Pseudogaps and strongly correlated superconductivity

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Outline

- Part I
 - Pseudogap
 - Mott physics
 - Precursor to LRO
- Part II
 - Strongly correlated superconductivity (BEDT)
 - Retardation



Part I

Pseudogap



Three broad classes of mechanisms for pseudogap

- Phase with a broken symmetry (discrete)
- Mott Physics + J
- Precursor of LRO (d = 2)
 - Mermin-Wagner allows a large fluctuation regime
 - Even with weak correlations



Influence of Mott transition away from half-filling

n = 1, d = 2 square lattice





Density of states





Density of states





U = 6.2 t Normal state. Density of states





Density of states



Khosaka et al. Science 315, 1380 (2007);



Spin susceptibility





Spin susceptibility



Julien et al. PRL 76, 4238 (1996)





G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012) P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)



c-axis resistivity







Plaquette eigenstates



Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges PRB 80, 064501 (2009)



The pseudogap in electron-doped cuprates



TPSC: Theory vs experiment



Our road map





Model parameters

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma} \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$





Weak coupling *U*<8*t*

n=1+x – electron filling



Hot spots from AFM quasi-static scattering

Mermin-Wagner



Vilk, A.-M.S.T (1997) Kyung, Hankevych, A.-M.S.T., PRL, 2004

Armitage et al. PRL 2001

d = 2

15% doping: EDCs along the Fermi surface TPSC



Fermi surface plots

Hubbard repulsion U has to...



Pseudogap temperature and QCP



 $\gg \Delta_{PG} \approx 10 k_B T^*$ comparable with optical measurements

Hankevych, Kyung, A.-M.S.T., PRL 2004 : Expt: Y. Onose et al., PRL (2001).



Thermal de Broglie wavelength

$\Delta \varepsilon \sim k_B T$

$$\nabla_{\mathbf{k}} \varepsilon \ \Delta k \sim k_B T$$

 $\xi_{th} \sim \frac{v_F}{T}$

$$\Delta k \sim \frac{k_B T}{\hbar v_F}$$

$$\frac{2\pi}{\xi_{th}} \sim \frac{k_B T}{\hbar v_F}$$



e-doped pseudogap

E. M. Motoyama et al.. Nature 445, 186–189 (2007).



Precursor of SDW state (dynamic symmetry breaking)

- Y.M. Vilk and A.-M.S. Tremblay, J. Phys. Chem. Solids **56**, 1769-1771 (1995).
- Y. M. Vilk, Phys. Rev. B 55, 3870 (1997).
- J. Schmalian, et al. Phys. Rev. B 60, 667 (1999).
- B.Kyung et al., PRB 68, 174502 (2003).
- Hankevych, Kyung, A.-M.S.T., PRL, sept 2004
- Kusko *et al.* PRB **66**, 140513 (2002).



Benchmarks for TPSC

Normal state



Benchmark comparison with QMC





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Proofs...





Calc. + QMC: Moukouri et al. P.R. B 61, 7887 (2000).







Theoretical difficulties



Theory without small parameter: How should we proceed?

- Identify important physical principles and laws to constrain non-perturbative approximation schemes
 - From weak coupling (kinetic)
 - From strong coupling (potential)
- Benchmark against "exact" (numerical) results.
- Check that weak and strong correlation approaches agree in intermediate range.
- Compare with experiment



TPSC: How it works

e-doped



TPSC: general ideas

- General philosophy
 - Drop diagrams
 - Impose constraints and sum rules
 - Conservation laws
 - Pauli principle ($< n_{\sigma}^2 > = < n_{\sigma} >$)
 - Local moment and local density sum-rules
- Get for free:
 - Mermin-Wagner theorem
 - Kanamori-Brückner screening
 - Consistency between one- and two-particle $\Sigma G =$

 $U < n_{\sigma} n_{-\sigma} >$ Vilk, AMT J. Phys. I France, 7, 1309 (1997);

Theoretical methods for strongly correlated electrons also (Mahan, 3rd)


TPSC: Single-particle properties

A better approximation for single-particle properties (Ruckenstein)



Y.M. Vilk and A.-M.S. Tremblay, J. Phys. Chem. Solids **56**, 1769 (1995). Y.M. Vilk and A.-M.S. Tremblay, Europhys. Lett. **33**, 159 (1996);

N.B.: No Migdal theorem



Crossing symmetry





TPSC approach: two steps

I: Two-particle self consistency

- 1. Functional derivative formalism (conservation laws)
 - (a) spin vertex: $U_{sp} = \frac{\delta \Sigma_{\uparrow}}{\delta G_{\downarrow}} \frac{\delta \Sigma_{\uparrow}}{\delta G_{\uparrow}}$
 - (b) analog of the Bethe-Salpeter equation:

$$\chi_{sp} = \frac{\delta G}{\delta \phi} = GG + GU_{sp}\chi_{sp}G$$

(c) self-energy: $\Sigma_{\sigma}(1,\overline{1};\{\phi\}) G_{\sigma}(\overline{1},2;\{\phi\}) = -U \left\langle c^{\dagger}_{-\sigma}(1^{+}) c_{-\sigma}(1) c_{\sigma}(1) c^{\dagger}_{\sigma}(2) \right\rangle_{\phi}$ $\approx A_{\{\phi\}} G^{(1)}_{-\sigma}(1,1^{+};\{\phi\}) G^{(1)}_{\sigma}(1,2;\{\phi\})$

2. Factorization -



TPSC...

$$U_{sp} = U \frac{\langle n_{\uparrow} n_{\downarrow} \rangle}{\langle n_{\uparrow} \rangle \langle n_{\downarrow} \rangle}$$
$$\chi_{sp}^{(1)}(q) = \frac{\chi_0(q)}{1 - \frac{1}{2} U_{sp} \chi_0(q)}$$

Kanamori-Brückner screening

3. The F.D. theorem and Pauli principle

$$\left\langle \left(n_{\uparrow} - n_{\downarrow}\right)^{2} \right\rangle = \left\langle n_{\uparrow} \right\rangle + \left\langle n_{\downarrow} \right\rangle - 2\left\langle n_{\uparrow} n_{\downarrow} \right\rangle \frac{T}{N} \sum_{q} \chi_{sp}^{(1)}(q) = n - 2\left\langle n_{\uparrow} n_{\downarrow} \right\rangle$$
II: Improved self energy

II: Improved self-energy

Insert the first step results

into exact equation: $\Sigma_{\sigma}(1,\overline{1};\{\phi\}) G_{\sigma}(\overline{1},2;\{\phi\}) = -U \langle c^{\dagger}_{-\sigma}(1^{+}) c_{-\sigma}(1) c_{\sigma}(1) c^{\dagger}_{\sigma}(2) \rangle_{\phi}$

Internal accuracy check

Internal accuracy check

$$\frac{1}{2} \operatorname{Tr} \left(\Sigma^{(2)} G^{(1)} \right) = U \left\langle n_{\uparrow} n_{\downarrow} \right\rangle \qquad \frac{1}{2} \operatorname{Tr} \left(\Sigma^{(2)} G^{(2)} \right)$$

f- sum rule (conservation law)

$$\int \frac{d\omega}{\pi} \omega \chi_{ch,sp}^{\prime\prime}(\mathbf{q},\omega) = \lim_{\eta \to 0} T \sum_{i\omega_n} \left(e^{-i\omega_n \eta} - e^{i\omega_n \eta} \right) i\omega_n \chi_{ch,sp}\left(\mathbf{q}, i\omega_n\right)$$
$$= \frac{1}{N} \sum_{\mathbf{k}\sigma} \left(\epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}-\mathbf{q}} - 2\epsilon_{\mathbf{k}} \right) n_{\mathbf{k}\sigma}$$



Main collaborators on TPSC



Liang Chen



Yury Vilk



Bumsoo Kyung



D. Poulin



S. Moukouri



F. Lemay









J.S. Landry V. Hankevych A.-M. Daré





Dominic Bergeron



Bahman Davoudi



Syed Hassan





Part II

Strongly correlated superconductivity







Charles-David Hébert

Patrick Sémon

Organics : Phase diagram, finite T

Made possible by algorithmic improvements

P. Sémon *et al.* PRB **85**, 201101(R) (2012) PRB **90** 075149 (2014); and PRB **89**, 165113 (2014)



Layered organics (κ -BEDT-X family)

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996), R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)

BEDT-TTF layer

Anion layer



Y. Shimizu, et al. Phys. Rev. Lett. **91**, 107001(2003)

 $t \approx 50 \text{ meV}$ $\Rightarrow U \approx 400 \text{ meV}$ $t'/t \sim 0.6 - 1.1$



Anisotropic triangular lattice



See: Poster Shaheen Acheche



Phase diagram for organics



Phase diagram at n = 1





Superconductivity near the Mott transition

n = 1, d = 2 square lattice





Superconductivity near the Mott transition

n = 1, d = 2 square lattice





Superconductivity near Mott transition (n = 1)



C.-D. Hébert, P. Sémon, A.-M.S. T PRB 92, 195112 (2015)



Doped Organics



Doped BEDT



H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL 114, 067002 (2015)



Doped organics





Doped organics

n = 1, d = 2 square lattice





First order and Widom line in organics





Compare: T. Watanabe, H. Yokoyama and M. Ogata JPS Conf. Proc. **3**, 013004 (2014)

C.-D. Hébert, P. Sémon, A.-M.S. T PRB 92, 195112 (2015)



Doped BEDT



H. Oike, K. Miyagawa, H. Taniguchi, K. Kanoda PRL 114, 067002 (2015)







Compare: T. Watanabe, H. Yokoyama and M. Ogata JPS Conf. Proc. **3**, 013004 (2014)



Generic case highly frustrated case





Summary : organics

- Agreement with experiment
 - SC: larger T_c and broader P range if doped
 - Larger frustration: Decrease T_N much more than T_c
 - Normal state metal to pseudogap crossover
- Predictions
 - First order transition at low *T* in normal state (B induced)
 - Crossovers in SC state associated with normal state.
- Physics
 - SC dome without an AFM QCP. Extension of Mott
 - SC from short range J.
 - T_c dome maximum near normal state 1st order



Pairing mechanism

Back to high T_c







A cartoon strong correlation picture

$$J\sum_{\langle i,j\rangle} \mathbf{S}_{i} \cdot \mathbf{S}_{j} = J\sum_{\langle i,j\rangle} \left(\frac{1}{2}c_{i}^{\dagger}\vec{\sigma}c_{i}\right) \cdot \left(\frac{1}{2}c_{j}^{\dagger}\vec{\sigma}c_{j}\right)$$
$$d = \langle \hat{d} \rangle = 1/N\sum_{\vec{k}} (\cos k_{x} - \cos k_{y}) \langle c_{\vec{k},\uparrow}c_{-\vec{k},\downarrow} \rangle$$
$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^{\dagger} c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^{\dagger}) + F_{0}$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson Science Miyake, Schmitt–Rink, and Varma 317, 1705 (2007)
 P.R. B 34, 6554-6556 (1986)
 More sophisticated Slave Boson: Kotliar Liu PRB 1988 SHERBROOKE

Im Σ_{an} and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)



The glue



The glue and neutrons



FIG. 3 (color online). **Q**-integrated dynamic structure factor $S(\omega)$ which is derived from the wide-*H* integrated profiles for LBCO 1/8 (squares), LSCO x = 0.25 (diamonds; filled for $E_i = 140 \text{ meV}$, open for $E_i = 80 \text{ meV}$), and x = 0.30 (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO x = 0.25 and 0.30 are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007); PRL (2004)



The glue in CDMFT and DCA

Th. Maier, D. Poilblanc, D.J. Scalapino, PRL (2008)
M. Civelli, PRL 103, 136402 (2009)
M. Civelli PRB 79, 195113 (2009)
E. Gull, A. J. Millis PRB 90, 041110(R) (2014)
S. Sakai, M. Civelli, M. Imada arXiv:1411.4365





Frequencies important for pairing

Bumsoo Kyung



David Sénéchal



Nearest-neighbor repulsion should destroy Tc?



Extended Hubbard model



$$\hat{\mathcal{H}} = -t \sum_{\langle i,j \rangle \sigma} \left(\hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + c.h \right) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_{\langle i,j \rangle} \hat{n}_{i} \hat{n}_{j} - \mu \sum_{i} \hat{n}_{i} \hat{n}_{i}$$

$$\underset{i \neq i}{\underbrace{\mathbb{N}}} \hat{n}_{i} \underbrace{\mathbb{N}}_{i \neq i} \hat{n}_{i}$$

$$\underset{i \neq i \neq i}{\underbrace{\mathbb{N}}} \hat{n}_{i} \underbrace{\mathbb{N}}_{i \neq i} \hat{n}_{i}$$

Resilience to near-neighbor repulsion V (Scalapino)

$$\hat{\mathcal{H}}_{Hubbard} = -\sum_{\langle i,j \rangle_{1,2,3}} \left(t_{ij} \hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + c.h \right) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_{\langle i,j \rangle} \hat{n}_{i} \hat{n}_{j} - \mu \sum_{i\sigma} \hat{n}_{i\sigma}$$

YBa₂**Cu**₃**O**₇: t = 1 t' = -0.3 t'' = 0.2

We expect superconductivity to disappear when:

 $V > \frac{U^2}{W} \qquad \text{In weakly correlated case} \qquad V > J \qquad \begin{array}{l} \text{In mean-field strongly} \\ \text{Correlated case} \\ V = 400 \text{ meV} \\ \text{In cuprates:} \\ U_c = V_c / [1 + N(0)V_c \ln(E_F/\omega_c)] \qquad \text{Anderson-Morel} \end{array}$

S. Onari, R. Arita, K. Kuroki et H. Aoki, PRB 70, 094523 (2004)

S. Raghu, E. Berg, A. V. Chubukov et S. A. Kivelson, PRB **85**, 024516 (2012) S. Sorella, et al. Phys. Rev. Lett. 88, 117002 (2002)



d-wave in mean-field

$$\begin{split} \hat{\mathcal{H}}_{mod\hat{e}le\ t-J} &= -t \sum_{\langle i,j \rangle \sigma} \hat{P}\left(\hat{c}_{i\sigma}^{\dagger}\hat{c}_{j\sigma} + c.h\right)\hat{P} + J \sum_{\langle i,j \rangle} \left(\hat{\vec{S}}_{i}.\hat{\vec{S}}_{j}\right) - \frac{1}{4}\hat{n}_{i}\hat{n}_{j}\right) \\ &= J(\hat{S}_{i}^{z}\hat{S}_{j}^{z} = J(\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow})(\hat{n}_{j\uparrow} - \hat{n}_{j\downarrow}) \\ &= J(\hat{c}_{i\uparrow}^{\dagger}\hat{c}_{i\uparrow} - \hat{c}_{i\downarrow}^{\dagger}\hat{c}_{i\downarrow})(\hat{c}_{j\uparrow}^{\dagger}\hat{c}_{j\uparrow} - \hat{c}_{j\downarrow}^{\dagger}\hat{c}_{j\downarrow}) \\ &= -J(\hat{c}_{i\uparrow}^{\dagger}\hat{c}_{i\downarrow}\hat{c}_{j\uparrow}\hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}^{\dagger}\hat{c}_{i\uparrow}\hat{c}_{j\downarrow}^{\dagger}\hat{c}_{j\downarrow}) + \dots \\ &= -J(\hat{c}_{j\uparrow}^{\dagger}\hat{c}_{i\downarrow}\hat{c}_{i\downarrow}\hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}^{\dagger}\hat{c}_{j\downarrow}\hat{c}_{j\downarrow}\hat{c}_{j\downarrow}) + \dots \\ &= -J(\hat{c}_{j\uparrow}^{\dagger}\hat{c}_{i\downarrow}\hat{c}_{i\downarrow}\hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}^{\dagger}\hat{c}_{j\downarrow}\hat{c}_{j\downarrow}\hat{c}_{j\downarrow}) + \dots \\ &= -J(\hat{c}_{j\uparrow}^{\dagger}\hat{c}_{i\downarrow}\hat{c}_{i\downarrow}\hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}\hat{c}_{j\uparrow}\hat{c}_{j\downarrow}\hat{c}_{j\downarrow}\hat{c}_{i\uparrow}) + \dots \\ &\text{Hartree-Fock :} \\ &\langle J\hat{S}_{i}^{z}\hat{S}_{j}^{z} \rangle = -2Jd^{*}d + \dots \end{split}$$

Miyake, Schmitt–Rink et Varma, PRB **34**, 6554-6556 (1986) Anderson, Baskaran, Zou et Hsu, PRL **58**, 26 (1987)




Resilience to near-neighbor repulsion

David Sénéchal

Alexandre Day





Vincent Bouliane

Sénéchal, Day, Bouliane, AMST PRB 87, 075123 (2013)



V also increases J







Binding aspects of V

$$J = \frac{4t^2}{U - V}$$

J increases with V explaining better pairing at low frequency

But V also induces more repulsion at high frequency, explaining the negative impact at high frequency on binding



Antagonistic effects of V at finite T





Summary

- Pseudogap in e-doped is a *d=2* precursor of AFM
- Normal state first-order transition from Mott & J is an organizing principle for
 - The normal and superconducting states
 - Cuprates and organics are examples
 - Predictions for organics
- Mechanism: *J* short-range is retarded and resilient to *V*



Open questions

- Why does T_c start to go down at such large filling?
- Effect of competition with other order.



Mammouth



Compute • calcul

High Performance Computing

CREATING KNOWLEDGE DRIVING INNOVATION BUILDING THE DIGITAL ECONOMY

Le calcul de haute performance

CRÉER LE SAVOIR ALIMENTER L'INNOVATION BÂTIR L'ÉCONOMIE NUMÉRIQUE Calcul Québec



Review: A.-M.S.T. arXiv: 1310.1481



A.-M.S. Tremblay "Strongly correlated superconductivity" Chapt. 10 : Emergent Phenomena in Correlated Matter Modeling and Simulation, Vol. 3, E. Pavarini, E. Koch, and U. Schollwöck (eds.) Verlag des Forschungszentrum Jülich, 2013