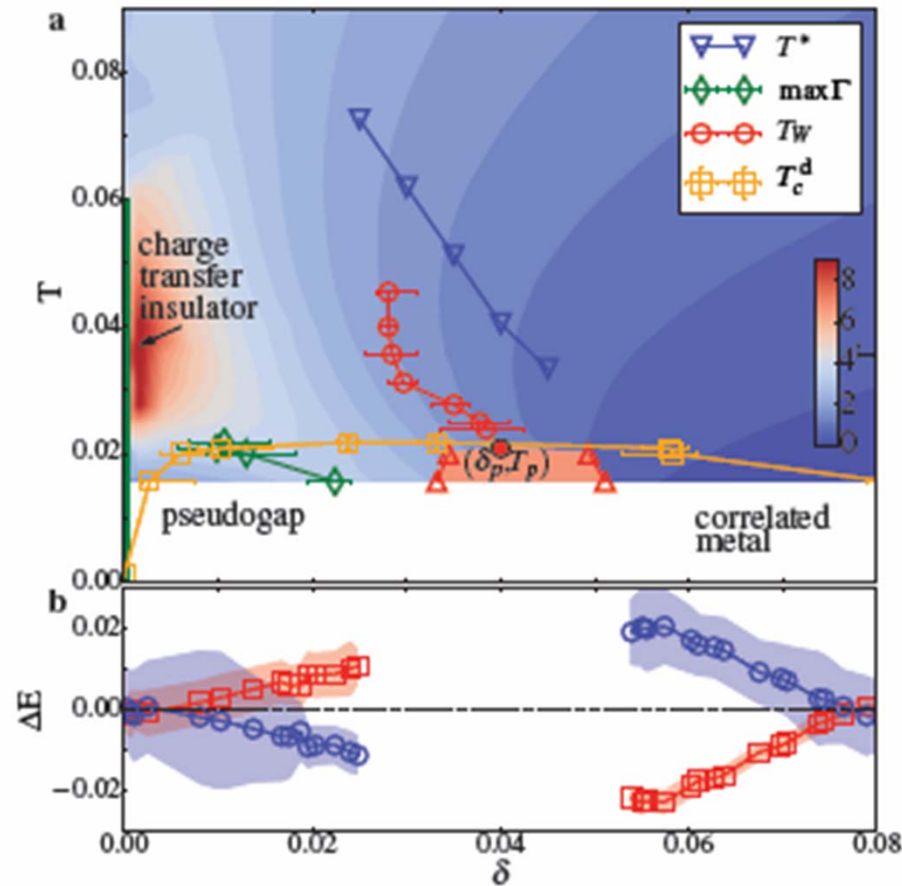


Lorenzo Fratino

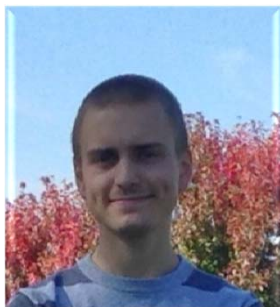


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3 bands, charge transfer insulator



Fratino et al. PRB 93, 245147 (2016)



Charles-David Hébert



Patrick Sémon

Organics : Phase diagram, finite T

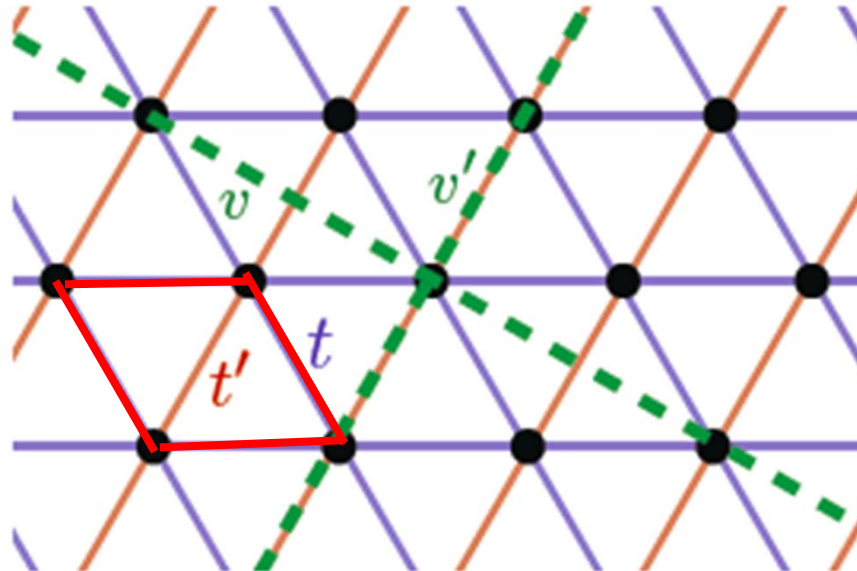
Made possible by algorithmic improvements

P. Sémon *et al.*
PRB **85**, 201101(R) (2012)
PRB **90** 075149 (2014);
and PRB **89**, 165113 (2014)



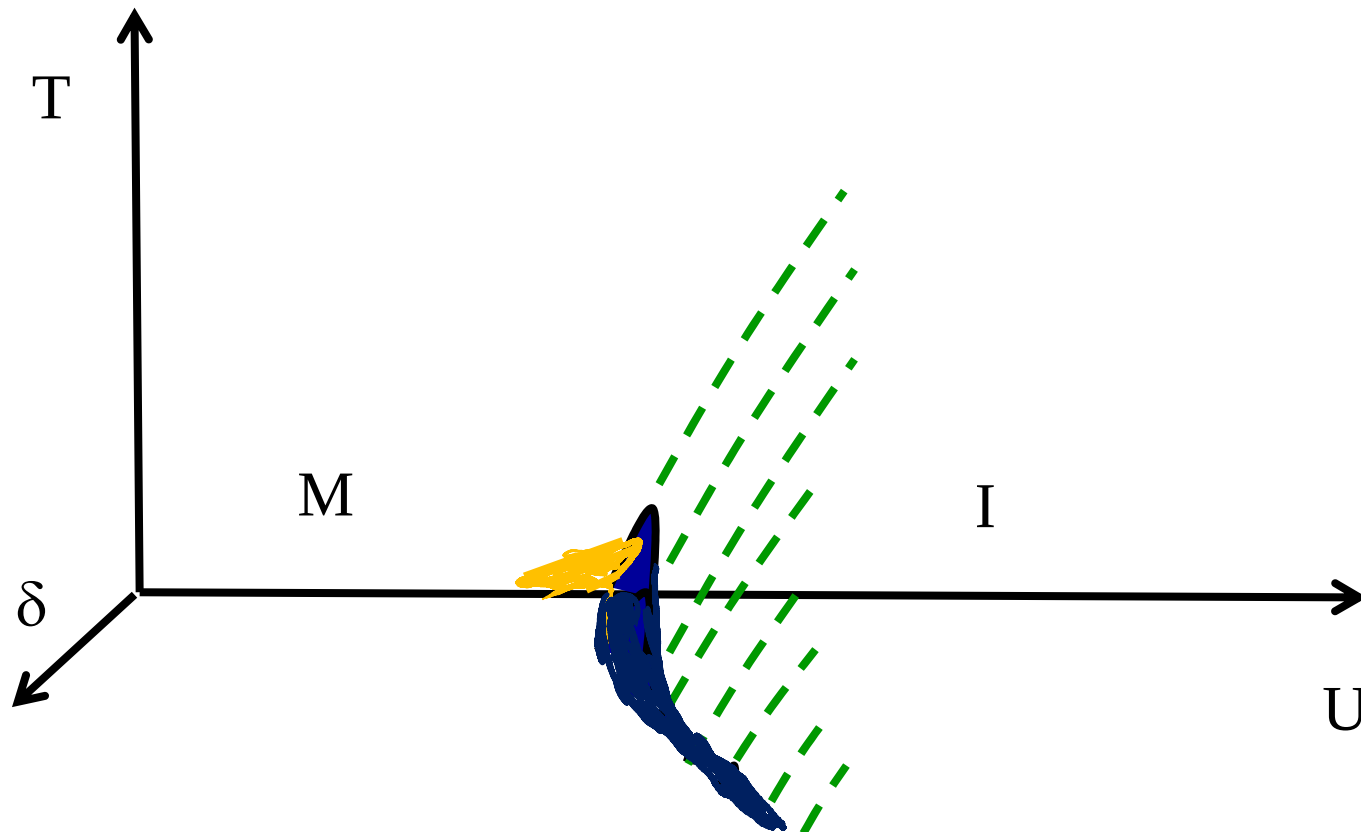
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Anisotropic triangular lattice

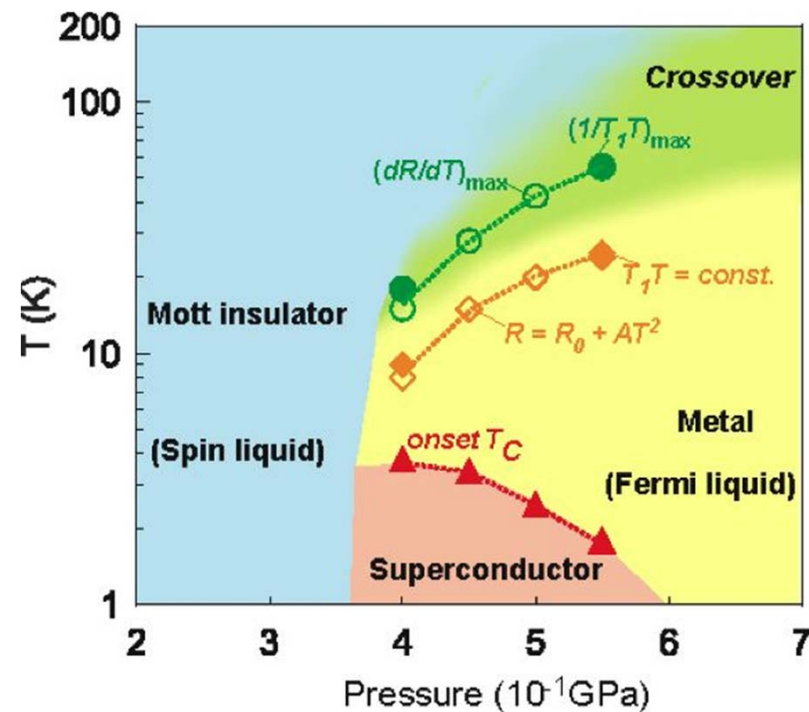


Superconductivity near the Mott transition

$n = 1, d = 2$ square lattice



Phase diagram at $n = 1$



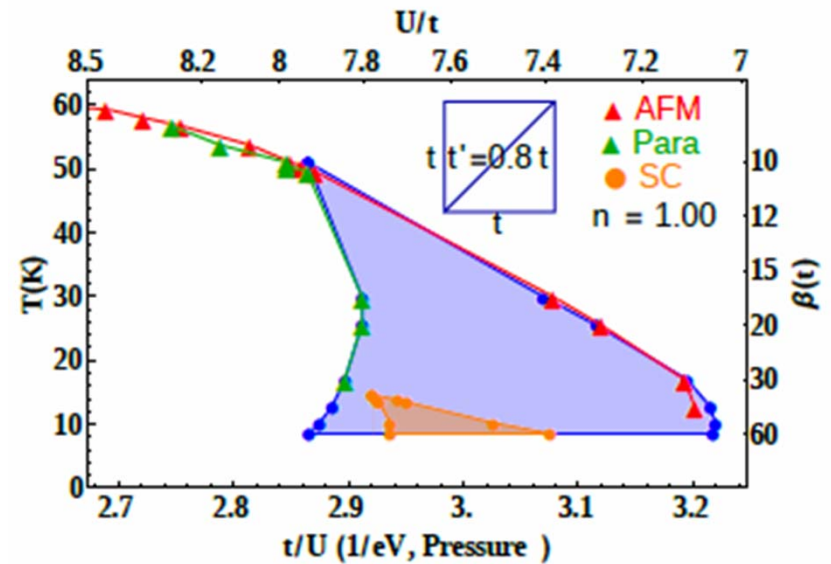
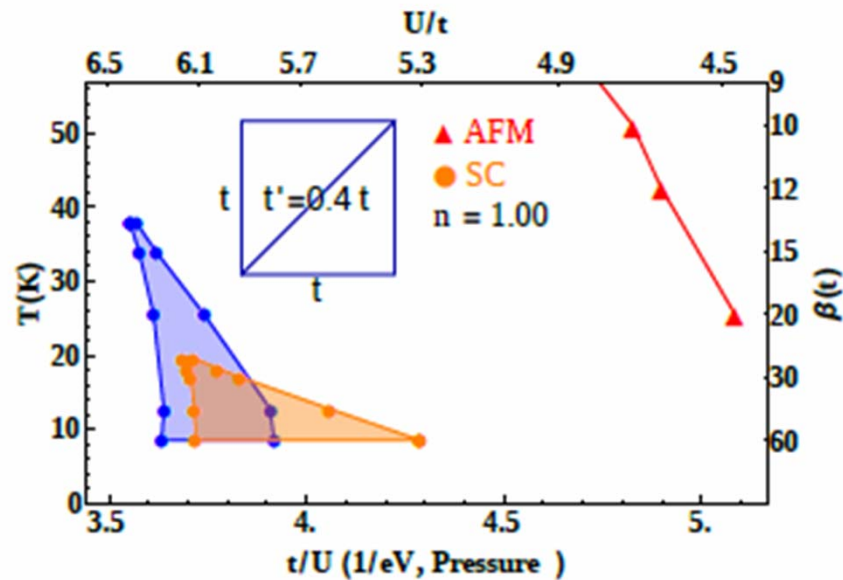
Y. Kurisaki, et al.

Phys. Rev. Lett. **95**, 177001(2005)

Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)



Effect of frustration on Mott transition ($n = 1$)

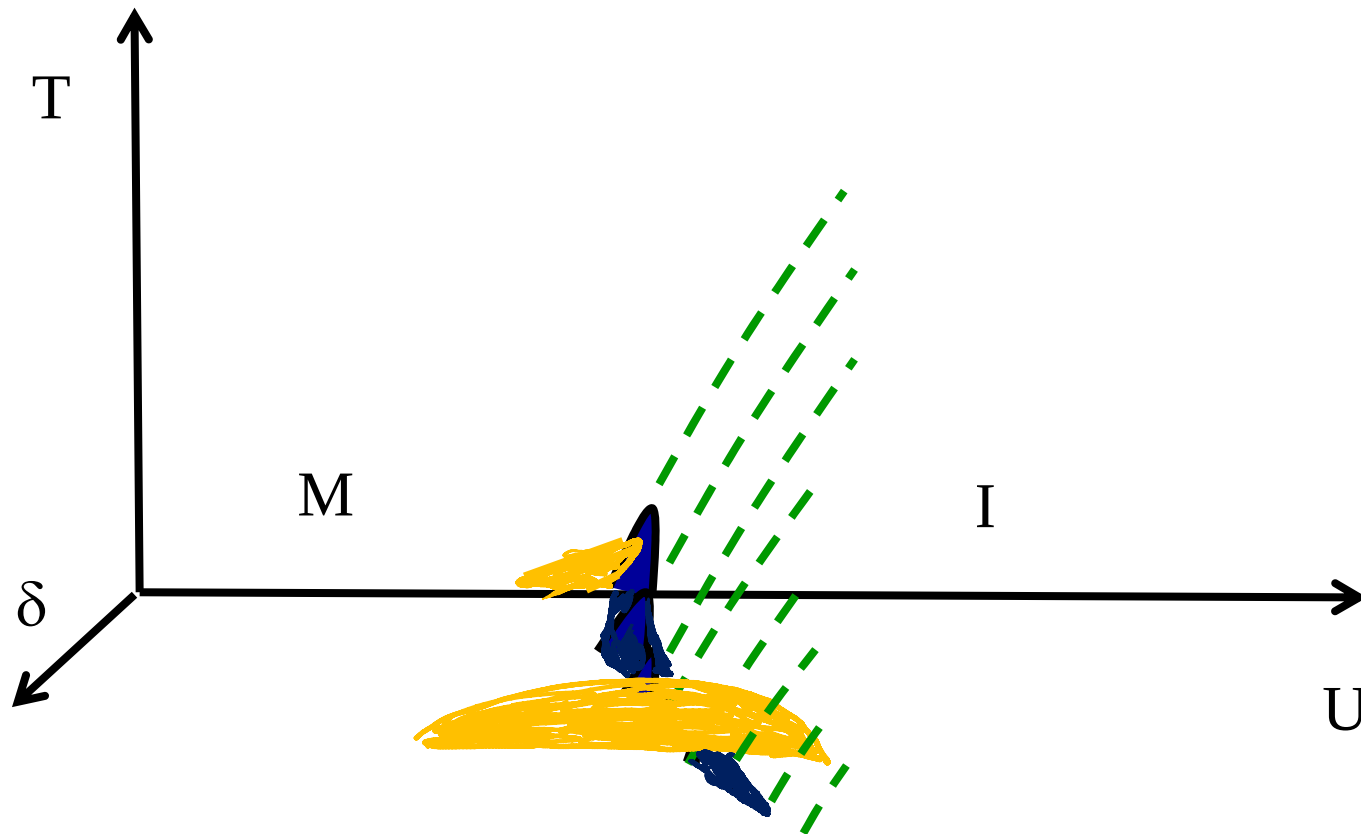


Doped Organics



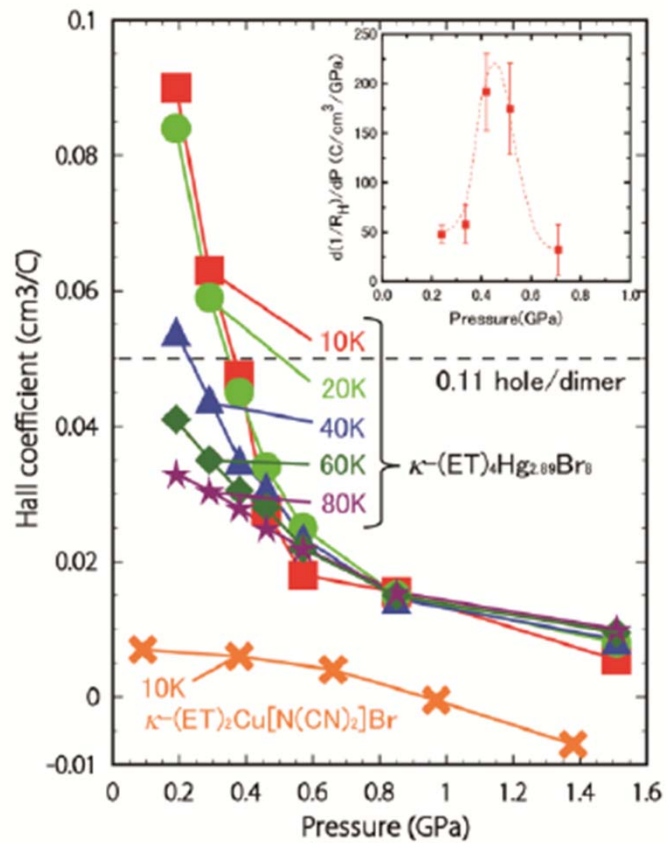
Doped organics

$n = 1, d = 2$ square lattice

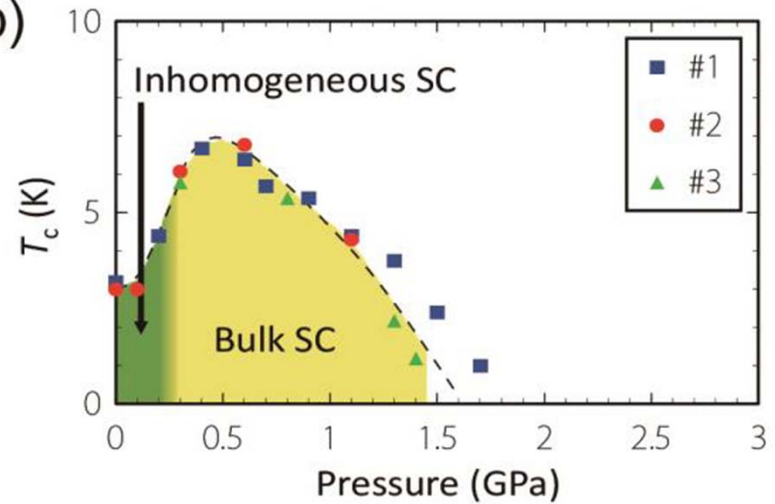


Doped BEDT

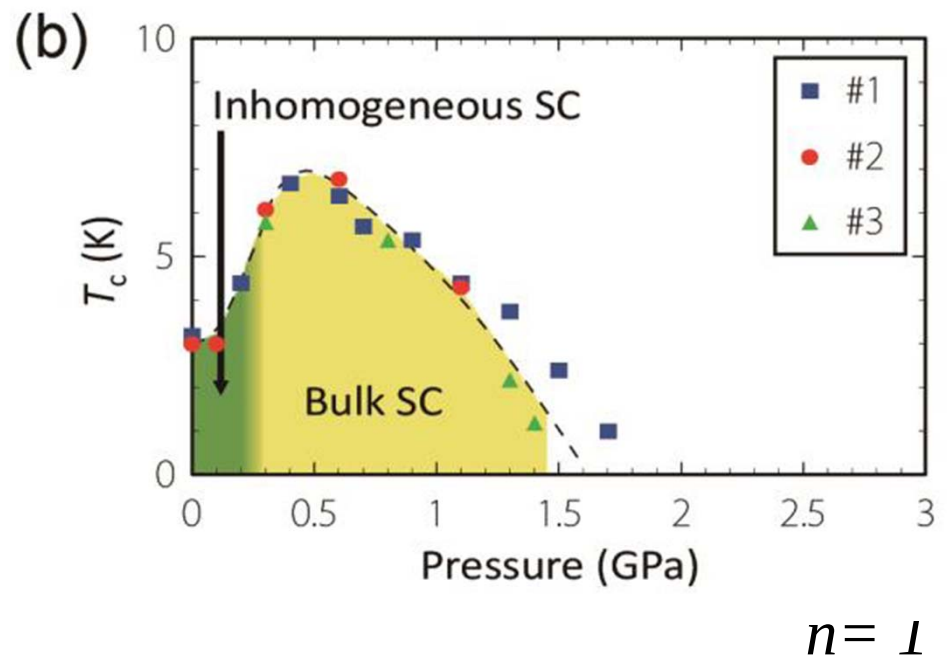
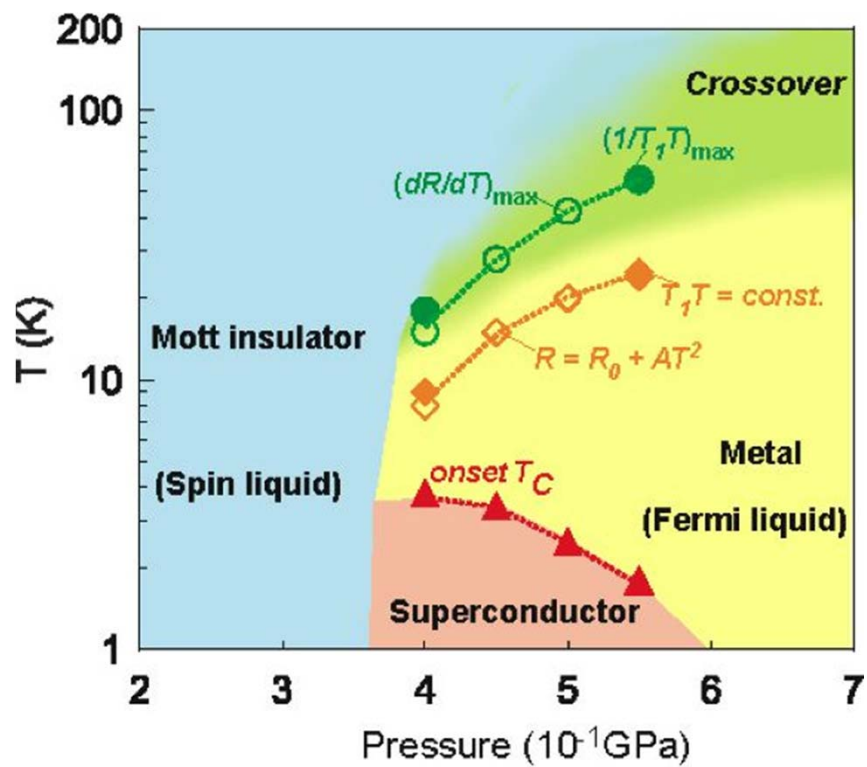
(b)



(b)



Doped BEDT

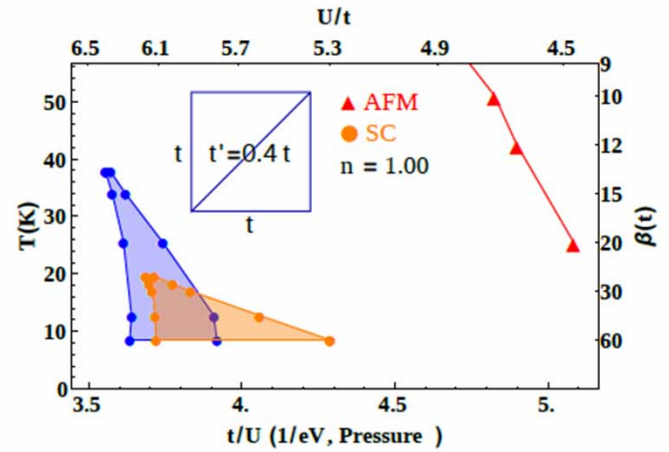
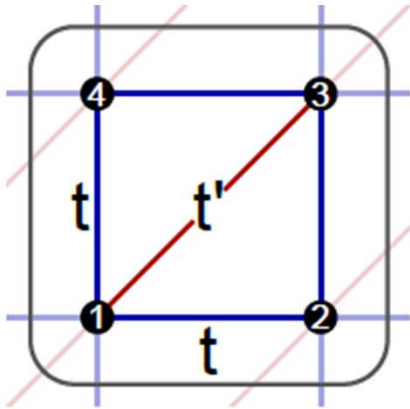


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)

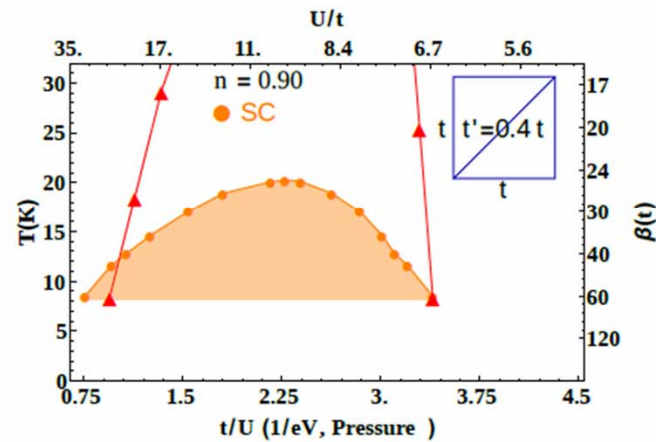


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$$t' = 0.4t$$



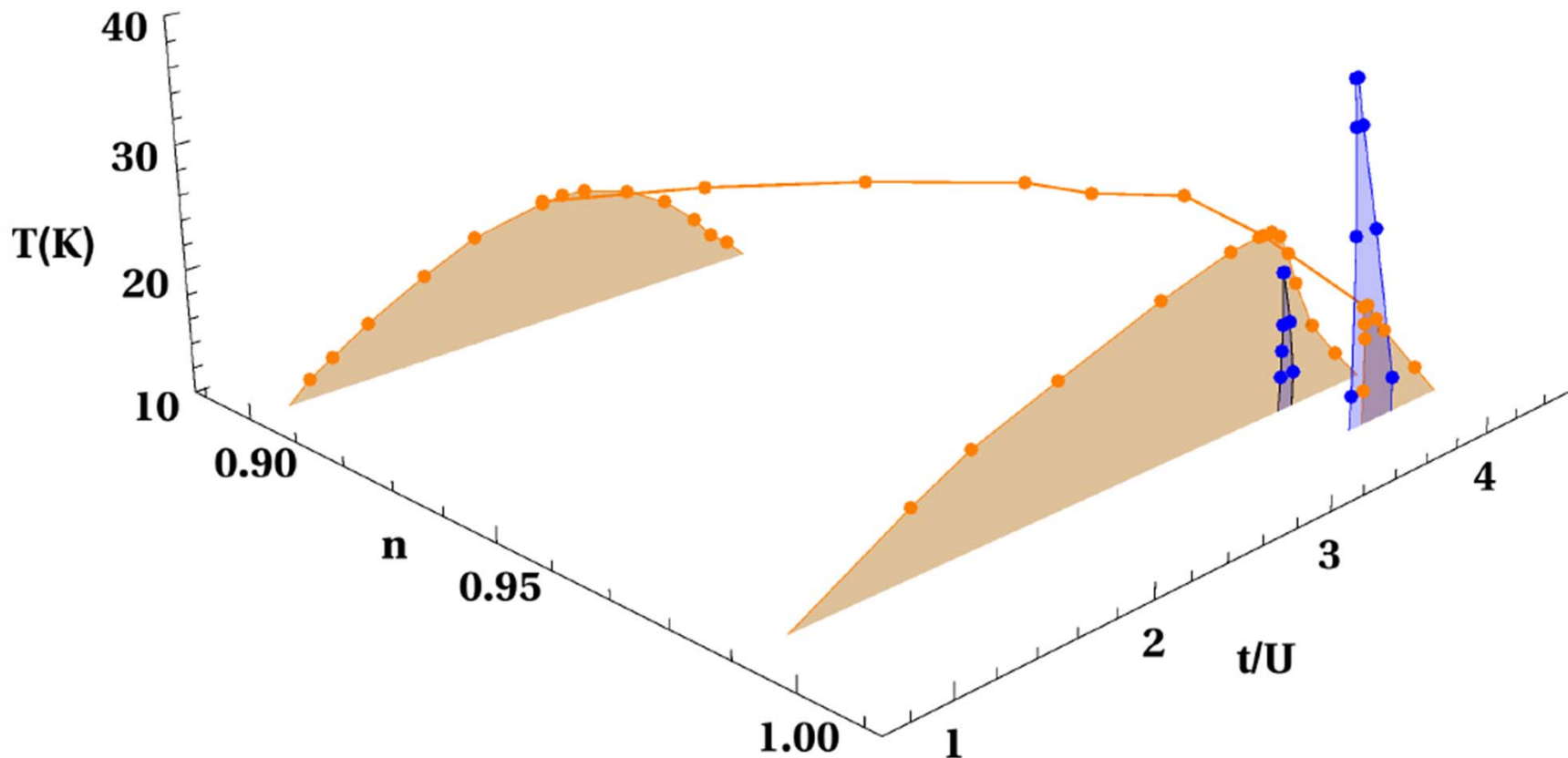
(a)



Compare: T. Watanabe, H. Yokoyama and M. Ogata
JPS Conf. Proc. 3, 013004 (2014)



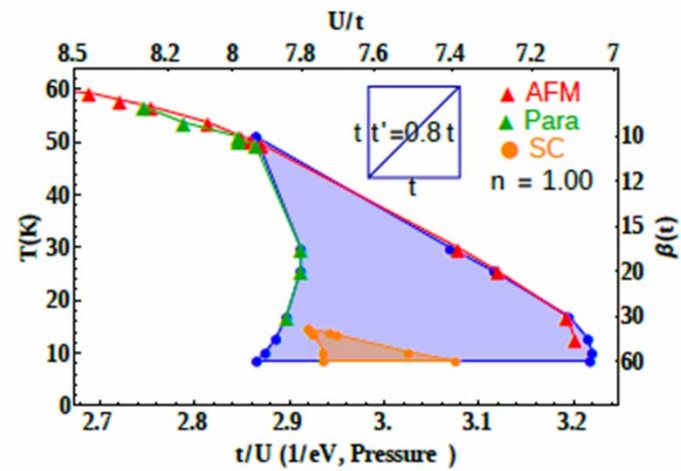
$t' = 0.4t$ overview



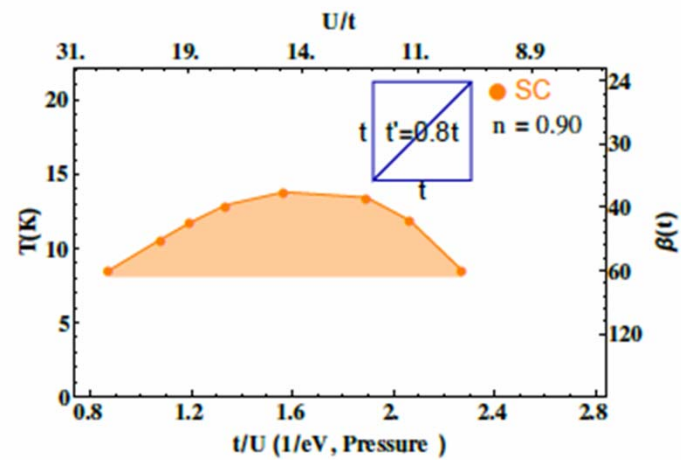
Compare: T. Watanabe, H. Yokoyama and M. Ogata
JPS Conf. Proc. 3, 013004 (2014)



$$t' = 0.8 t$$

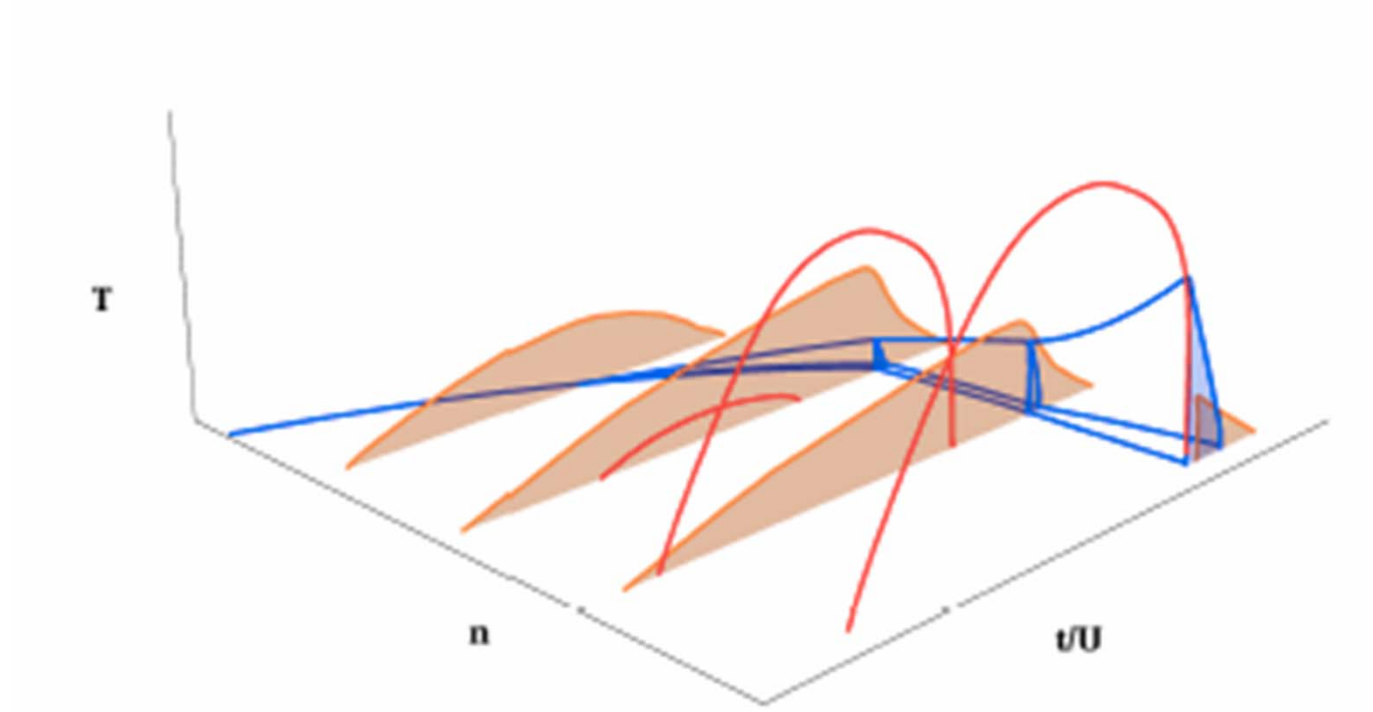


(a)



(b)

Generic case highly frustrated case



Summary : organics

- Agreement with experiment
 - SC: larger T_c and broader P range if doped
 - Larger frustration: Decrease T_N *much more* than T_c
 - Normal state metal to pseudogap crossover
- Predictions
 - First order transition at low T in normal state (B induced)
 - Crossovers in SC state associated with normal state.
- Physics
 - SC dome without an AFM QCP. Extension of Mott
 - SC from short range J .
 - T_c dome maximum near normal state 1st order

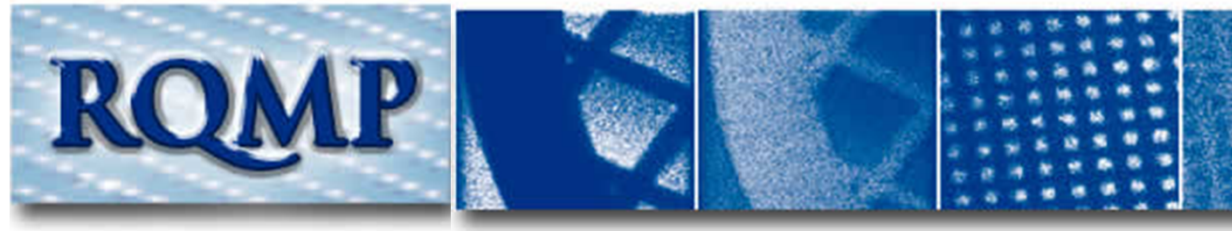
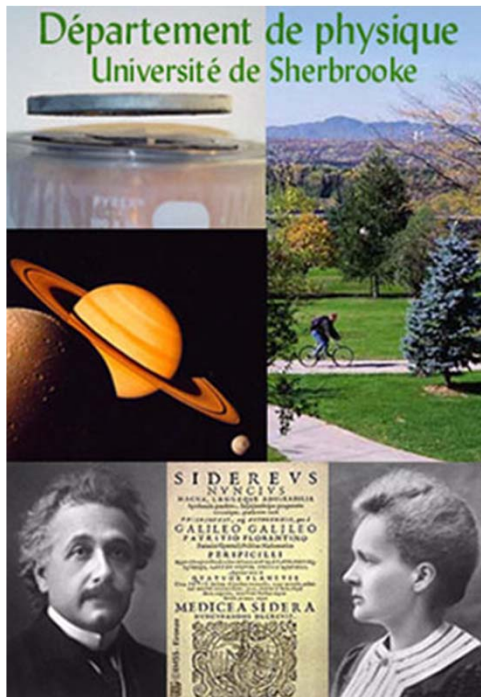


Summary

- AFM QCP for a heavy-fermion model
- No QCP: First order transition that extends Mott physics away from half-filling
- Is an organizing principle for
 - The normal and superconducting states
 - Cuprates and organics are examples
 - Predictions for organics
- Mechanism: J short-range (not shown)



André-Marie Tremblay



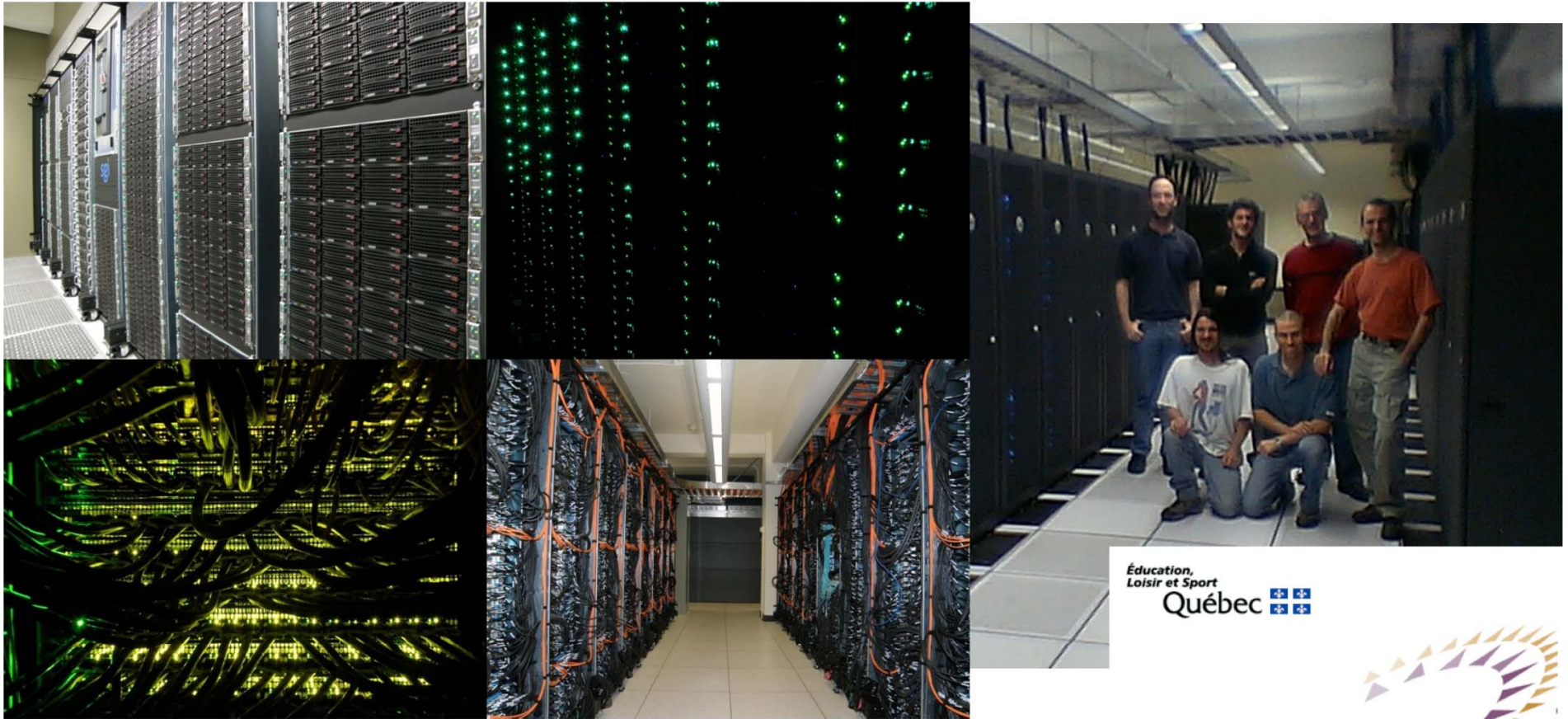
Le regroupement québécois sur les matériaux de pointe



Sponsors:



Mammoth



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Review: A.-M.S.T. [arXiv: 1310.1481](https://arxiv.org/abs/1310.1481)



A.-M.S. Tremblay

“Strongly correlated superconductivity”

Chapt. 10 : *Emergent Phenomena in Correlated Matter Modeling and Simulation*, Vol. 3, E. Pavarini, E. Koch, and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich, 2013