

Supraconductivité avec et sans point critique quantique antiferromagnétique

A.-M. Tremblay

Institut quantique

**Réunion inaugurale, LIA
Jouvence 27 octobre 2016**



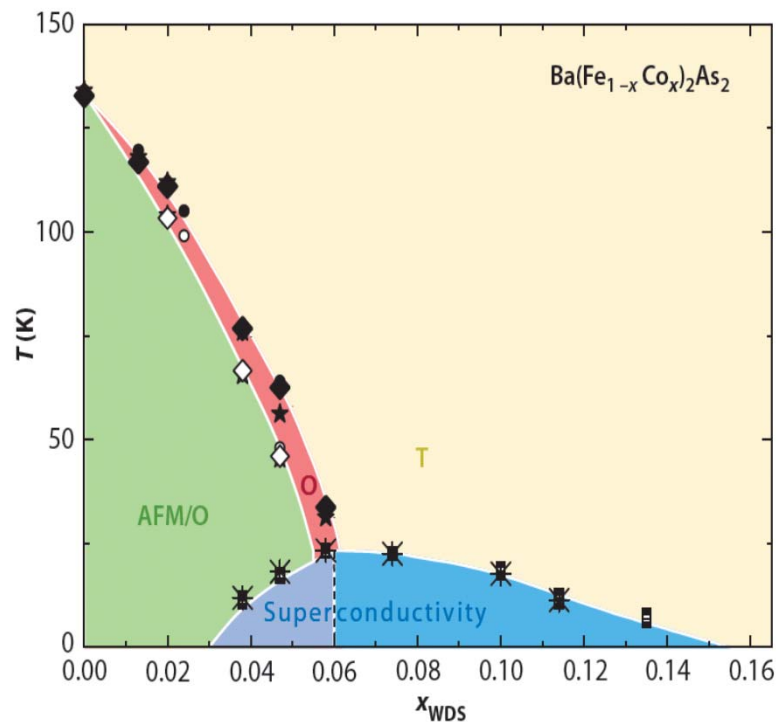
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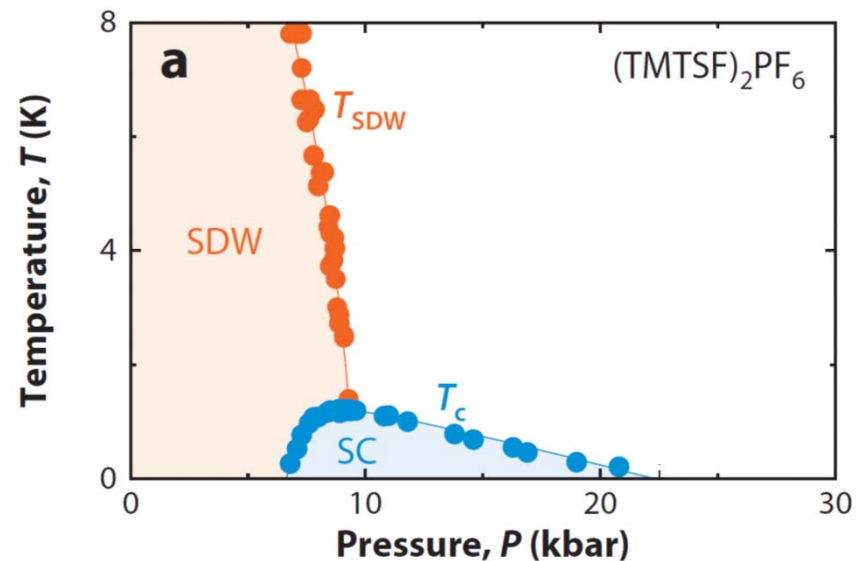
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Pnictides and organics

Pnictides



Bechgaard salts



The Archetype

Magnetic superconductivity

Nicolas Doiron-Leyraud, Bourbonnais, Taillefer 2010

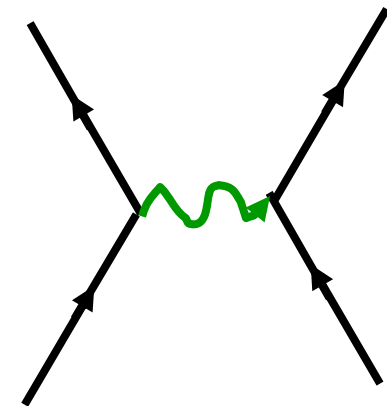
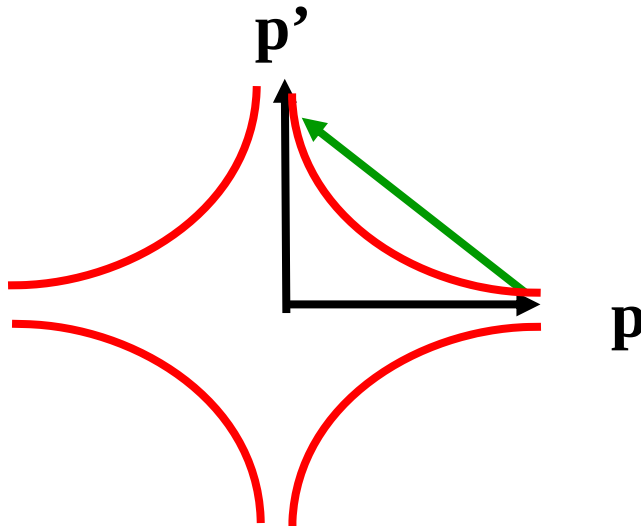
Canfield et al. (2010)



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Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?
Kohn-Luttinger

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).

T_c with pressure

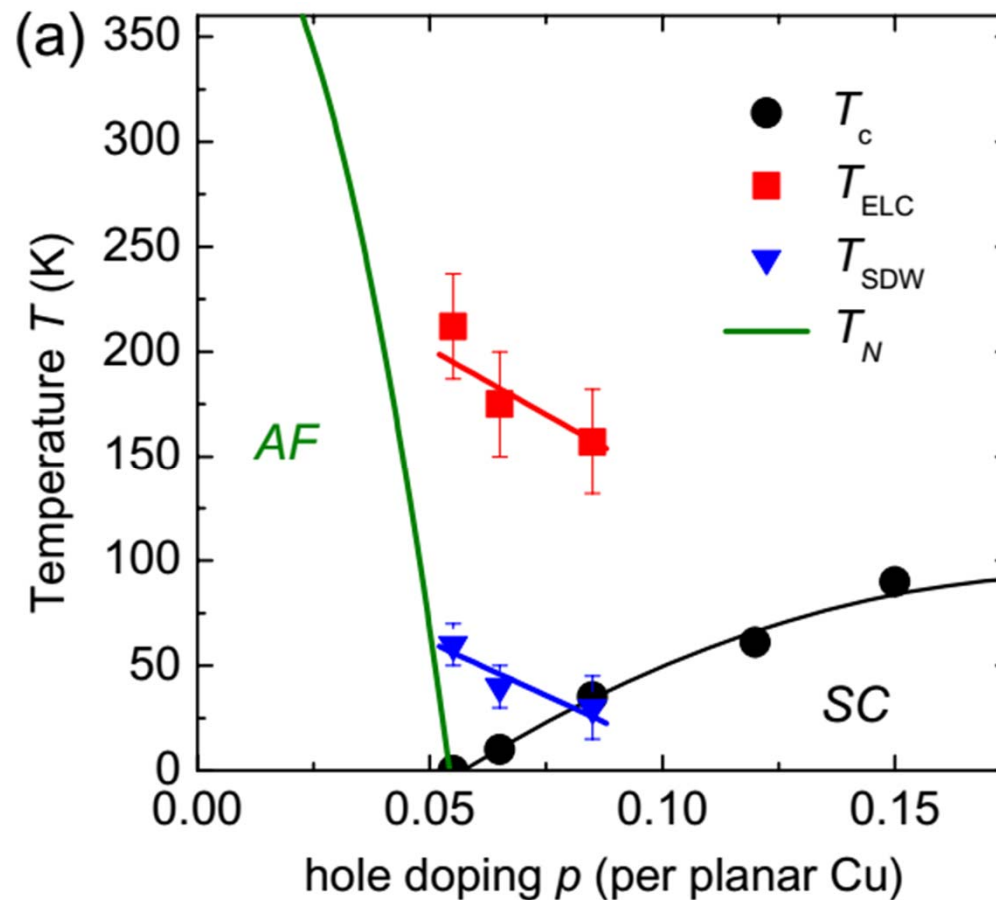
Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)

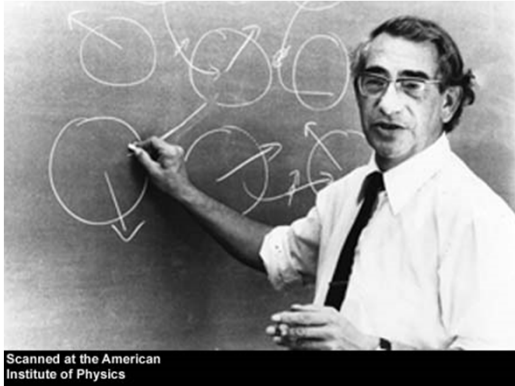


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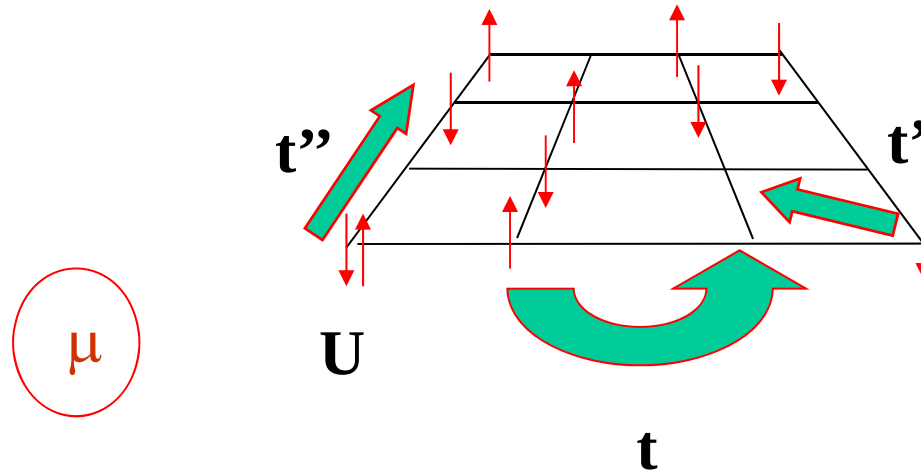
High temperature superconductors



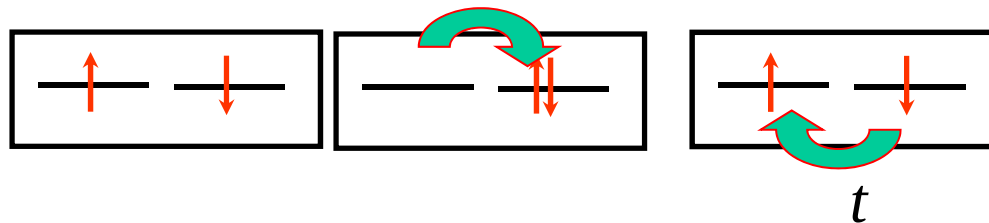
Hubbard model



1931-1980



$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Effective model, Heisenberg: $J = 4t^2 / U$

Attn: Charge transfer insulator



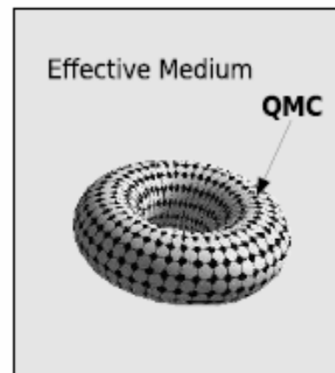
Method for strongly correlated matter

Dynamical Mean Field Theory (+ clusters)

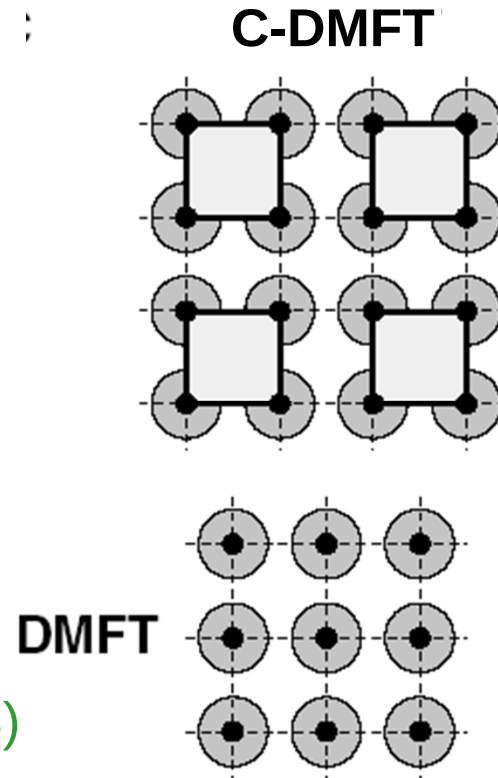
Concept: atomic localized correlations
consistent with delocalized aspect



2d Hubbard: Quantum cluster method



DCA



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff *et al.* PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

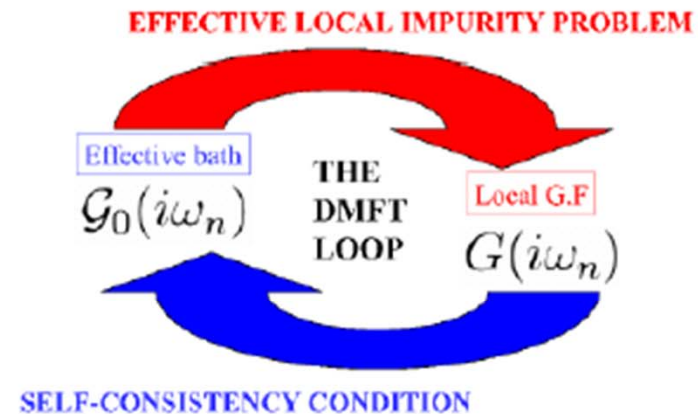
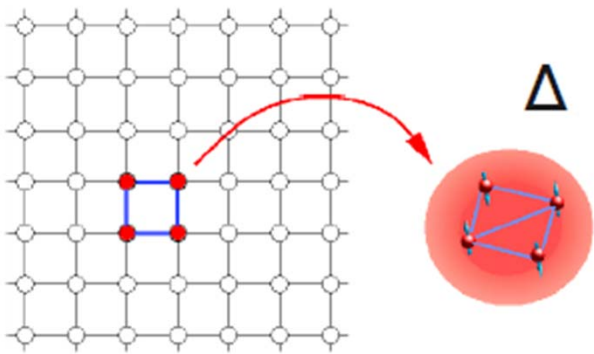
AMST *et al.* LTP (2006)



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Impurity solver

$$Z = \int D[d^\dagger, d] \exp \left[-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_i [d_i^\dagger(\tau) \Delta_{i'i}(\tau, \tau') d_{i'}(\tau')] \right]$$



Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006
P. Werner, PRB 2007
K. Haule, PRB 2007

$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$= \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

+ and -

- Long range order:
 - Allow symmetry breaking in the bath (mean-field)
- Included:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations



Groups using these methods for cuprates

- Europe:
 - Georges, Parcollet, Ferrero, Civelli, (Paris)
 - de Medici (Grenoble) Capone (Italy)
- USA:
 - Gull (Michigan) Millis (Columbia)
 - Kotliar, Haule (Rutgers)
 - Jarrell (Louisiana)
 - Maier, Okamoto (Oakridge)
- Japan
 - Imada (Tokyo) Sakai





Wei Wu

Superconductivity around an AFM quantum critical point

A heavy fermion example

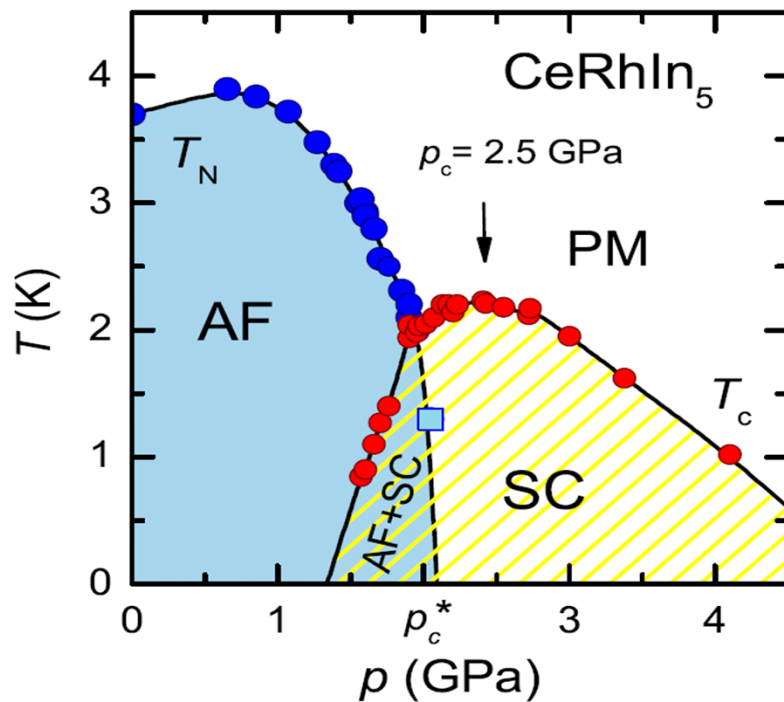
W. Wu A.-M.S.T. Phys. Rev. X, 2015



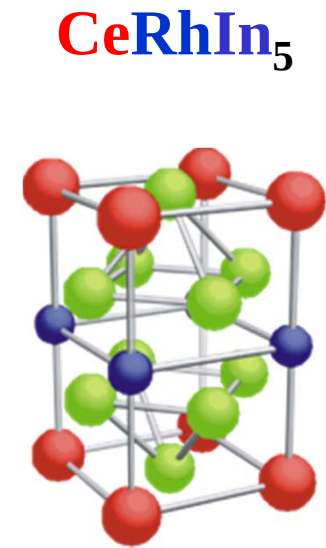
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Heavy fermions

Heavy fermions 3D metals tuned by pressure, field or concentration



Knebel et al. (2009)



**Magnetic
superconductivity**

Quantum criticality

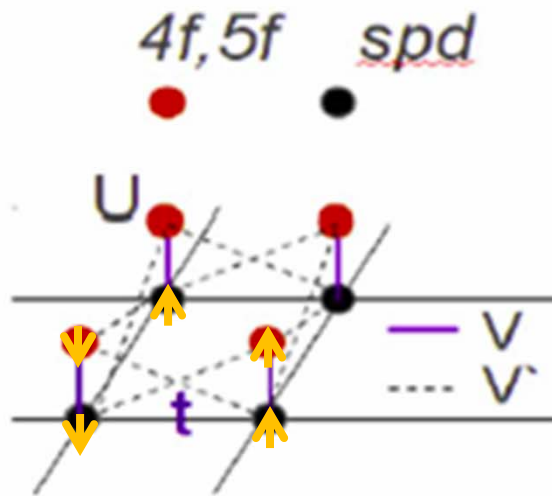
Mathur et al., Nature 1998



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Heavy fermions

$$H = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{k,\sigma} \epsilon_f^f f_{k,\sigma}^\dagger f_{k,\sigma} \\ + \sum_{k,\sigma} V_k (f_{k,\sigma}^\dagger c_{k,\sigma} + \text{H.c.}) + \sum_i U \left(n_f^\uparrow - \frac{1}{2} \right) \left(n_f^\downarrow - \frac{1}{2} \right)$$



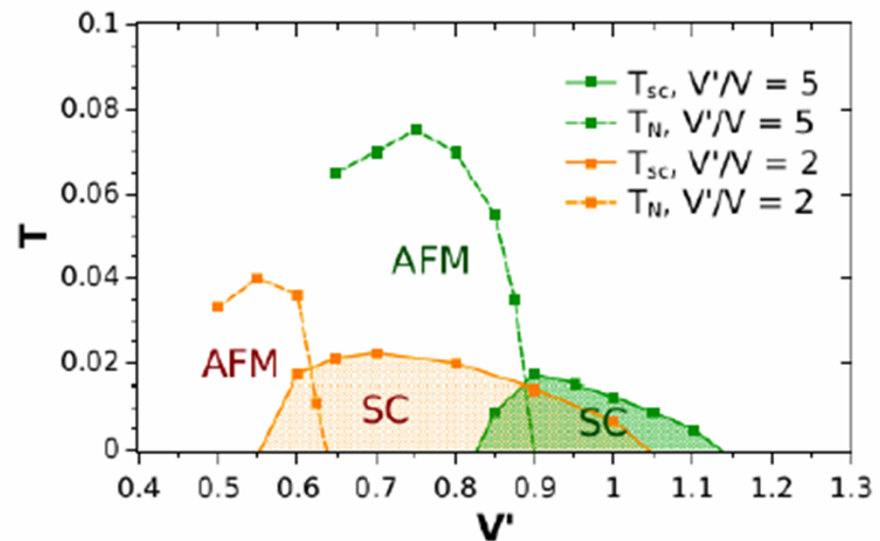
W. Wu A.-M.S.T. Phys. Rev. X, 2015

Phase diagram

$$U=4$$

AFM: antiferro-magnetism
SC: superconducting

$V'/V = 2$: more frustrated case
 $V'/V = 5$: less frustrated case



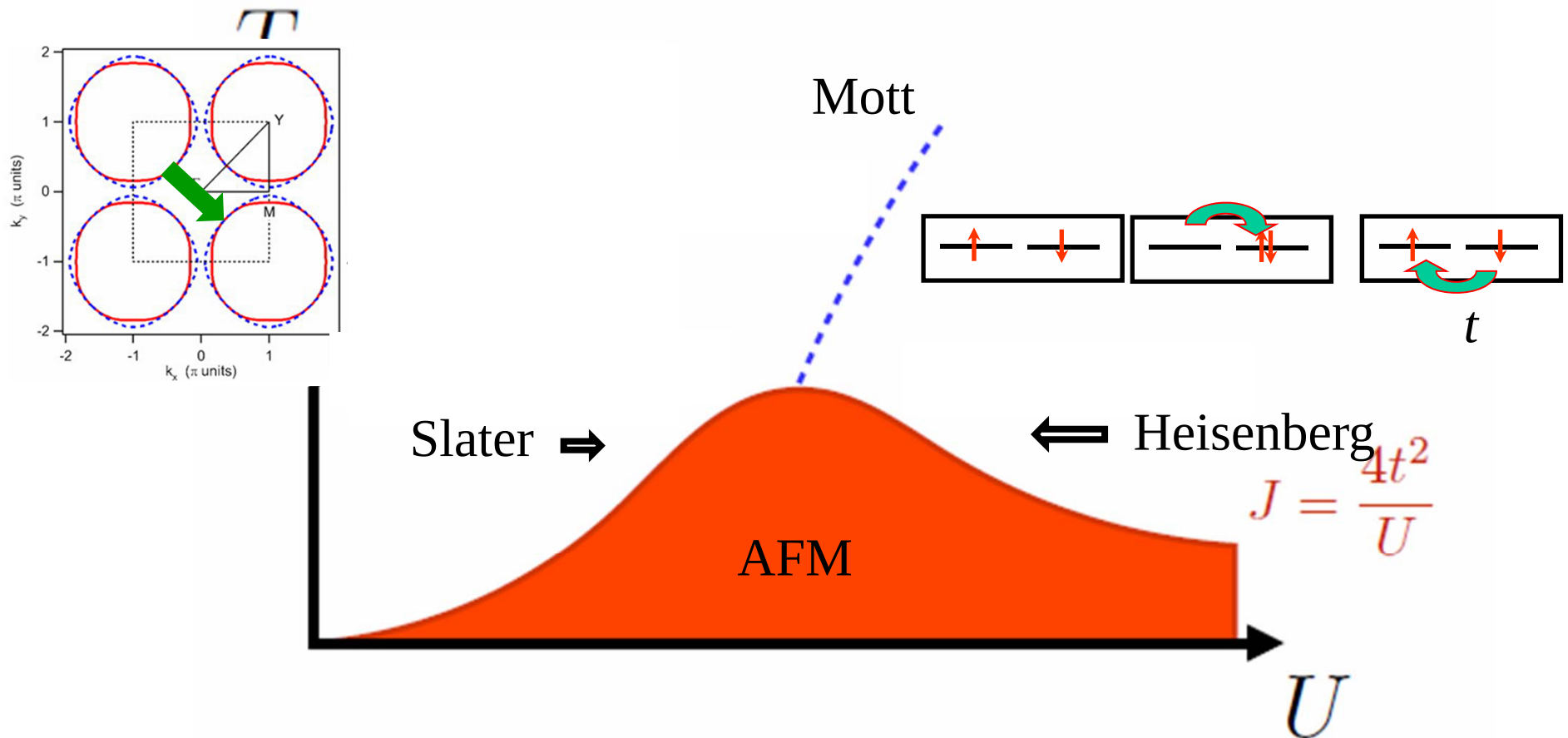
Weakly vs strongly correlated superconductivity

Analog to weakly and strongly
correlated antiferromagnets



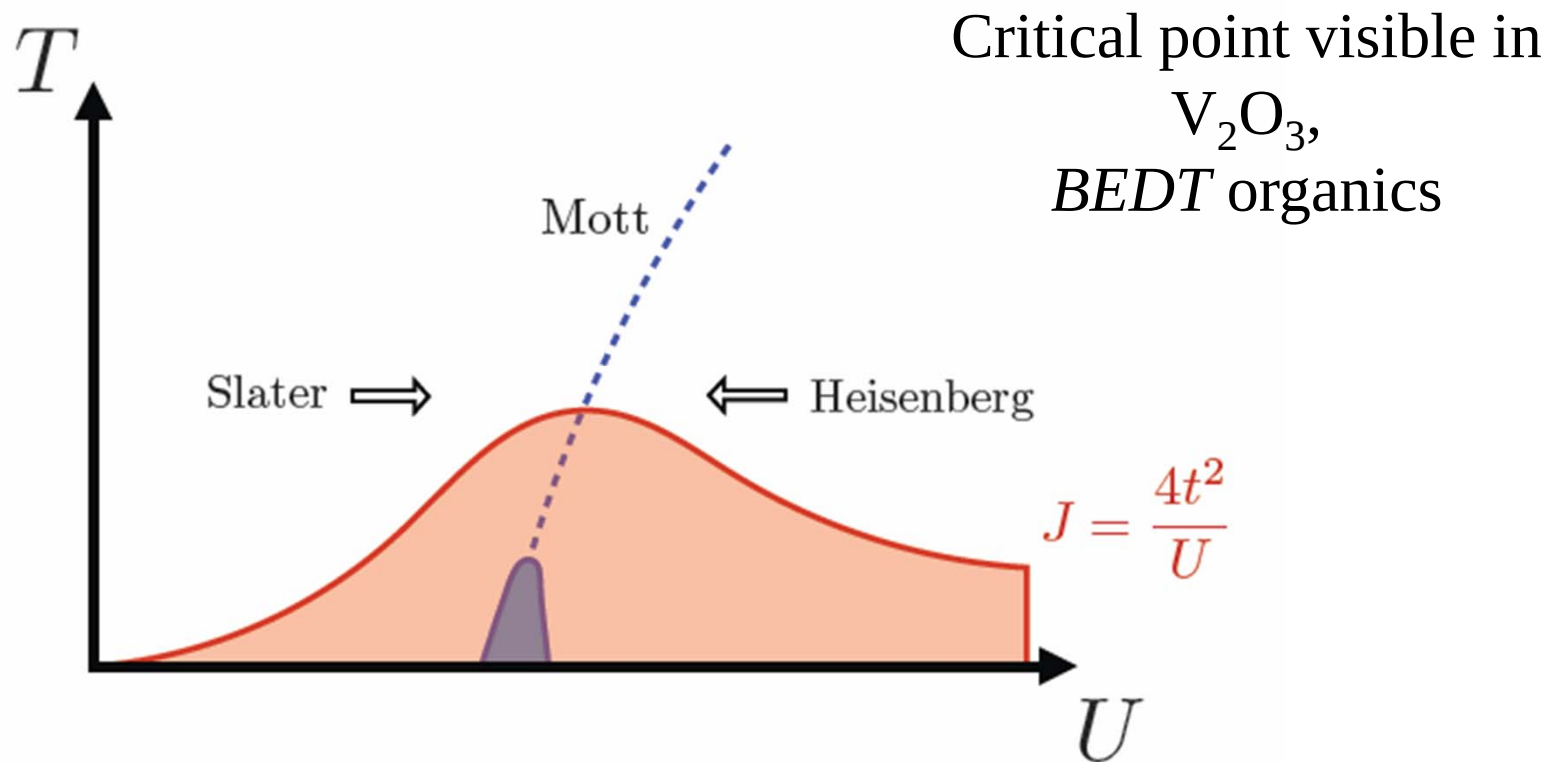
Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



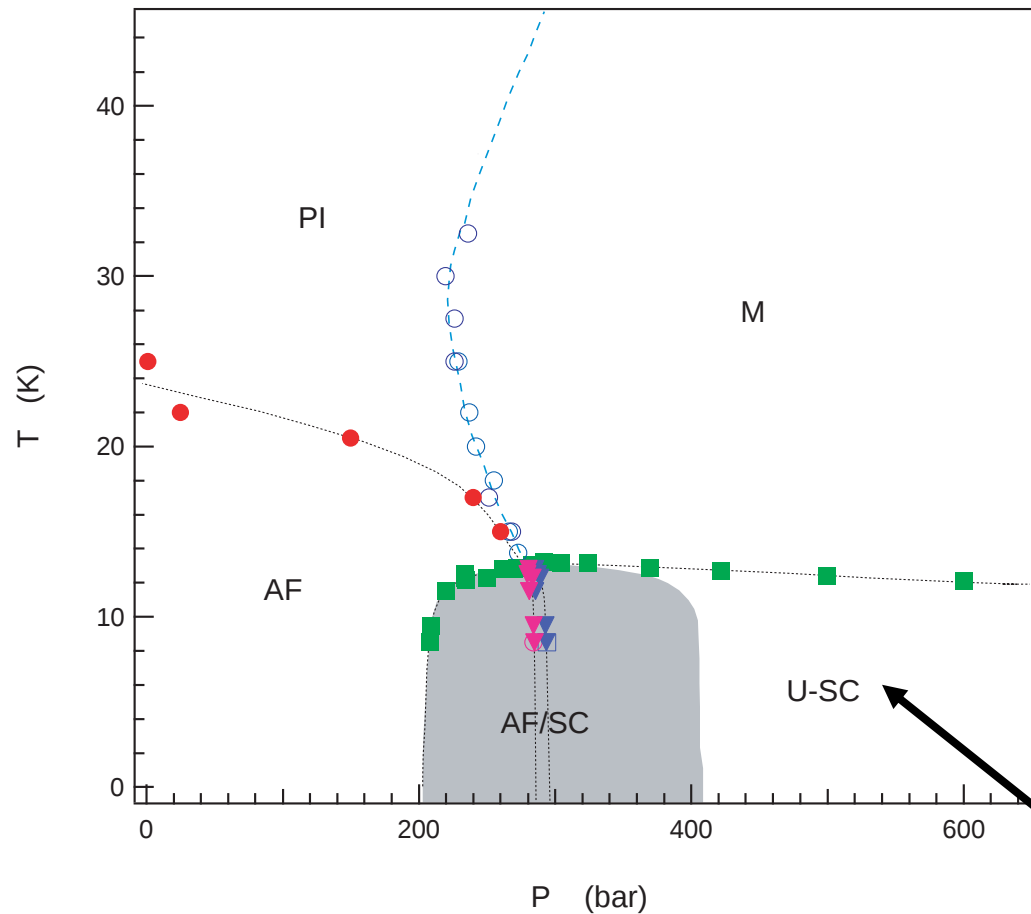
Local moment and Mott transition

$n = 1, d = 2$ square lattice



Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

Phase diagram for organics



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

B_g for C_{2h} and B_{2g} for D_{2h}

Powell, McKenzie cond-mat/0607078

Phase diagram ($X = \text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

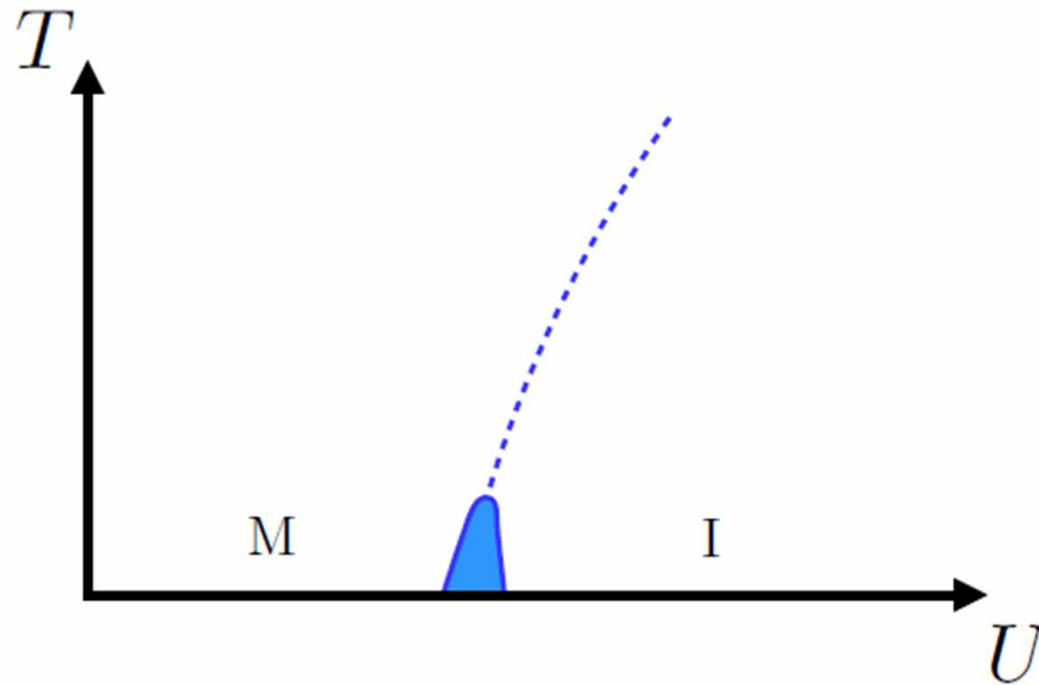
S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)



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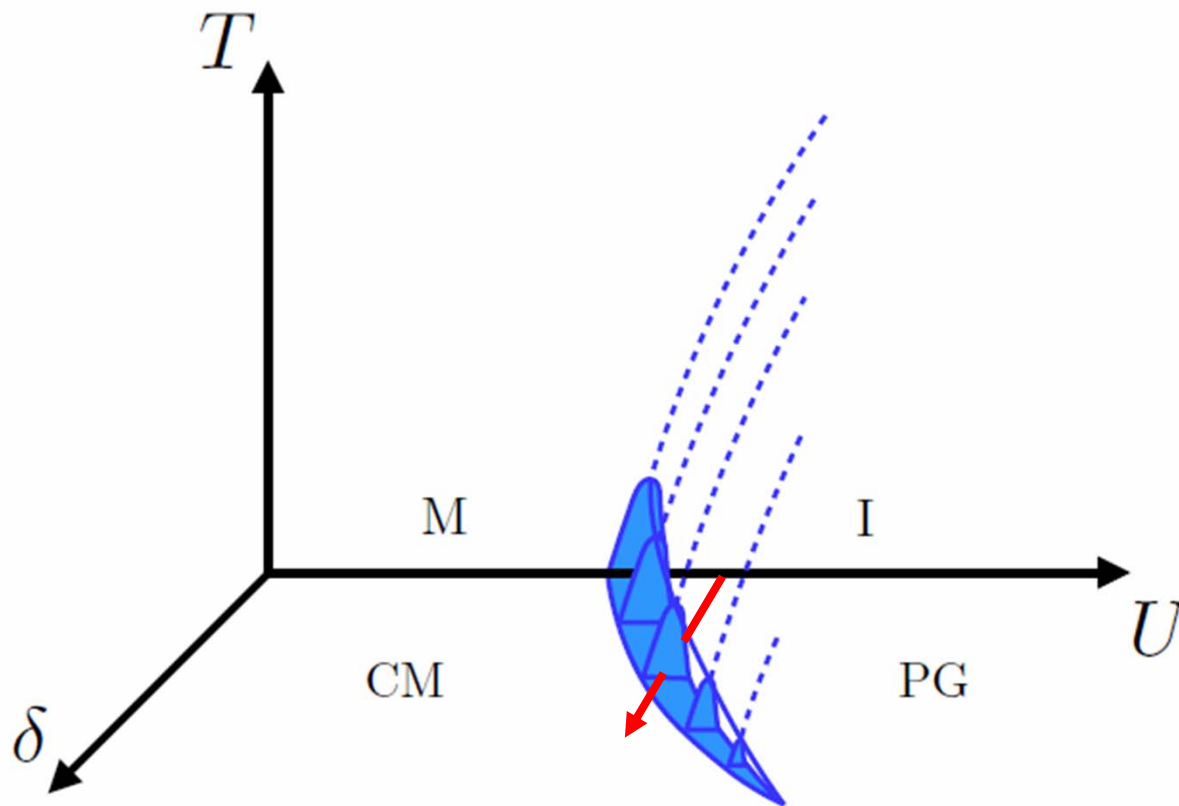
Influence of Mott transition away from half-filling

$n = 1, d = 2$ square lattice

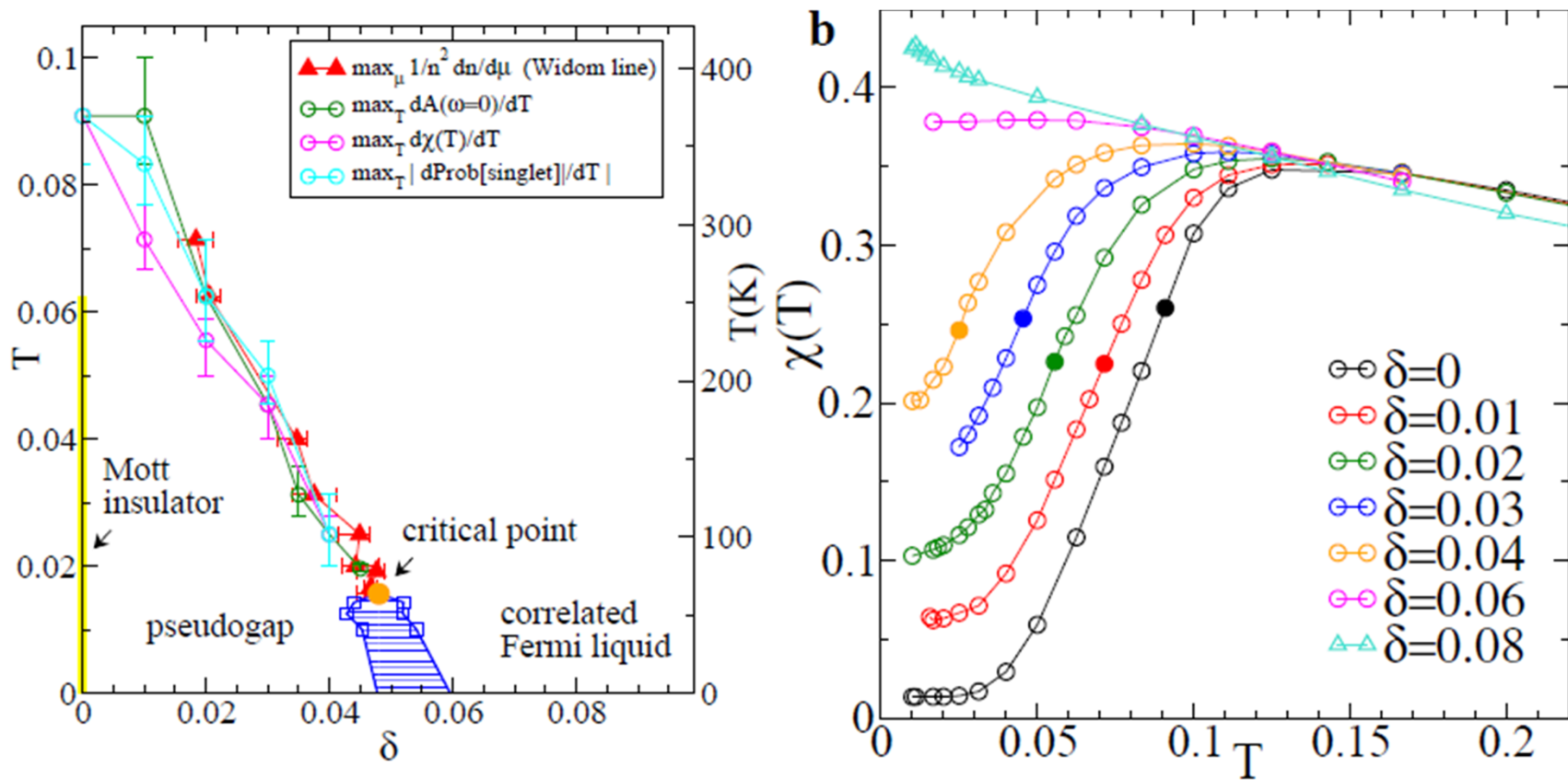


Influence of Mott transition away from half-filling

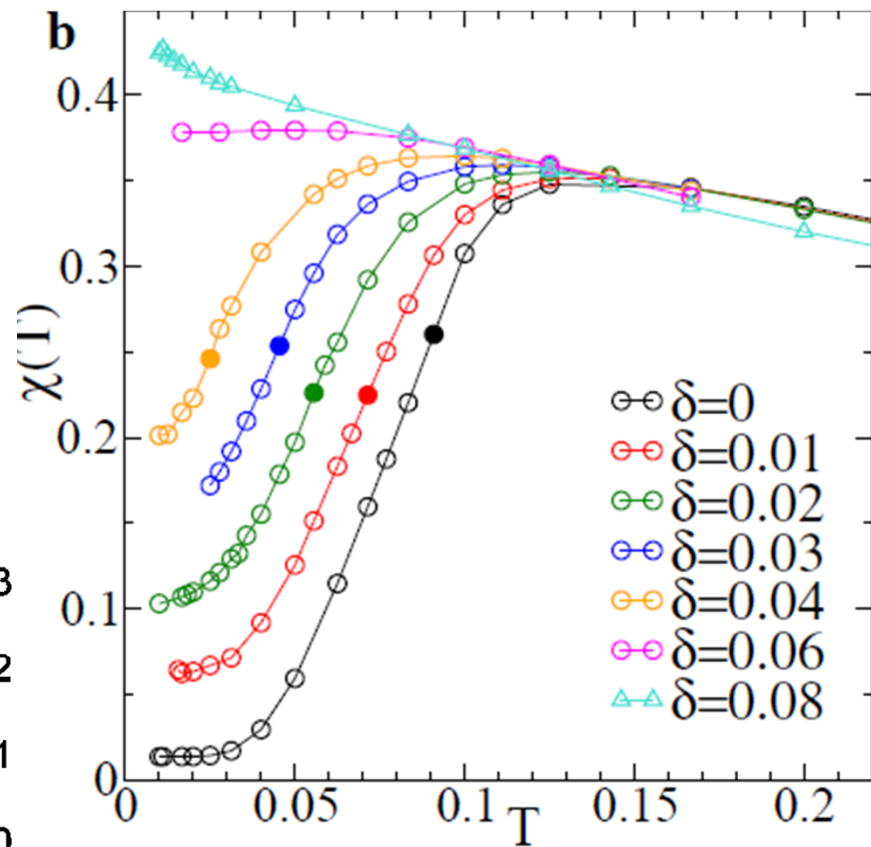
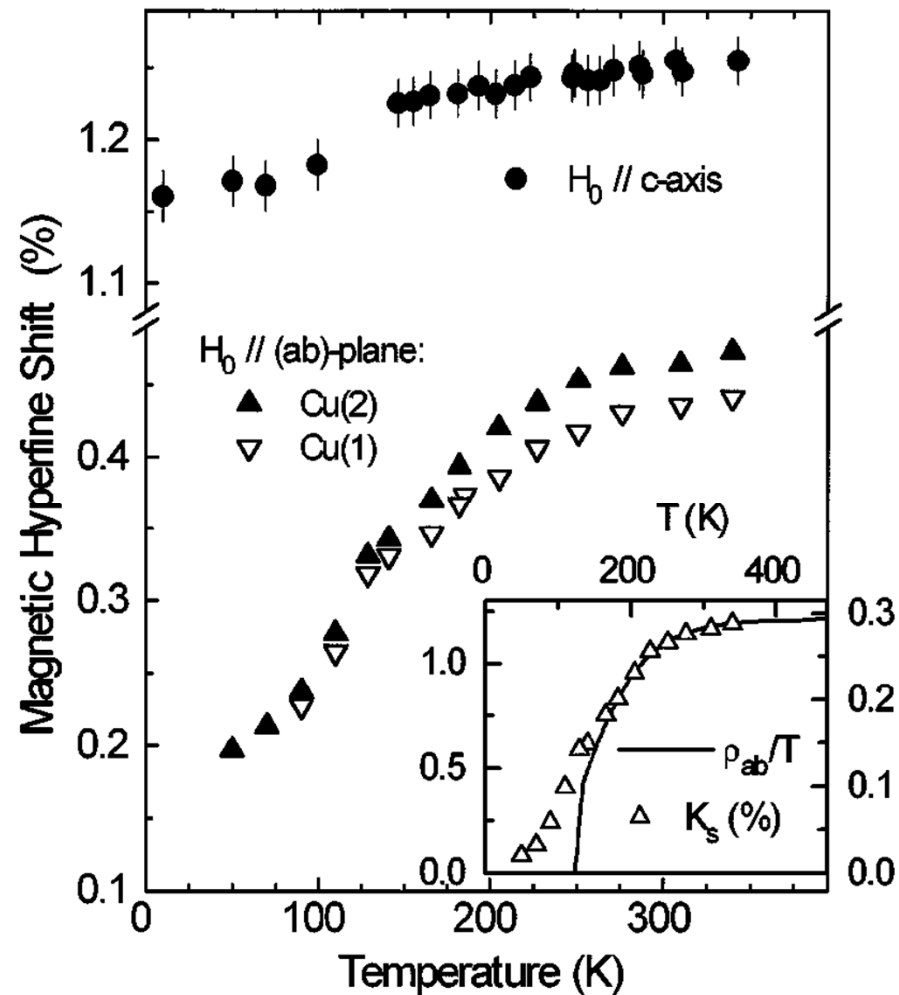
$n = 1, d = 2$ square lattice



Spin susceptibility



Spin susceptibility



Underdoped Hg1223

Julien et al. PRL **76**, 4238 (1996)



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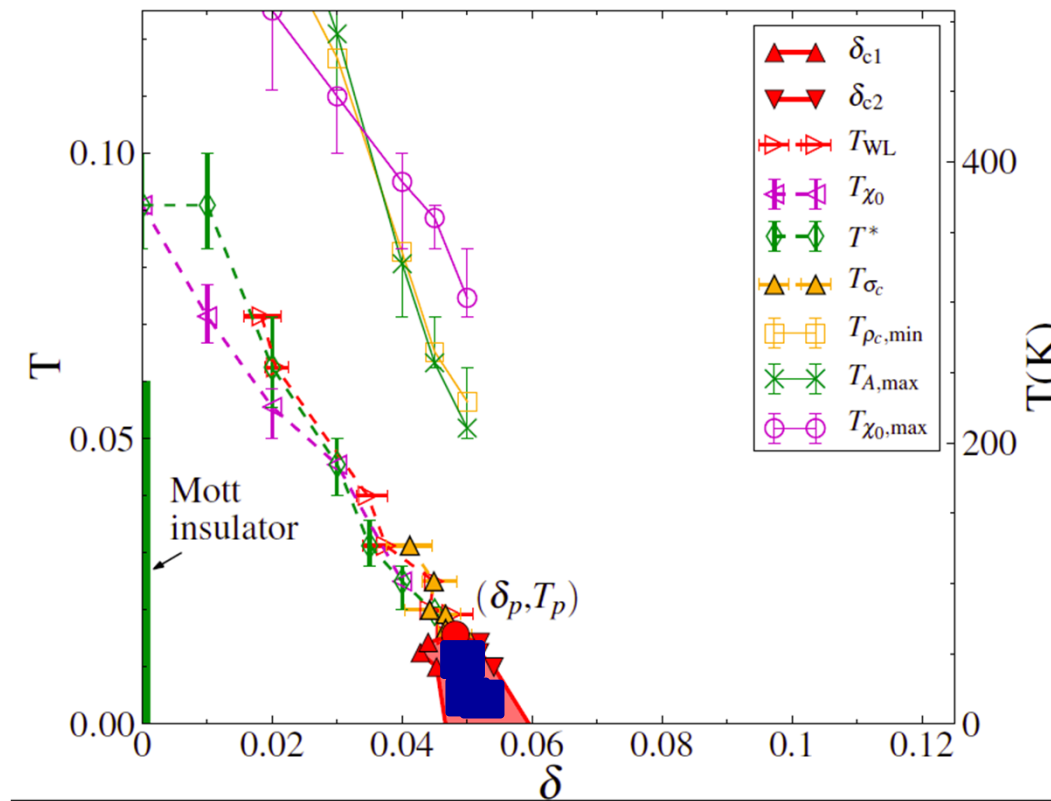


Giovanni Sordi

Two crossover lines



Patrick Sémon



G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)



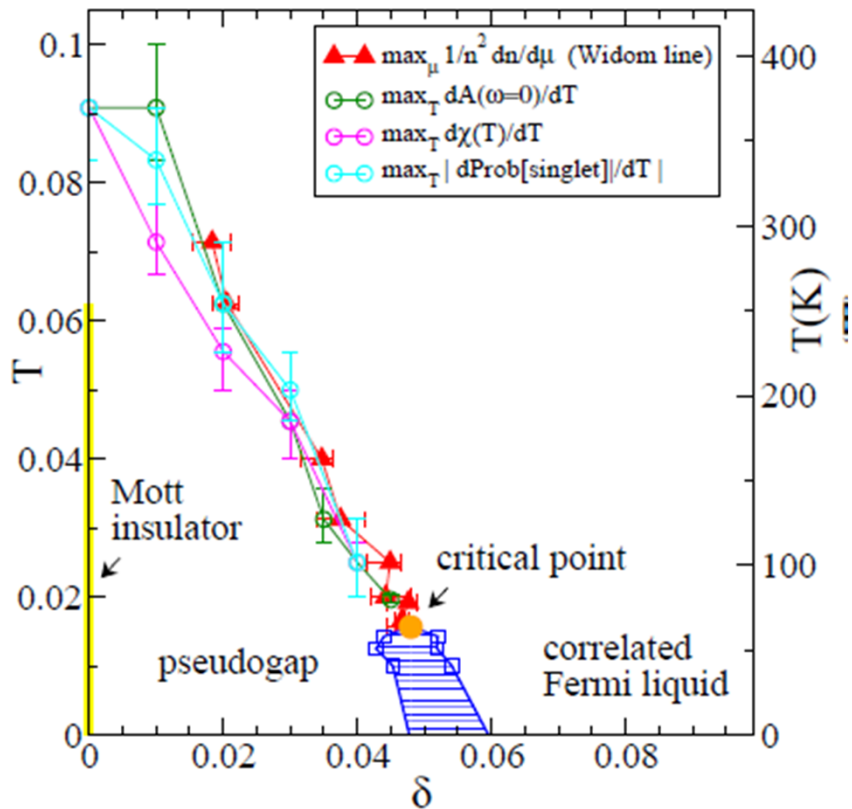
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Physics



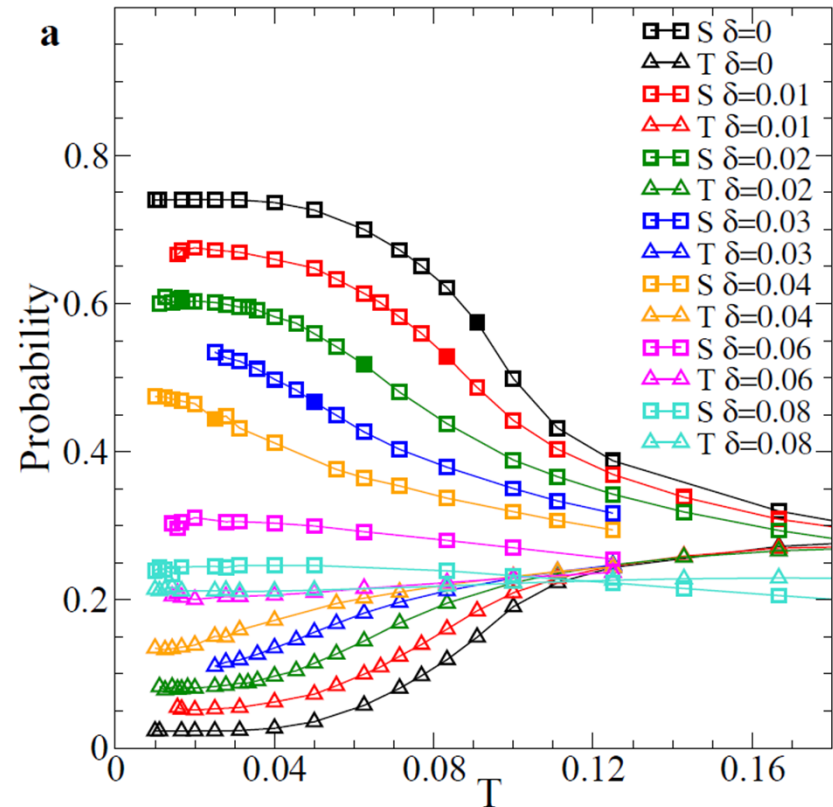
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Plaquette eigenstates



See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges
PRB **80**, 064501 (2009)





Giovanni Sordi



Patrick Sémon



Lorenzo Fratino

Finite T phase diagram Superconductivity

Sordi et al. PRL **108**, 216401 (2012)

Fratino et al.

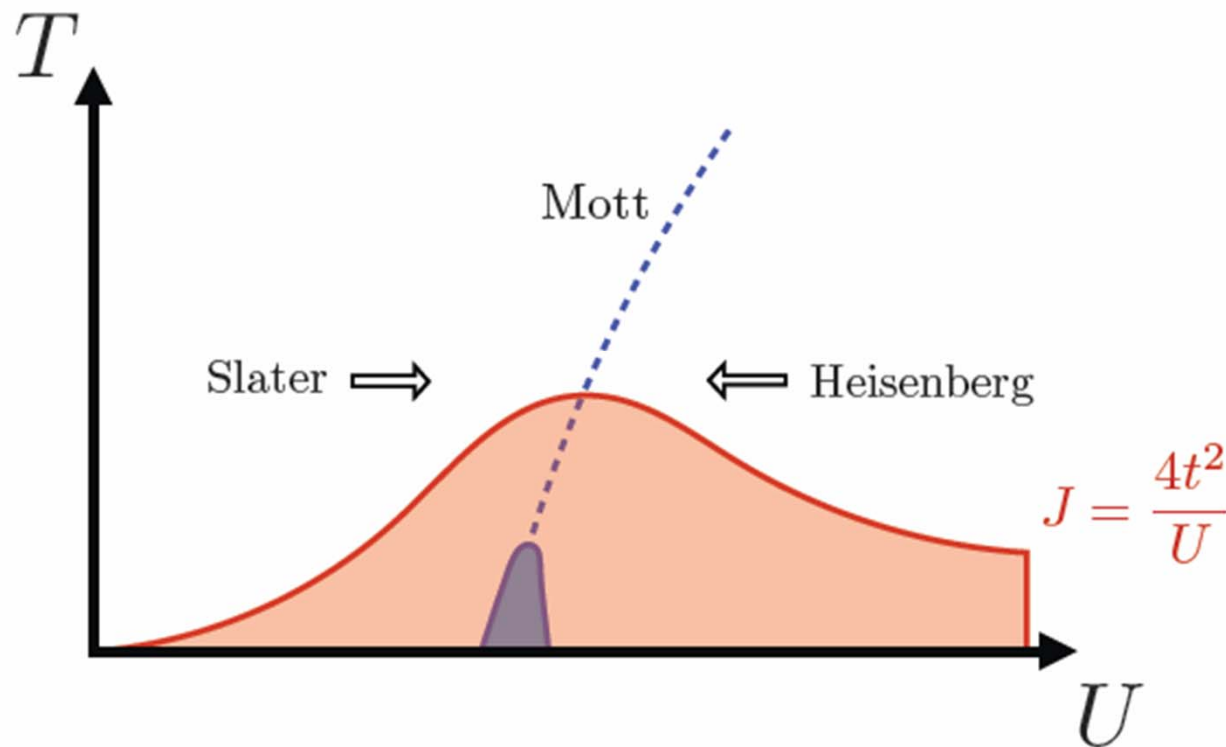
Sci. Rep. **6**, 22715 (2016)



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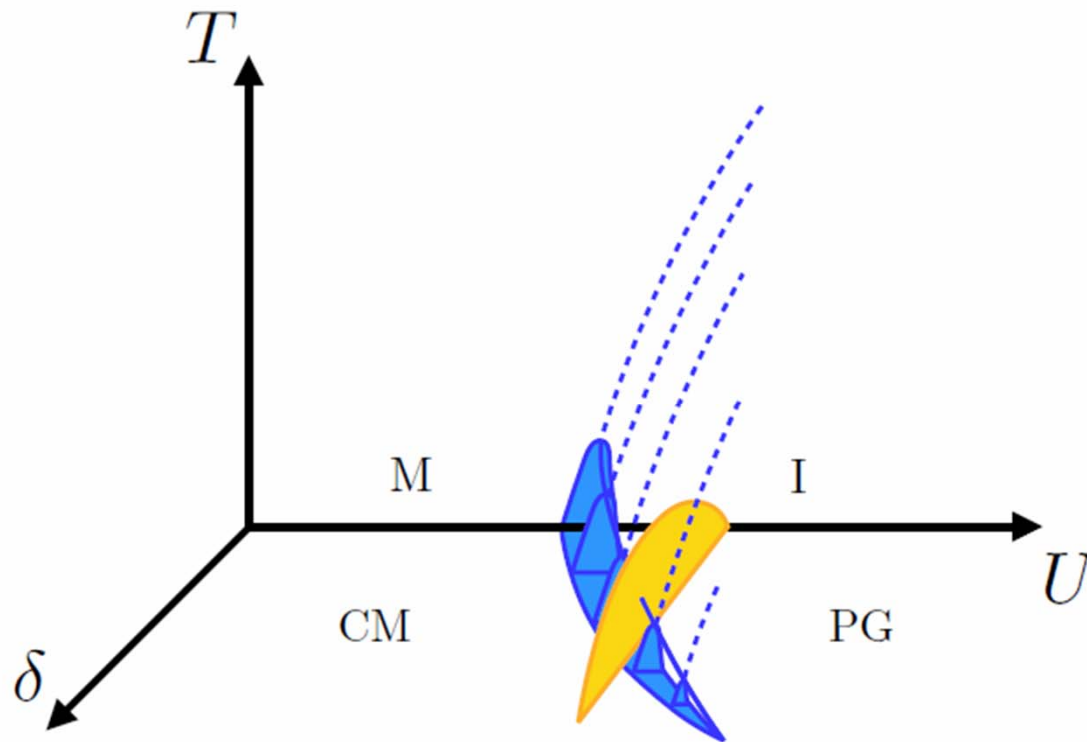
Crossovers inside the AFM phase

$n = 1$, unfrustrated $d = 3$ cubic lattice



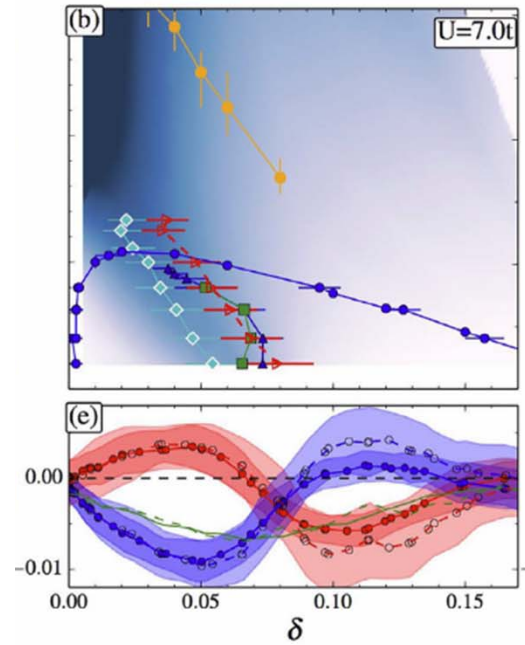
Superconductivity in Doped Mott insulator

$n = 1, d = 2$ square lattice



An organizing principle

Fratino et al.
Sci. Rep. **6**, 22715



3 bands, charge transfer insulator

Fratino et al. PRB **93**, 245147 (2016)

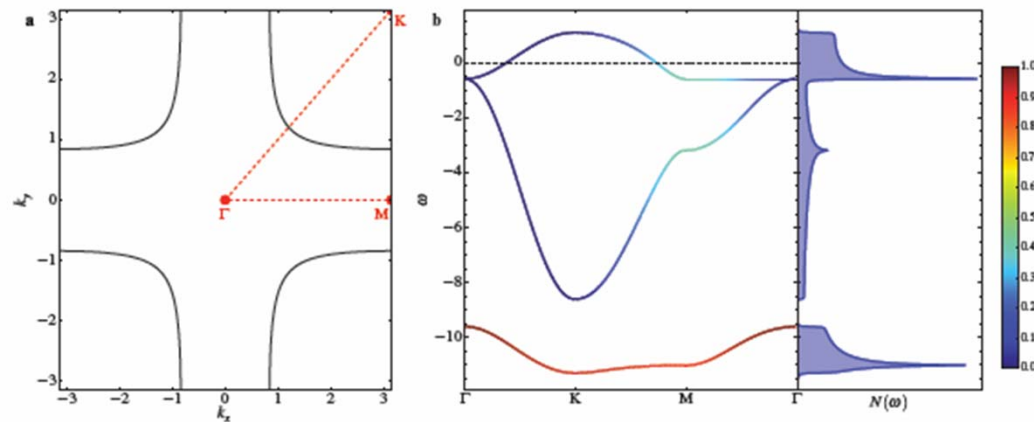
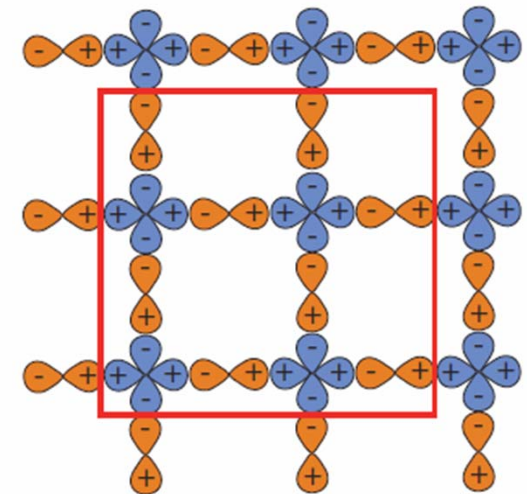


FIG. 2. (a) Noninteracting Fermi surface for the model parameter investigated in Fig. 1a of main text, namely $\epsilon_p = 9$, $t_{pp} = 1$, $t_{pd} = 1.5$, which gives a total occupation n_{tot} equal to five. (b) Non-interacting band structure for the same model parameter along with the resulting total density of states. Color corresponds to the d-character of the hybridised bands. The band crossing the Fermi level has mostly oxygen character.



Giovanni Sordi

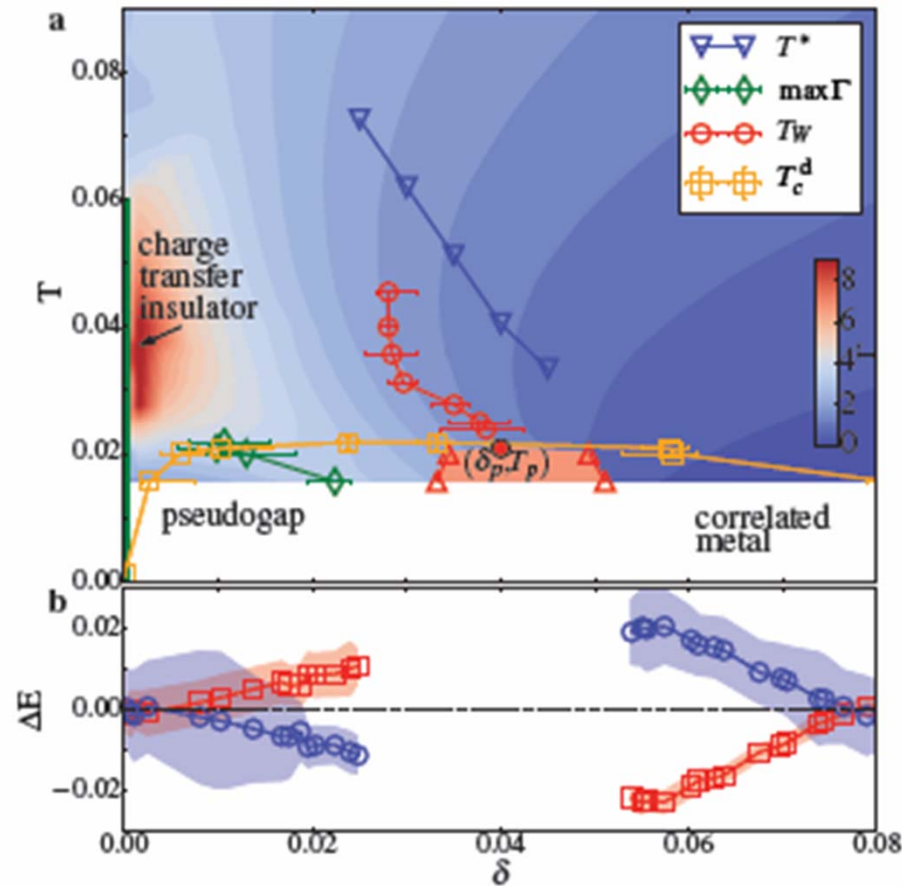


Lorenzo Fratino

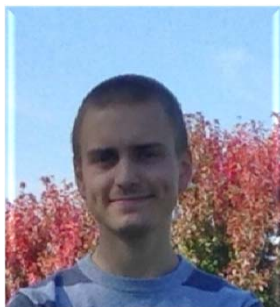


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3 bands, charge transfer insulator



Fratino et al. PRB 93, 245147 (2016)



Charles-David Hébert



Patrick Sémon

Organics : Phase diagram, finite T

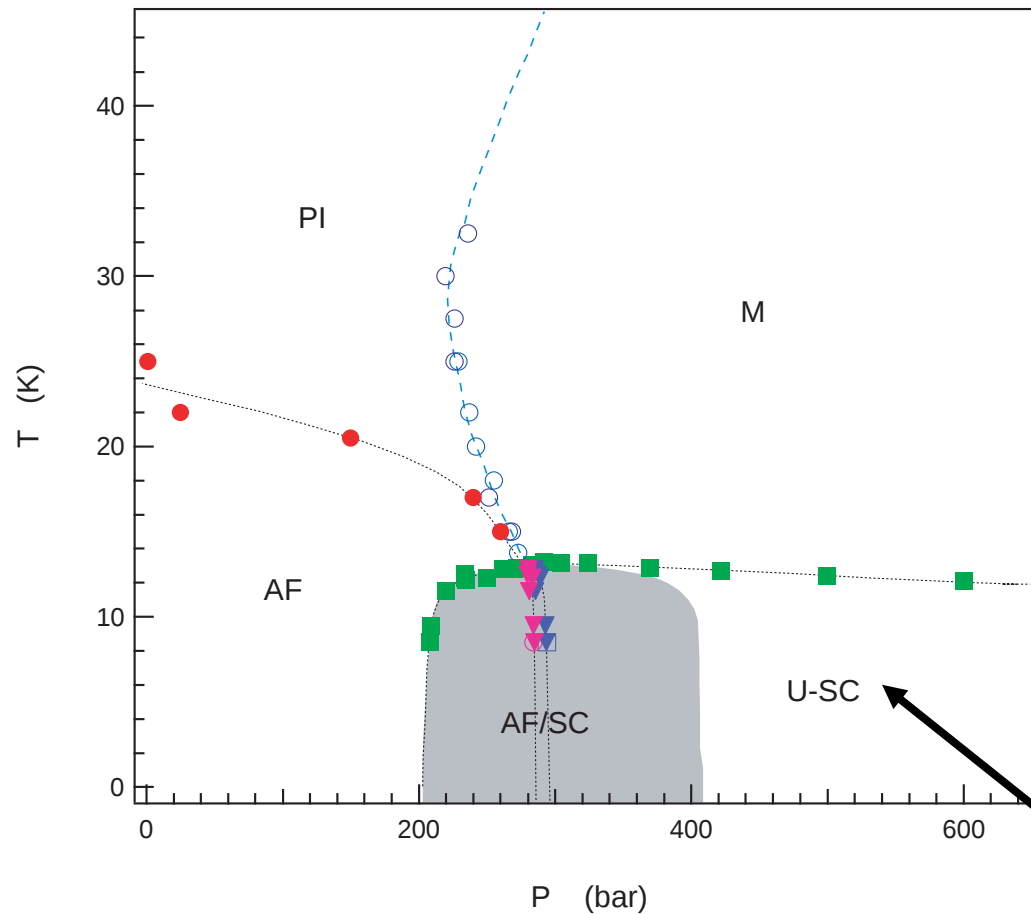
Made possible by algorithmic improvements

P. Sémon *et al.*
PRB **85**, 201101(R) (2012)
PRB **90** 075149 (2014);
and PRB **89**, 165113 (2014)



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Phase diagram for organics



F. Kagawa, K. Miyagawa, + K. Kanoda
PRB **69** (2004) + Nature **436** (2005)

B_g for C_{2h} and B_{2g} for D_{2h}

Powell, McKenzie cond-mat/0607078

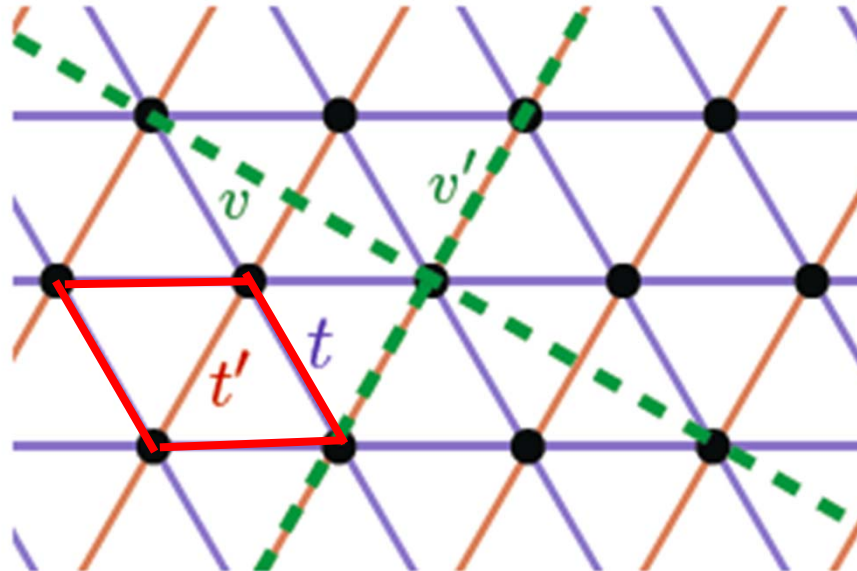
Phase diagram ($X = \text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)



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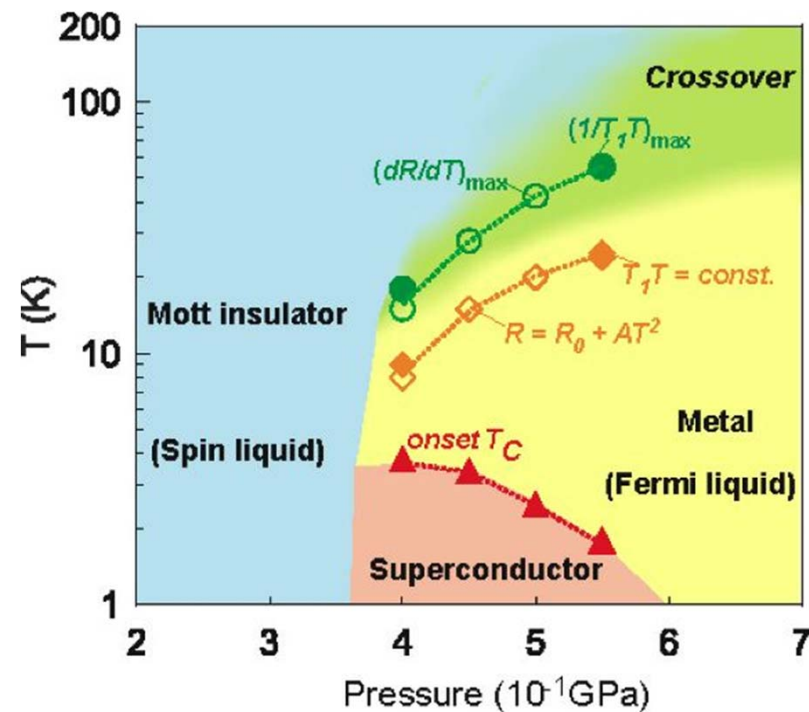
Anisotropic triangular lattice



See: Poster Shaheen Acheche



Phase diagram at $n = 1$



Y. Kurisaki, et al.

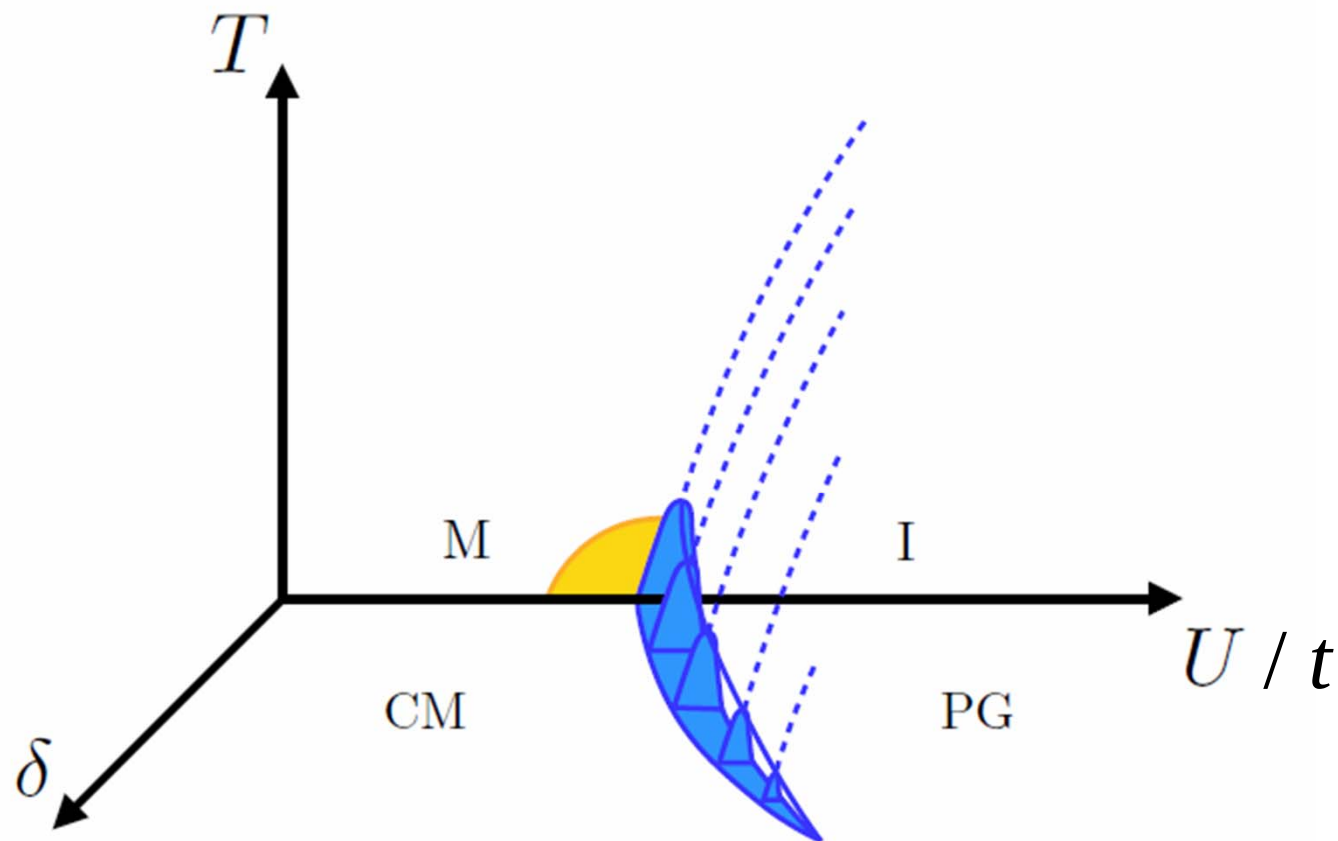
Phys. Rev. Lett. **95**, 177001(2005)

Y. Shimizu, et al. Phys. Rev. Lett. **91**, (2003)



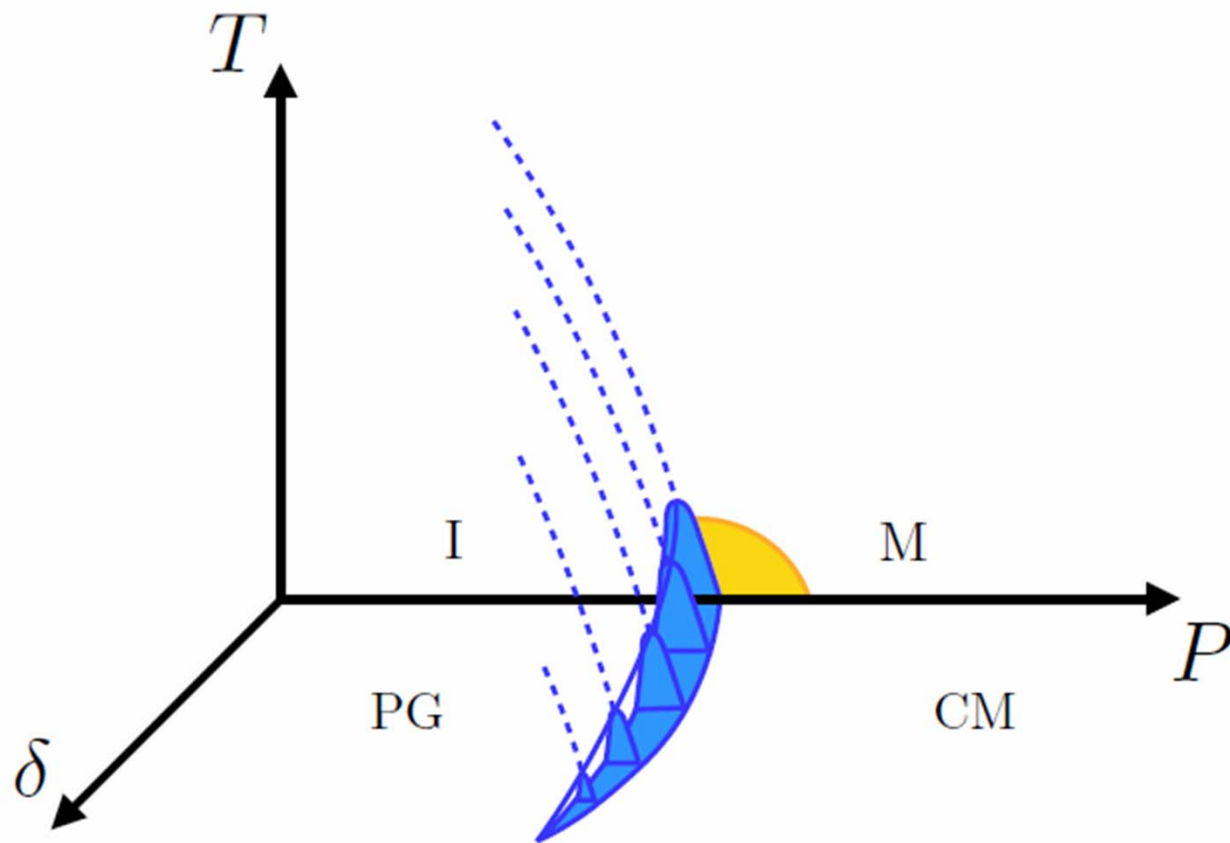
Superconductivity near the Mott transition

$n = 1, d = 2$ square lattice

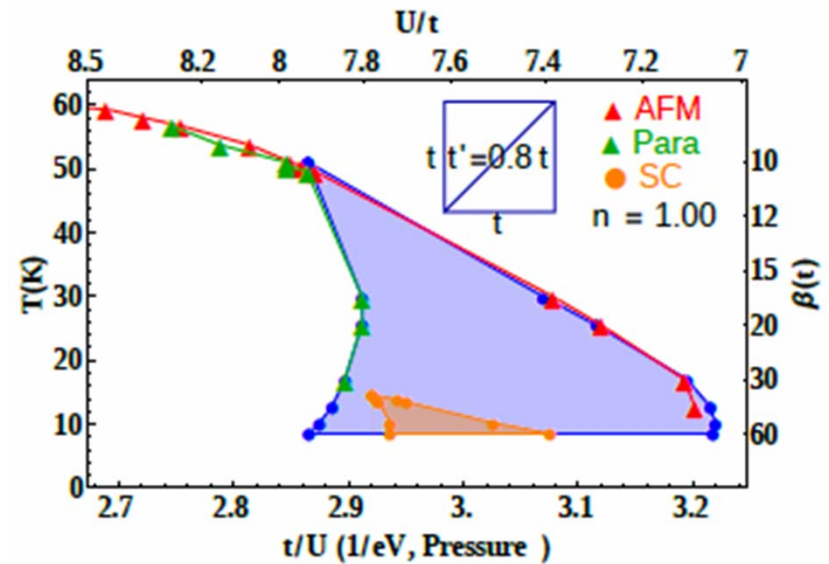
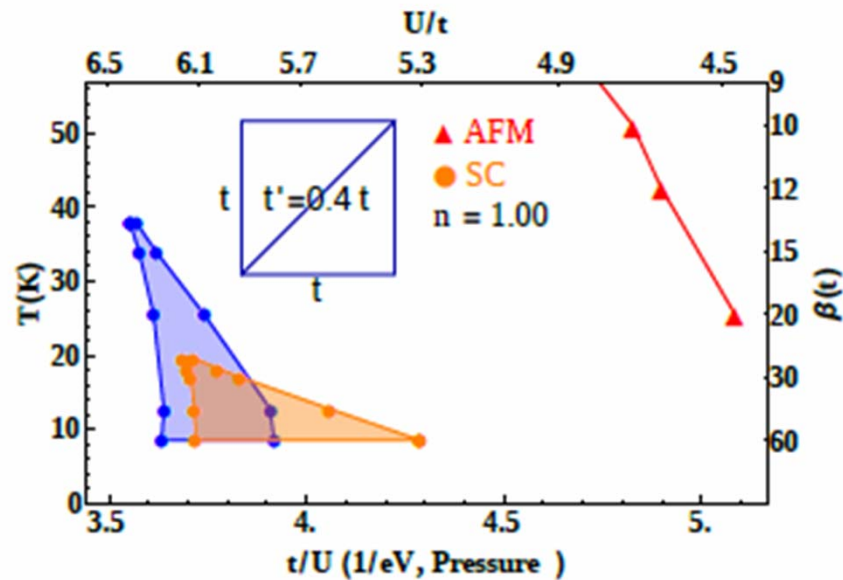


Superconductivity near the Mott transition

$n = 1, d = 2$ square lattice



Superconductivity near Mott transition ($n = 1$)



C.-D. Hébert, P. Sémon, A.-M.S. T PRB **92**, 195112 (2015)



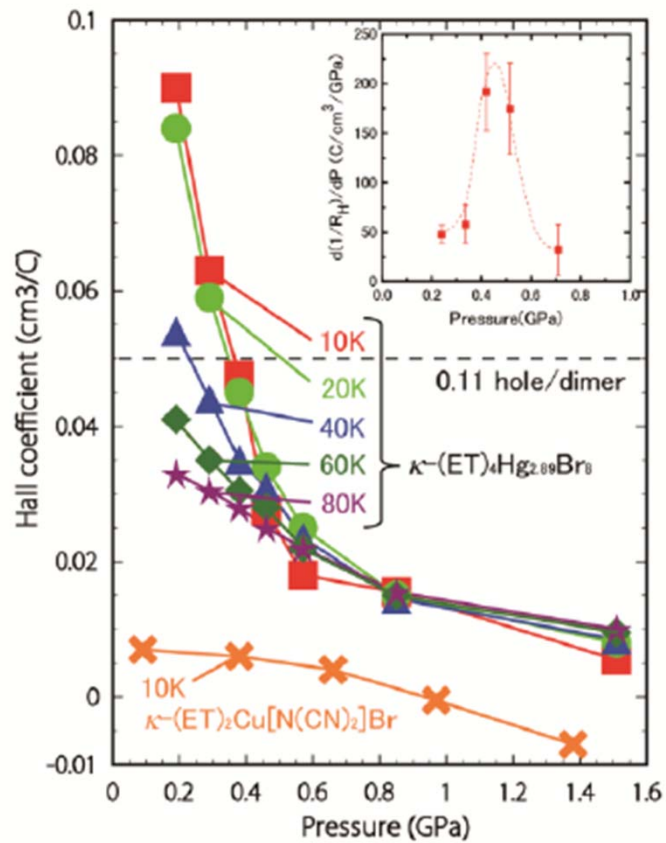
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Doped Organics

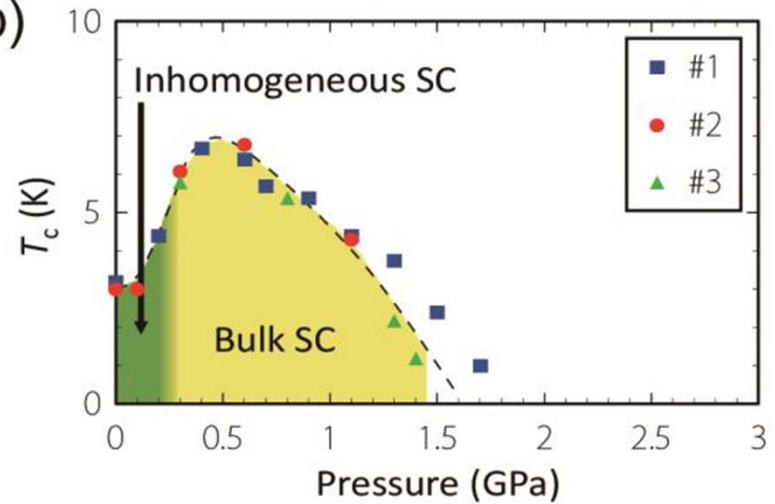


Doped BEDT

(b)

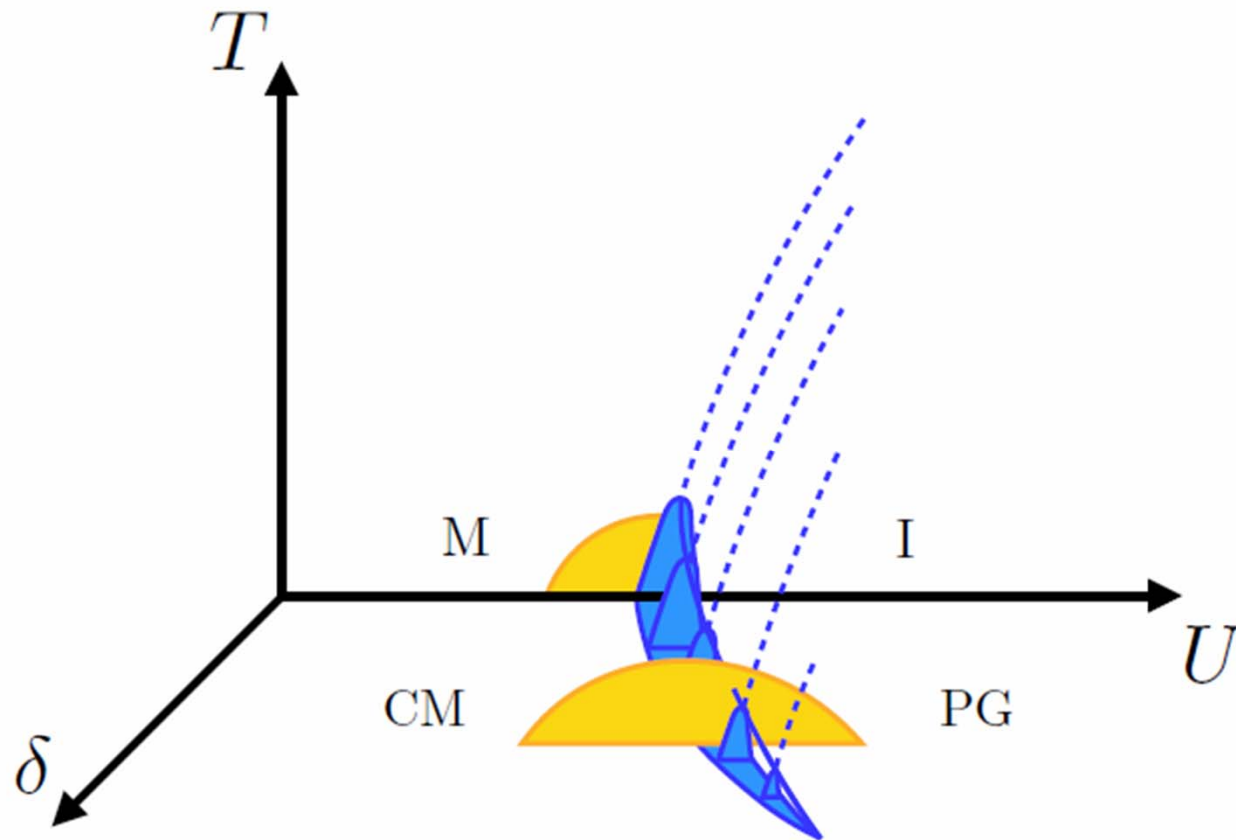


(b)



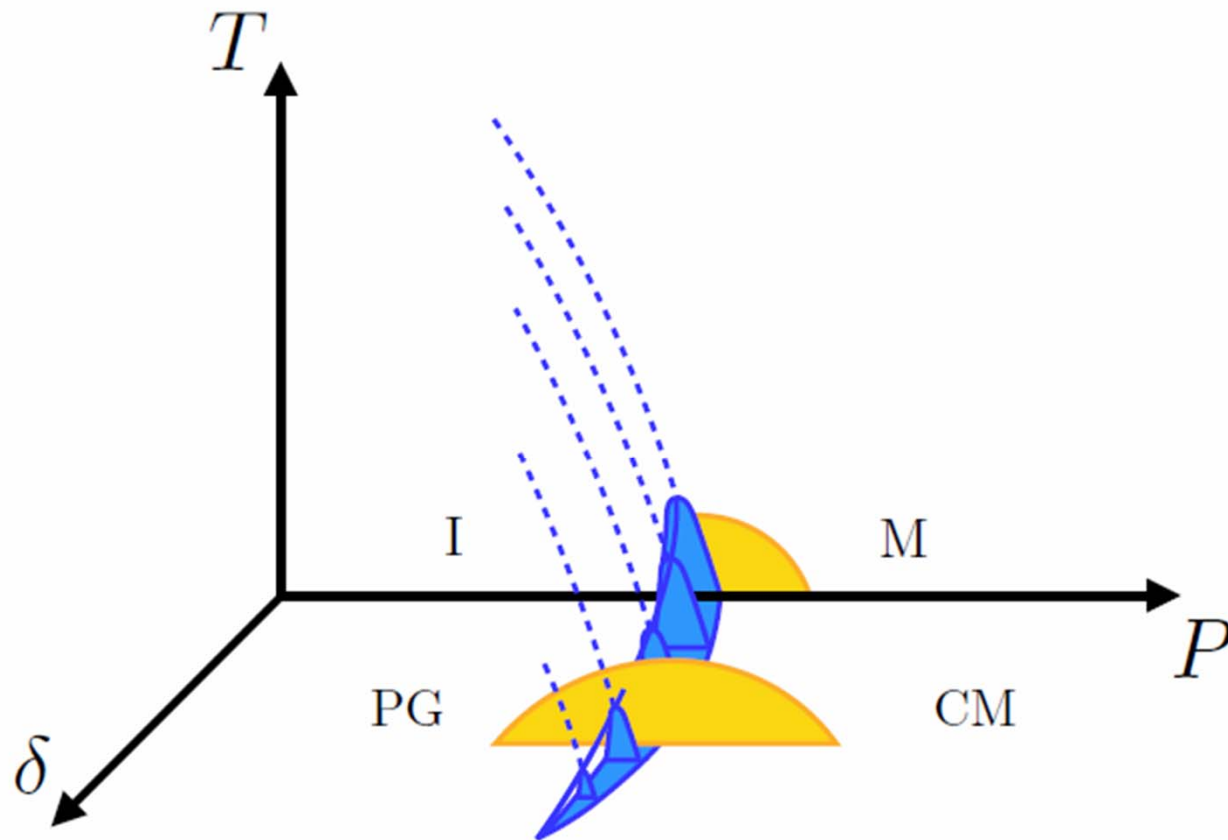
Doped organics

$n = 1, d = 2$ square lattice



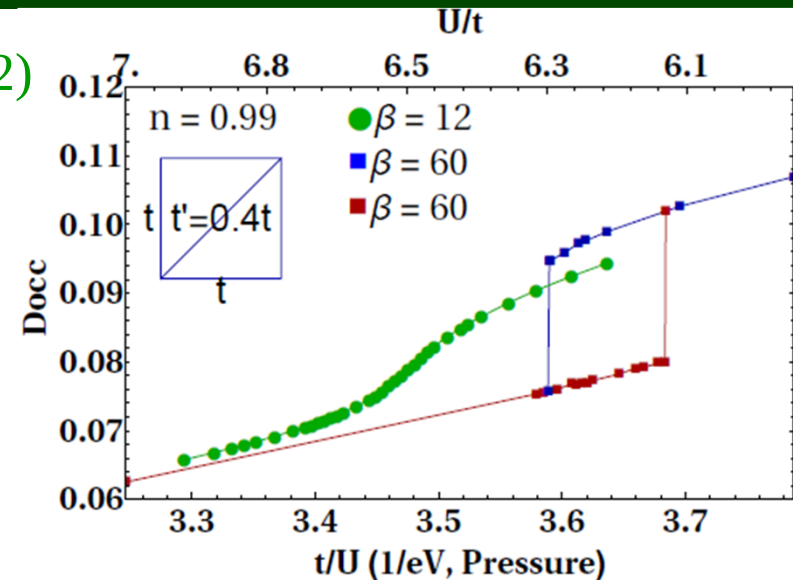
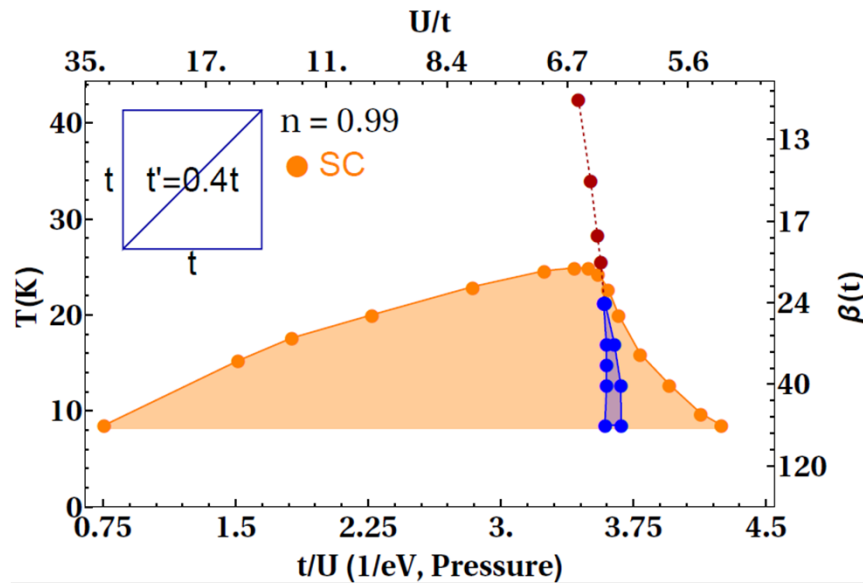
Doped organics

$n = 1, d = 2$ square lattice



First order and Widom line in organics

G. Sordi *et al.* Scientific Reports, 2, 547 (2012)



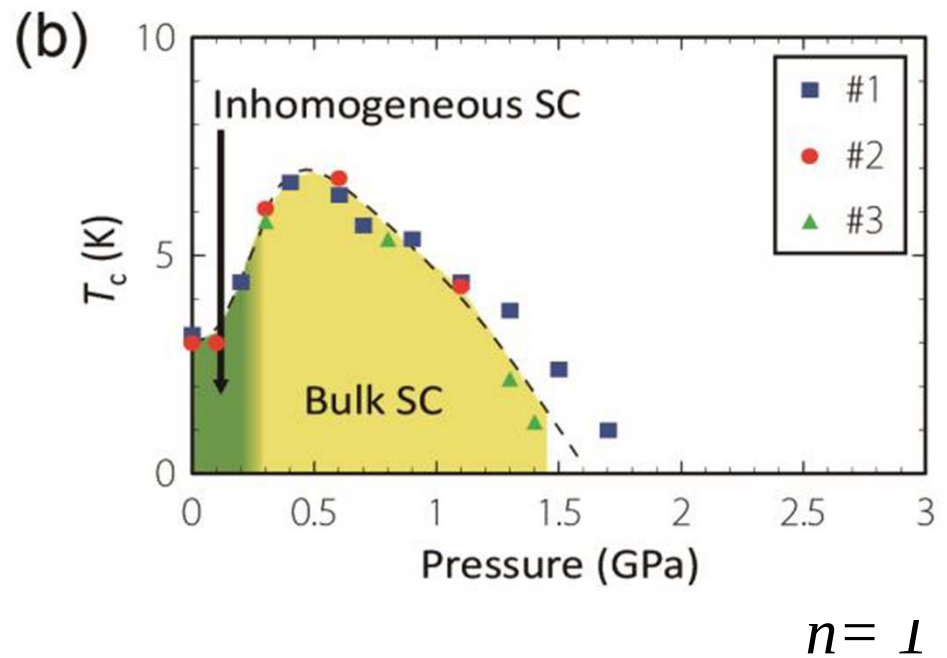
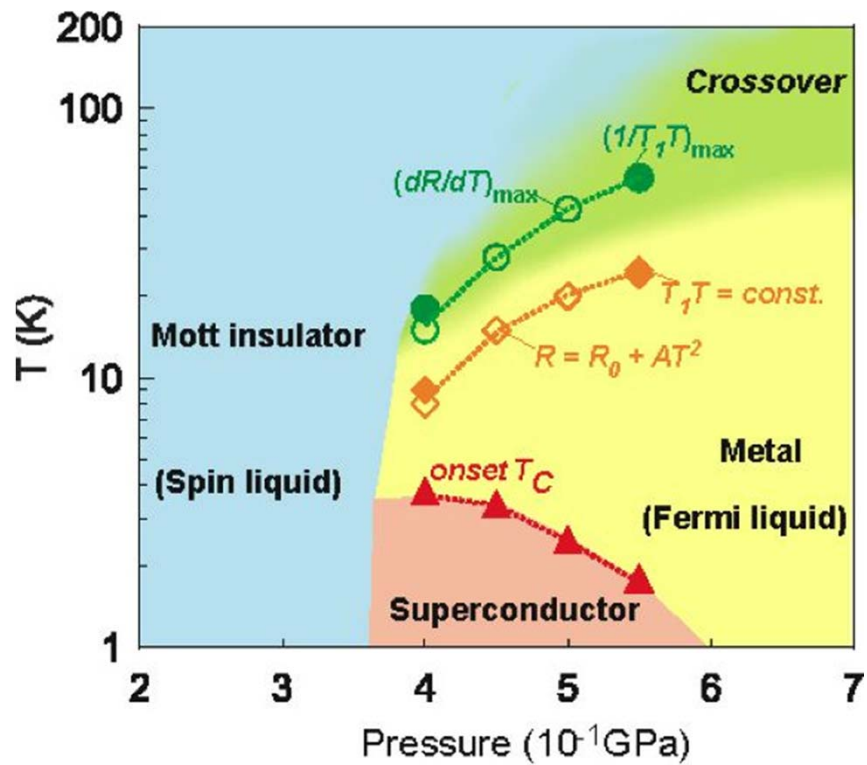
Compare: T. Watanabe, H. Yokoyama
and M. Ogata
JPS Conf. Proc.
3, 013004 (2014)

C.-D. Hébert, P. Sémon, A.-M.S. T PRB 92, 195112 (2015)



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Doped BEDT

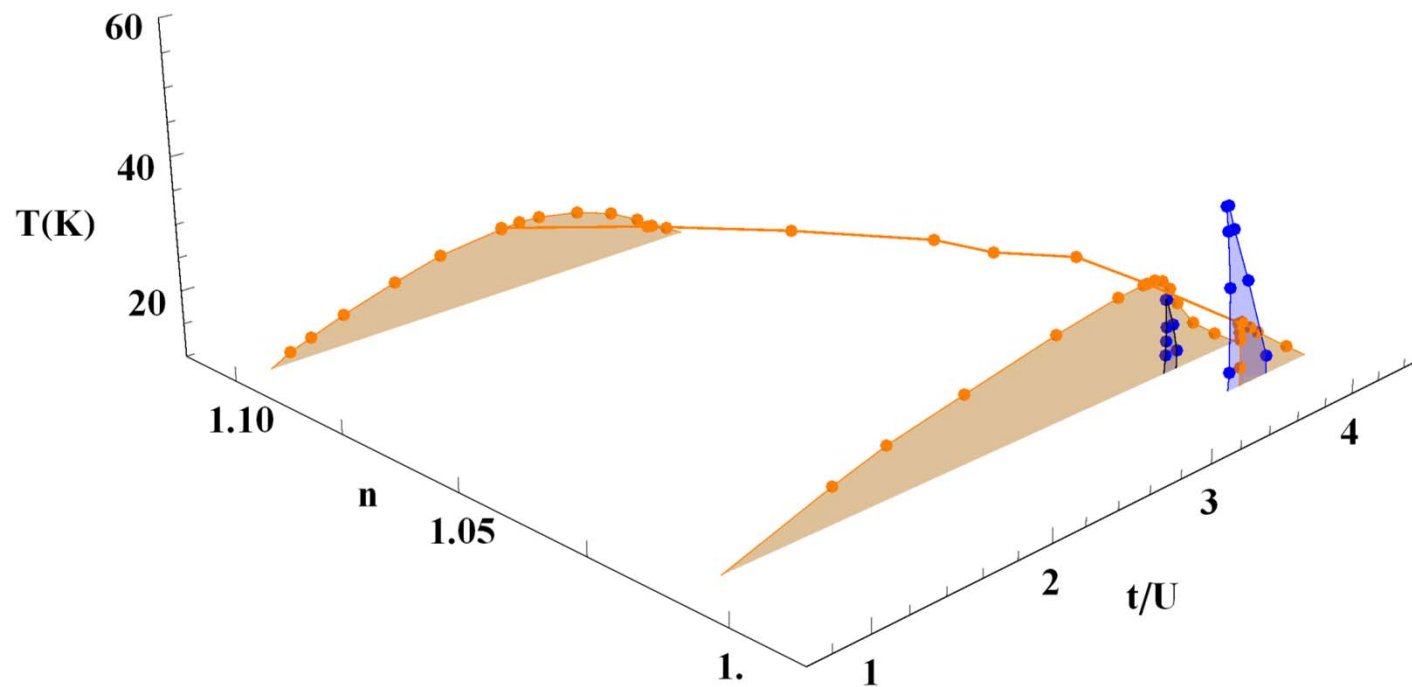


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



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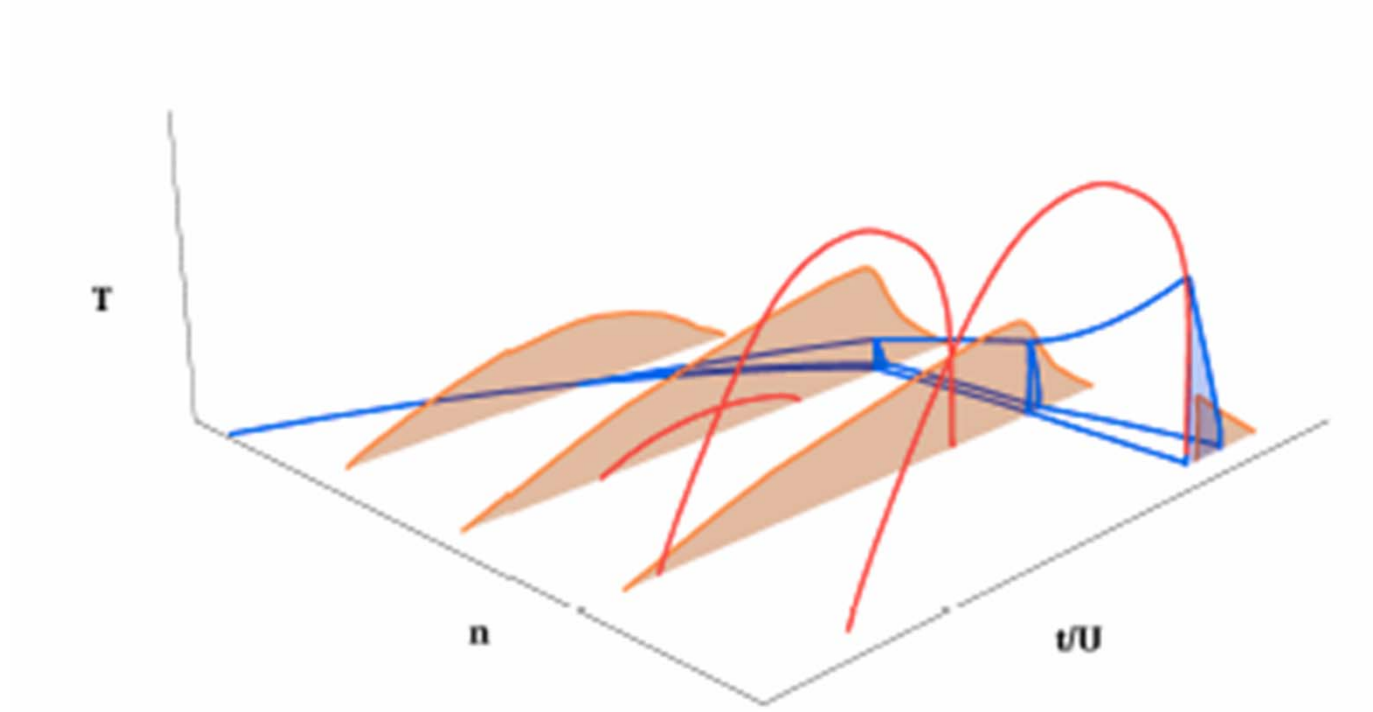
$t' = 0.4t$ overview



Compare: T. Watanabe, H. Yokoyama and M. Ogata
JPS Conf. Proc. 3, 013004 (2014)



Generic case highly frustrated case



Summary : organics

- Agreement with experiment
 - SC: larger T_c and broader P range if doped
 - Larger frustration: Decrease T_N *much more* than T_c
 - Normal state metal to pseudogap crossover
- Predictions
 - First order transition at low T in normal state (B induced)
 - Crossovers in SC state associated with normal state.
- Physics
 - SC dome without an AFM QCP. Extension of Mott
 - SC from short range J .
 - T_c dome maximum near normal state 1st order



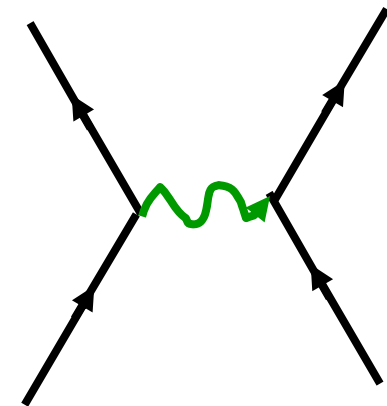
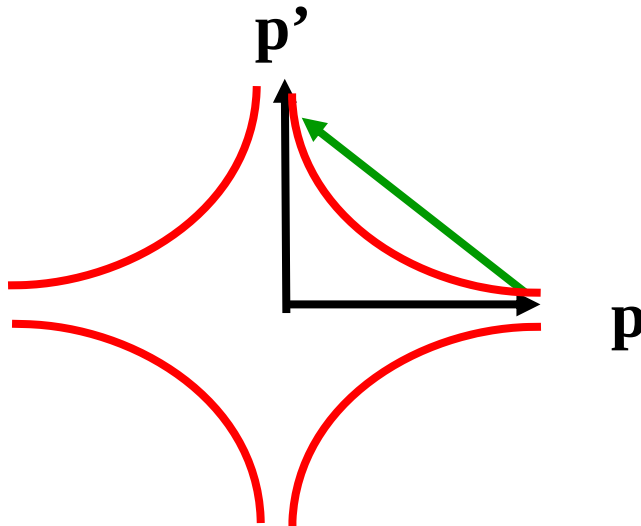
Pairing mechanism

Back to high T_c



Cartoon « BCS » weak-coupling picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?
Kohn-Luttinger

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).

T_c with pressure

Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)



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A cartoon strong coupling picture

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson Science
317, 1705 (2007)

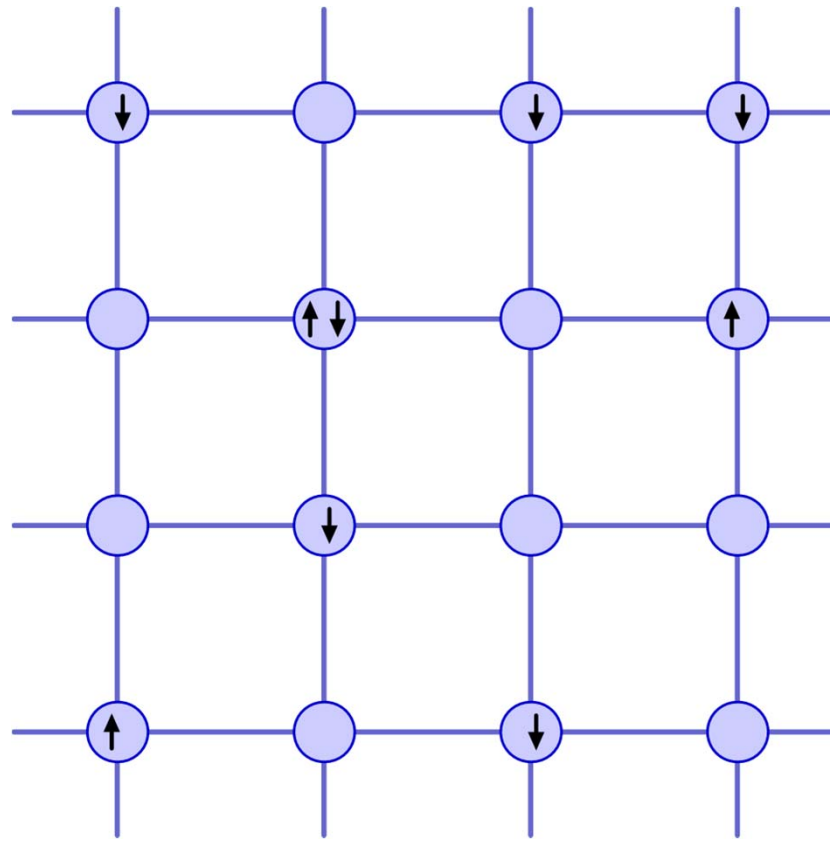
Miyake, Schmitt–Rink, and Varma
P.R. B 34, 6554-6556 (1986)

More sophisticated Slave Boson: Kotliar Liu PRB 1988



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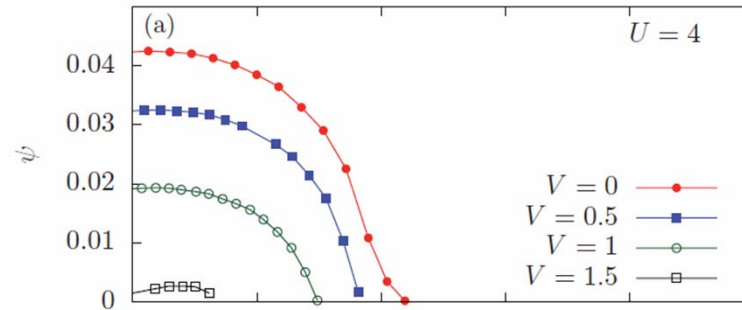
Extended Hubbard model



$$\hat{\mathcal{H}} = -t \sum_{\langle i,j \rangle \sigma} \left(\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + c.h \right) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j - \mu \sum_i \hat{n}_i$$



Strongly correlated: From J , yet retarded



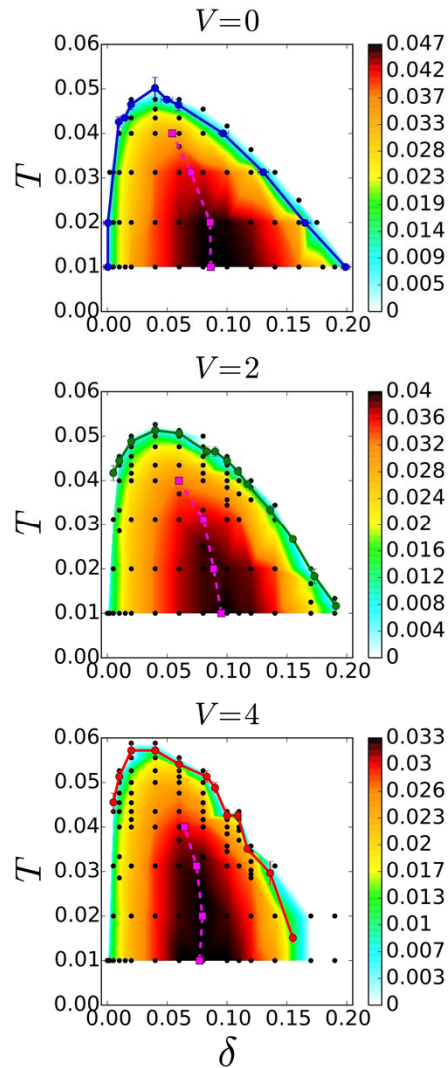
x

Sénéchal, Day, Bouliane, AMST, Phys. Rev. B **87**, 075123 (2013)

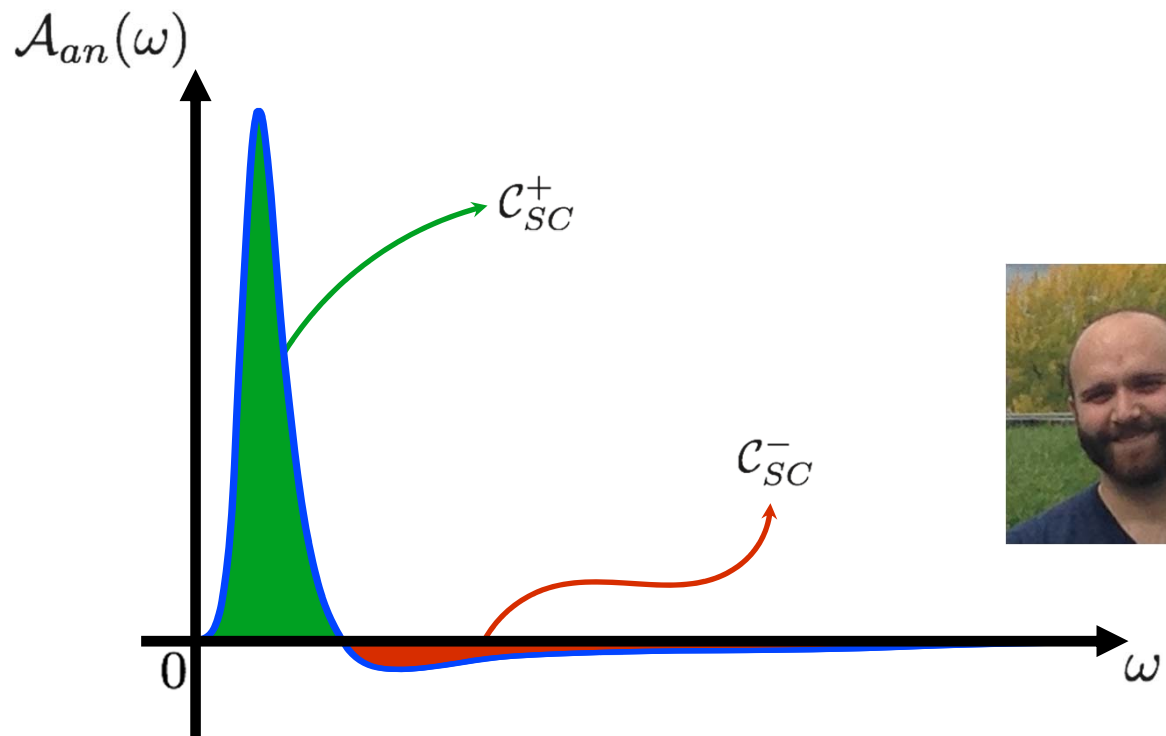


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Antagonistic effects of V at finite T



$$J = \frac{4t^2}{U - V}$$



A. Reymbaut *et al.* PRB **94** 155146 (2016)



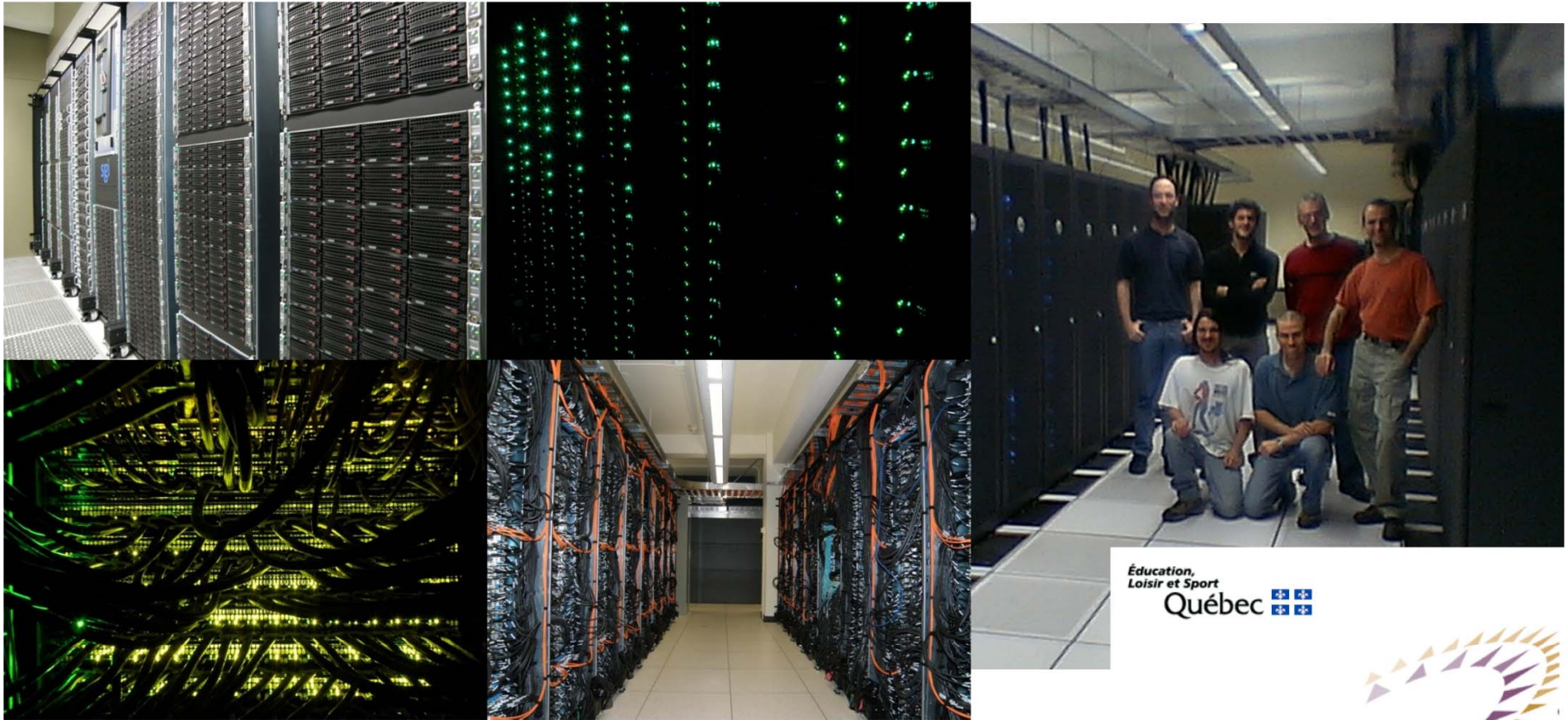
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Summary

- AFM QCP for a heavy-fermion model
- No QCP: First order transition that extends Mott physics away from half-filling
- Is an organizing principle for
 - The normal and superconducting states
 - Cuprates and organics are examples
 - Predictions for organics
- Mechanism: J short-range



Mammoth



Éducation,
Loisir et Sport
Québec



Canada Foundation for Innovation
Fondation canadienne pour l'innovation


Calcul Québec

 **compute + calcul**
CANADA

High Performance Computing

CREATING KNOWLEDGE
DRIVING INNOVATION
BUILDING THE DIGITAL ECONOMY

Le calcul de haute performance

CRÉER LE SAVOIR
ALIMENTER L'INNOVATION
BÂTIR L'ÉCONOMIE NUMÉRIQUE



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Review: A.-M.S.T. [arXiv: 1310.1481](https://arxiv.org/abs/1310.1481)



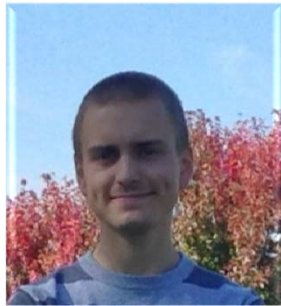
A.-M.S. Tremblay

“Strongly correlated superconductivity”

Chapt. 10 : *Emergent Phenomena in Correlated Matter Modeling and Simulation*, Vol. 3, E. Pavarini, E. Koch, and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich, 2013

Collaborators for this work



Charles-David Hébert



Patrick Sémon



Wei Wu



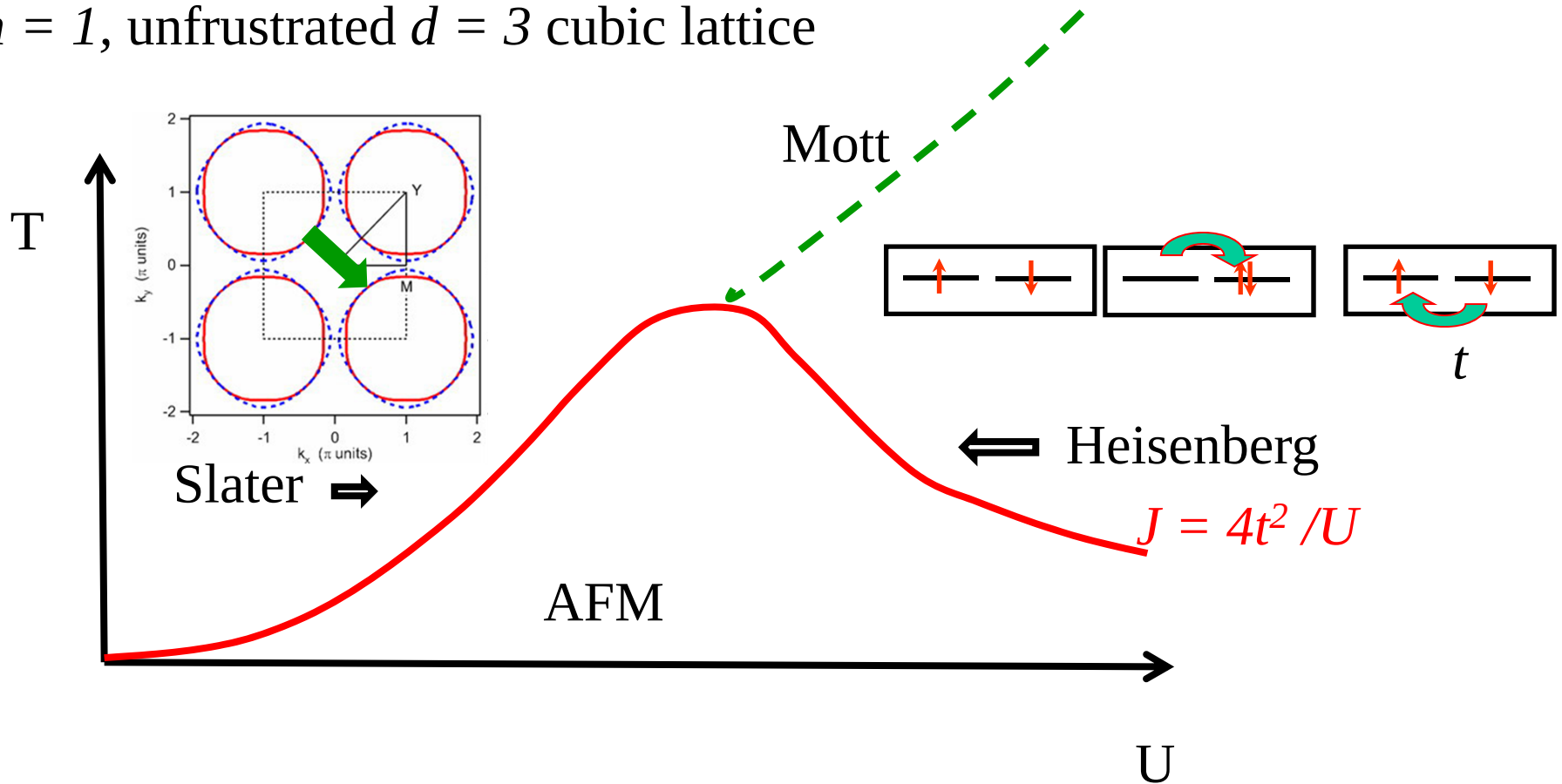
Weakly vs strongly correlated superconductivity

Analog to weakly and strongly
correlated antiferromagnets



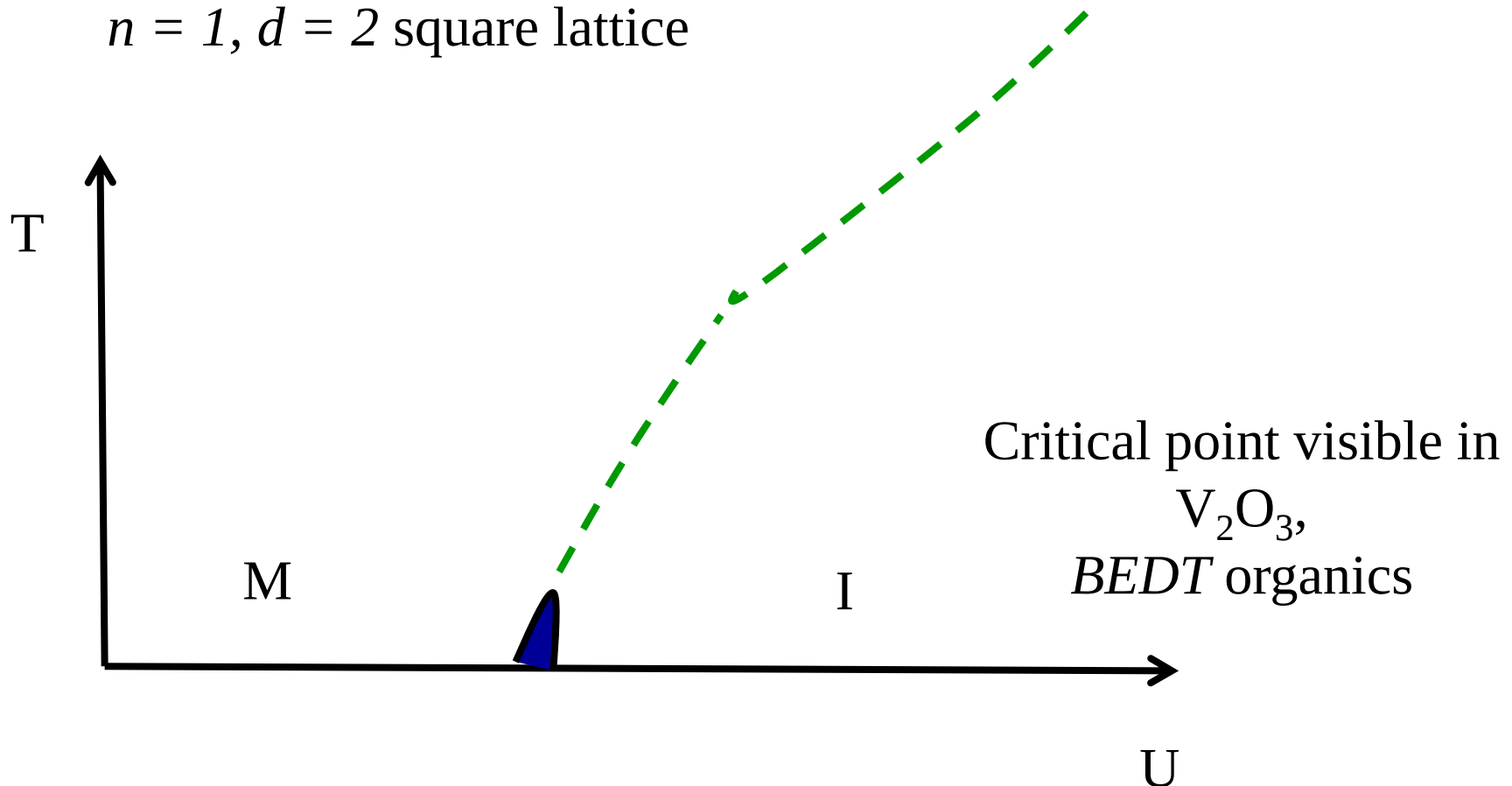
Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



Local moment and Mott transition

$n = 1, d = 2$ square lattice



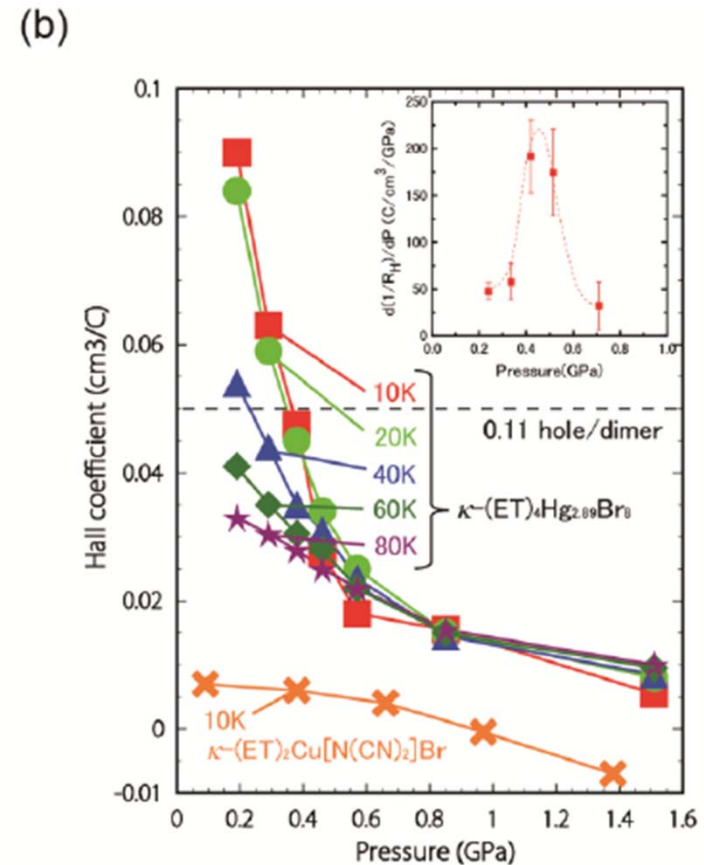
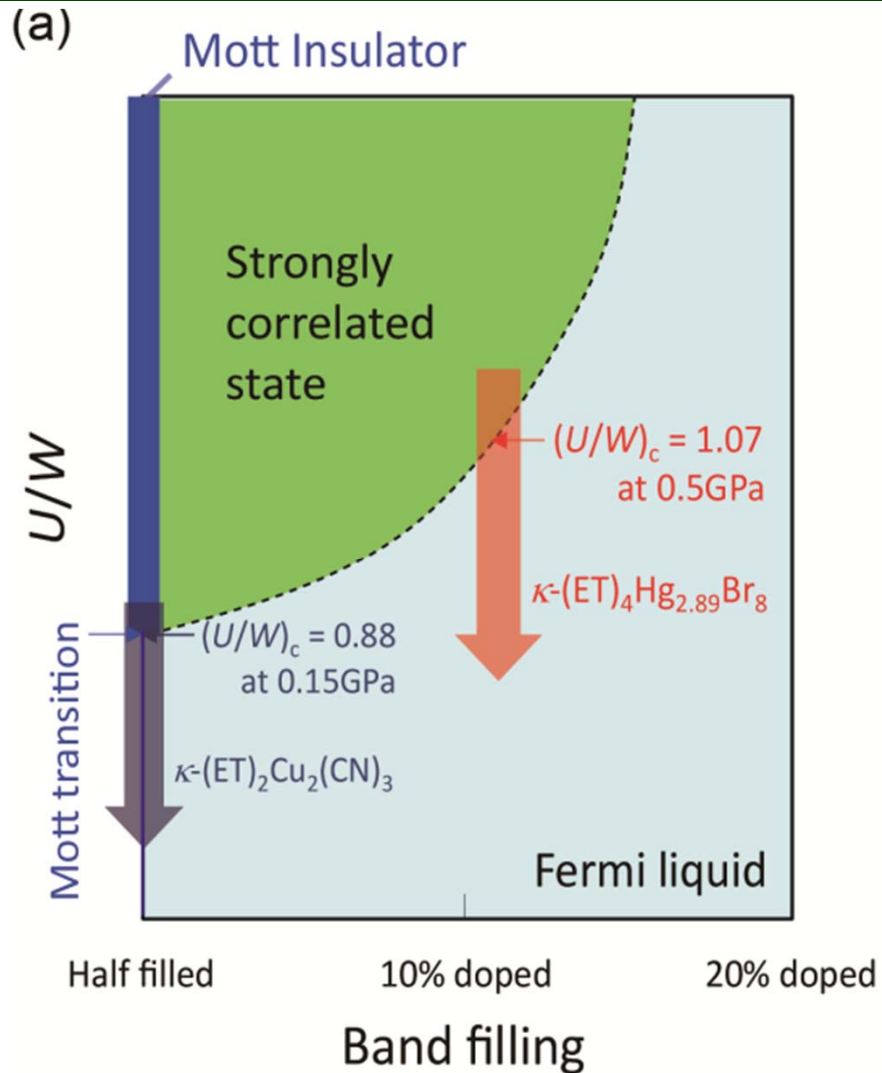
Understanding finite temperature phase from a *mean-field theory* down to $T = 0$



Doped organic: experiment



Doped BEDT

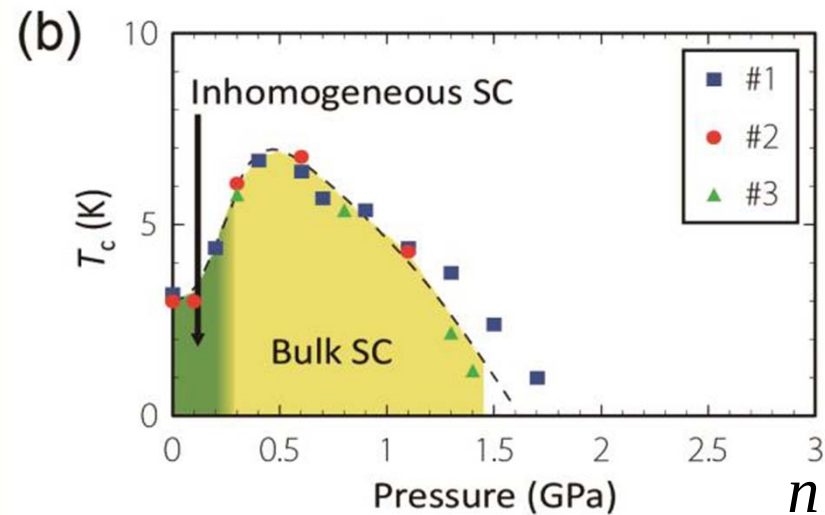
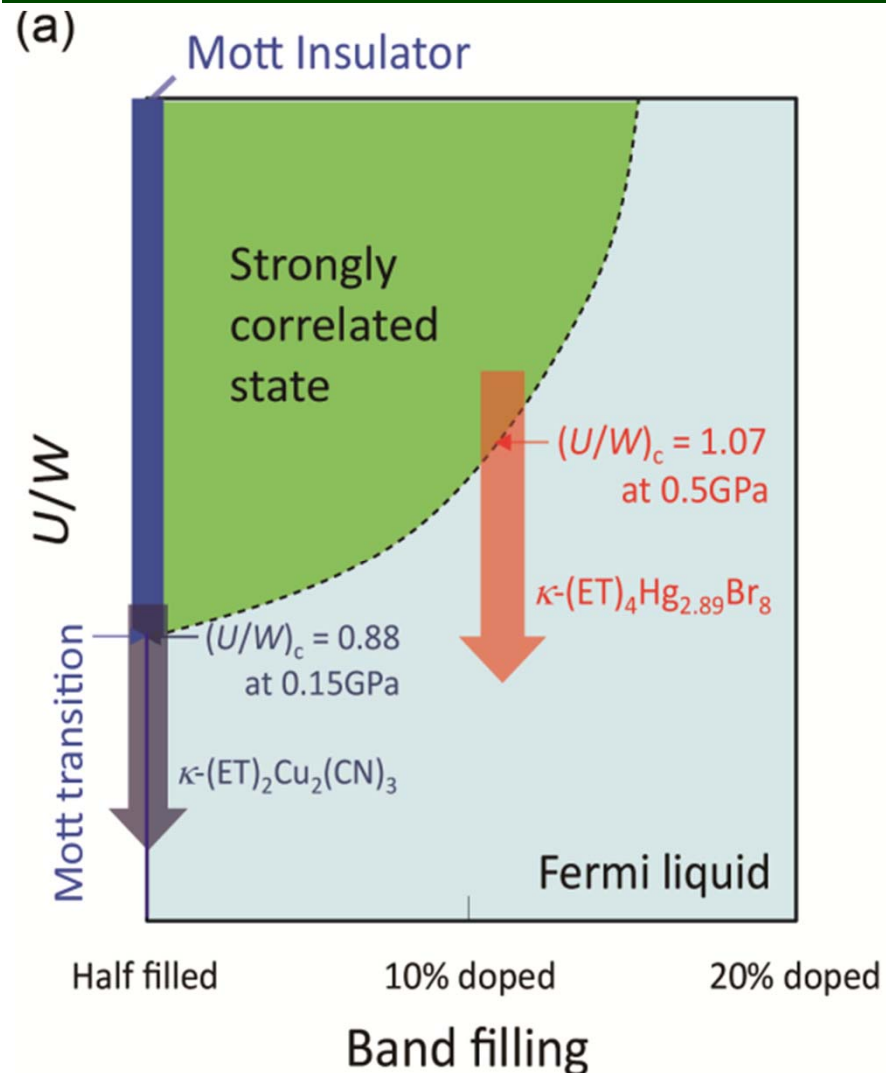
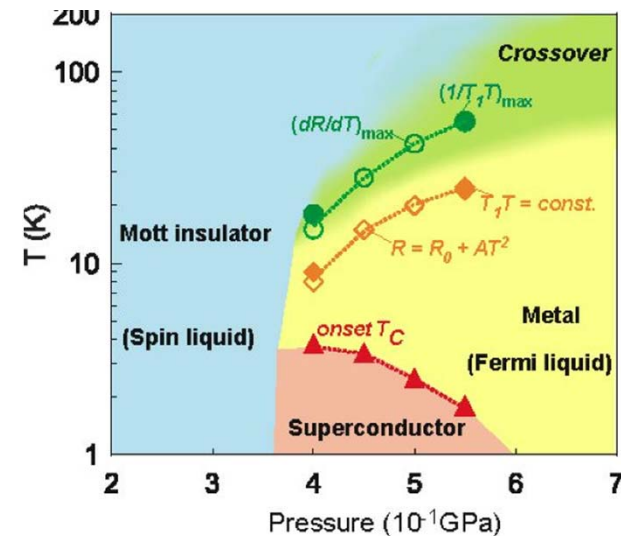


H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



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Doped BEDT


$$n = 0.90$$

$$n=1$$

H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL **114**, 067002 (2015)



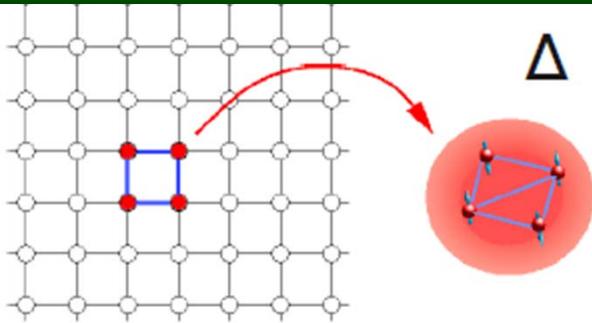
Method

Concept: Cluster - DMFT

Tools: Impurity solver



CTQMC impurity solver (tool) (T finite)



$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger(\tau) \Delta_{\mathbf{k}}(\tau, \tau') \psi_{\mathbf{k}}(\tau')}$$

Mean-field is not a trivial problem! Many impurity solvers.

Here: continuous time QMC

P. Werner, PRL 2006

P. Werner, PRB 2007

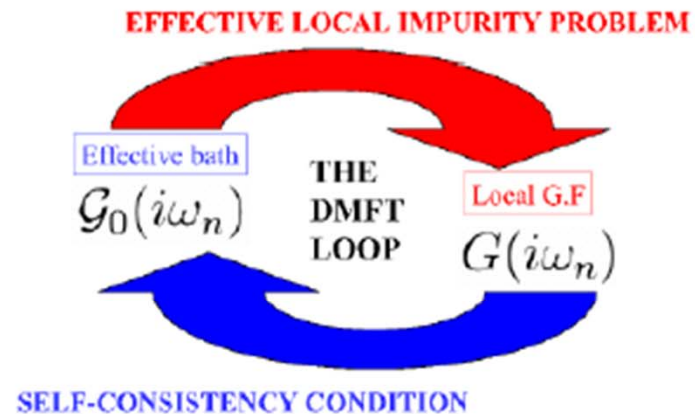
K. Haule, PRB 2007

P. Sémon *et al.*

PRB **85**, 201101(R) (2012)

PRB **90** 075149 (2014);

and PRB **89**, 165113 (2014)



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$- \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$