

Strongly correlated superconductivity and quantum criticality

A.-M. Tremblay



March Meeting of the American Physical Society
New Orleans - C23.00003



Atomic structure

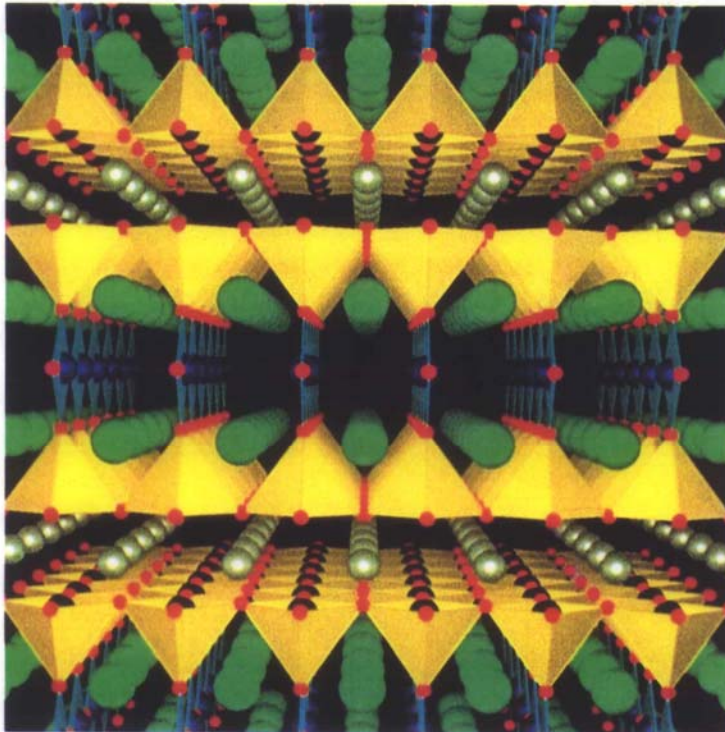
SCIENTIFIC AMERICAN

JUNE 1988
\$3.50

How nonsense is deleted from genetic messages.

R_x for economic growth: aggressive use of new technology.

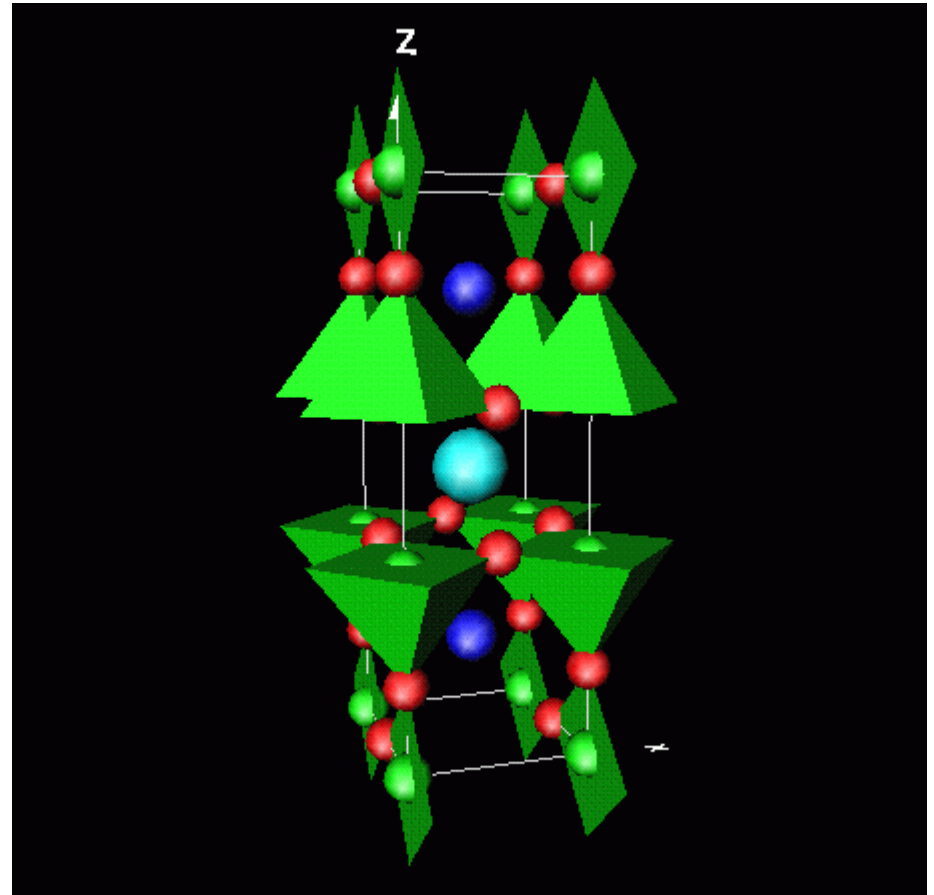
Can particle physics test cosmology?



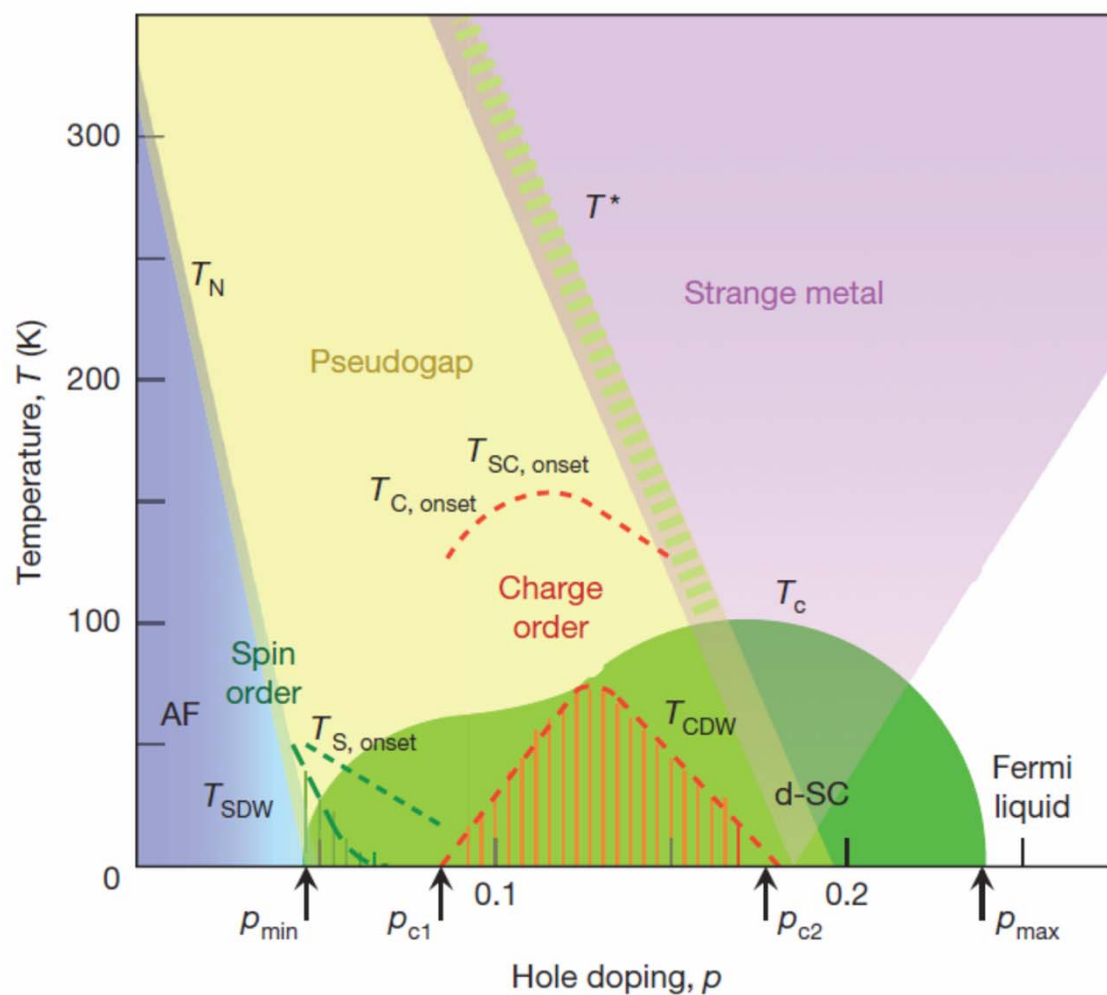
High-Temperature Superconductor belongs to a family of materials that exhibit exotic electronic properties.



92-37

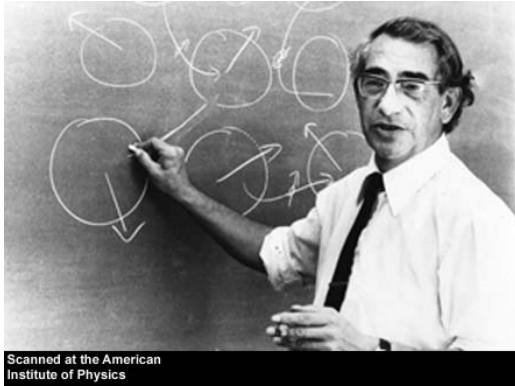


Phase diagram $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$

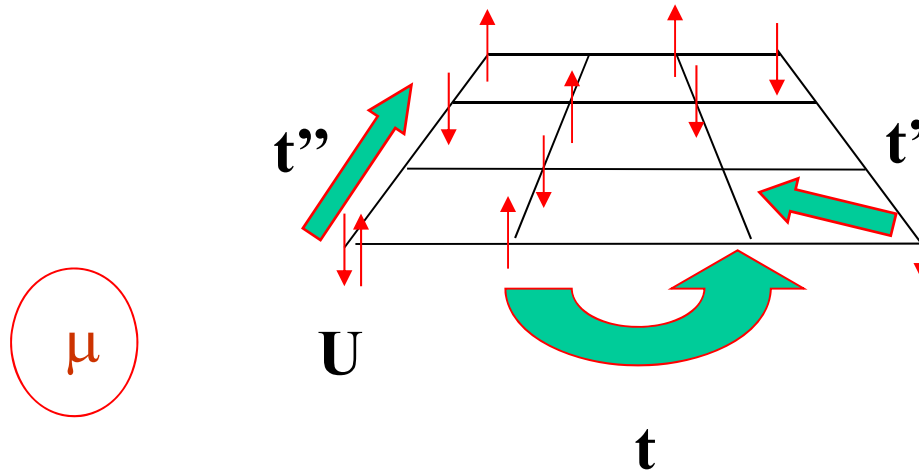


Keimer et al., Nature 518, 179 (2015)

Hubbard model



1931-1980



$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} \left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$t = 1, k_B = 1, \hbar = 1$$

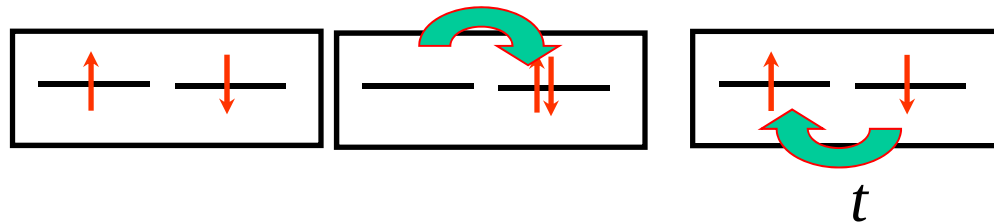
Attn: Charge transfer insulator

Interesting in the general case

No mean-field factorization for d-wave superconductivity

$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

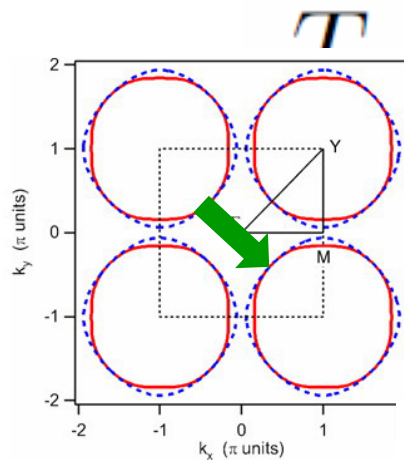
Interaction-induced metal-insulator (Mott) transition



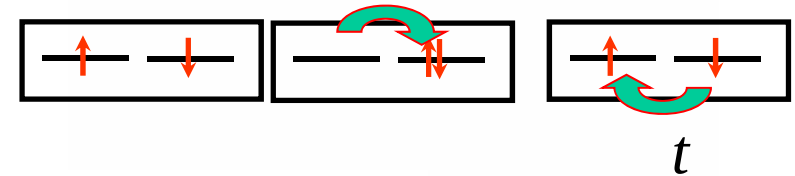
Effective model, Heisenberg: $J = 4t^2 / U$

Weak vs Strong correlations

$n = 1$, unfrustrated $d = 3$ cubic lattice



Mott



Slater $\Rightarrow$

$\Leftarrow$ Heisenberg

AFM

U/t

2. Method for strongly correlated matter

Dynamical Mean Field Theory (+ clusters)

Concept: atomic-like localized correlations
consistent with delocalized aspect

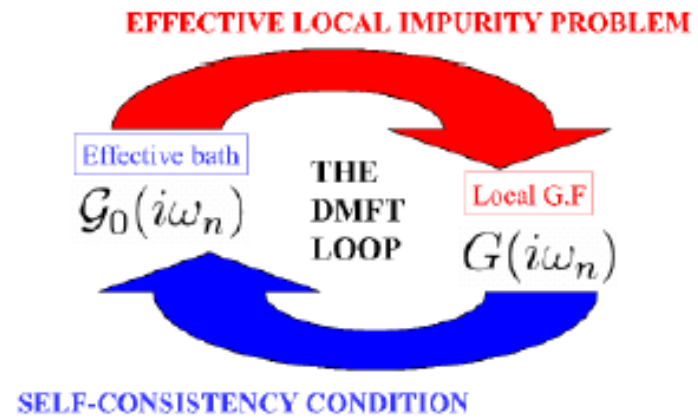
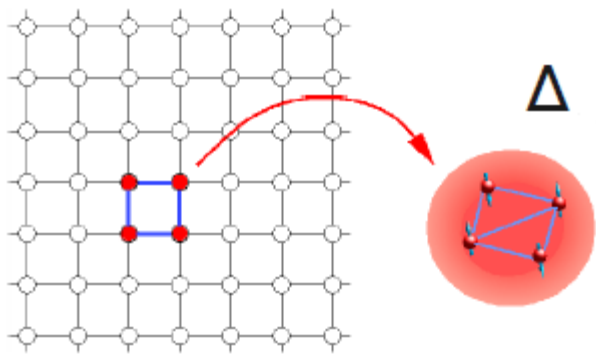
REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)

Method



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$- \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

+ and -

- Long range order:
 - No mean-field factorization on the cluster
 - Symmetry breaking allowed in the bath (mean-field)
- Included exactly:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations

Some groups using these methods for cuprates

- Europe:
 - Georges, Parcollet, Ferrero, Civelli, (Paris)
 - de Medici (Grenoble) Capone (Italy)
- USA:
 - Gull (Michigan) Millis (Columbia)
 - Kotliar, Haule (Rutgers)
 - Jarrell (Louisiana)
 - Maier, Okamoto (Oakridge)
- Japan
 - Imada (Tokyo) Sakai, Tsunetsugu, Motome

Outline

- The model
- The method
- Part I: Weakly and strongly correlated electrons
- Part II: Strongly correlated superconductivity
 - Cuprates
 - Organics

Part I

Weakly vs strongly correlated electrons
Normal and antiferromagnetic state



Giovanni Sordi



Lorenzo Fratino



Maxime Charlebois

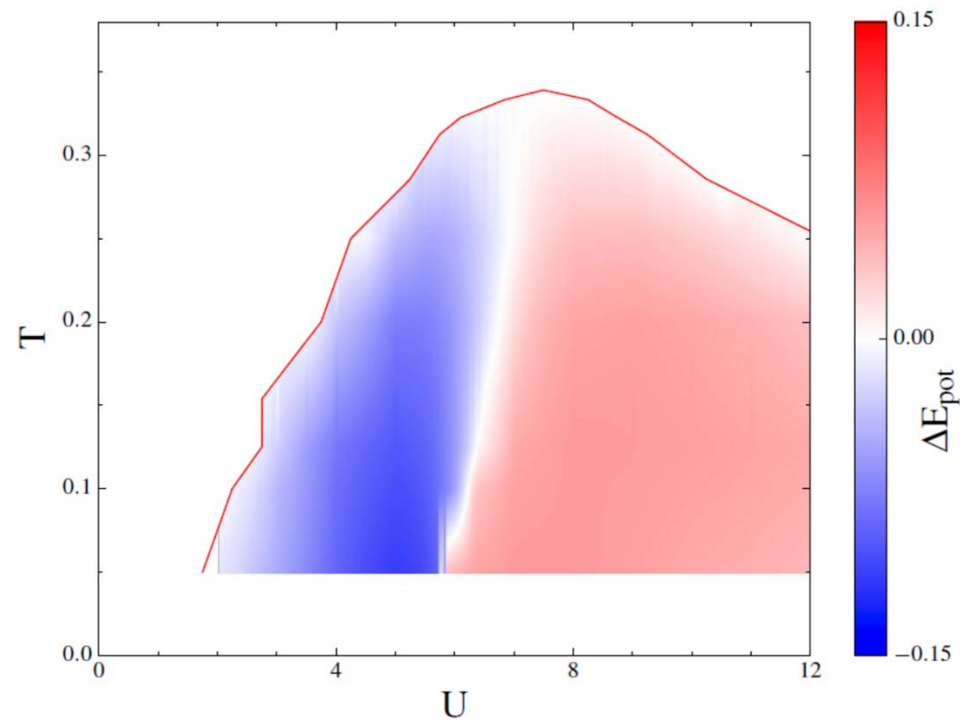


Patrick Sémon

Mott transition as an organizing principle

Influence of the underlying normal
state on the ordered state

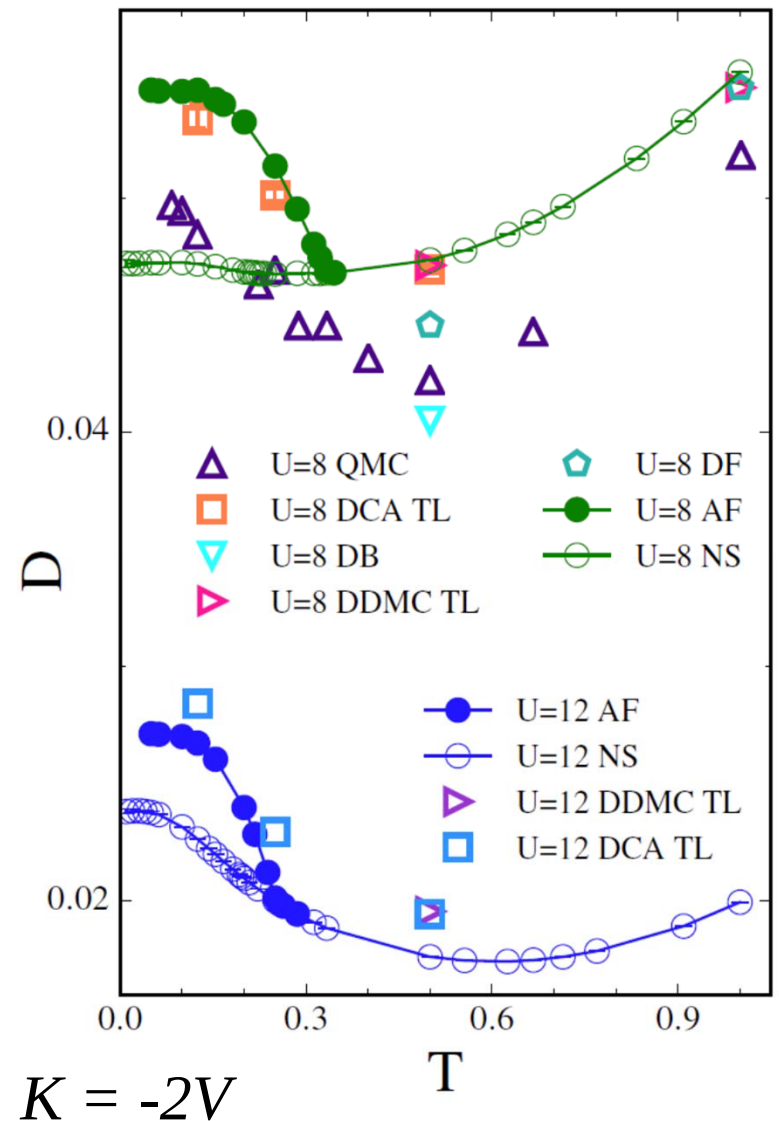
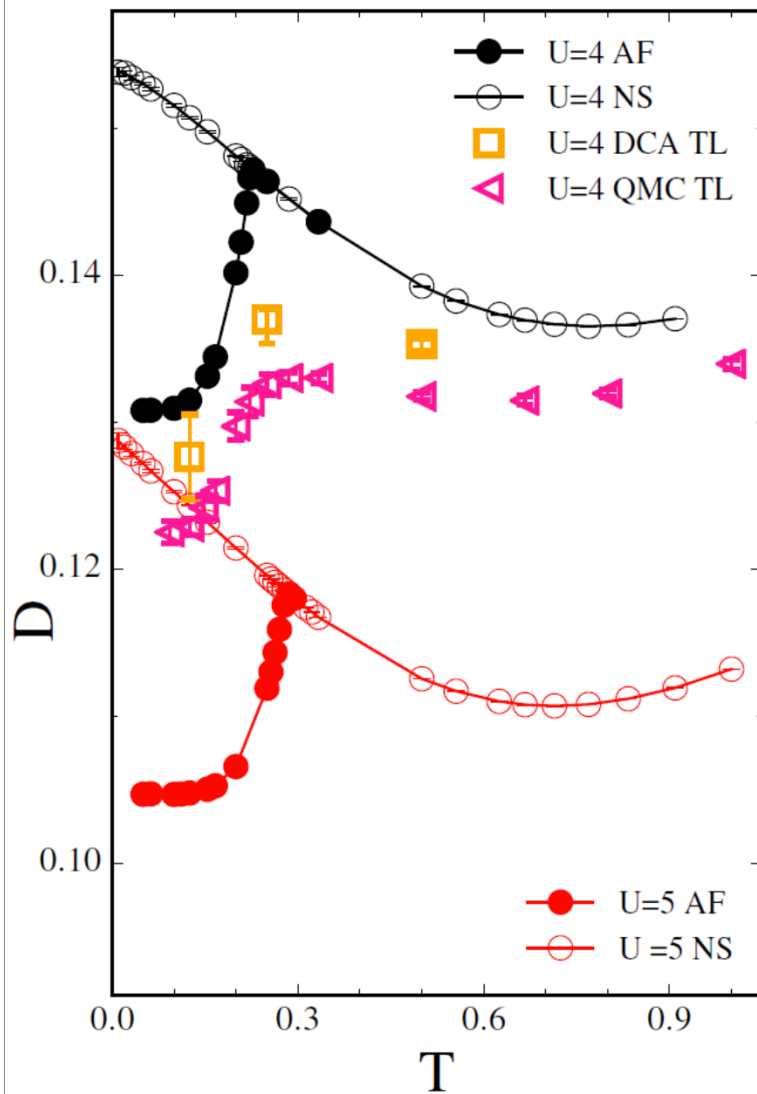
Change in potential energy due to large ξ



L. Fratino,¹ P. Sémon,² M. Charlebois,² G. Sordi,¹ and A.-M. S. Tremblay^{2,3}
[arXiv:1702.01821](https://arxiv.org/abs/1702.01821)

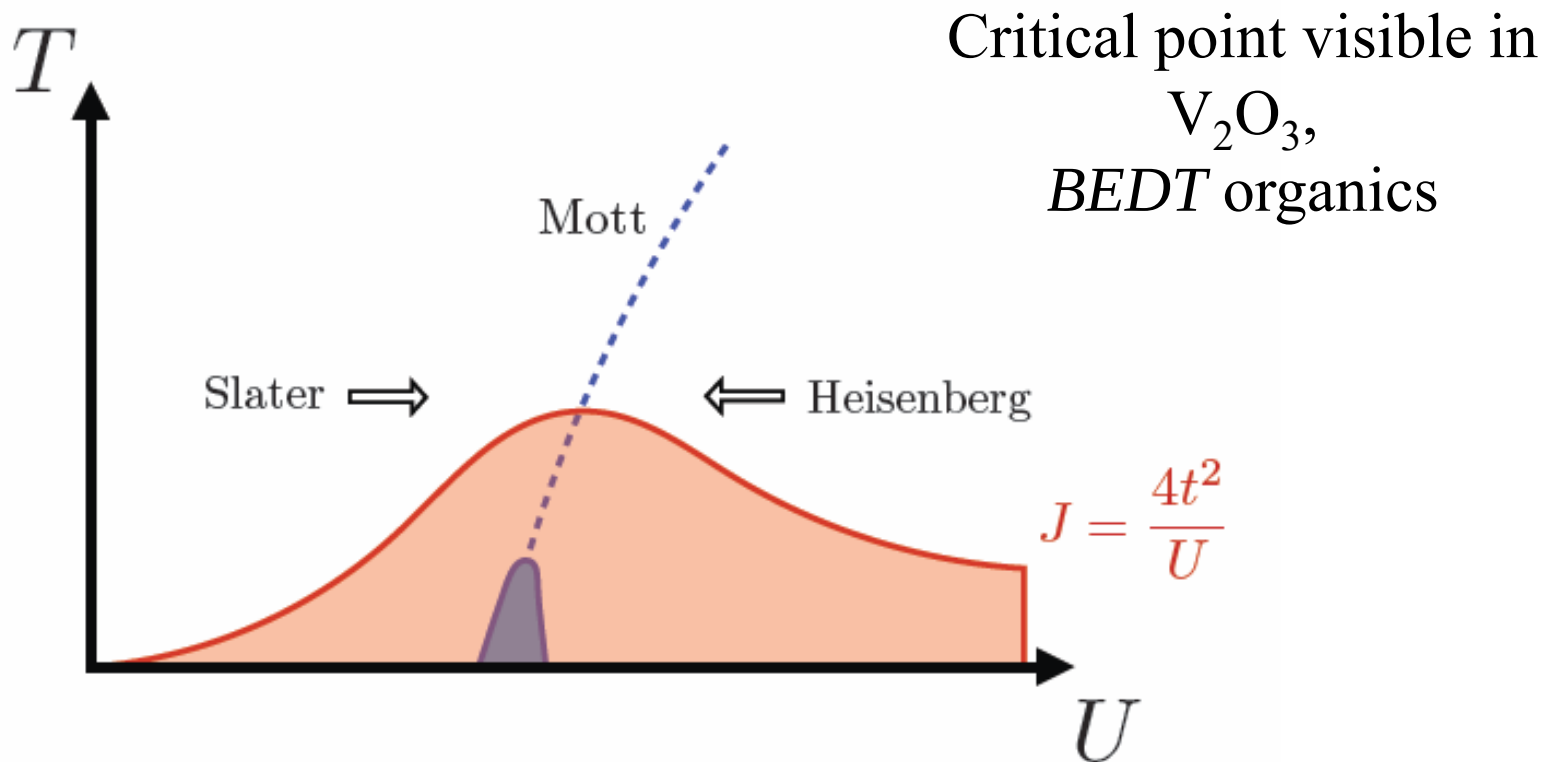
L. Fratino: **Competing order in correlated electrons**
B37b.00007

Double occupancy at weak and strong interactions: benchmarks



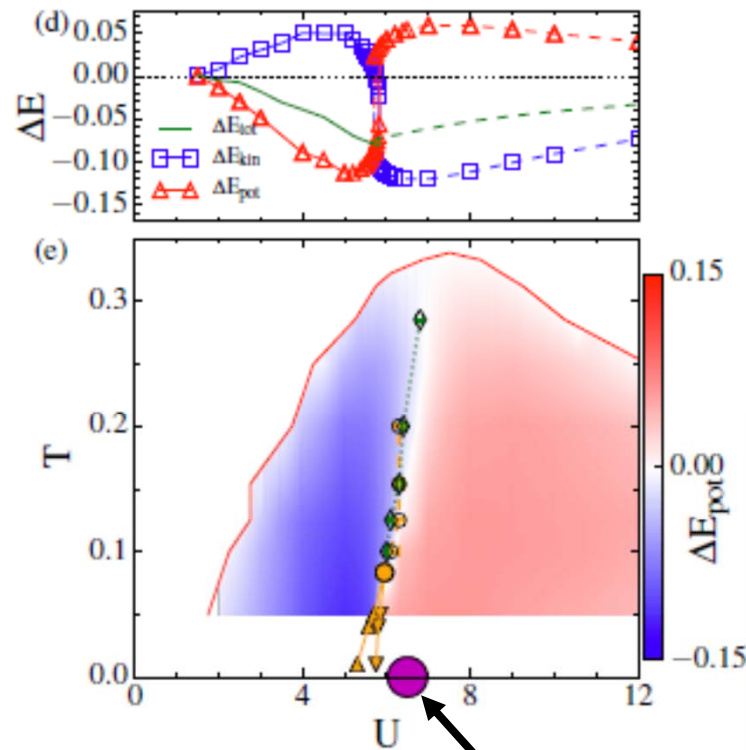
Underlying Mott transition

$n = 1, d = 2$ square lattice



Understanding finite temperature phase from a *mean-field theory* down to $T = 0$

Change in mechanism for stability of the AFM



L. Fratino,¹ P. Sémon,² M. Charlebois,² G. Sordi,¹ and A.-M. S. Tremblay^{2,3}
arXiv:1702.01821

L. F. Tocchio, F. Becca, and S. Sorella, Phys. Rev. B 94, 195126 (2016).

L. Fratino: **Competing order in correlated electrons**

B37b.00007



Giovanni Sordi

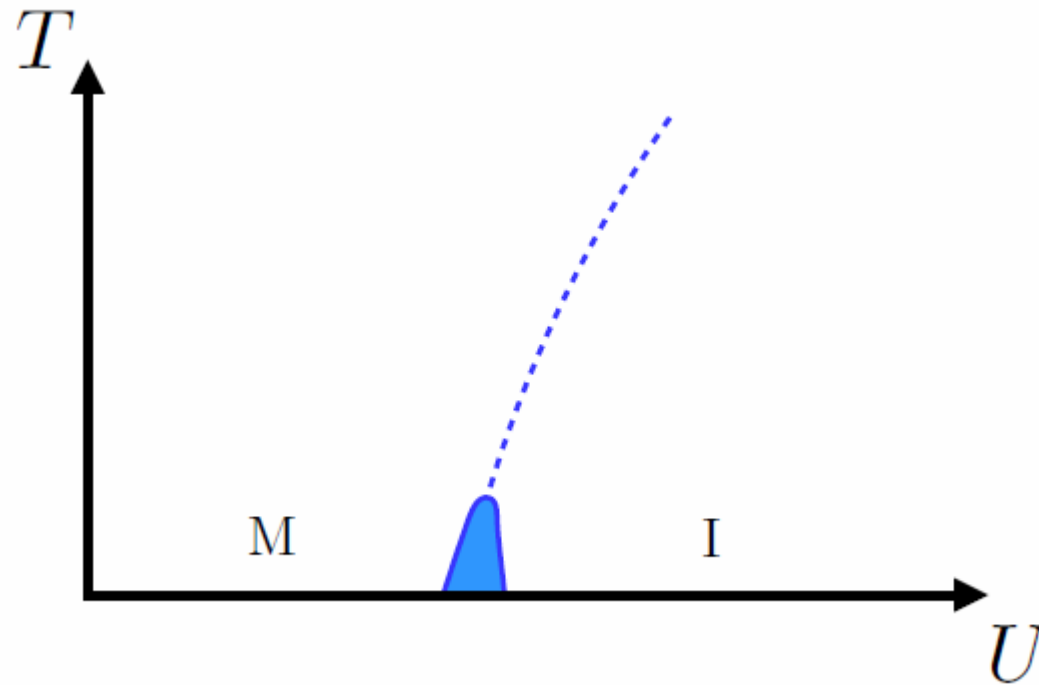


Kristjan Haule

Influence of the Mott transition away from half-filling

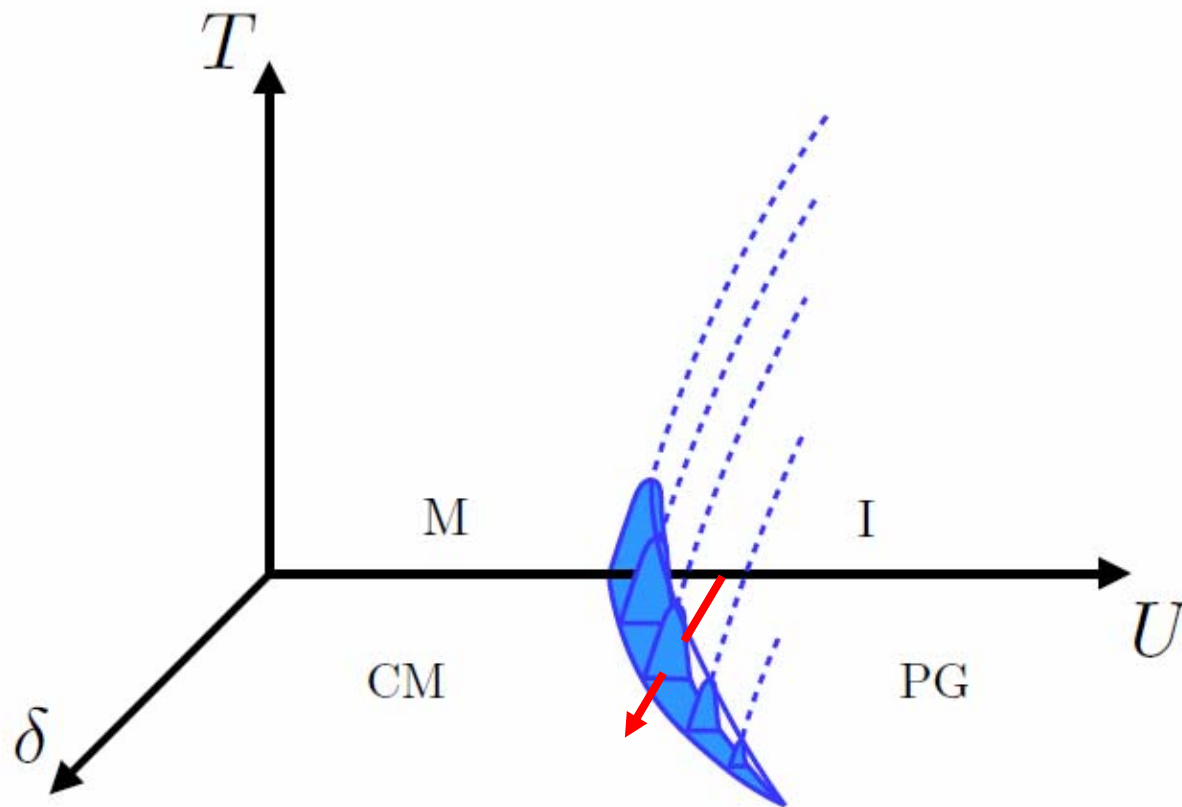
Influence of Mott transition away from half-filling

$n = 1, d = 2$ square lattice

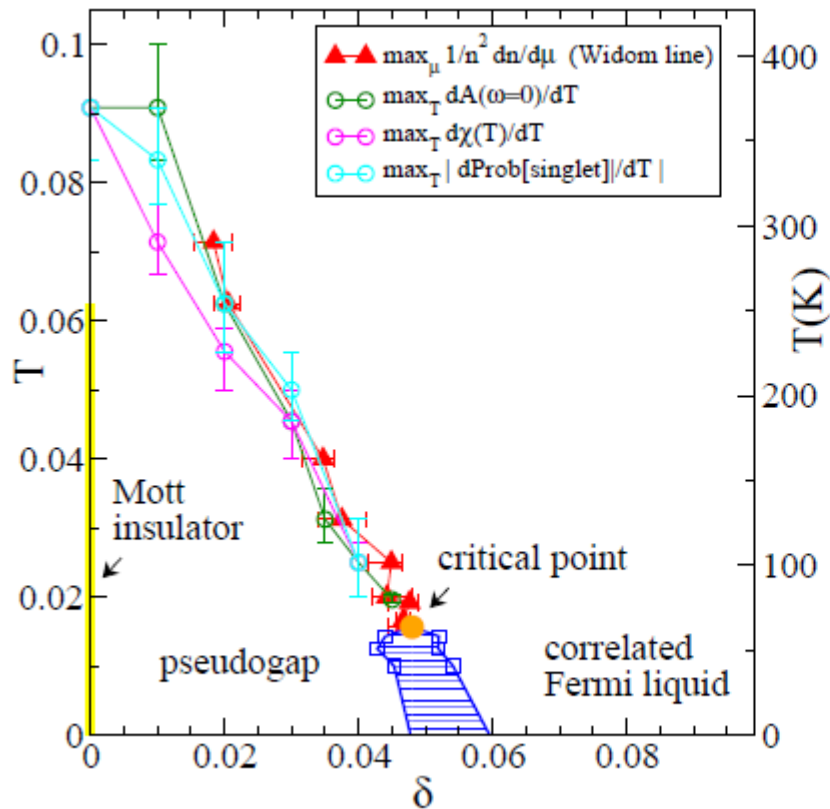


Influence of Mott transition away from half-filling

$n = 1, d = 2$ square lattice

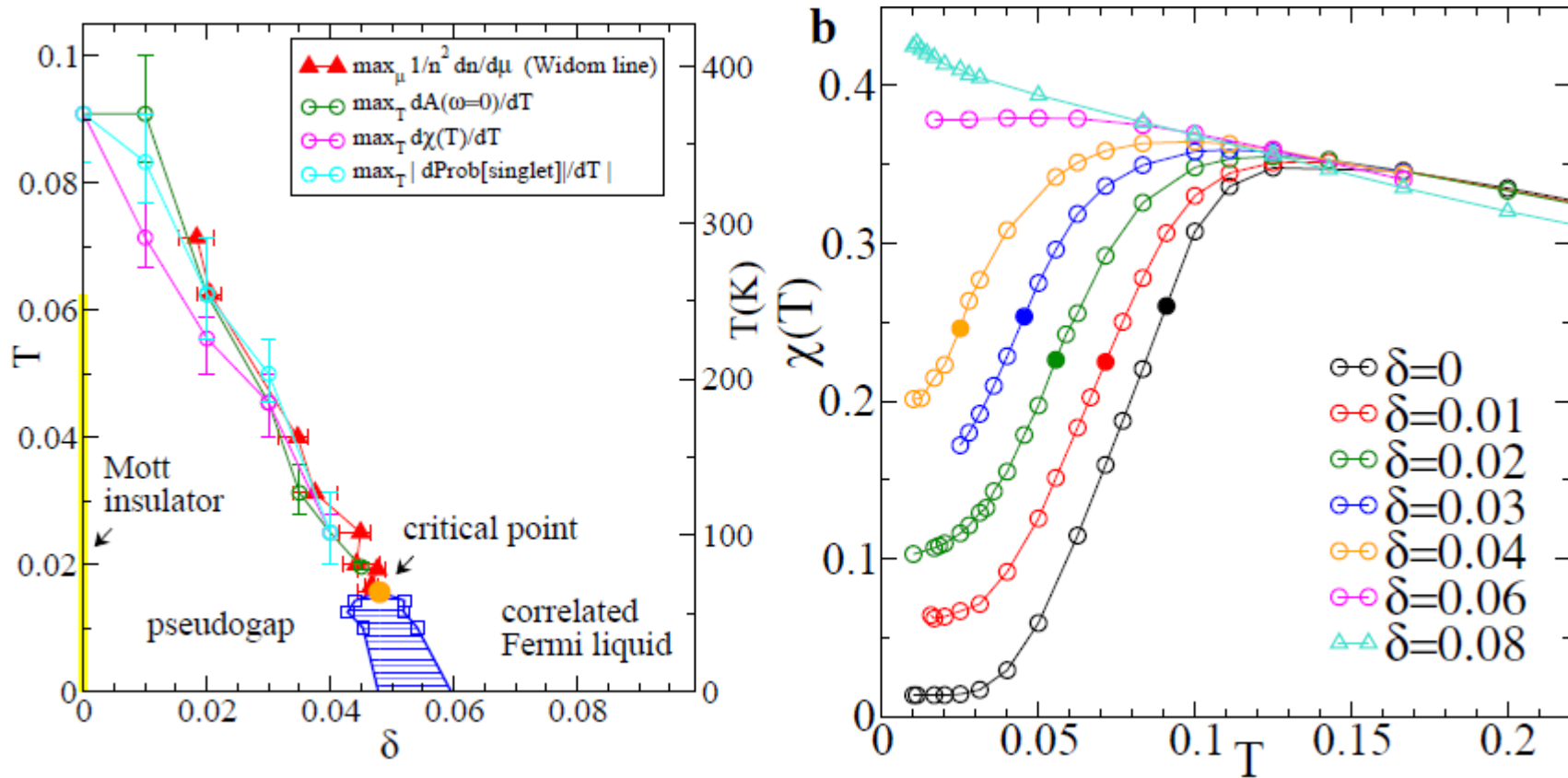


Spin susceptibility



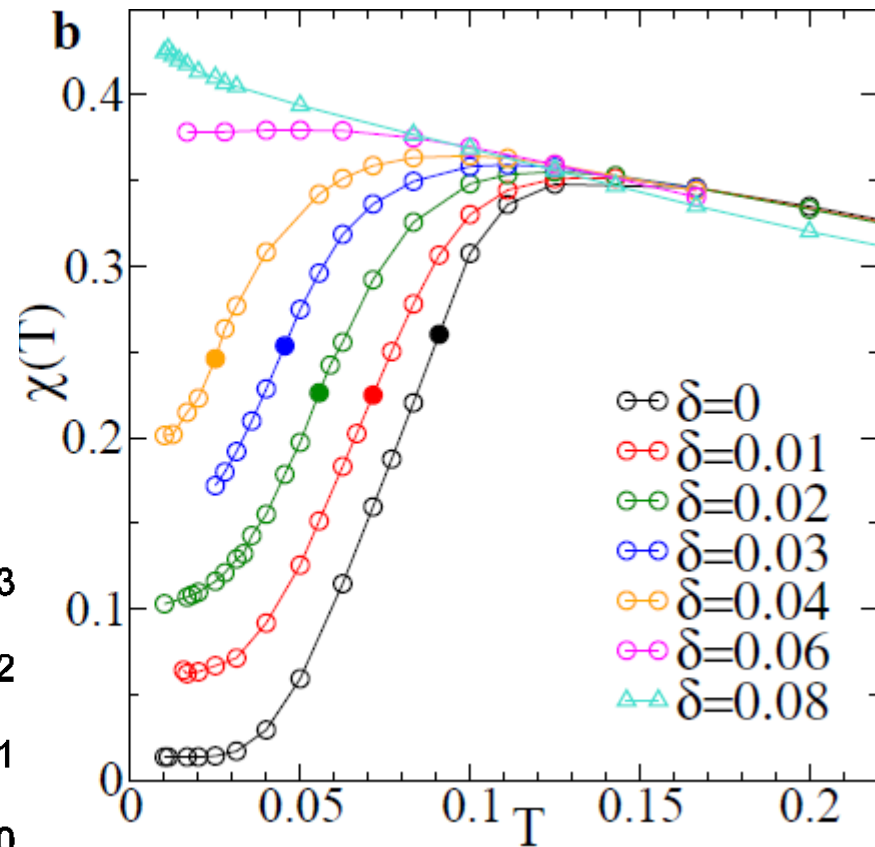
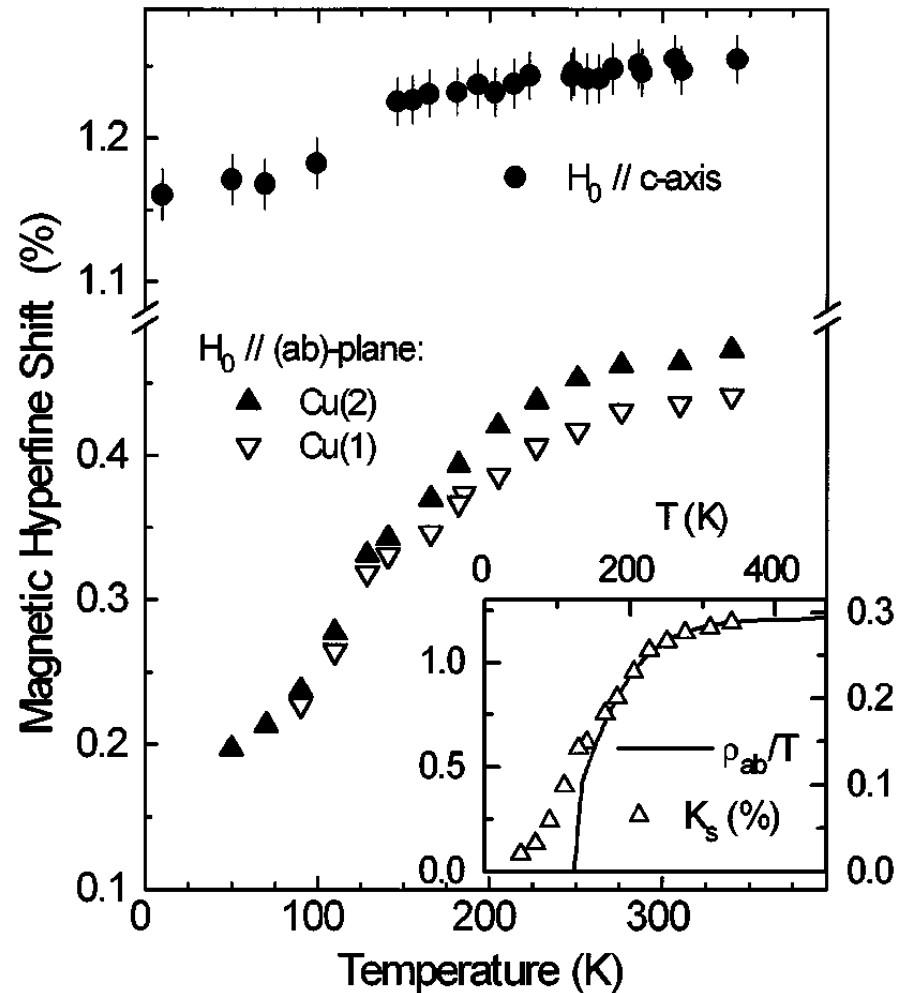
G. Sordi, *et al.* Scientific Reports 2, 547 (2012)

Spin susceptibility



G. Sordi, *et al.* Scientific Reports 2, 547 (2012)

Spin susceptibility



Underdoped Hg1223

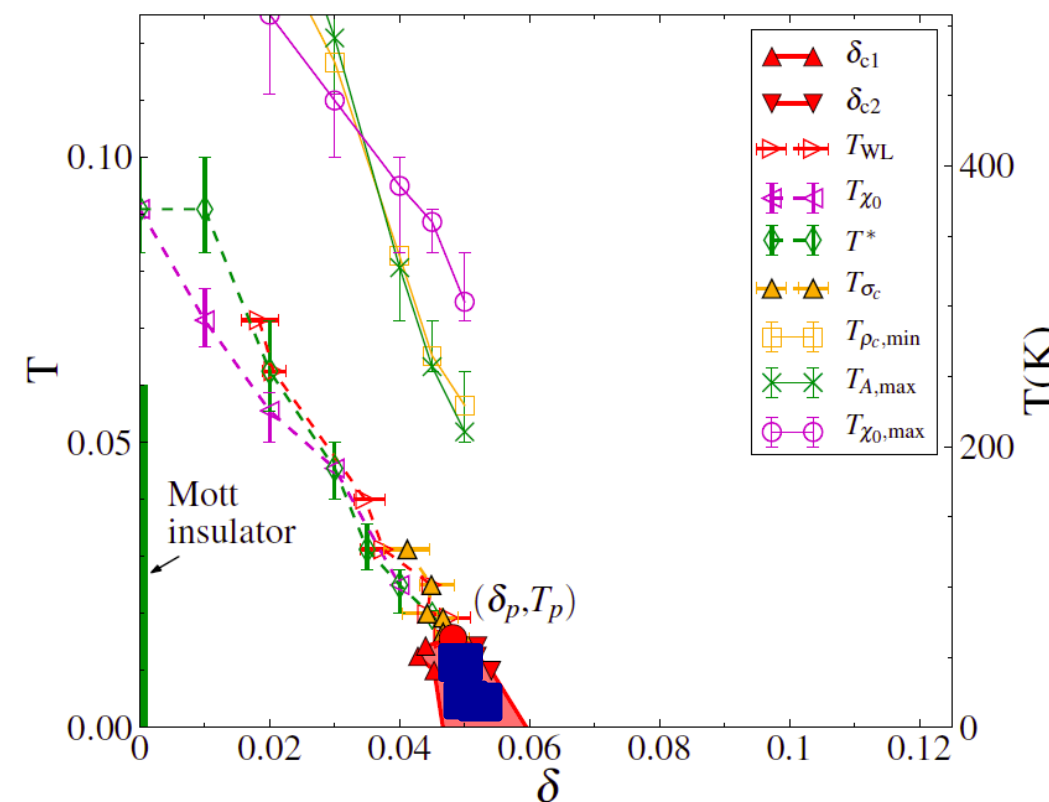


Giovanni Sordi

Two crossover lines



Patrick Sémon

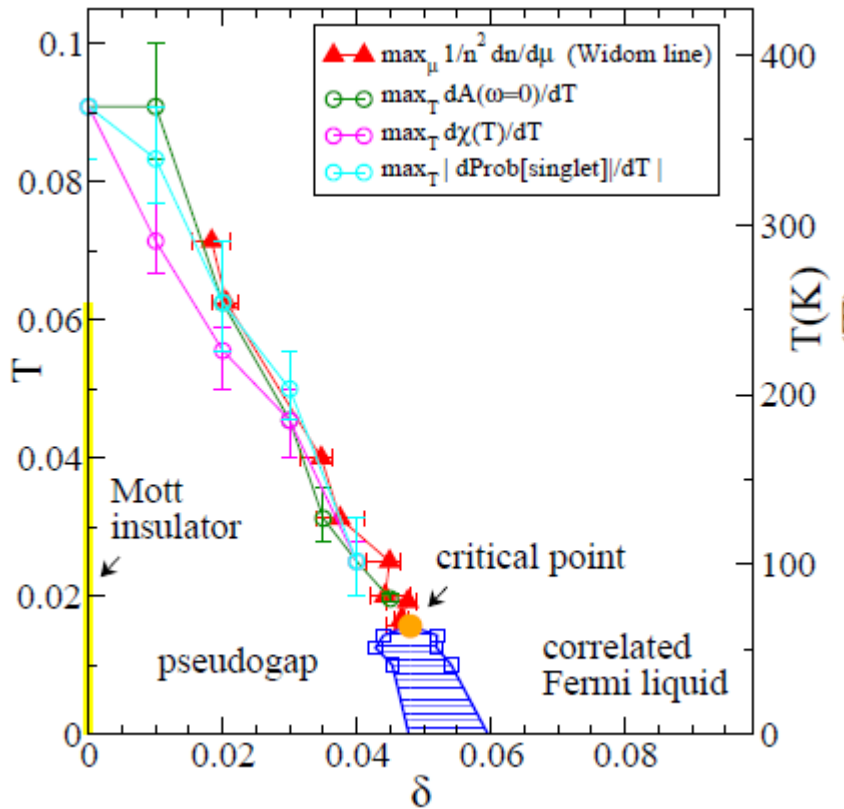


G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B **89**, 165113/1-6 (2014)

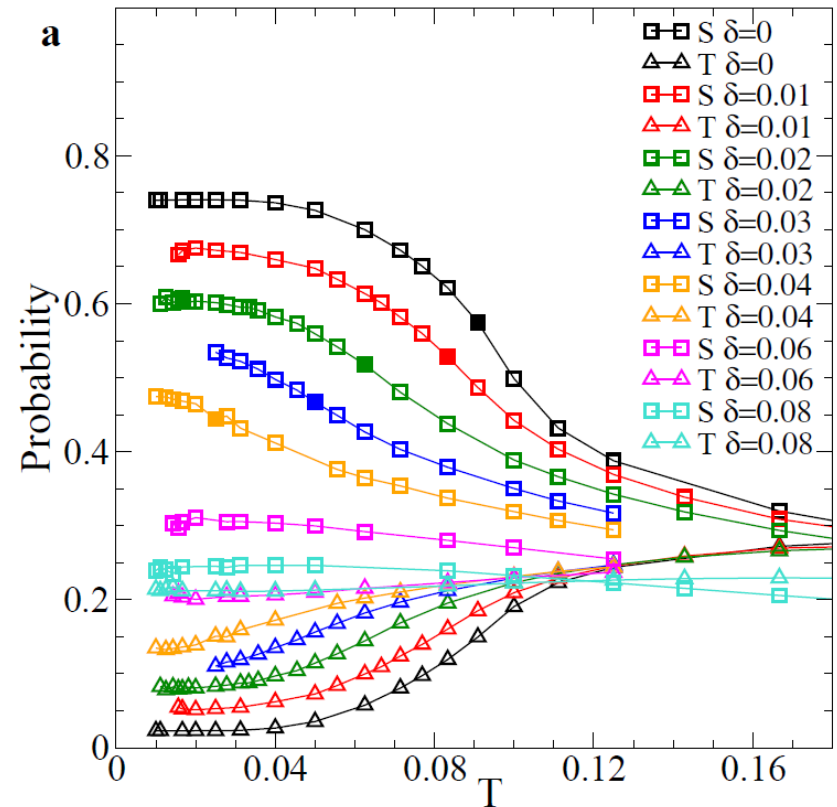
Physics

Plaquette eigenstates



See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges
 PRB **80**, 064501 (2009)





Giovanni Sordi



Patrick Sémon



Lorenzo Fratino

Part II

Strongly correlated Superconductivity

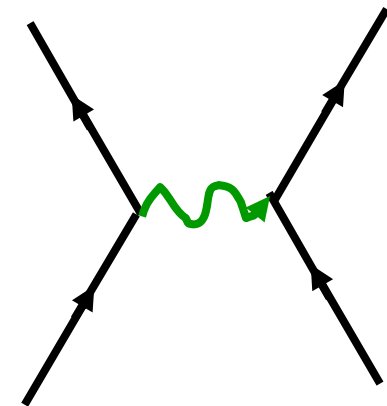
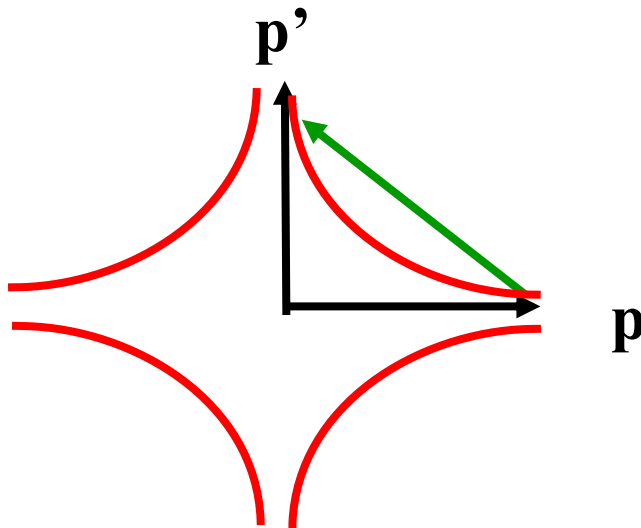
Sordi et al. PRL **108**, 216401 (2012)

Fratino et al.

Sci. Rep. **6**, 22715 (2016)

Cartoon « BCS » weak-correlation picture

$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Béal–Monod, Bourbonnais, Emery
P.R. B. **34**, 7716 (1986).

Exchange of spin waves?
Kohn-Luttinger

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch
P.R. B **34**, 8190-8192 (1986).

T_c with pressure

Kohn, Luttinger, P.R.L. **15**, 524 (1965).

Q IN THE QUANTIQUE **P.W. Anderson Science 317, 1705 (2007)**

A cartoon strong correlation picture

$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left(\frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left(\frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

Pair state orthogonal to repulsive core of Coulomb interaction

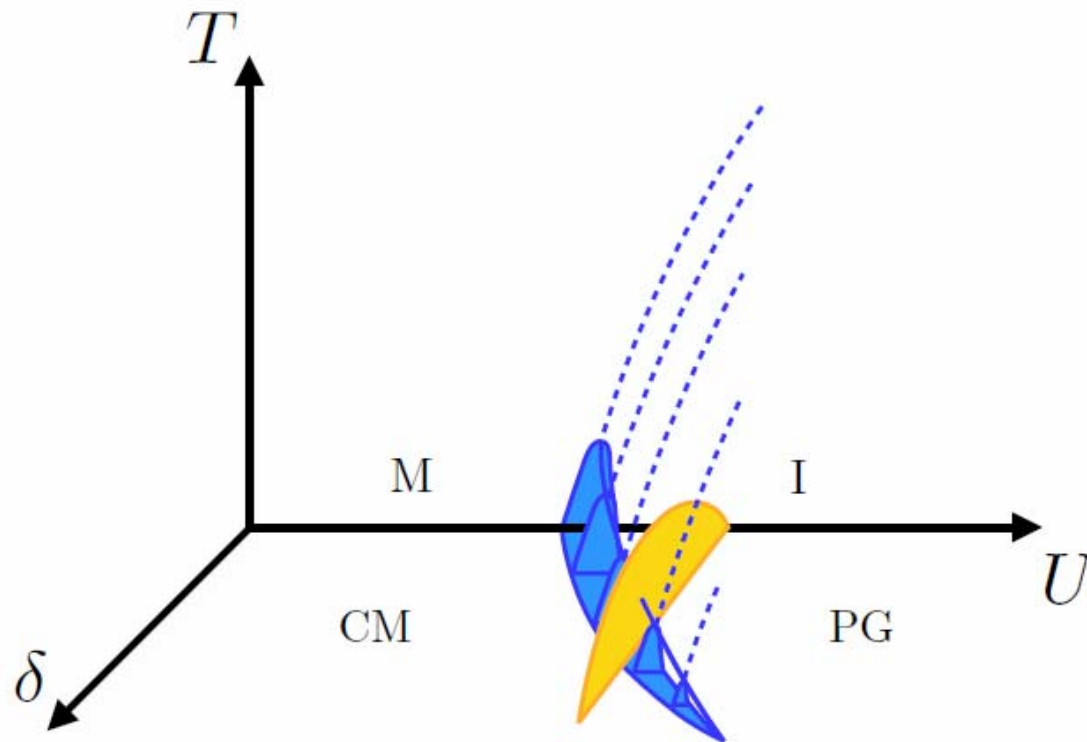
P.W. Anderson *Science*
317, 1705 (2007)

Miyake, Schmitt–Rink, and Varma
P.R. B 34, 6554-6556 (1986)

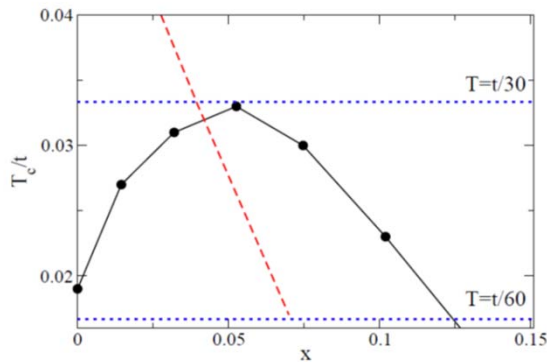
 More sophisticated Slave Boson: Kotliar Liu *PRB* 1988

Superconductivity in Doped Mott insulator

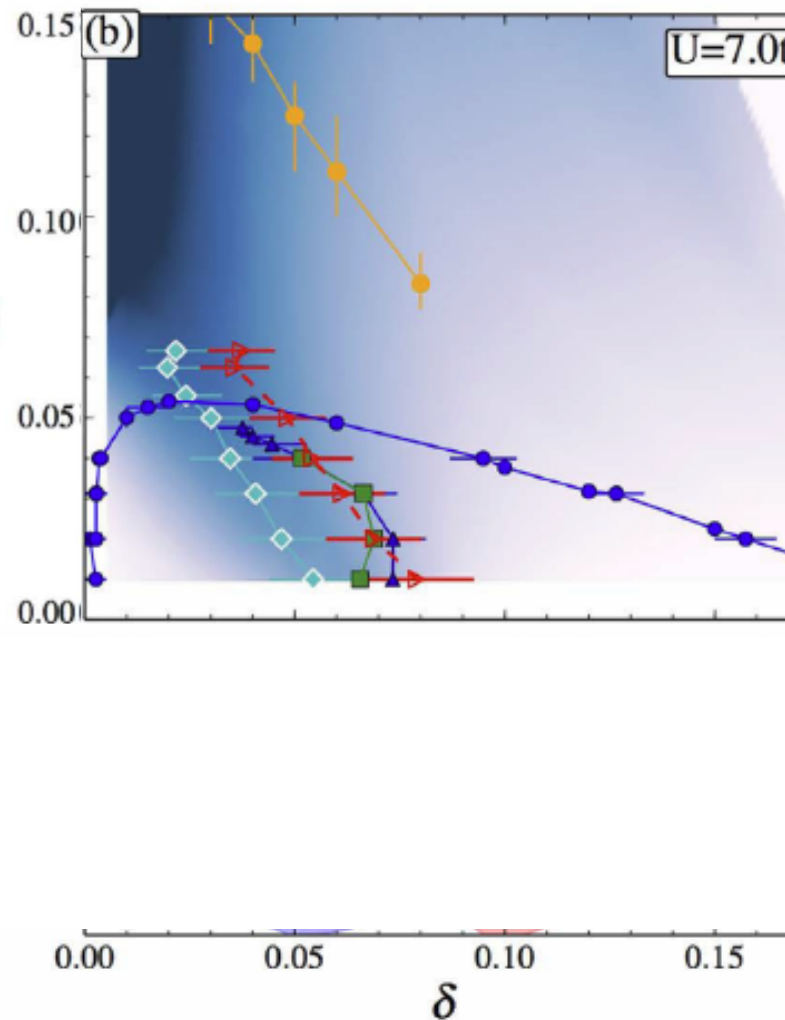
$n = 1, d = 2$ square lattice



An organizing principle



E. Gull and A. J. Millis
Phys. Rev. B 88, 075127

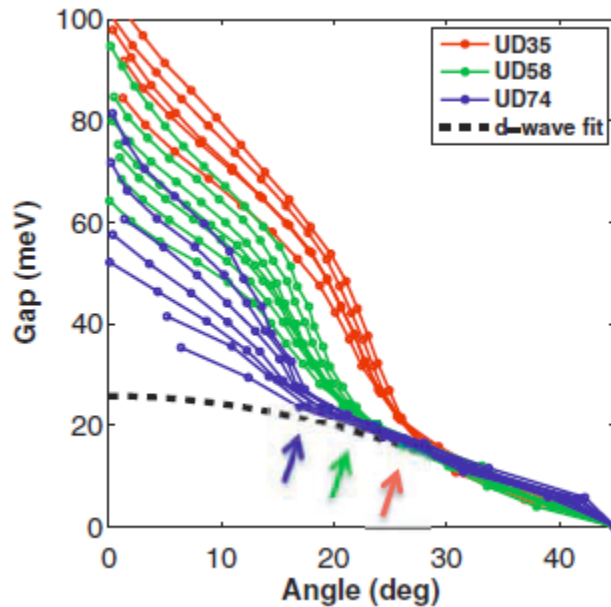


Fratino et al.
Sci. Rep. 6, 22715

Theory, see also
Jarrel PRL
(2004), Gull
Millis PRB
(2014)

Experiments:
Bontemps, Van
der Marel ...

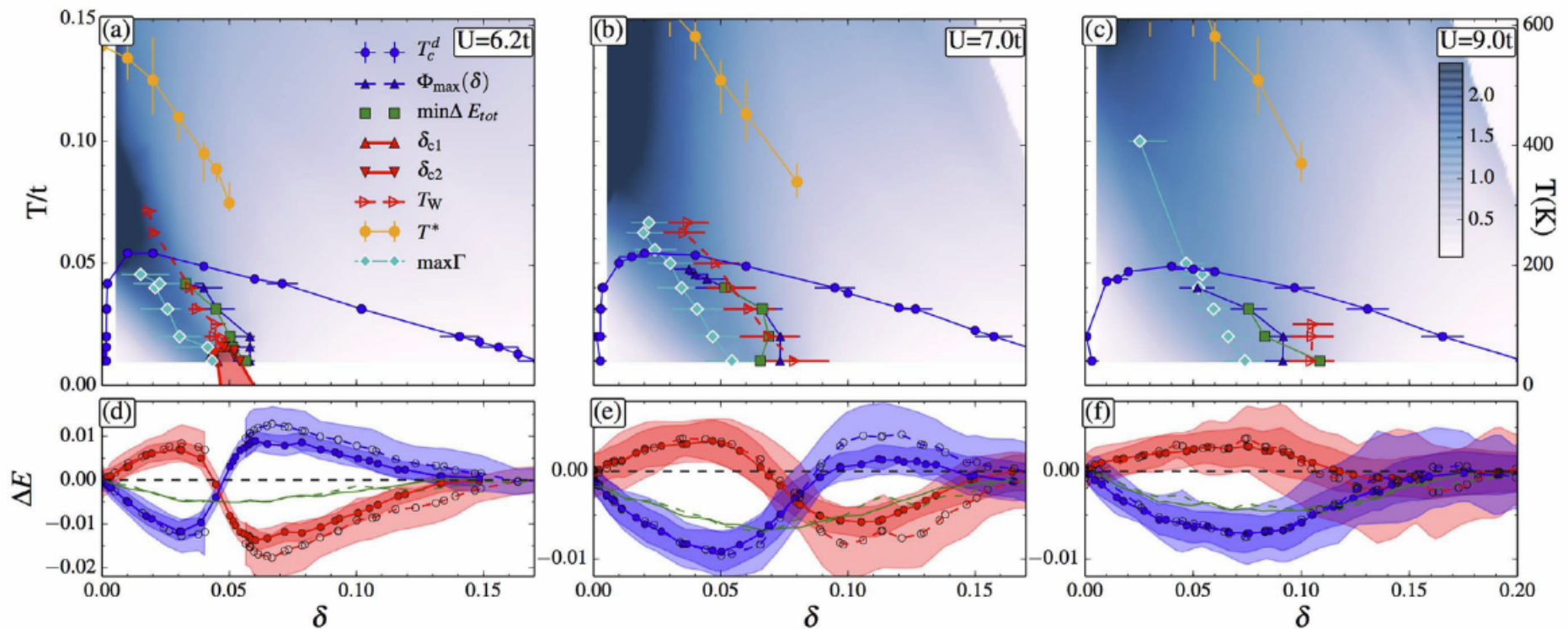
Meaning of T_c^d : Local pair formation



A. Pushp, Parker, ... A. Yazdani,
Science **364**, 1689 (2009)

However, our measurements demonstrate that the nodal gap does not change with reduced doping. The pairing strength does not get weaker or stronger as the Mott insulator is approached; rather, it saturates.

An organizing principle



Fratino et al.
Sci. Rep. 6, 22715



Giovanni Sordi



Lorenzo Fratino



Patrick Sémon

h-doped as charge-transfer insulator

3 bands, charge transfer insulator

Fratino et al. PRB **93**, 245147 (2016)

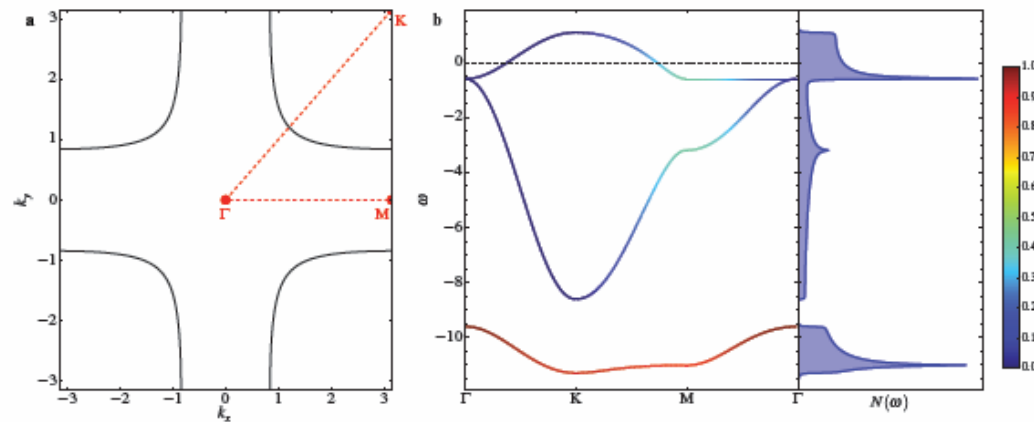
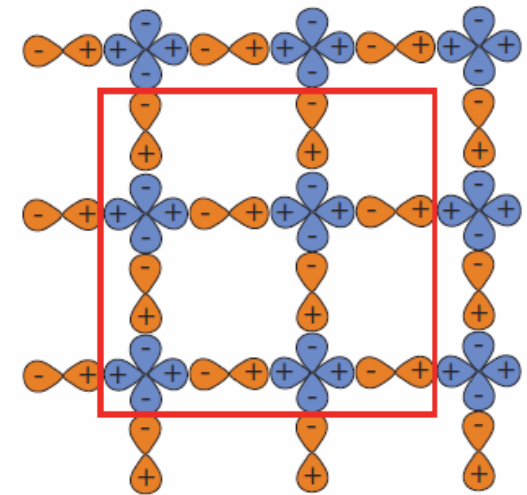
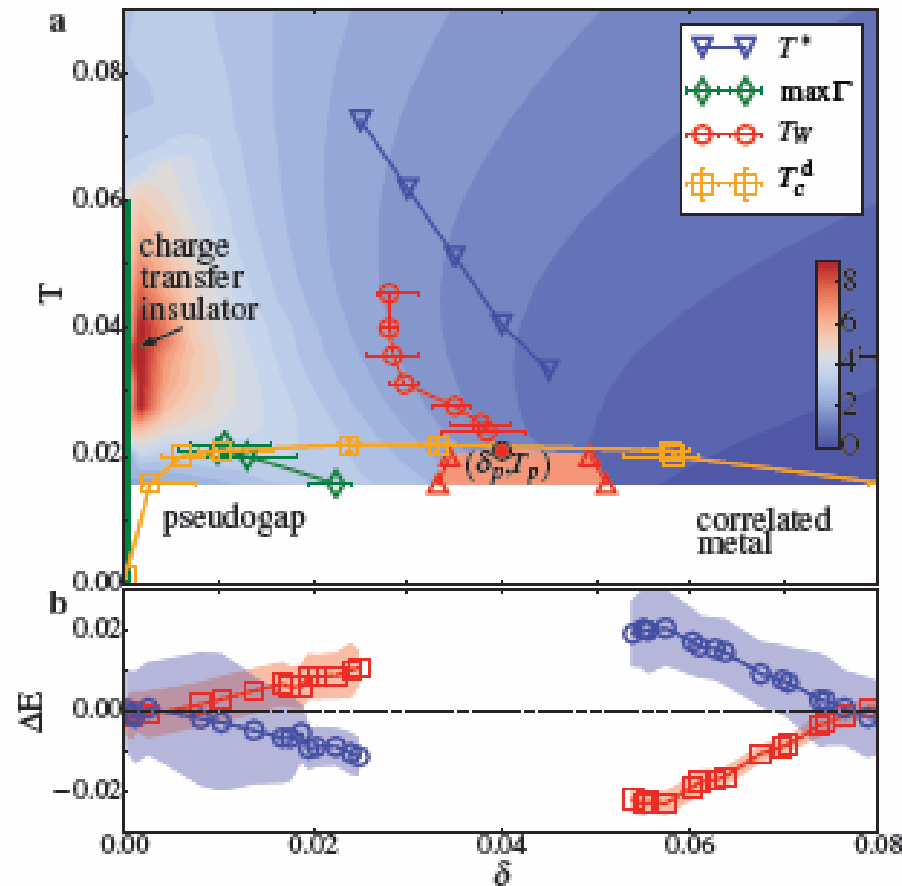


FIG. 2. (a) Noninteracting Fermi surface for the model parameter investigated in Fig. 1a of main text, namely $\epsilon_p = 9$, $t_{pp} = 1$, $t_{pd} = 1.5$, which gives a total occupation n_{tot} equal to five. (b) Non-interacting band structure for the same model parameter along with the resulting total density of states. Color corresponds to the d-character of the hybridised bands. The band crossing the Fermi level has mostly oxygen character.



No interaction on oxygen

3 bands, charge transfer insulator



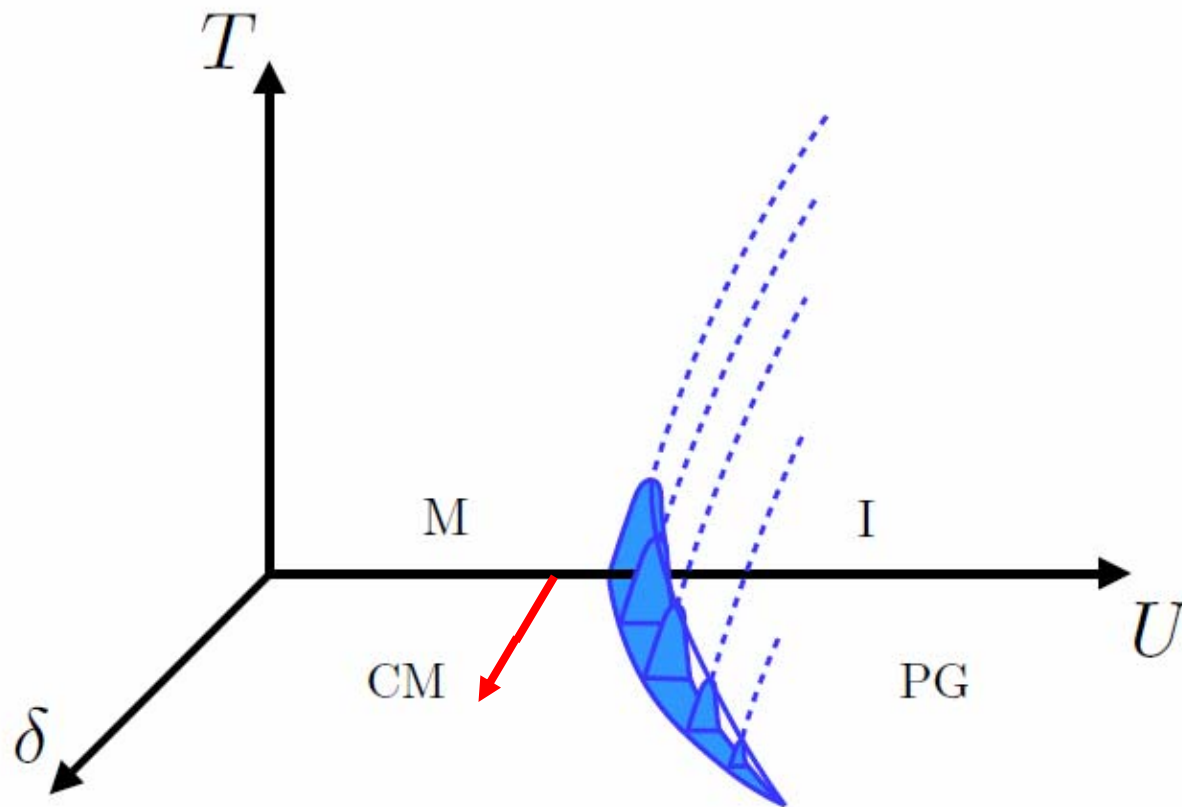
Fratino et al. PRB 93, 245147 (2016)

Other materials

e-doped cuprates

Influence of Mott transition away from half-filling

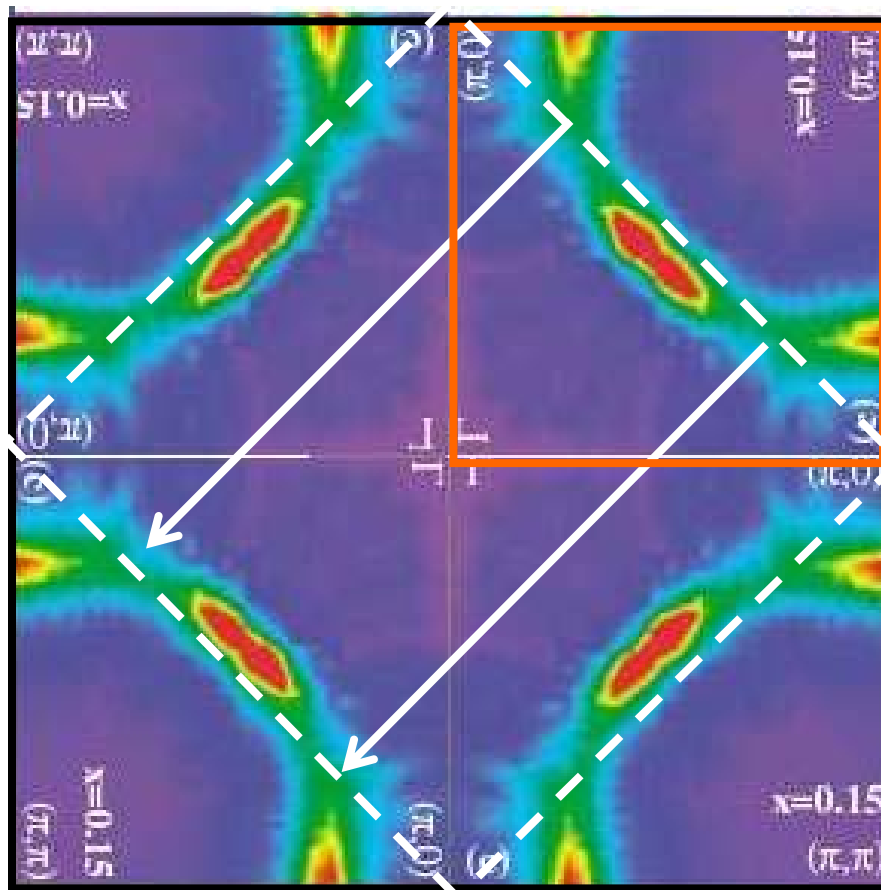
$n = 1, d = 2$ square lattice



Hot spots from AFM quasi-static scattering

Mermin-Wagner

$d = 2$

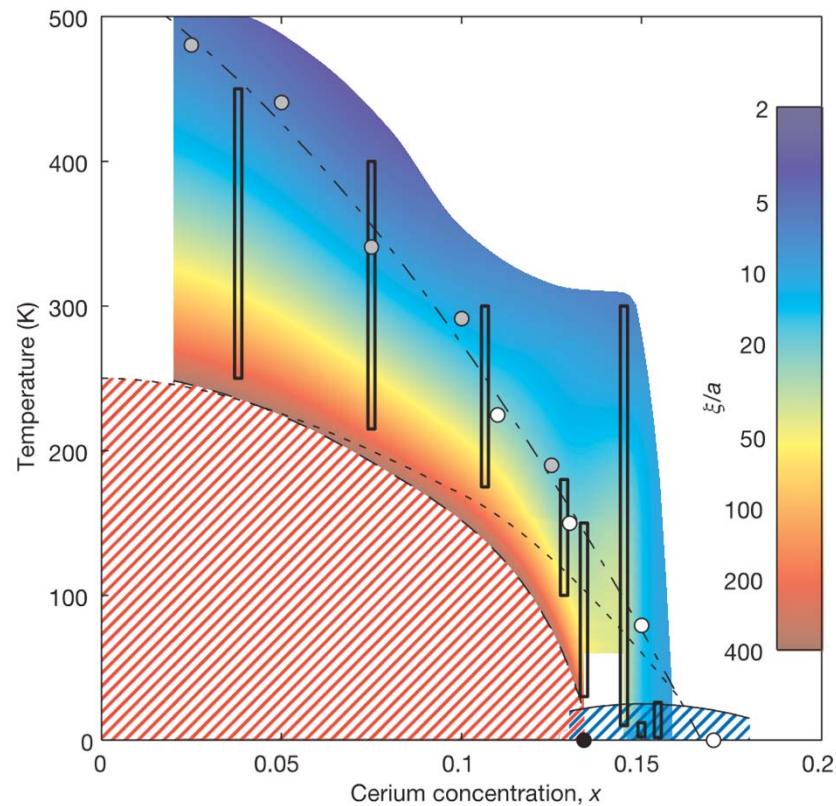


Vilk, A.-M.S.T (1997)
Kyung, Hankevych,
A.-M.S.T., PRL, 2004

Armitage et al. PRL 2001

e-doped pseudogap

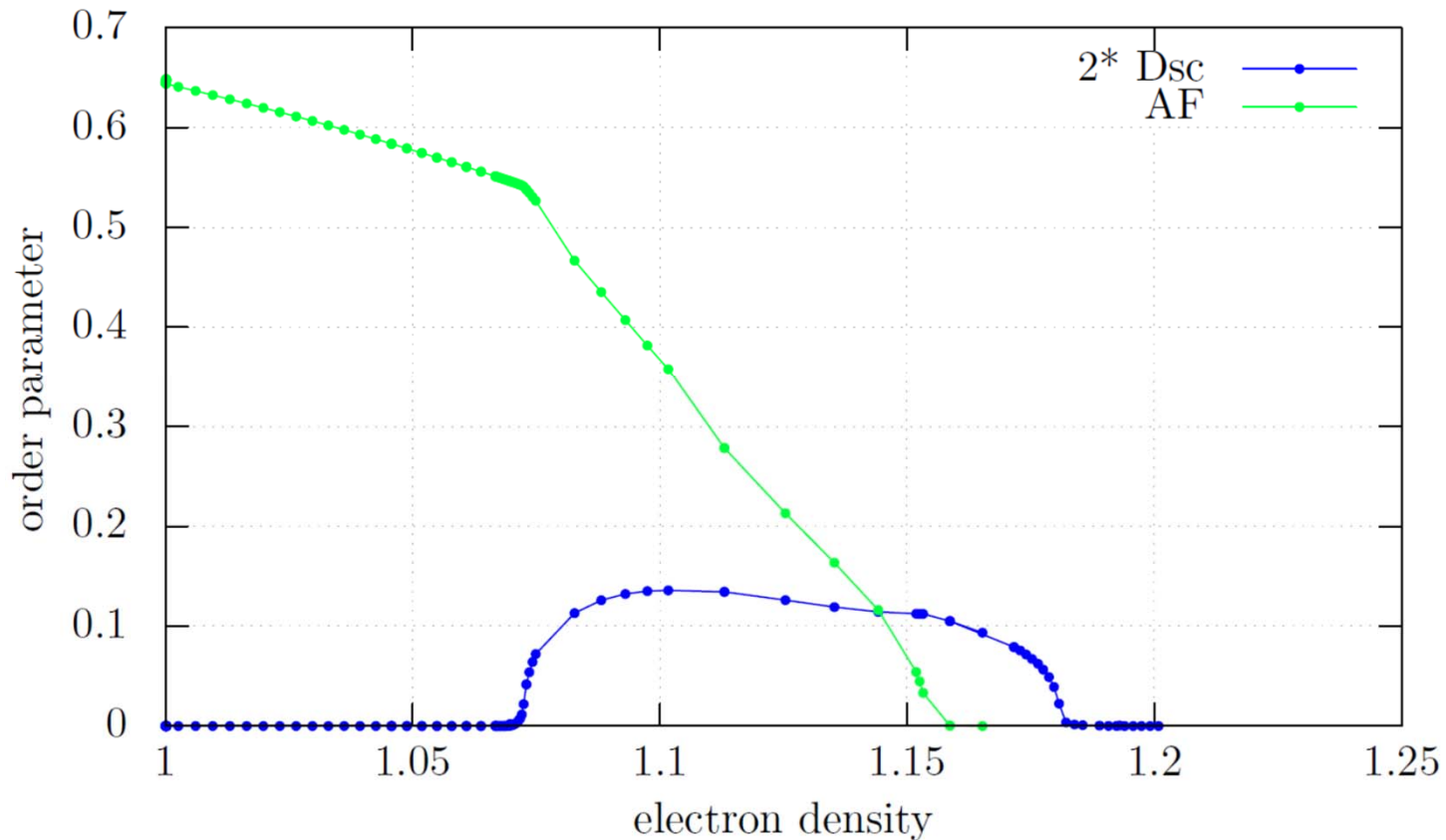
E. M. Motoyama et al.. Nature 445, 186–189 (2007).



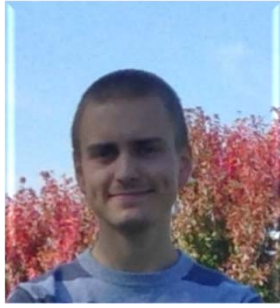
Vilk criterion $\xi^* = 2.6(2) \xi_{th}$

e-doped: Ground State AFM-QCP and dSC

Competing phase diagram for the electron-doped Hubbard model with NCCO and $U=5$ parameters



A. Foley and D. Sénéchal, unpublished



Charles-David Hébert



Patrick Sémon

Organics : Phase diagram, finite T

Made possible by algorithmic improvements

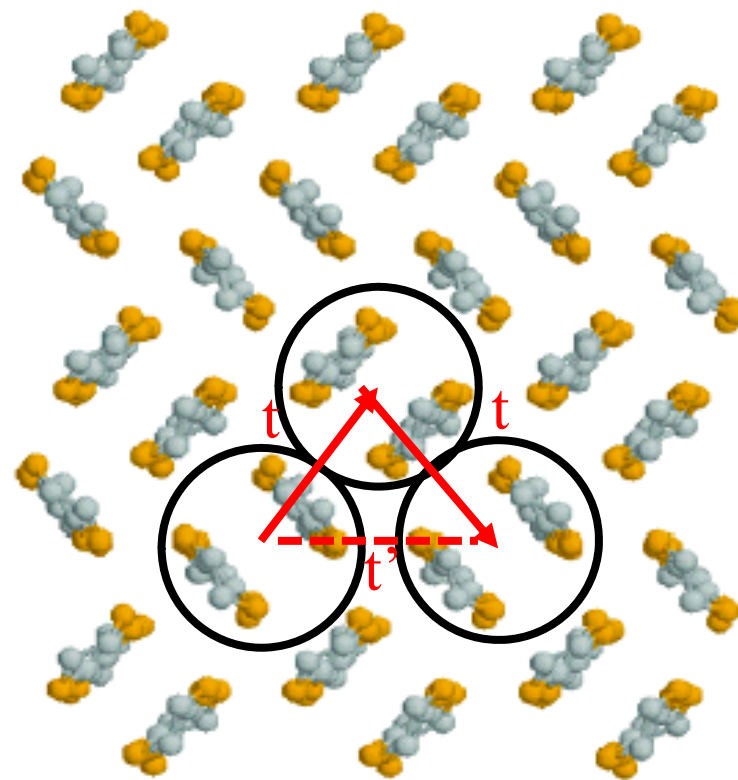
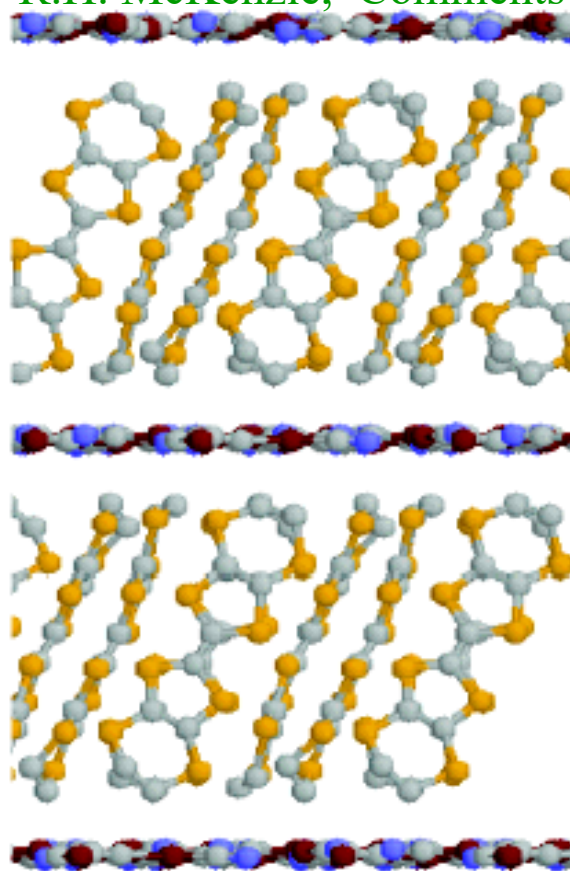
P. Sémon *et al.*
PRB **85**, 201101(R) (2012)
PRB **90** 075149 (2014);
and PRB **89**, 165113 (2014)

Layered organics (κ -BEDT-X family)

H. Kino + H. Fukuyama, J. Phys. Soc. Jpn **65** 2158 (1996),
R.H. McKenzie, Comments Condens Mat Phys. **18**, 309 (1998)

BEDT-TTF
layer

Anion layer



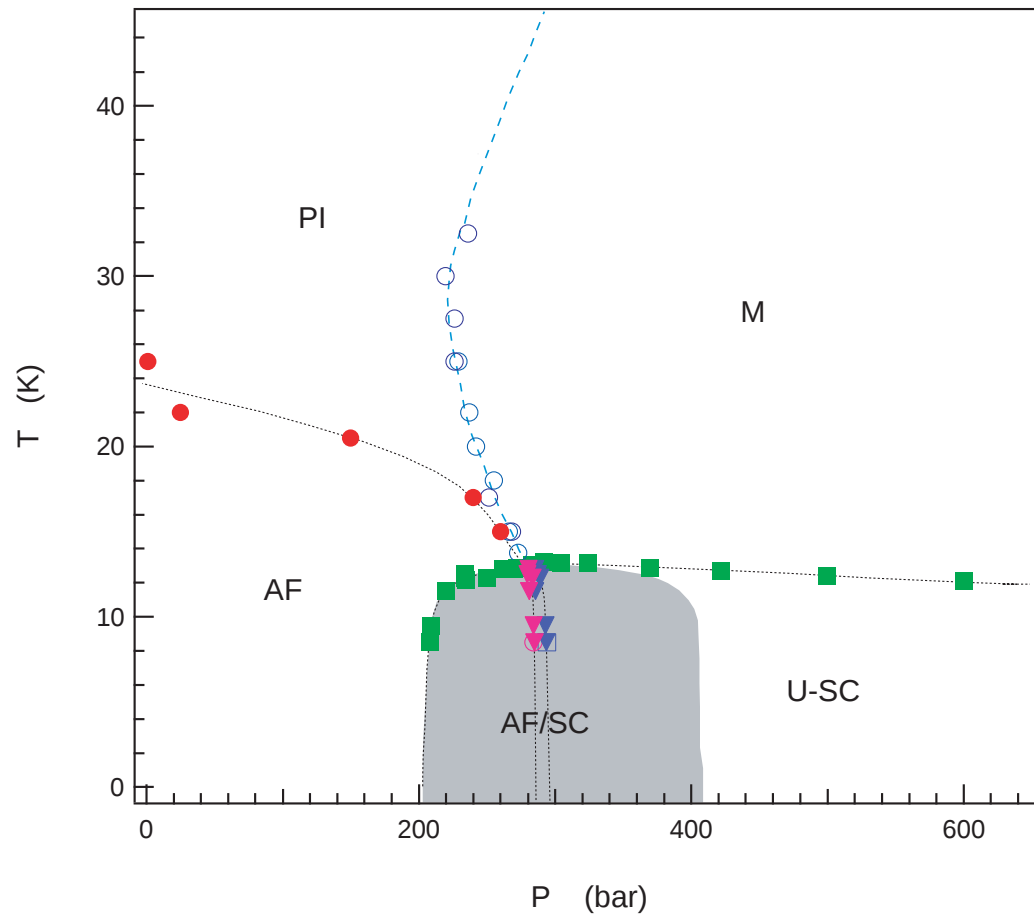
$$t \approx 50 \text{ meV}$$

$$\Rightarrow U \approx 400 \text{ meV}$$

$$t'/t \sim 0.6 - 1.1$$

Y. Shimizu, et al. Phys. Rev. Lett. **91**,
107001(2003)

Phase diagram for organics

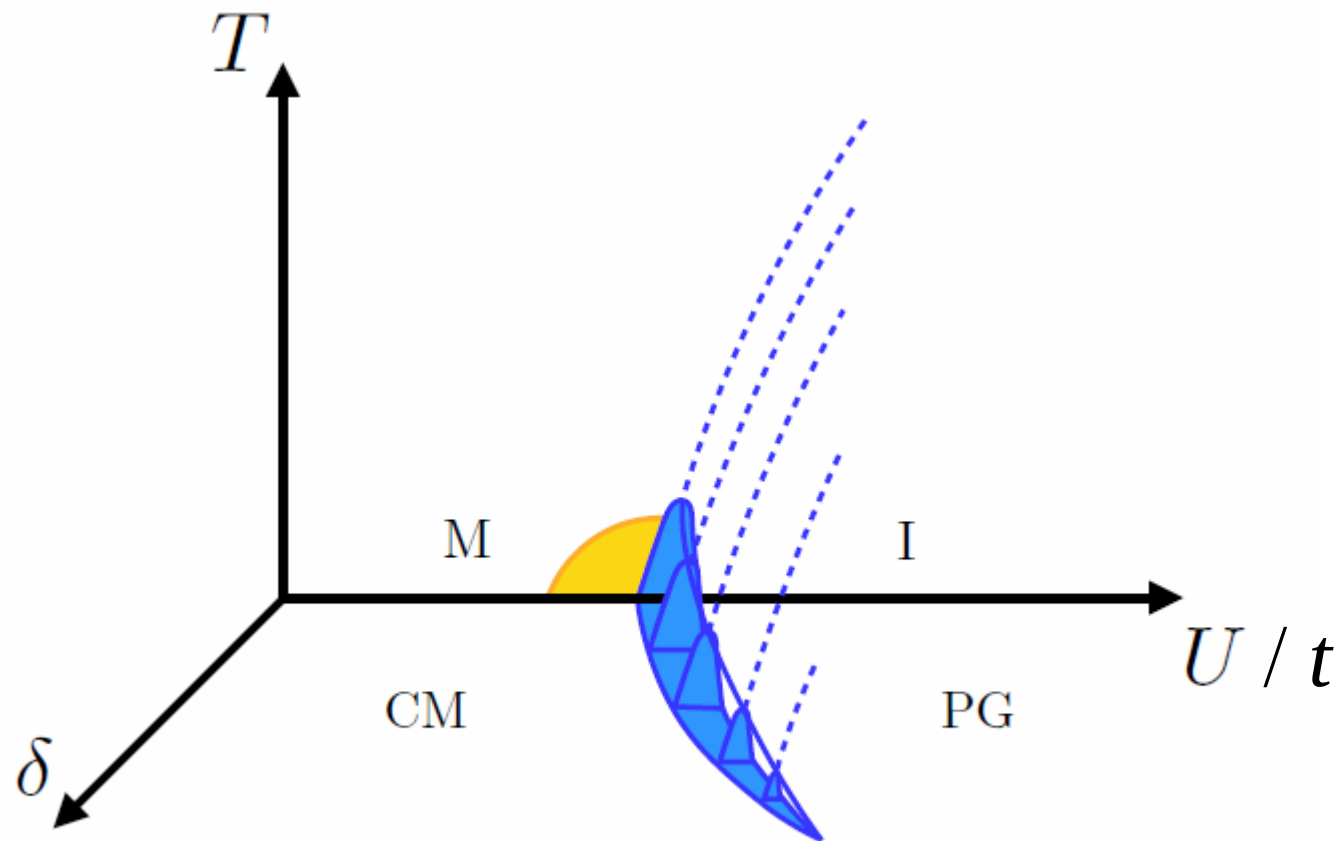


Phase diagram ($X=\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$)

S. Lefebvre et al. PRL **85**, 5420 (2000), P. Limelette, et al. PRL **91** (2003)

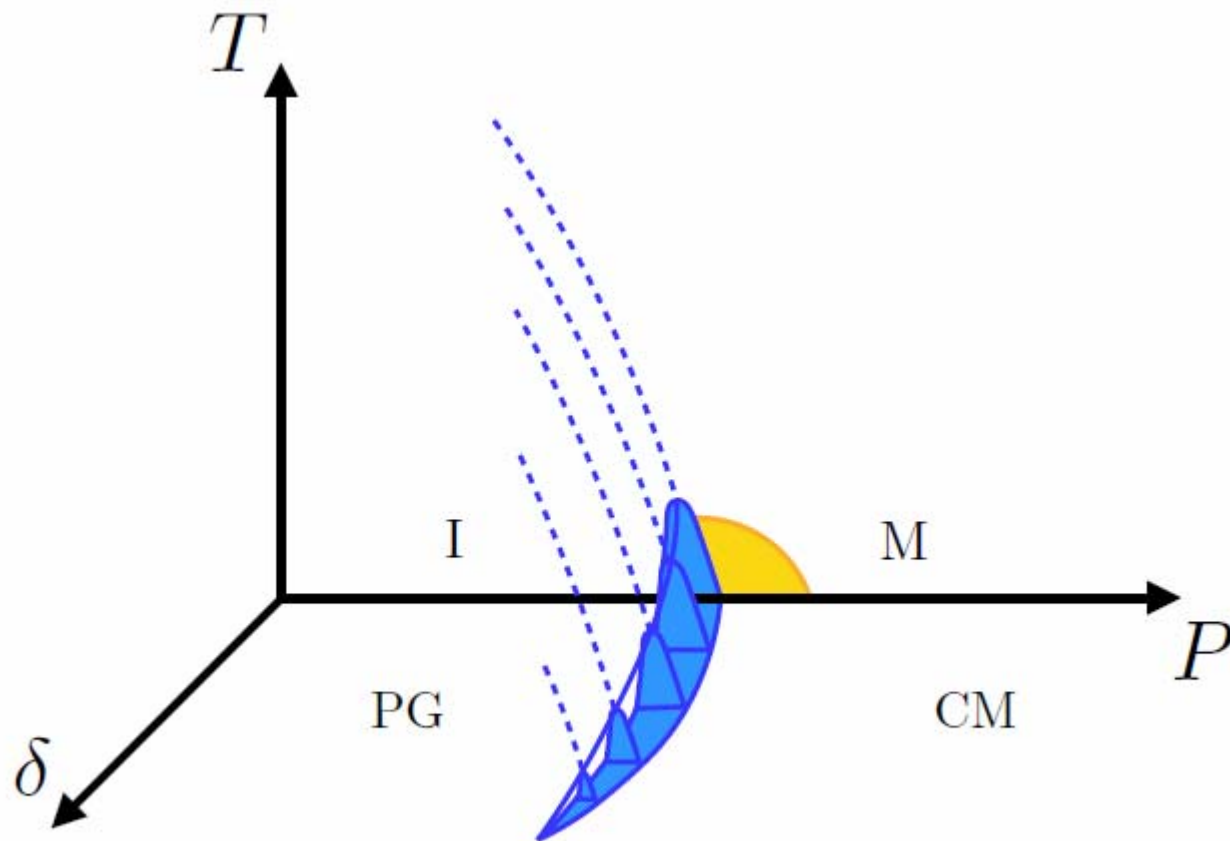
Superconductivity near the Mott transition

$n = 1, d = 2$ square lattice

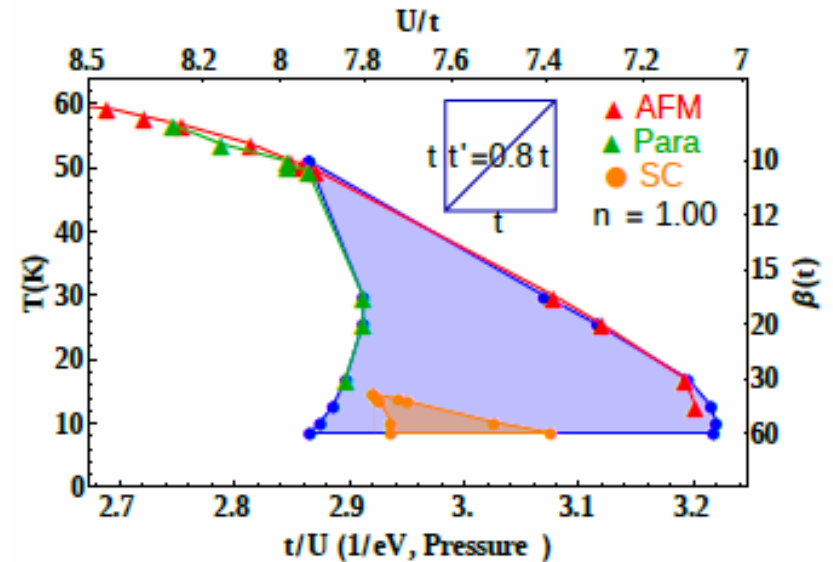
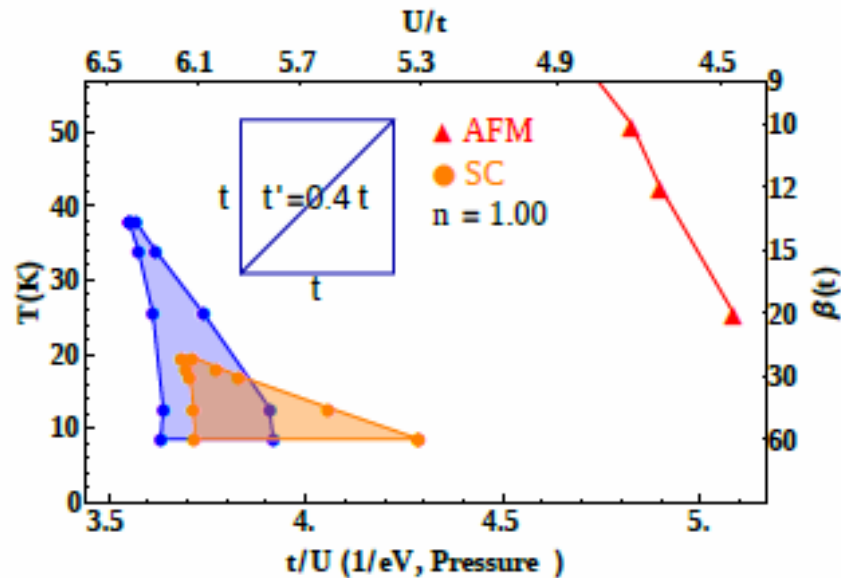


Superconductivity near the Mott transition

$n = 1, d = 2$ square lattice



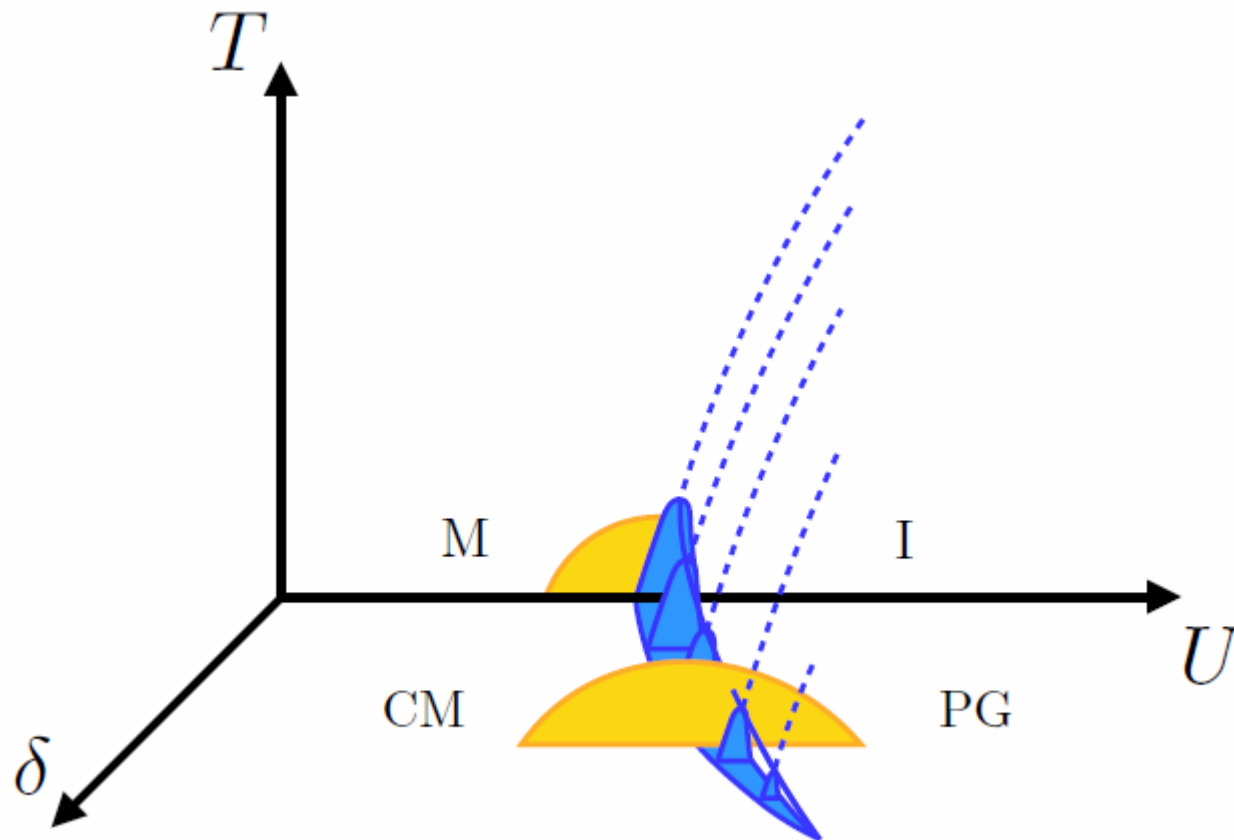
Superconductivity near Mott transition ($n = 1$)



Doped Organics

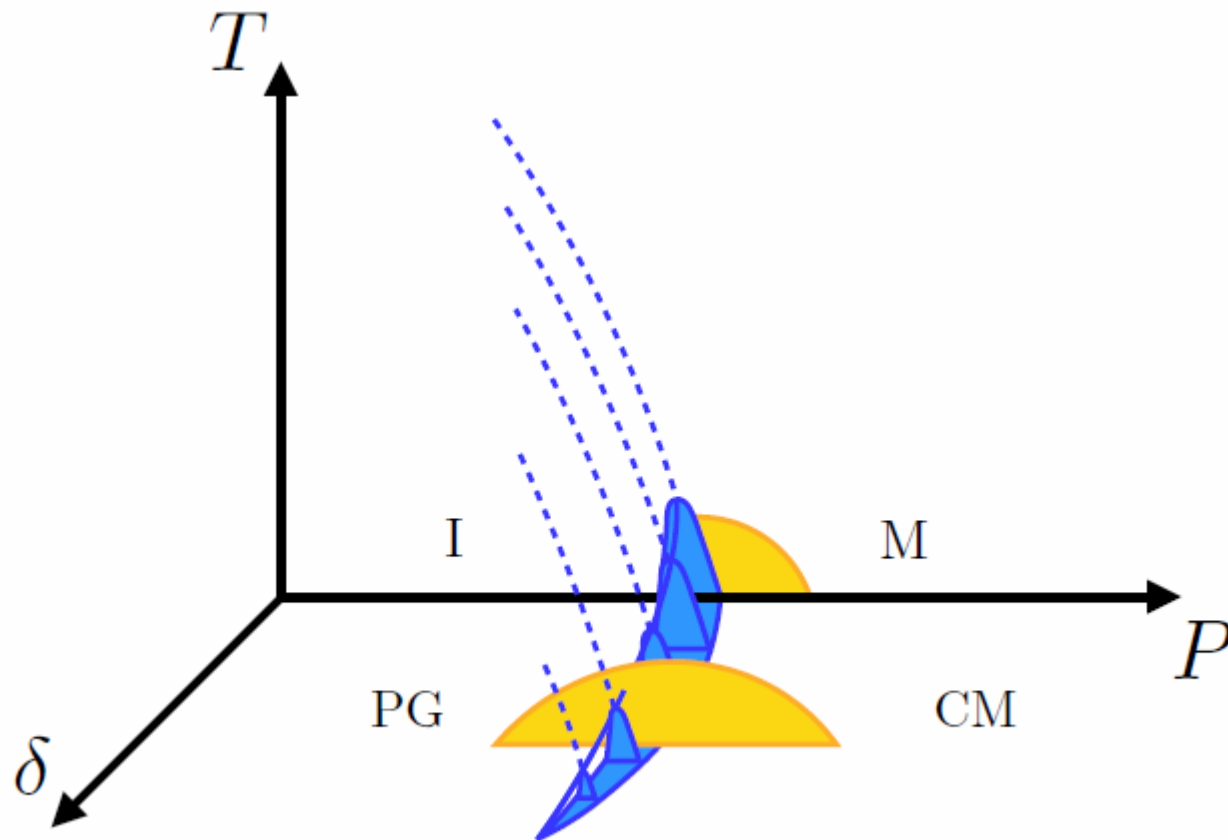
Doped organics

$n = 1, d = 2$ square lattice

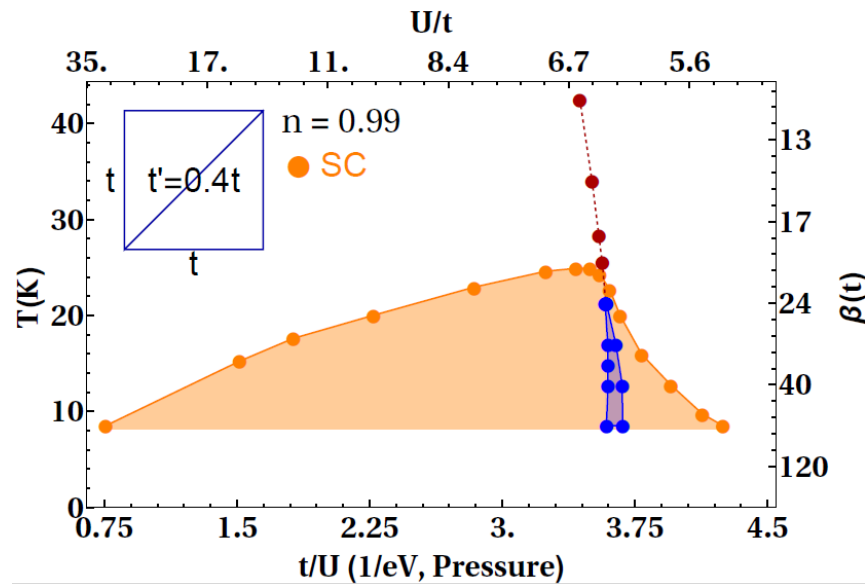


Doped organics

$n = 1, d = 2$ square lattice



First order and Widom line in organics

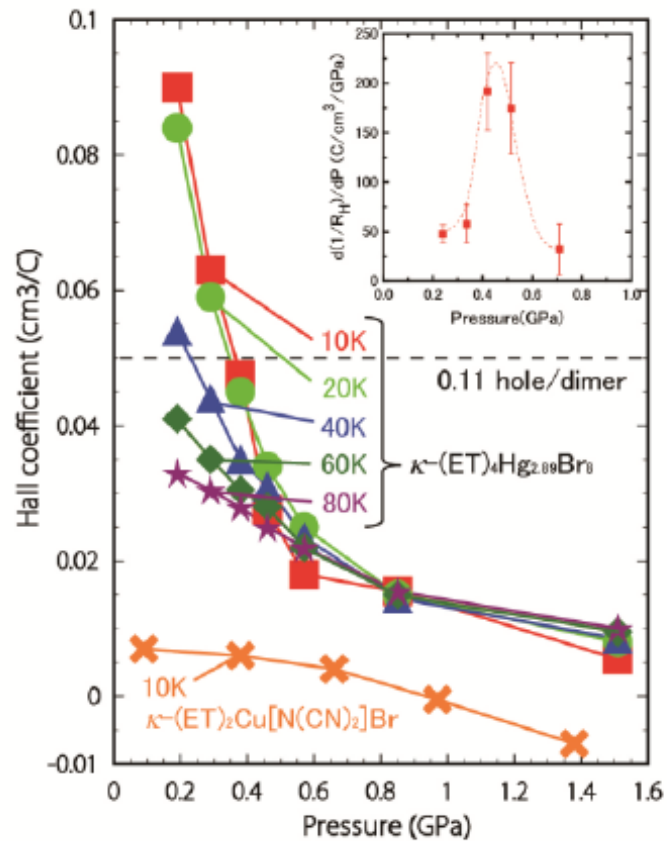


Compare: T. Watanabe, H. Yokoyama
and M. Ogata
JPS Conf. Proc.
3, 013004 (2014)

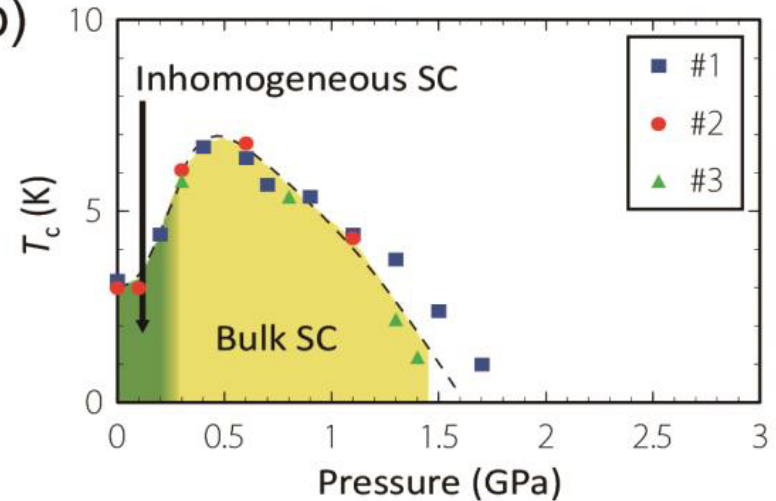
C.D. Hébert, P. Sémon, A.-M.S. T PRB **92**, 195112 (2015)

Doped BEDT

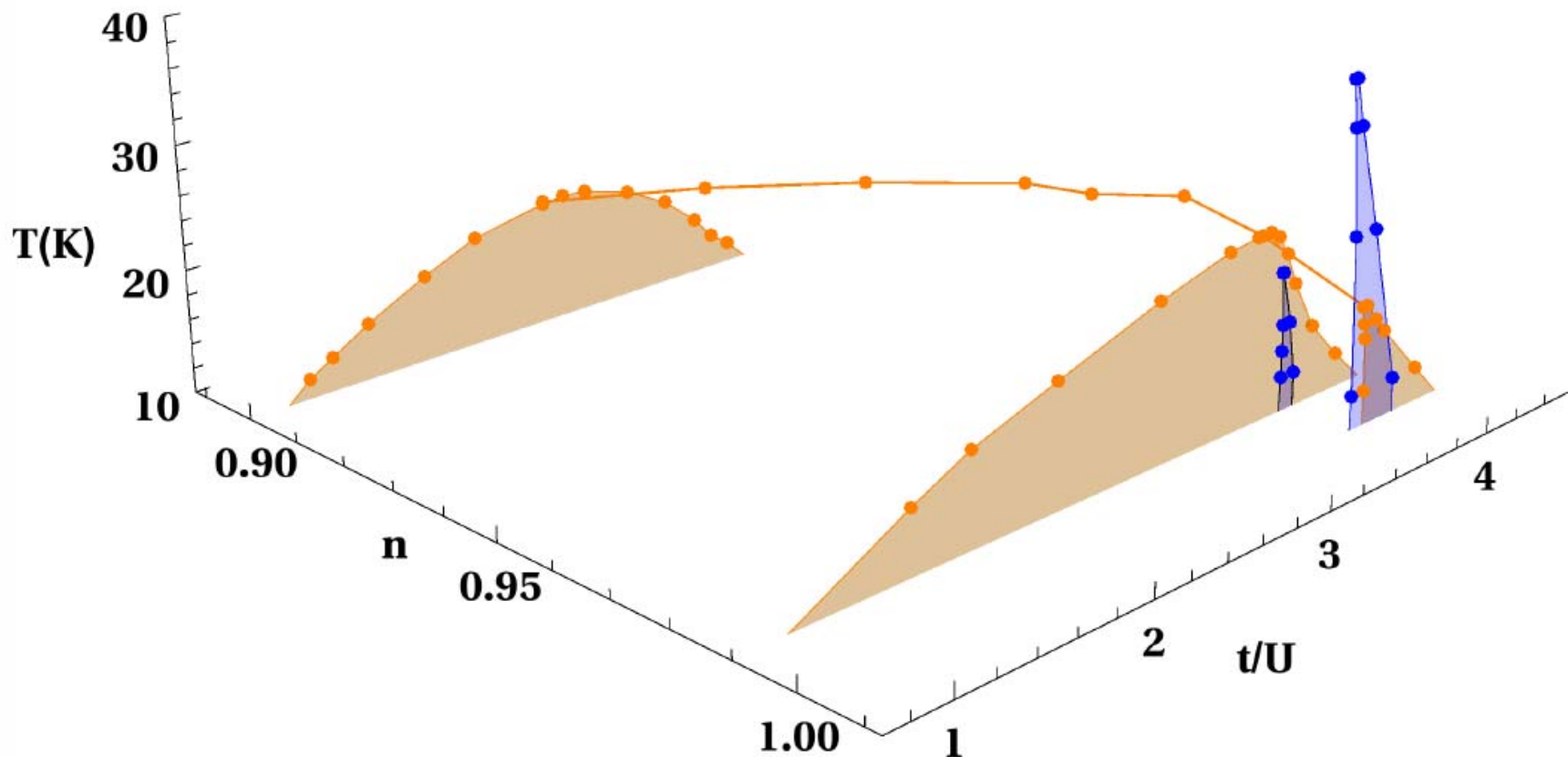
(b)



(b)

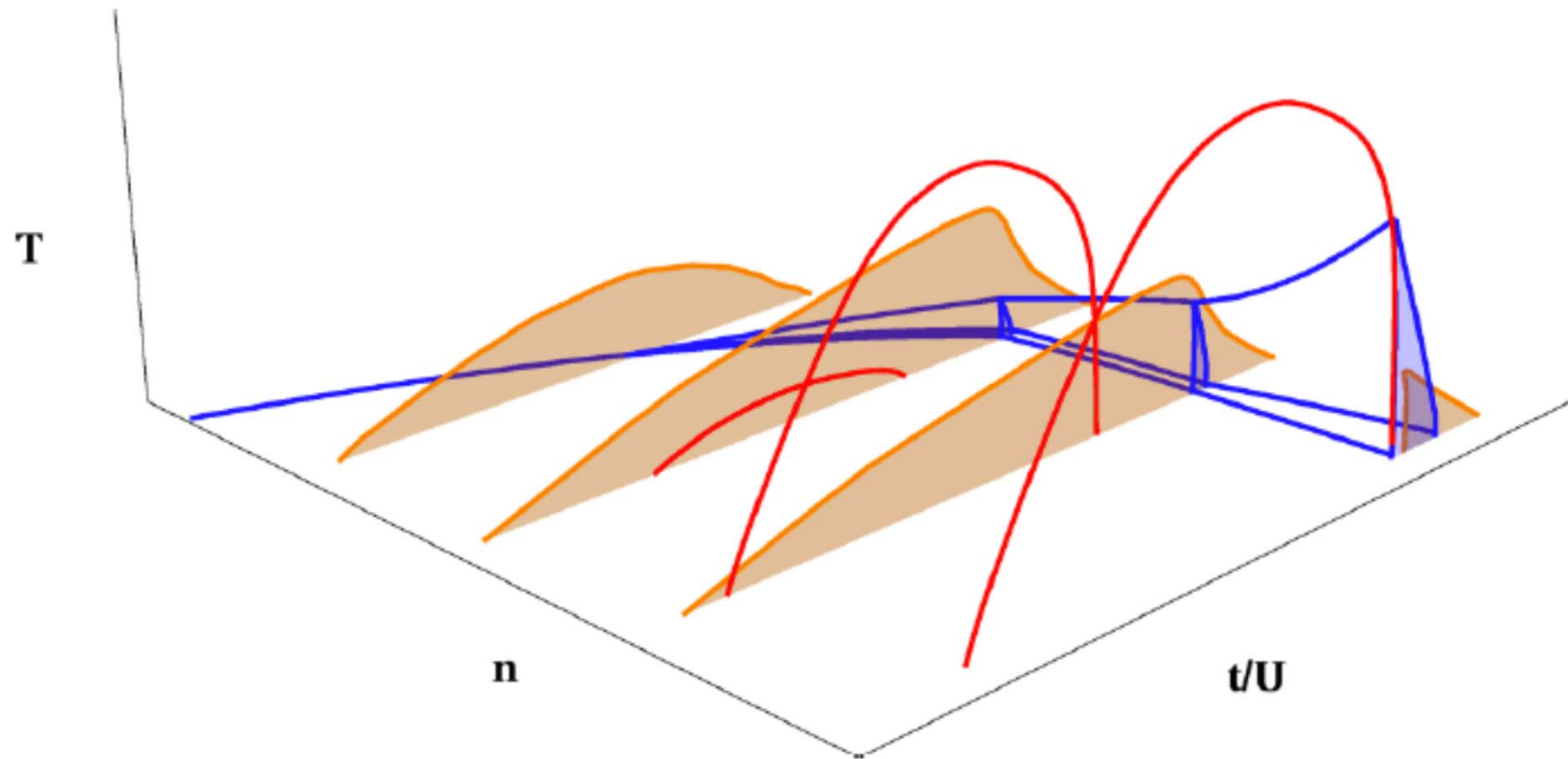


$t' = 0.4t$ overview



Compare: T. Watanabe, H. Yokoyama and M. Ogata
JPS Conf. Proc. **3**, 013004 (2014)

Generic case highly frustrated case

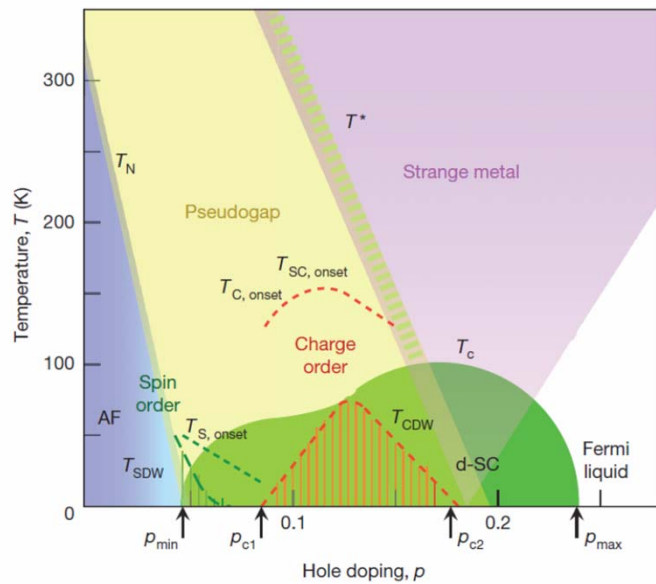


Conclusion

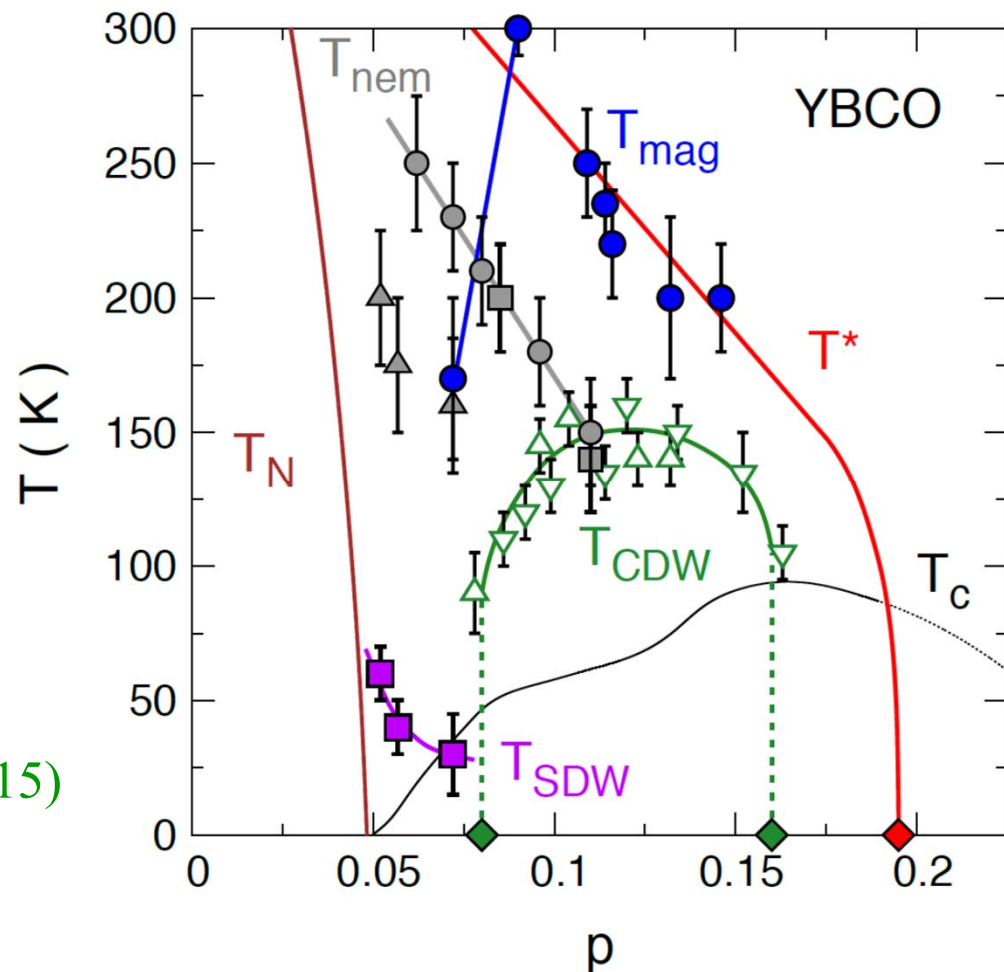
- Even within a single phase, there can be qualitative differences between the strong and weak correlation limits
- The underlying normal state organizes the ordered state
- Mott + J suffices for many qualitative properties of both pseudogap and d-SC
- A quantum critical point is not a necessary condition to have a dome shape

Prologue

Phase diagram $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$

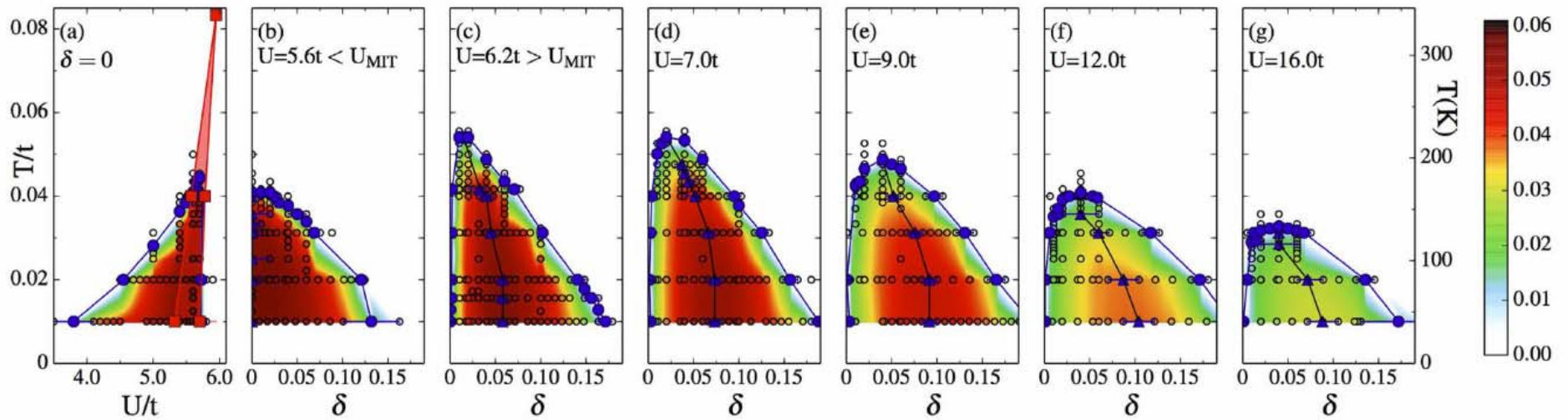


Keimer et al., Nature 518, 179 (2015)



Cyr-Choinières... Taillefer (unpublished)

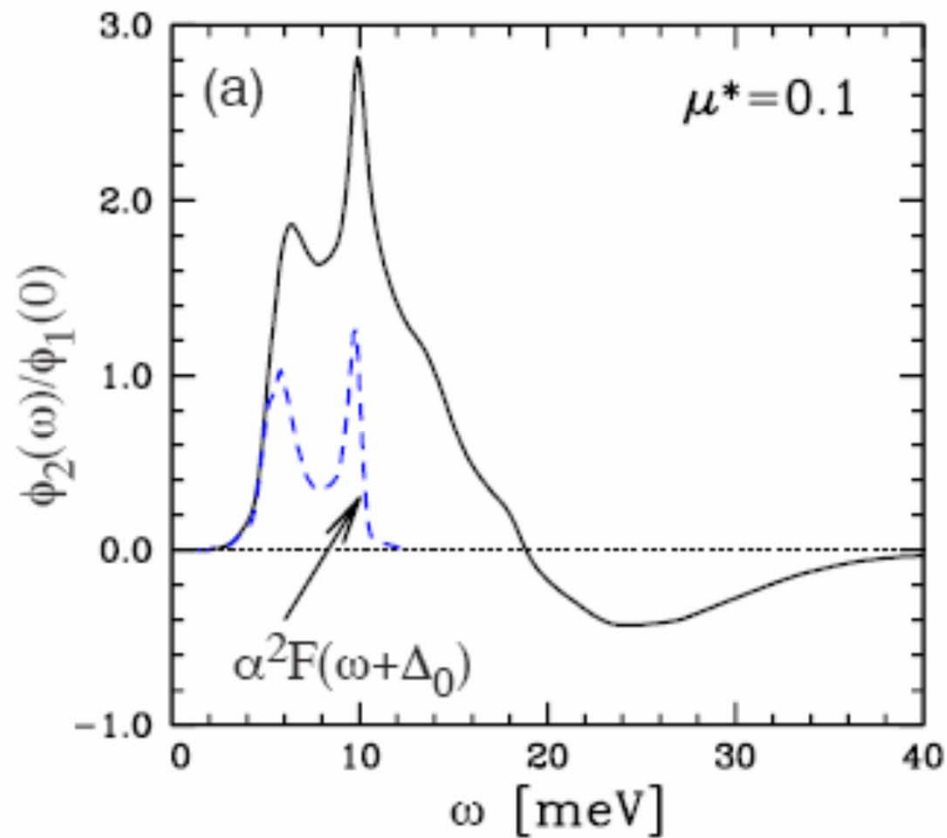
Pairing mechanism



Fratino et al.
Sci. Rep. 6, 22715

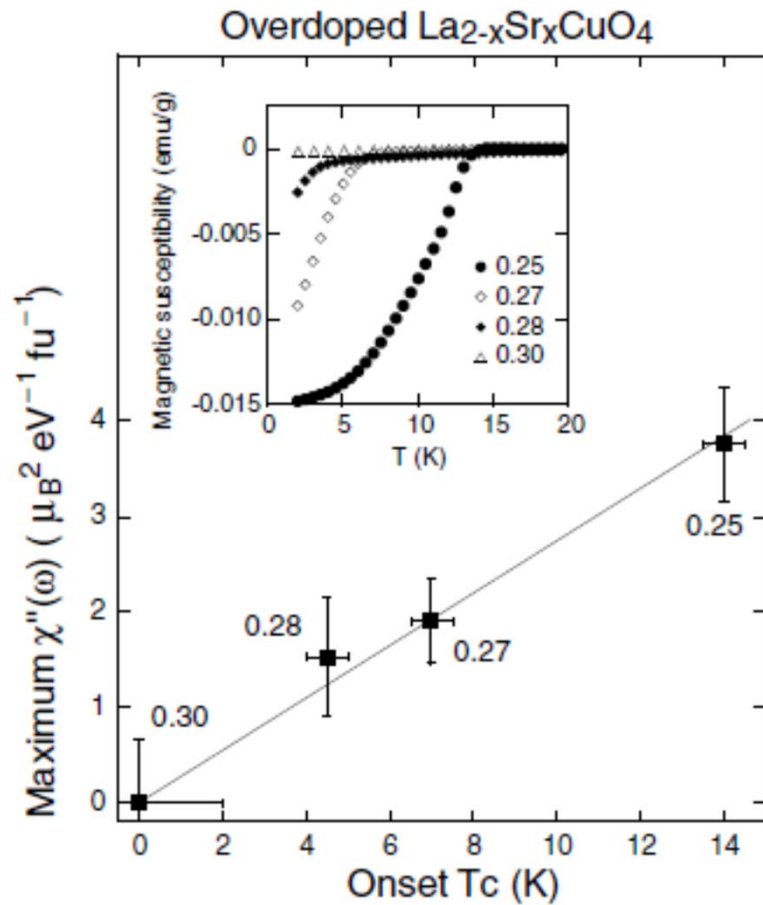
$\text{Im } \Sigma_{\text{an}}$ and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

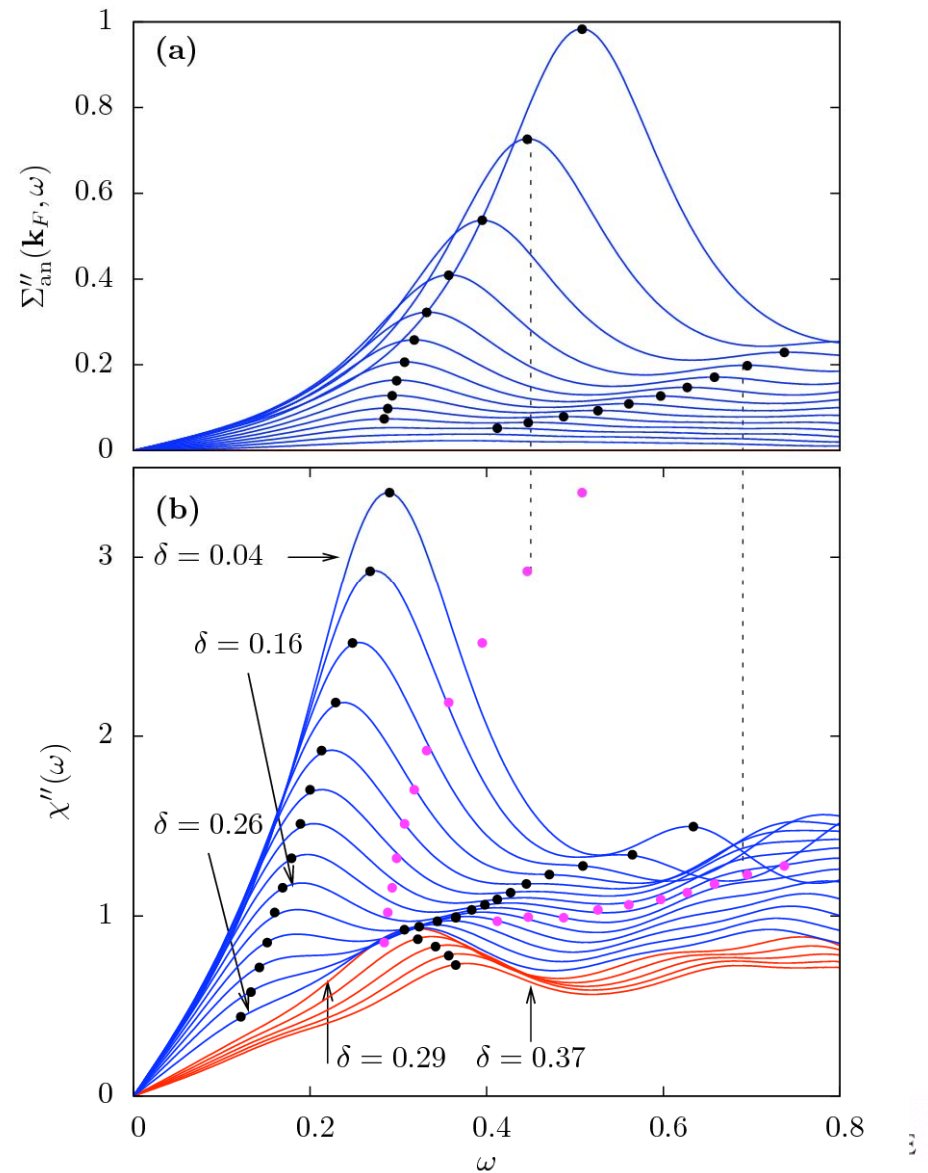


The glue

Kyung, S  n  chal, Tremblay, Phys. Rev. B
80, 205109 (2009)



Wakimoto ... Birgeneau
PRL (2004)



The glue and neutrons

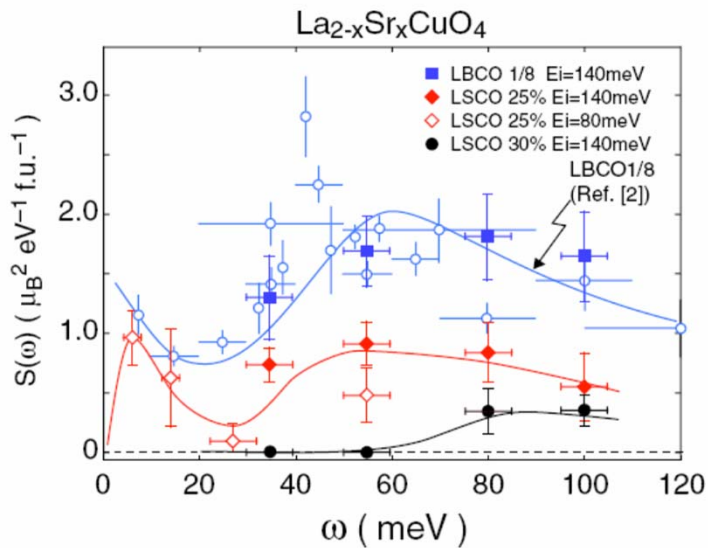
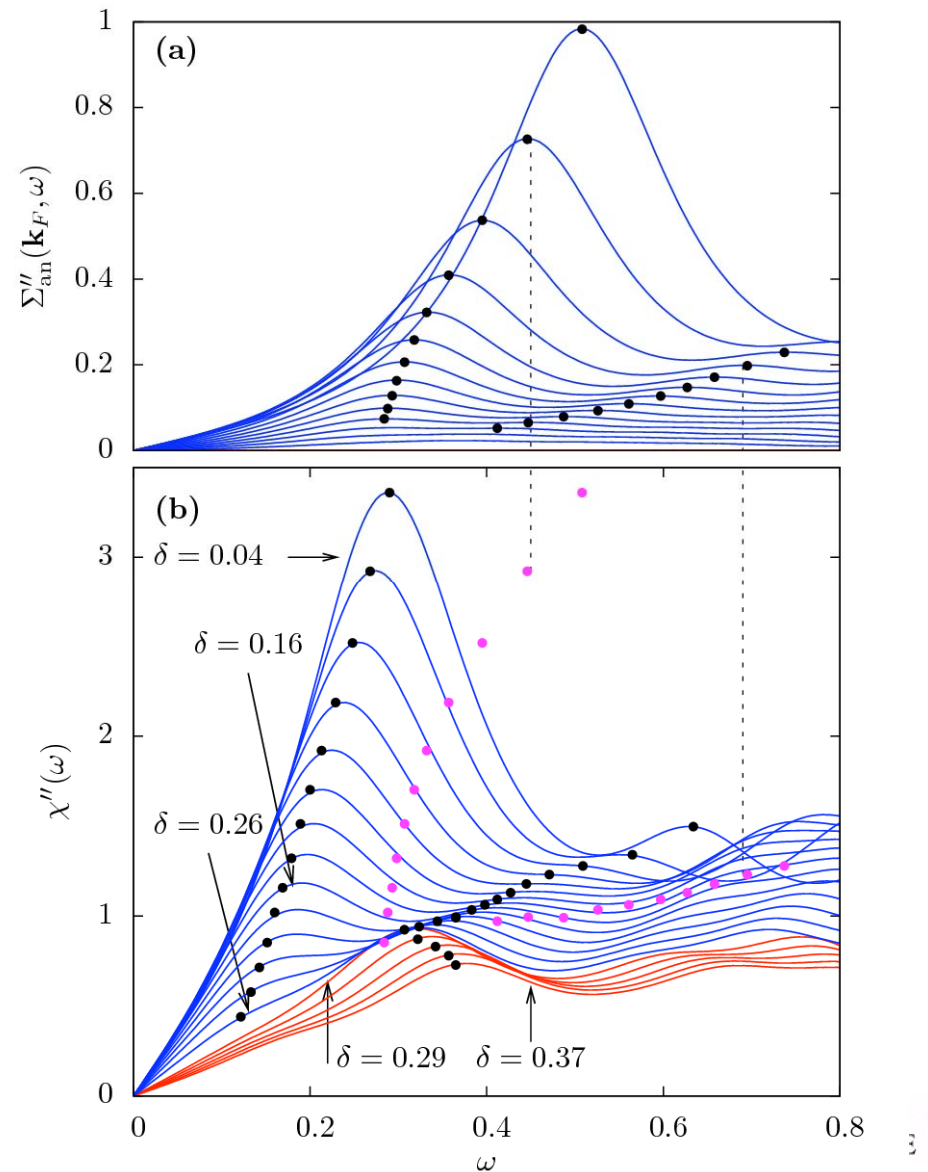
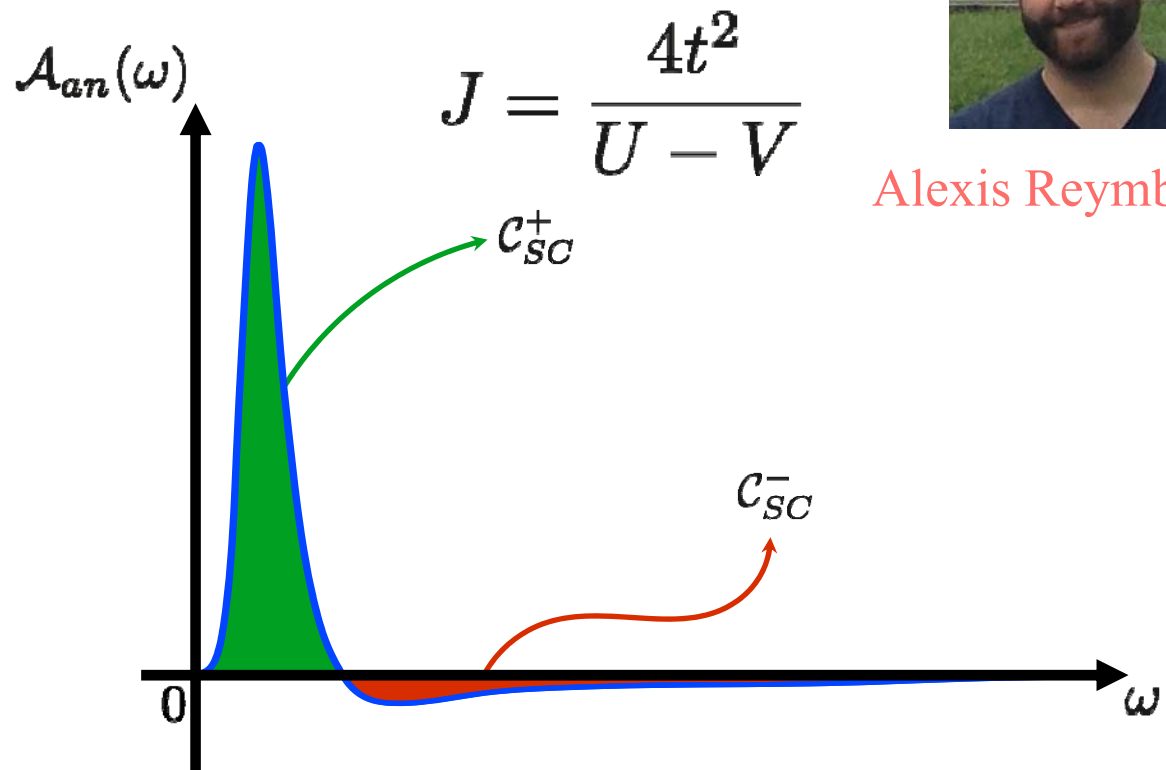
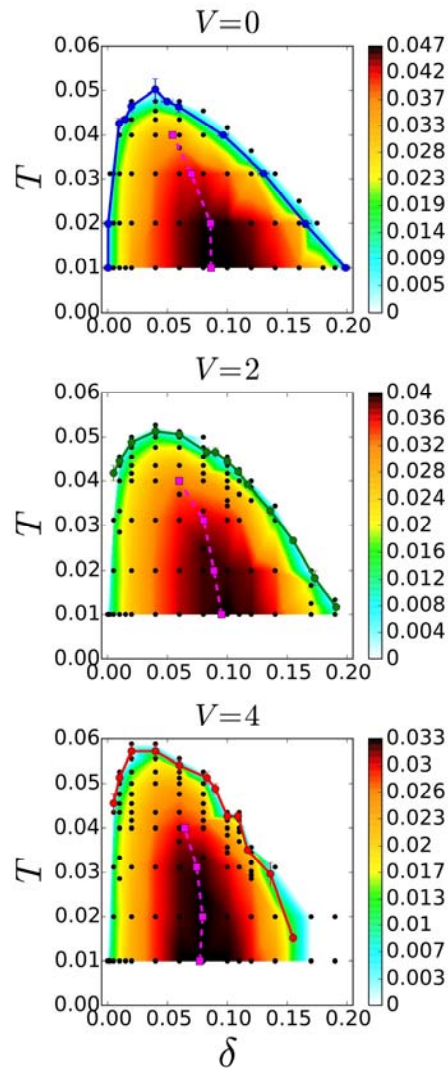


FIG. 3 (color online). $\mathbf{Q}$ -integrated dynamic structure factor $S(\omega)$ which is derived from the wide- H integrated profiles for LBCO 1/8 (squares), LSCO $x = 0.25$ (diamonds; filled for $E_i = 140 \text{ meV}$, open for $E_i = 80 \text{ meV}$), and $x = 0.30$ (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO $x = 0.25$ and 0.30 are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007);
PRL (2004)



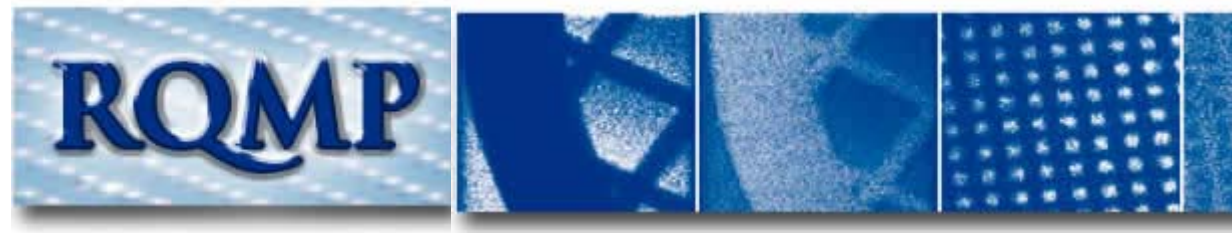
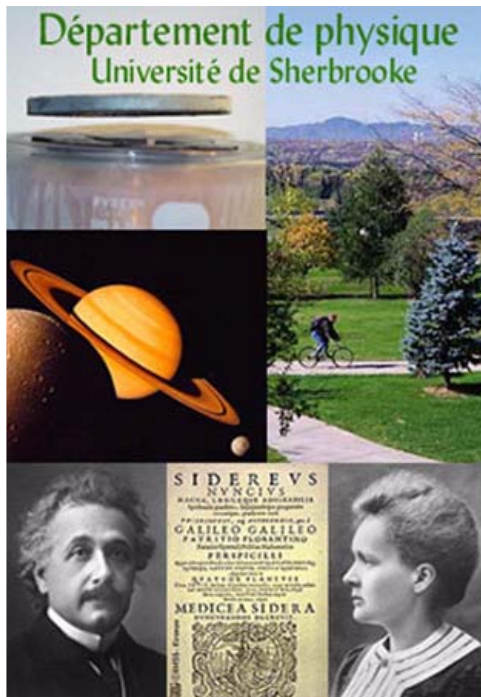
Antagonistic effects of V at finite T



Alexis Reymbaut

A. Reymbaut *et al.* PRB 94 155146 (2016)

André-Marie Tremblay



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Review: A.-M.S.T. [arXiv: 1310.1481](https://arxiv.org/abs/1310.1481)



A.-M.S. Tremblay

“Strongly correlated superconductivity”

Chapt. 10 : *Emergent Phenomena in Correlated Matter Modeling and Simulation*, Vol. 3, E. Pavarini, E. Koch, and U. Schollwöck (eds.)

Verlag des Forschungszentrum Jülich, 2013