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technologies  
Québec



# Superconductivity in Doped Mott Insulators: A Dynamical Mean Field Perspective

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S. Bergeron, Maxime Charlebois, L. Fratino,  
A. Foley, Charles-David Hébert, A. Reymbaut, D. Sénéchal  
O. Simard, G. Sordi, Patrick Sémon, M. Thénault

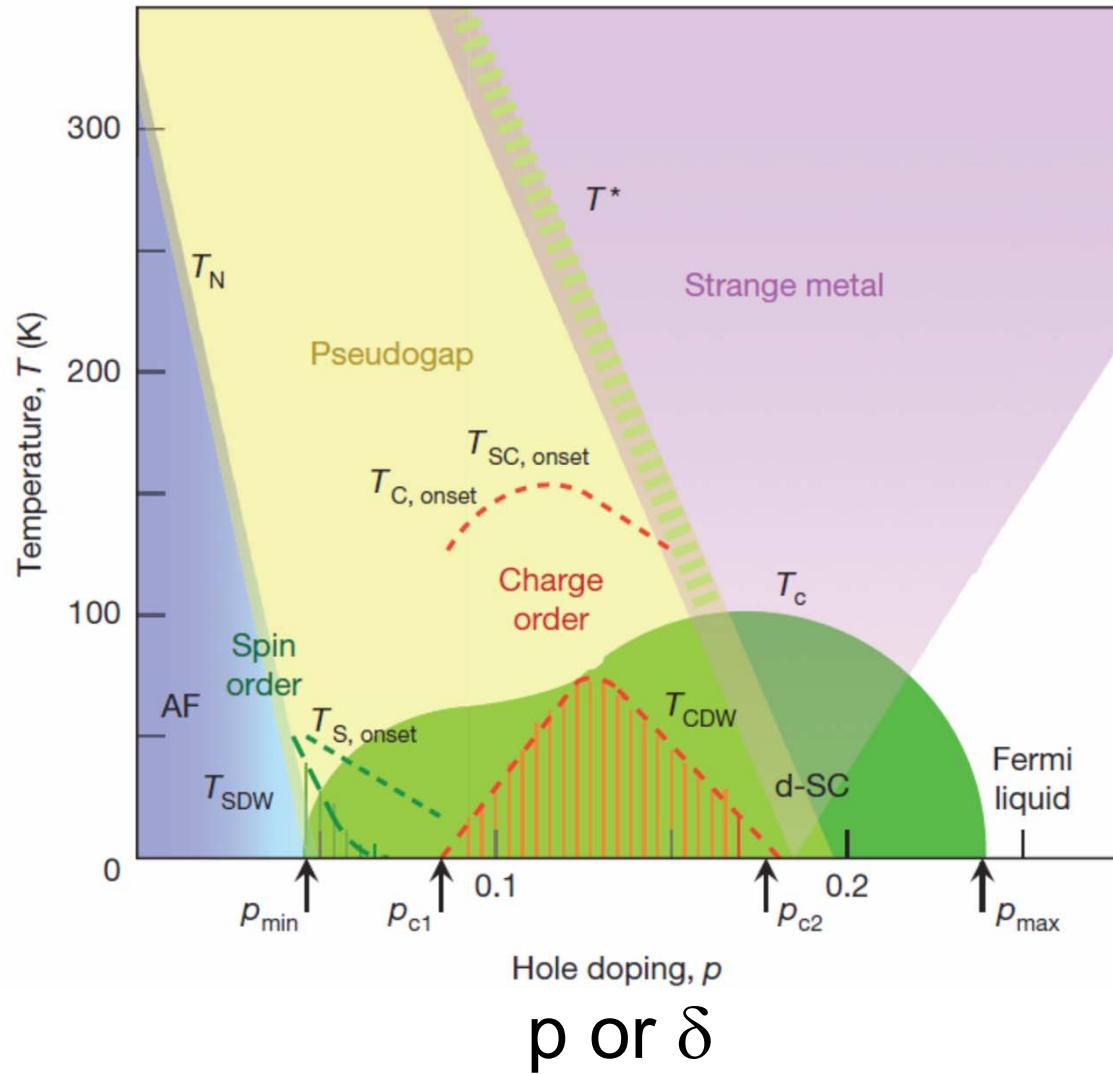
**M2S Beijing, 23 August 2018**  
**Th-S50 Mott Physics-2**

**Room 6, 14h00-15h55**

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# Phase diagram $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$

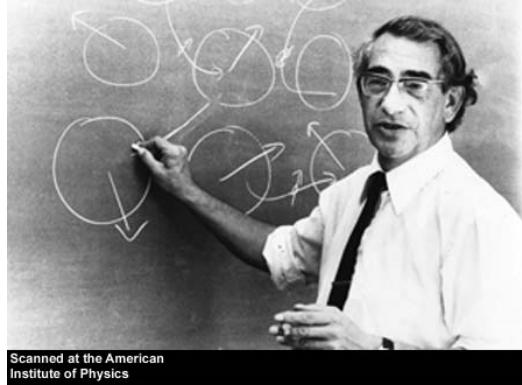


Keimer et al., Nature 518, 179 (2015)

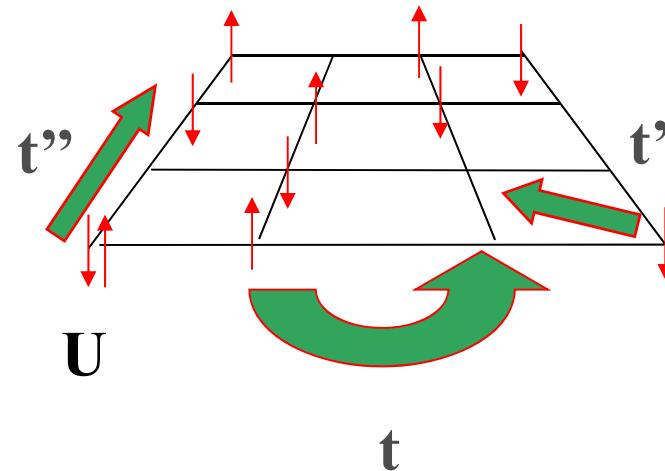
# Model



# Hubbard Model



$\mu$



1931-1980

$$H = -\sum_{\langle ij \rangle \sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$t = 1, k_B = 1, \hbar = 1$$

Attn: Charge transfer insulator

# Method

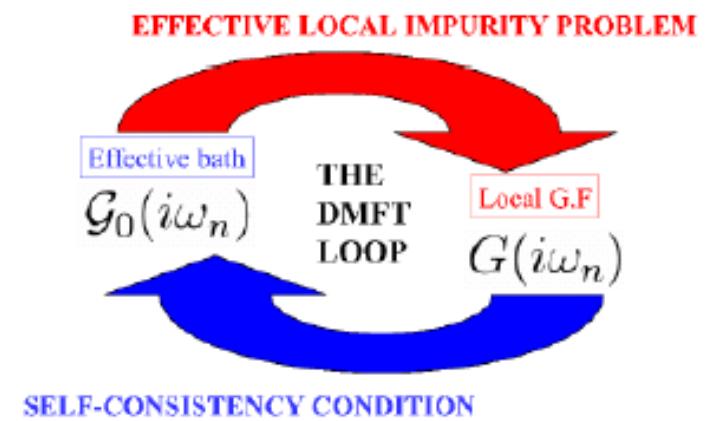
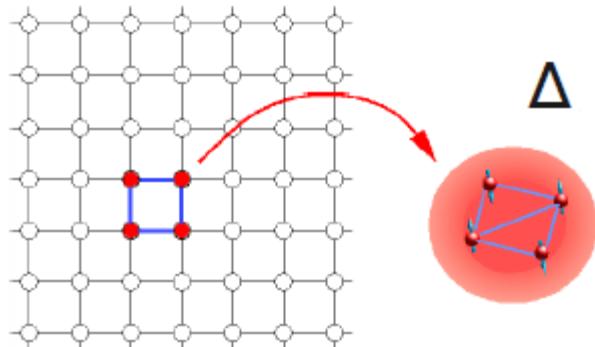
- Dynamical Mean Field Theory
  - clusters
- Concept: atomic-like localized correlations consistent with delocalized aspect

## REVIEWS

Maier, Jarrell et al., RMP. (2005)  
Kotliar et al. RMP (2006)  
AMST et al. LTP (2006)

Hettler et al, PRB 1998  
Lichtenstein et al., PRB 2000  
Kotliar et al., PRB 2000  
M. Potthoff, EJP 2003

# Cellular DMFT + CT-QMC



$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n)$$

$$- \left[ \sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)} \right]^{-1}$$

## + and -

- Long range order:
  - No mean-field factorization on the cluster
  - Symmetry breaking allowed in the bath (mean-field)
- Included exactly:
  - Short-range dynamical and spatial correlations
- Missing:
  - Long wavelength p-h and p-p fluctuations

# Some groups using these methods for cuprates

- Europe:
  - Georges, Parcollet, Ferrero, Civelli, Wu (Paris)
  - Lichtenstein, Potthoff, (Hamburg) Aichhorn (Graz), Liebsch (Jülich) de Medici (Grenoble) Capone (Italy)
- USA:
  - Gull (Michigan) Millis (Columbia)
  - Kotliar, Haule (Rutgers)
  - Jarrell (Louisiana)
  - Maier, Okamoto (Oakridge)
- Japan
  - Imada (Tokyo) Sakai, Tsunetsugu, Motome

# Outline

- The model
- The method
- Part I: Half-filling (Mott insulator)
- Part II: The pseudogap (doped Mott insulator)
- Part III: Strongly correlated superconductivity

# Part I

## Half-filling





Giovanni Sordi



Lorenzo Fratino



Maxime Charlebois



Patrick Sémon

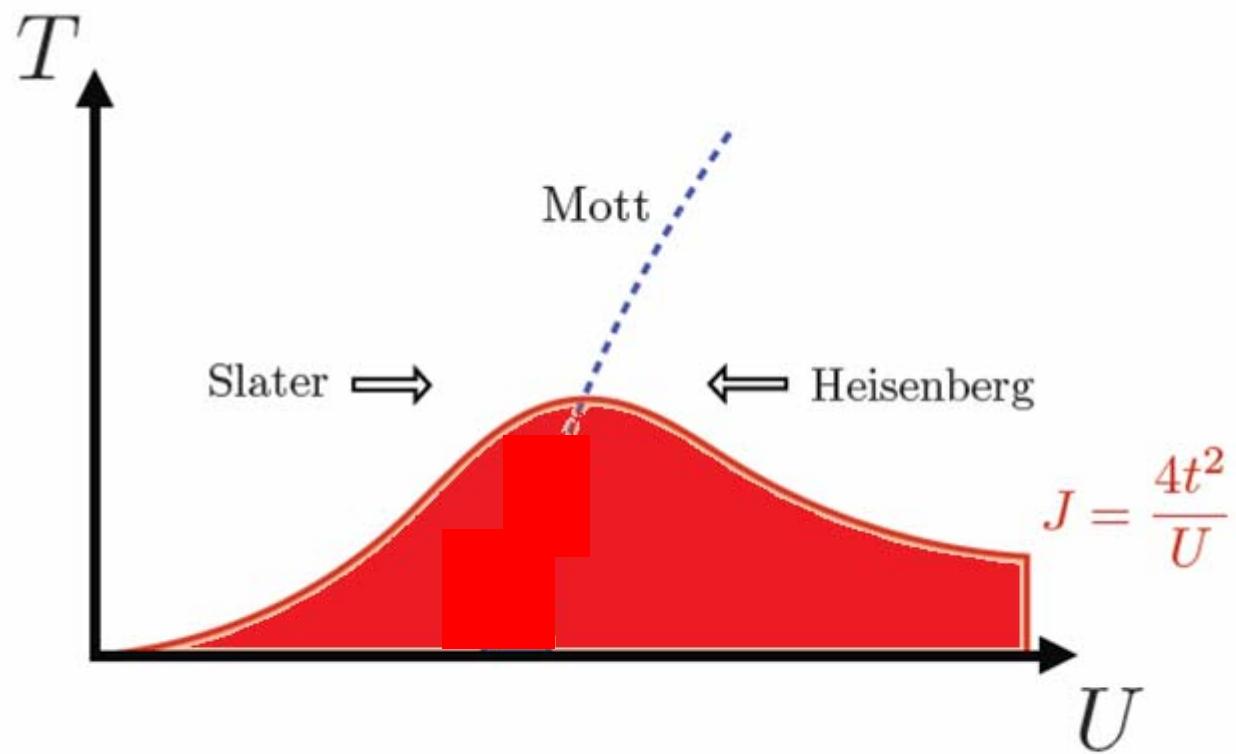
# Mott transition and antiferromagnetism

Influence of the underlying normal state  
on the ordered state

# Underlying Mott transition

10

$n = 1, d = 2$  square lattice

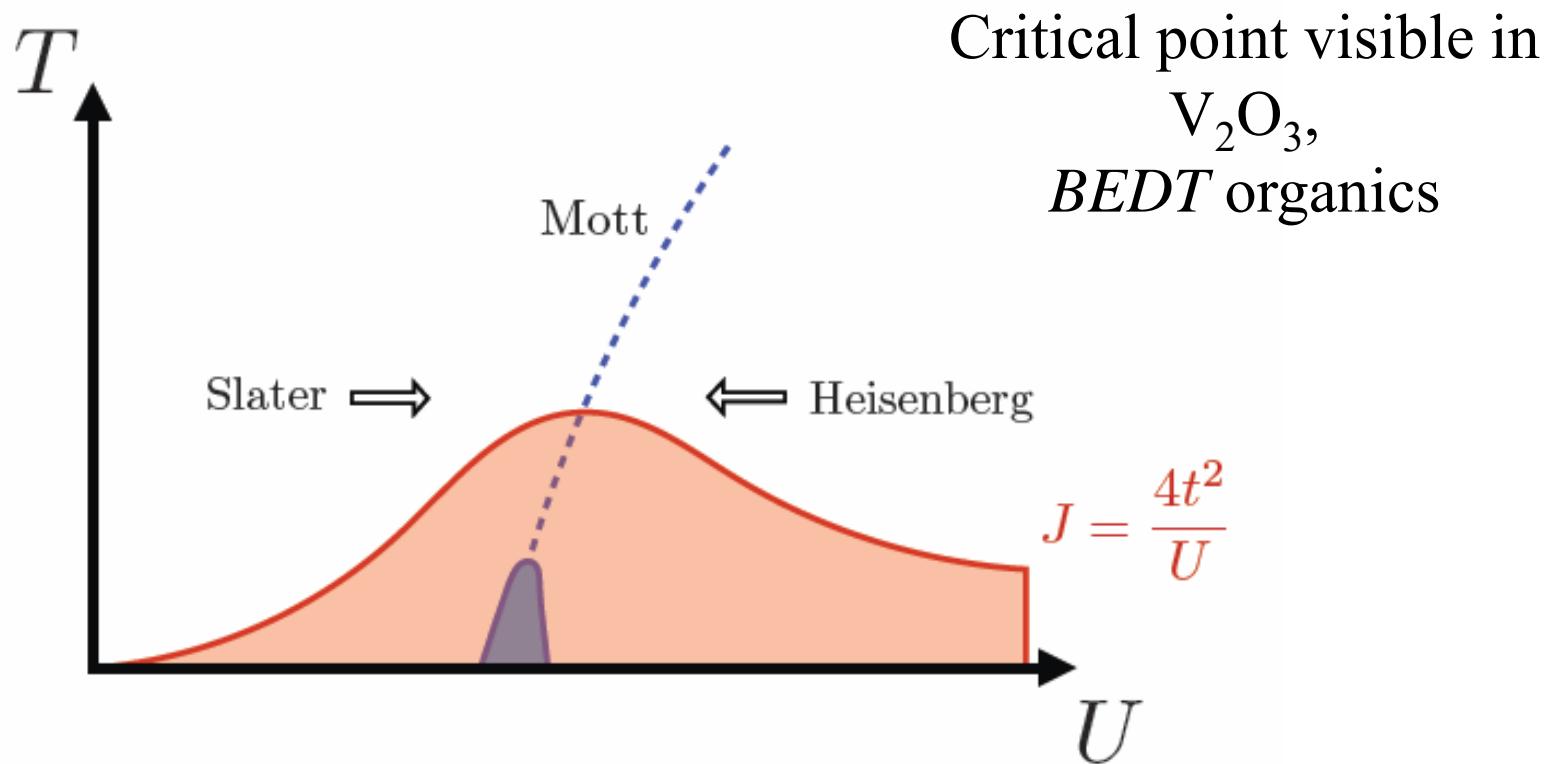


Understanding finite temperature phase from a *mean-field theory* down to  $T = 0$

# Underlying Mott transition

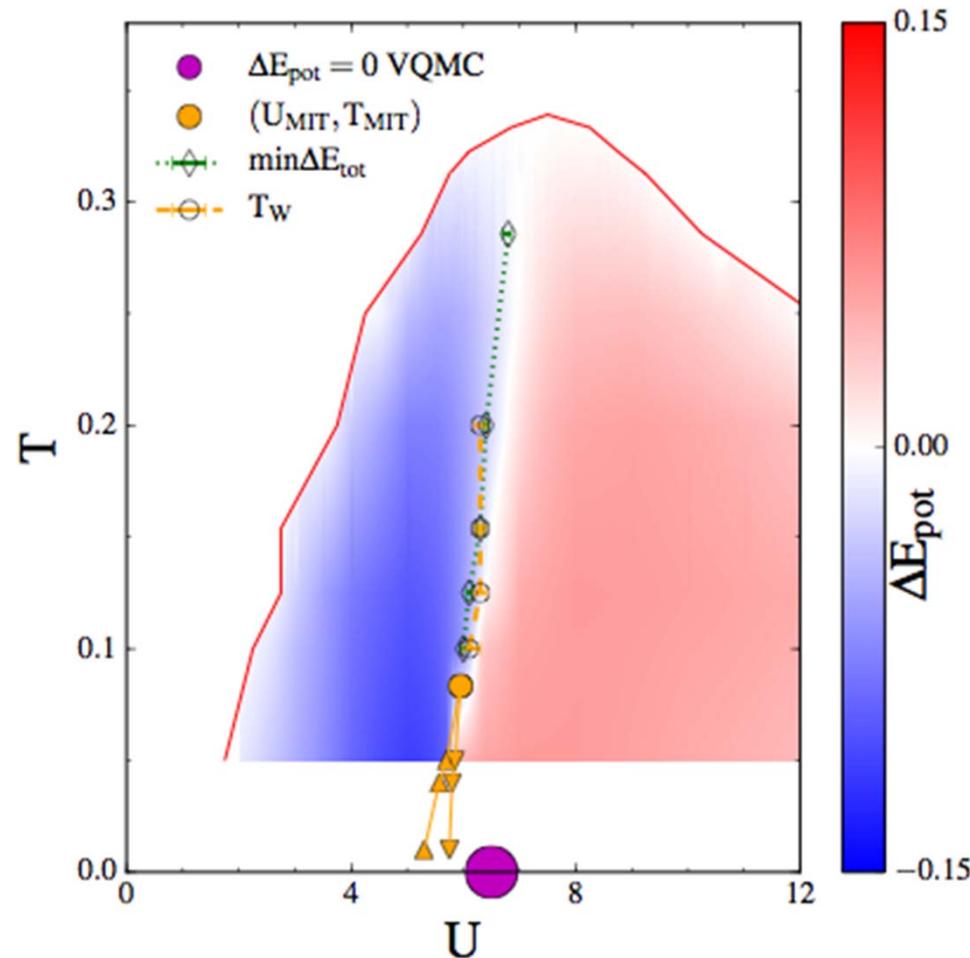
10

$n = 1, d = 2$  square lattice



Understanding finite temperature phase from a *mean-field theory* down to  $T = 0$

# Change in mechanism for stability of the AFM



L. F. Tocchio, F. Becca, and S. Sorella, Phys. Rev. B **94**, 195126 (2016).

## Part II:

The pseudogap  
(doped Mott insulator)





Simon Bergeron



Maxime Charlebois



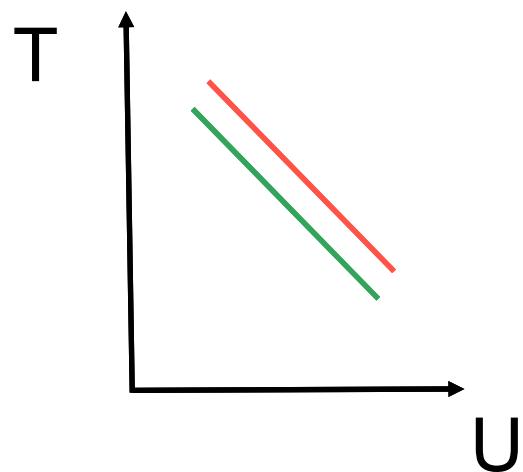
Patrick Sémon



Alexis Reymbaut



## The pseudogap from the calculated Knight shift



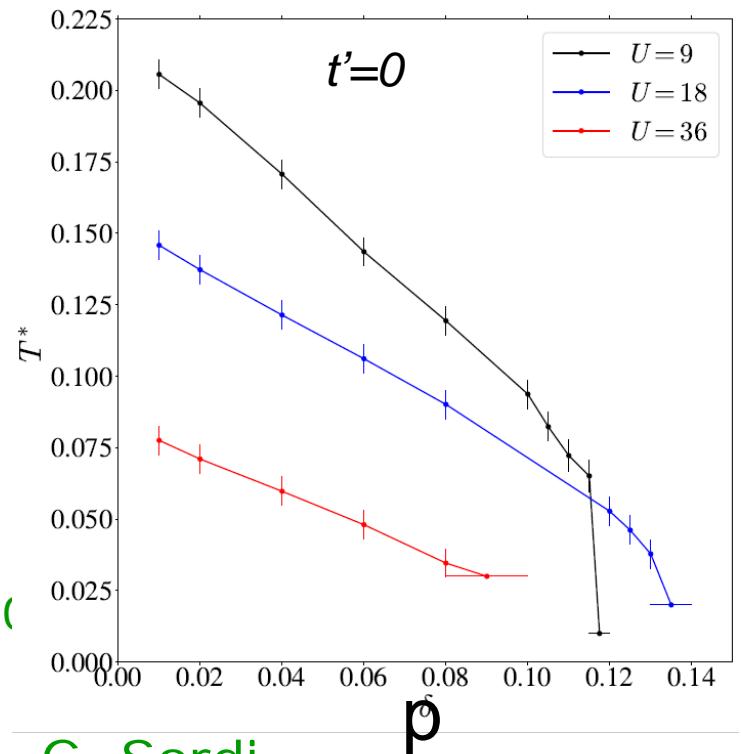
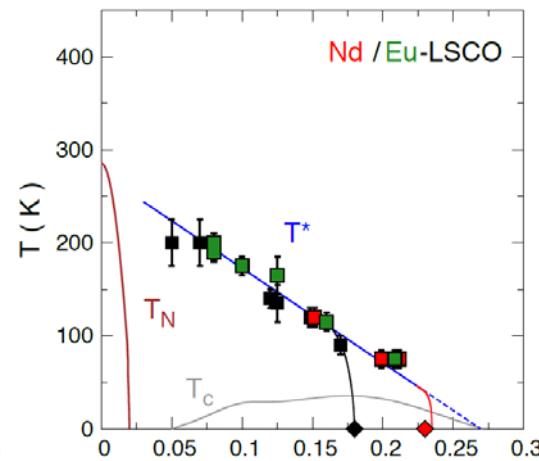
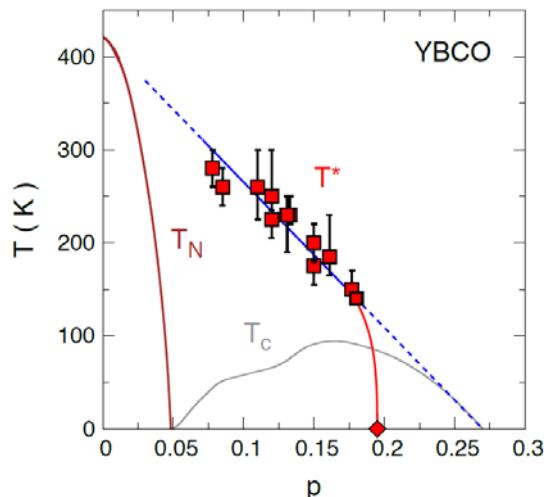
Marion Thénault

# Results $T^*$

DOI

$$k_B T^* \sim J$$

$$J = 4t^2/U$$

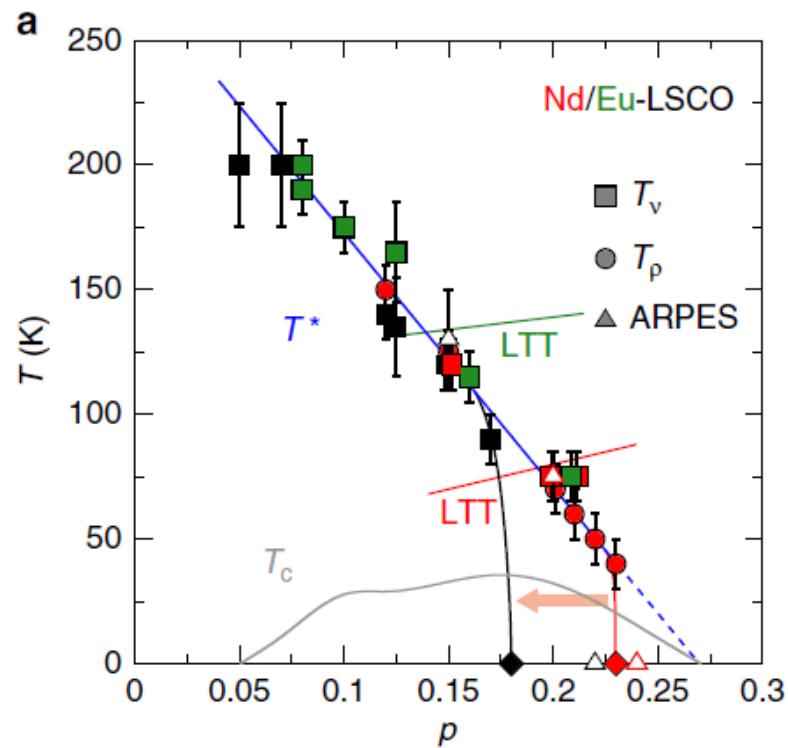


Cyr-Choinières et al. Phys. Rev. B **97**, 064506 (2018)

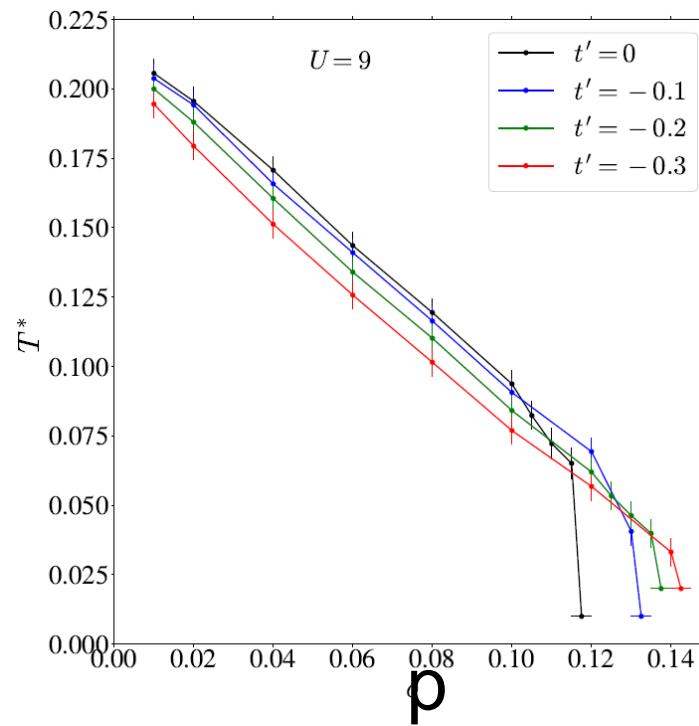
A. Reymbaut, M. Thénault, L. Fratino, G. Sordi,  
P. Sémon, AMT, unpublished

W Wu, A Georges, M Ferrero - arXiv preprint arXiv:1707.06602  
Bragança, Sakai, Aguiar, Civelli, PRL **120**, 067002 (2018)

# Results : effect of $t'$ on $T^*$

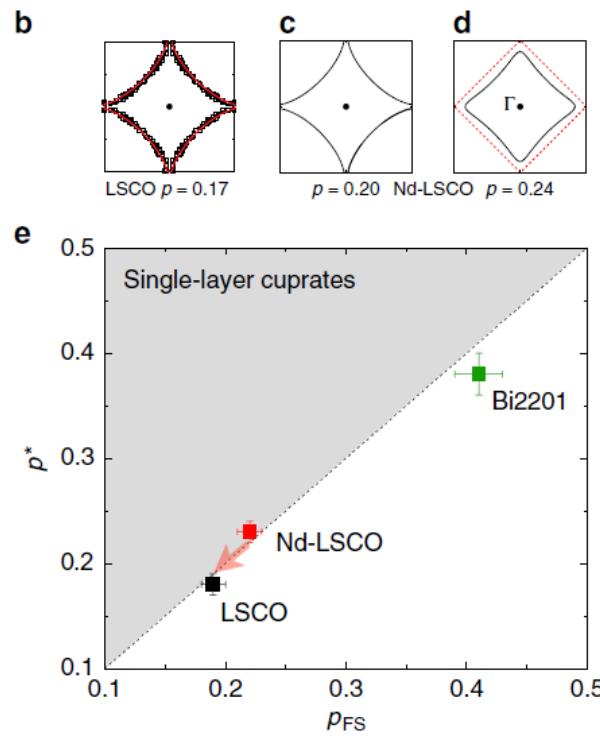


Doiron-Leyraud *et al.*  
Nature Comm. **8** 2044

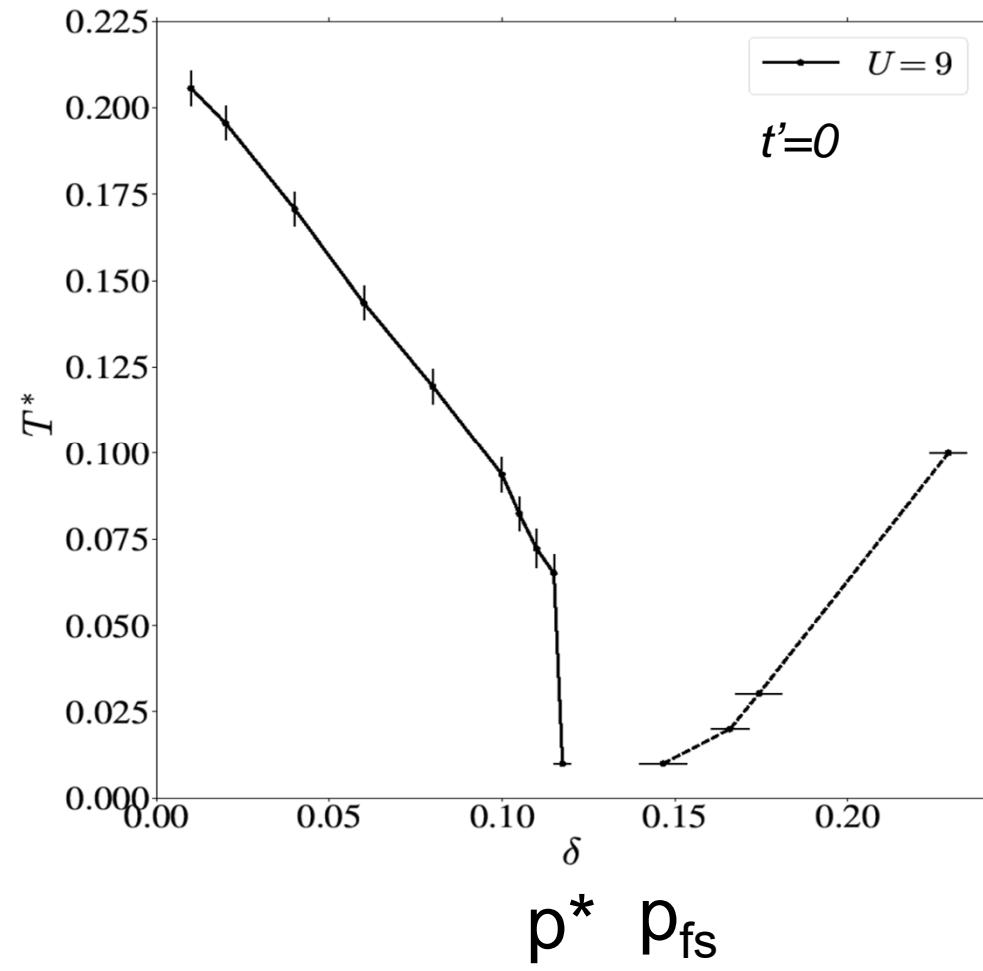


A. Reymbaut, M. Thénault, L. Fratino,  
G. Sordi, P. Sémon, AMT, unpublished

# Results: van Hove singularity



$$p^* < p_{fs}$$





Giovanni Sordi



Kristjan Haule

# Pseudogap from the influence of the Mott transition away from half-filling

Sordi et al., PRL 104, 226402 (2010)

Sordi et al., PRB 84, 075161 (2011)

Fratino et al., PRB 93, 245147 (2016) [Emery model]

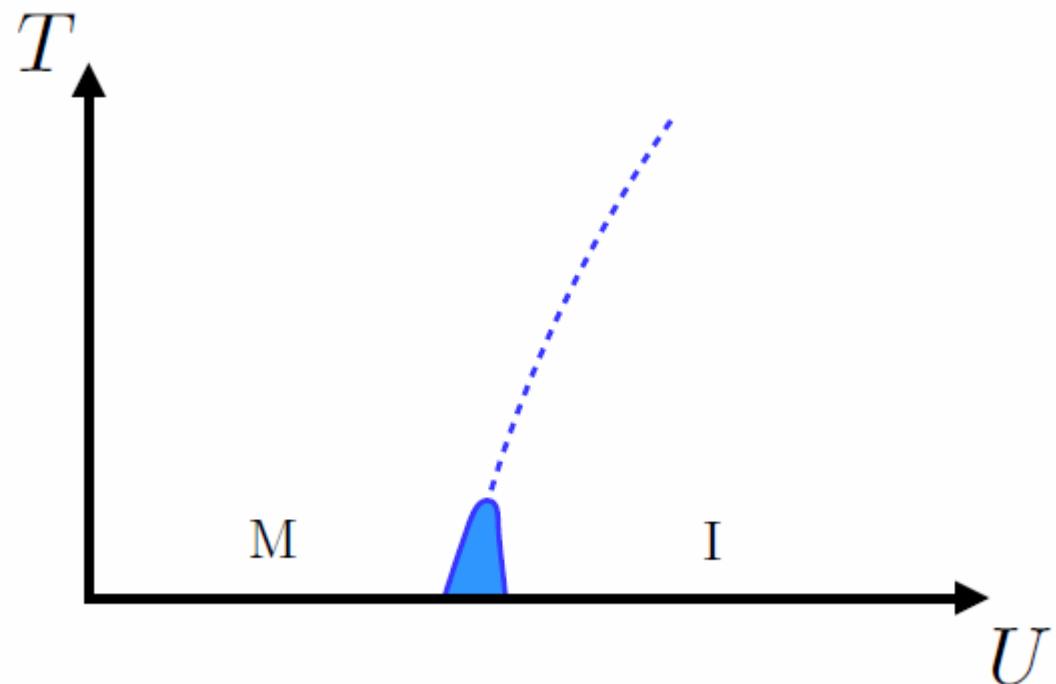
Sordi et al., Sci. Rep. 2 547 (2012);

Sordi et al., PRB 87, 041101(R) (2013)

Fratino et al., PRB 93, 245147 (2016) [Emery model]

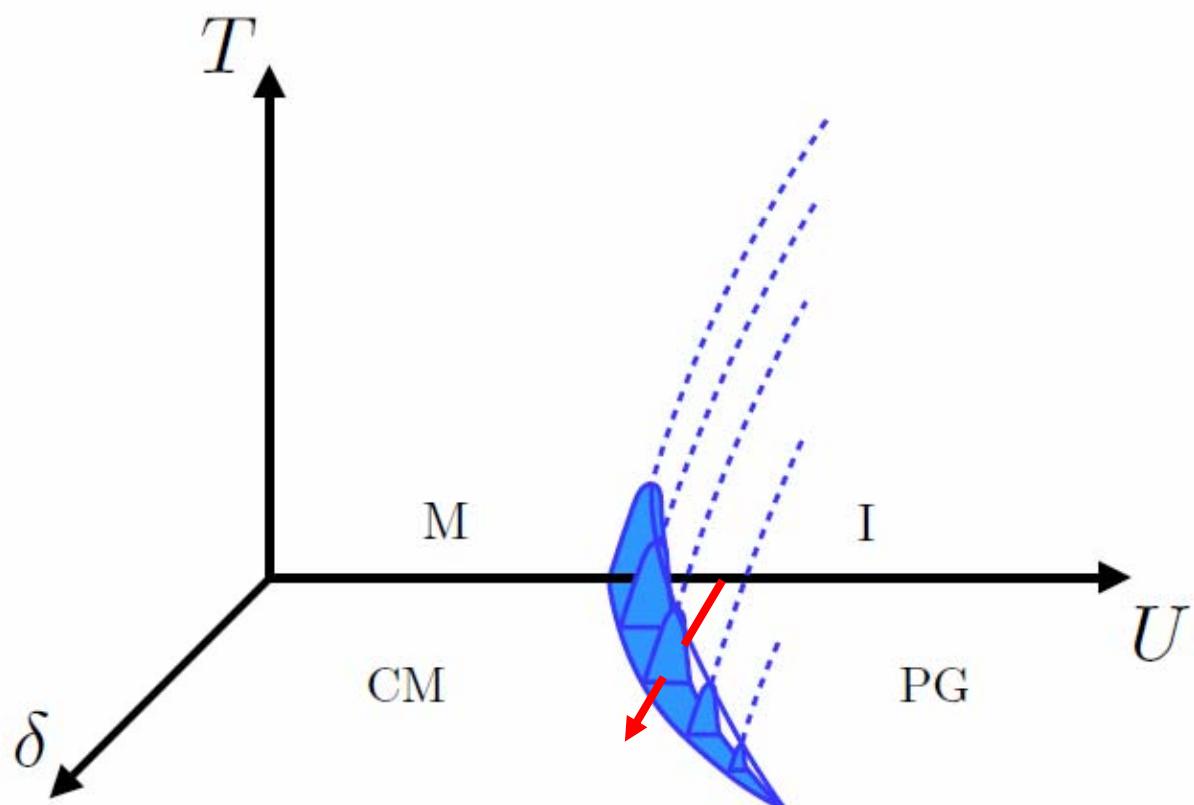
# Influence of Mott transition away from half-filling

$n = 1$ ,  $d = 2$  square lattice



# Influence of Mott transition away from half-filling

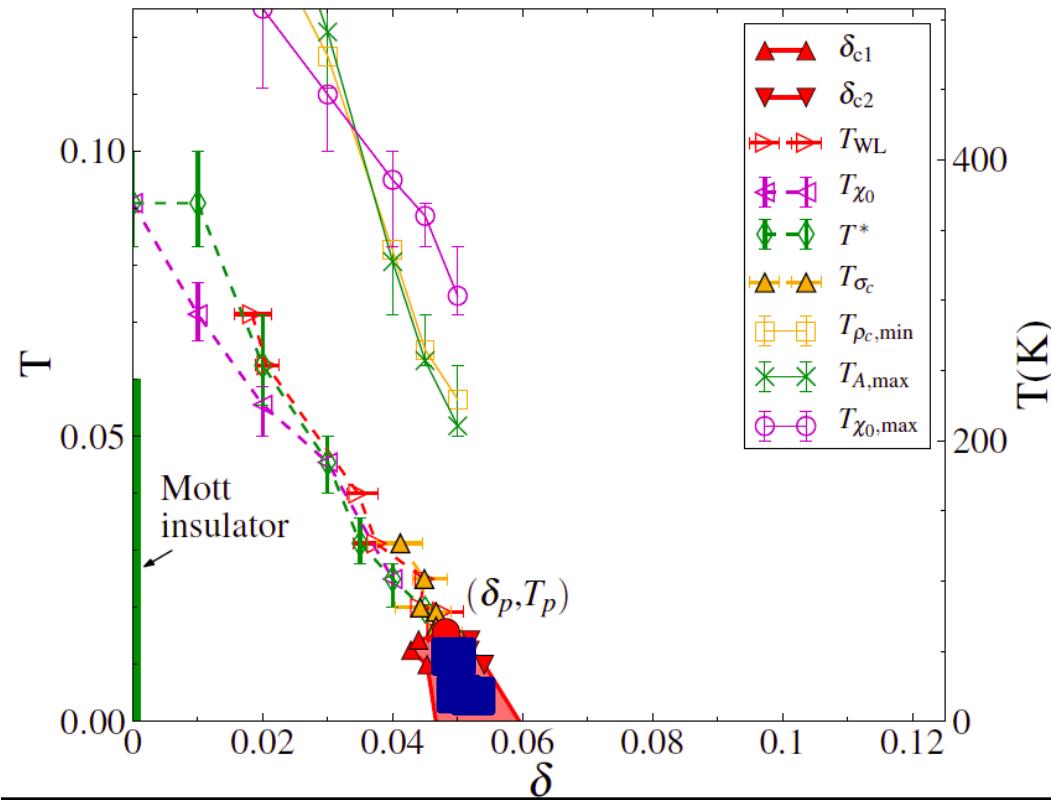
$n = 1, d = 2$  square lattice



# Two crossover lines



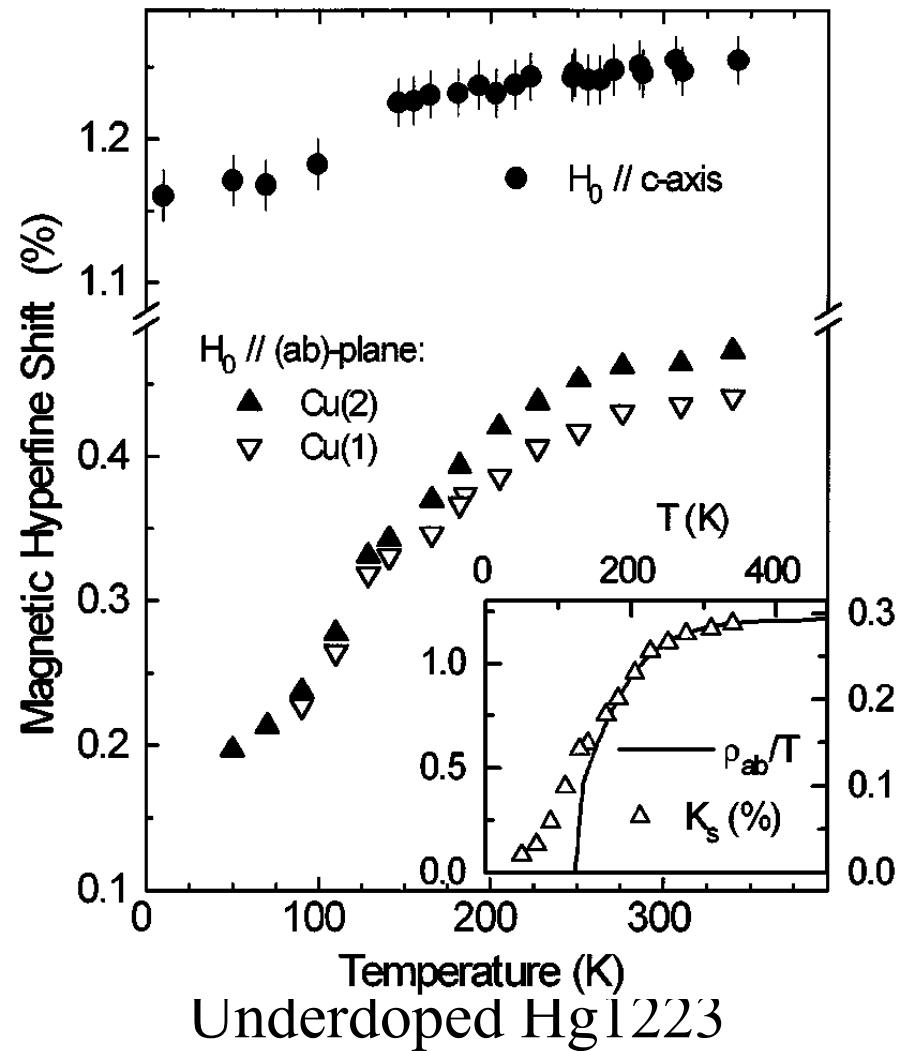
Giovanni Sordi



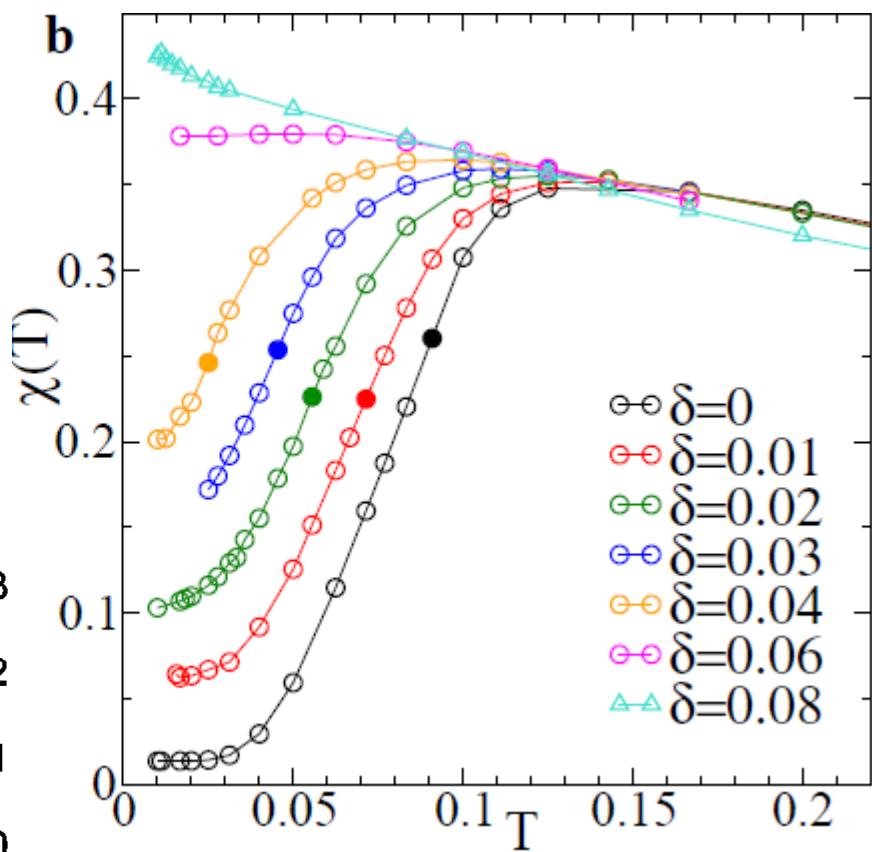
Patrick Sémon

G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)  
P. Sémon, G. Sordi, A.-M.S.T., Phys. Rev. B 89, 165113/1-6 (2014)

# Spin susceptibility



Julien et al. PRL 76, 4238 (1996)



## Part III:

# Strongly correlated superconductivity





Giovanni Sordi



Patrick Sémon



Lorenzo Fratino

# Superconductivity in a doped Mott insulator

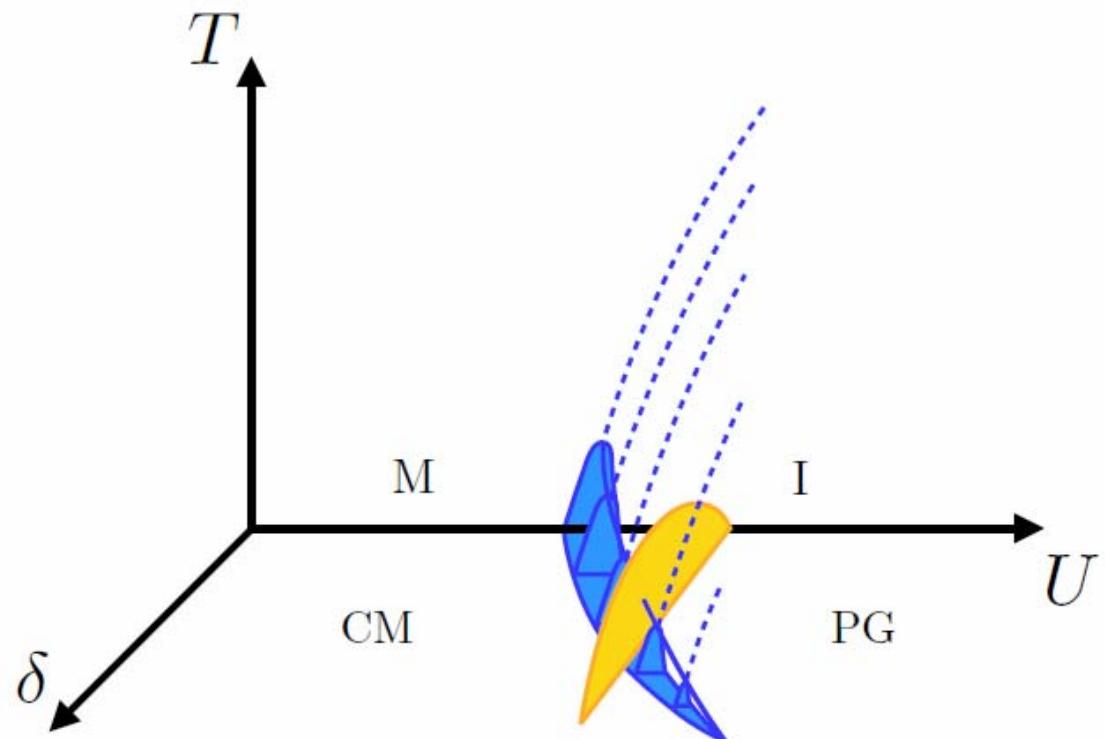
Sordi et al. PRL **108**, 216401 (2012)

Fratino et al.

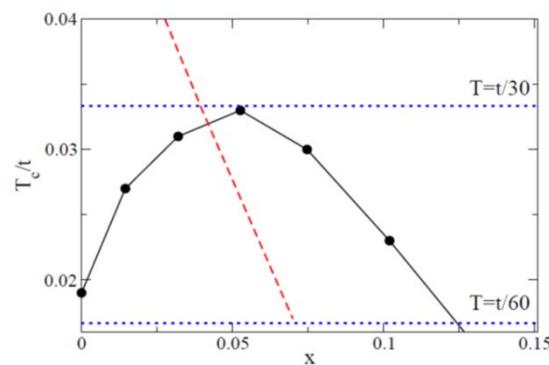
Sci. Rep. **6**, 22715 (2016)

# Superconductivity in Doped Mott insulator

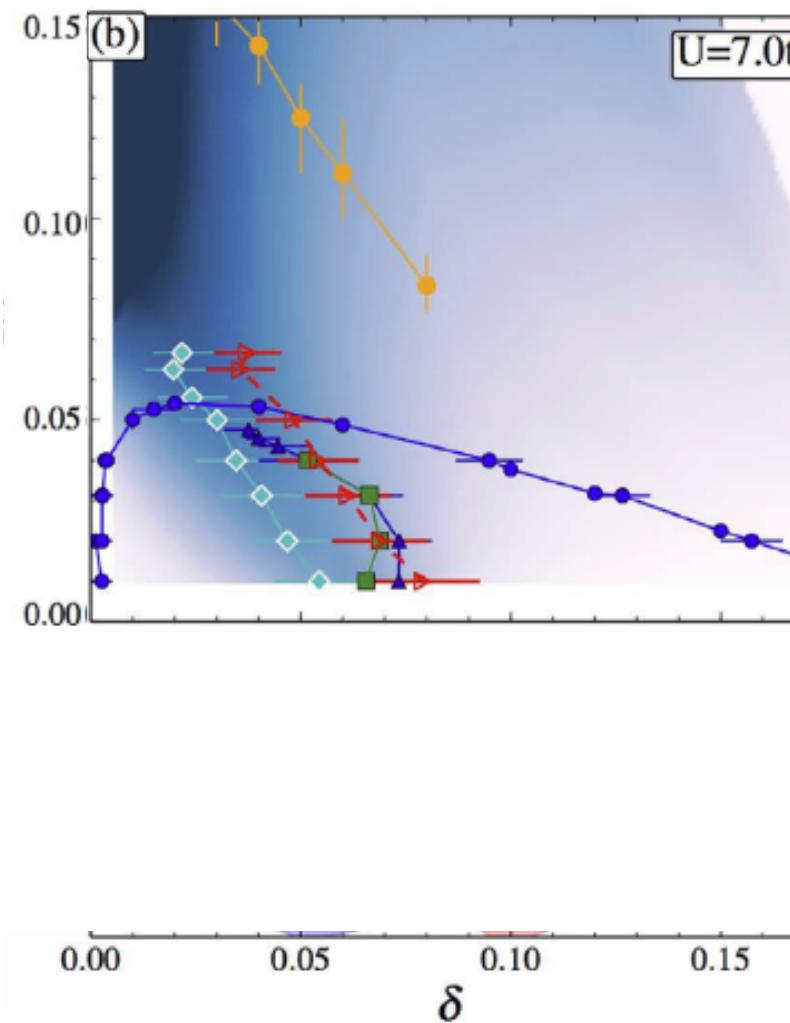
$n = 1, d = 2$  square lattice



# An organizing principle



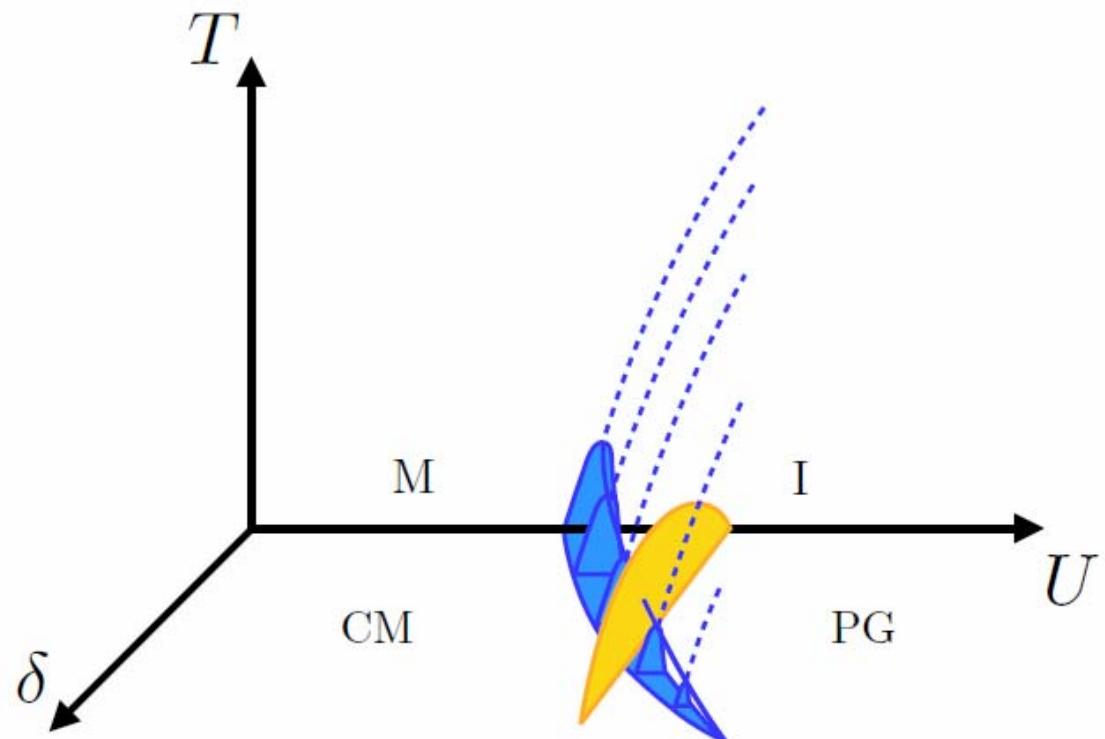
E. Gull and A. J. Millis  
Phys. Rev. B 88, 075127



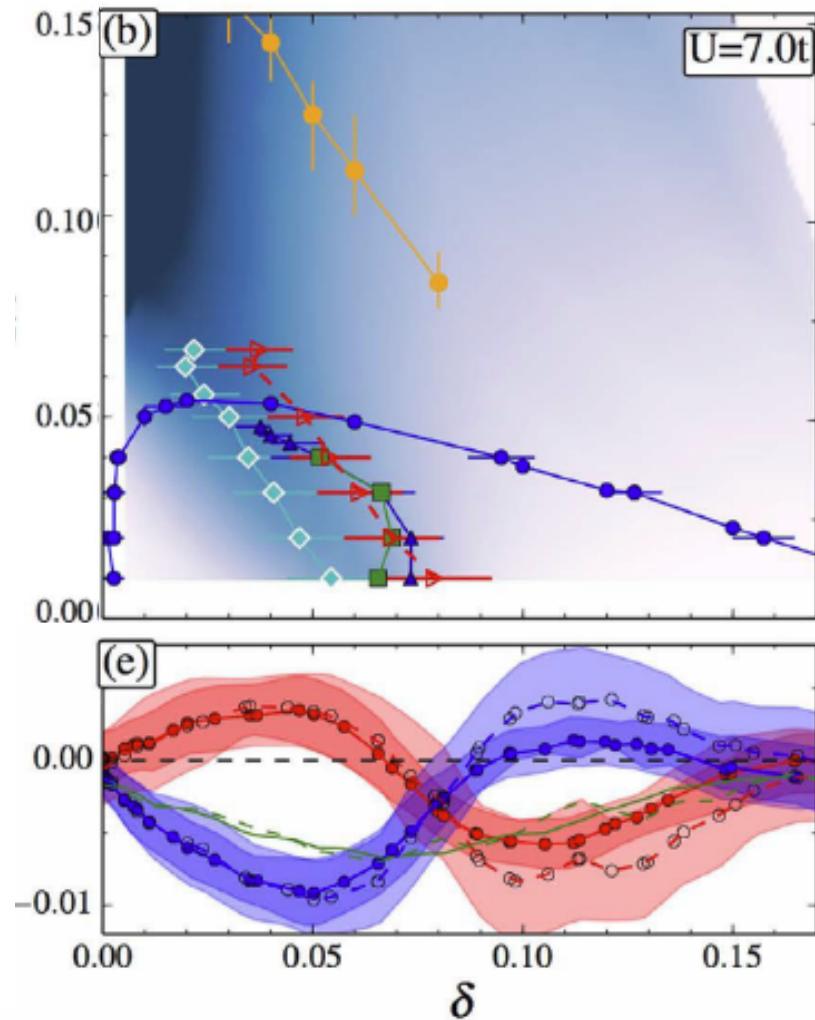
Fratino et al.  
Sci. Rep. 6, 22715

# Superconductivity in Doped Mott insulator

$n = 1, d = 2$  square lattice



# An organizing principle

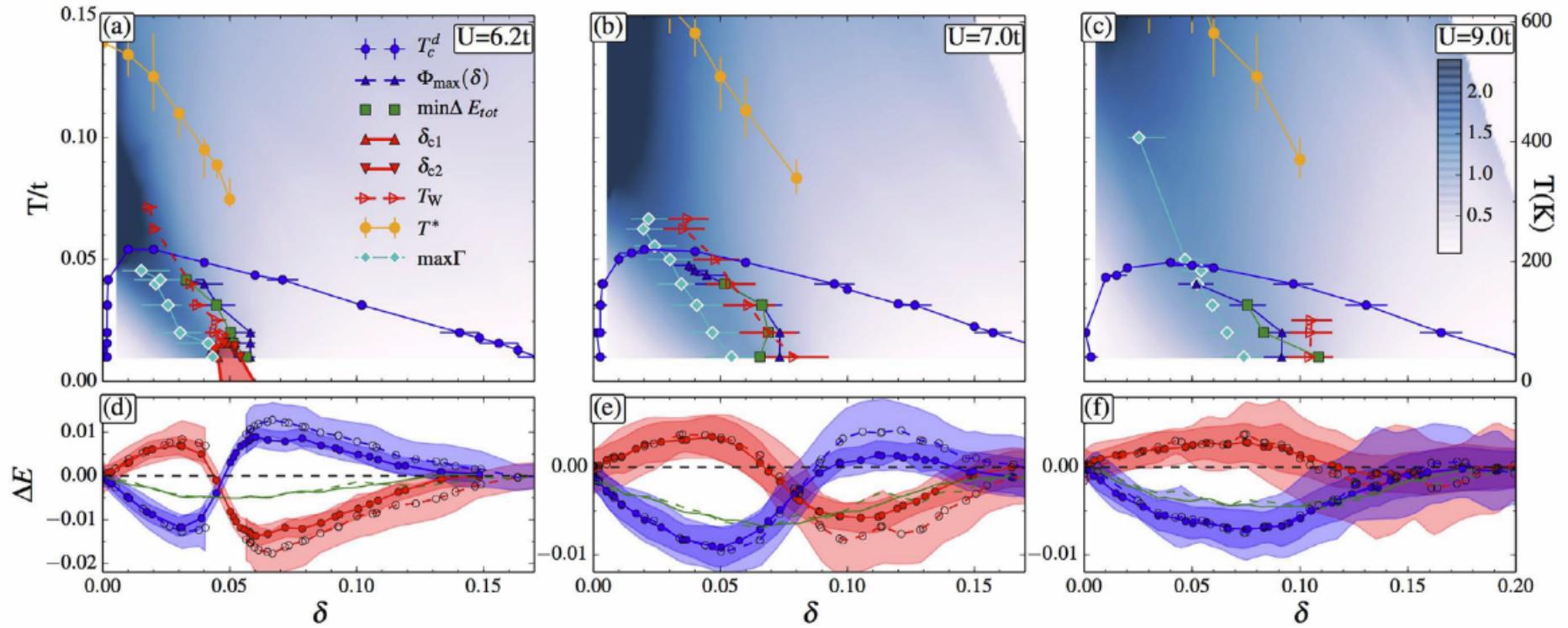


Fratino et al.  
Sci. Rep. 6, 22715

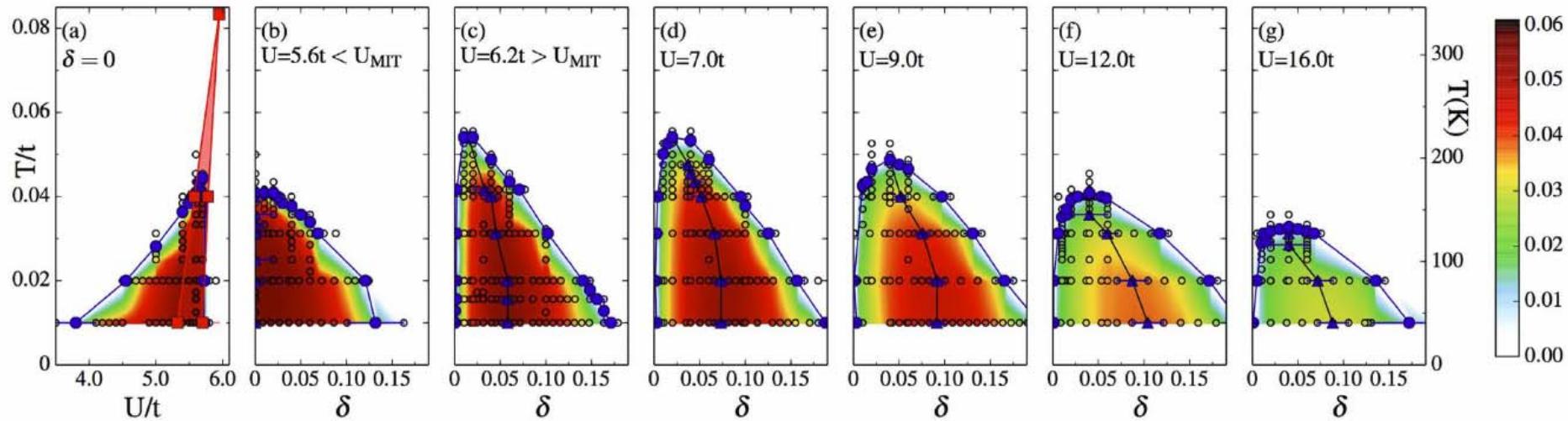
Theory, see also  
Jarrel PRL  
(2004), Gull  
Millis PRB  
(2014)

Experiments:  
Bontemps,  
Santander-Syro  
Van der Marel ...

# An organizing principle



# $T_c$ controlled by $J$



Fratino et al.  
Sci. Rep. 6, 22715

Some experiments that suggest  $T_c < T_{\text{pair}} < T^*$

T. Kondo *et al.* PRL 111 (2013)

Kondo, Takeshi, et al. Kaminski Nature Physics 2011, 7, 21-25

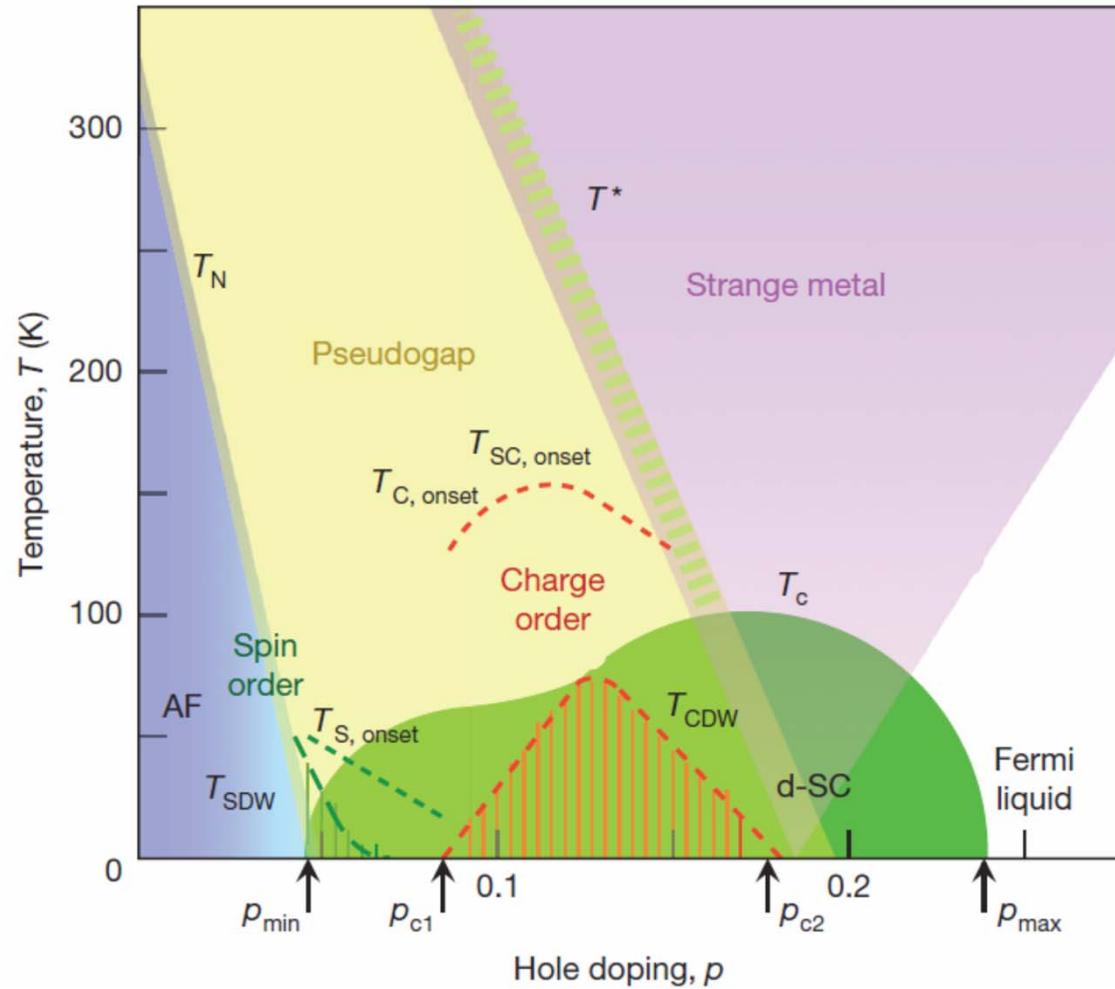
A. Pushp, Parker, ... A. Yazdani, Science 364, 1689 (2009)

Lee ... Tajima (Osaka) <https://arxiv.org/pdf/1612.08830>

Patrick M. Rourke, et al. Hussey Nature Physics 7, 455–458 (2011)

Lee et al. J. Phys. Soc. Jpn. 86, 023701 (2017)

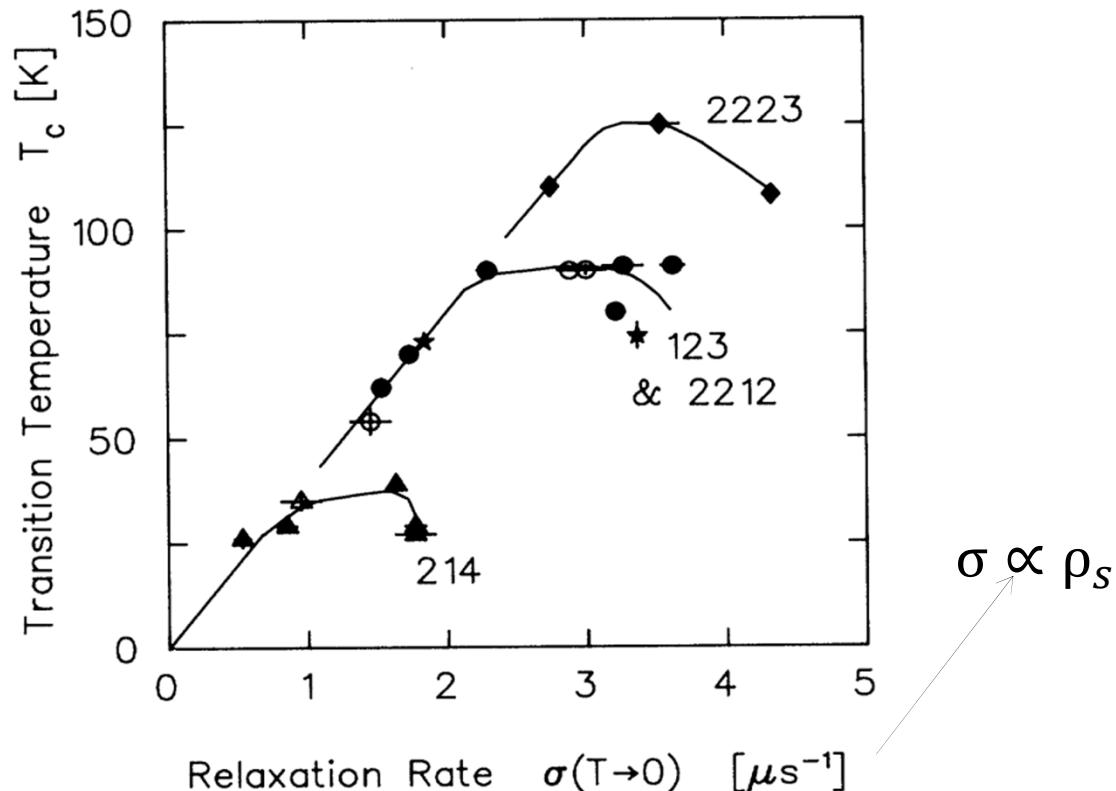
# Phase diagram $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$



Keimer et al., Nature 518, 179 (2015)

# What causes $T_c$ to drop?

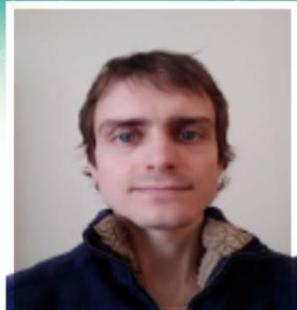
Phase fluctuations? Emery Kivelson Nature 374 (1995)



Uemura, Y.J. *et al.*, PRL vol.62, (1989)  
Tallon *et al.*, PRB 68, 180501(R) (2003)



Olivier Simard



Charles-David Hébert



Alexandre Foley



David Sénéchal

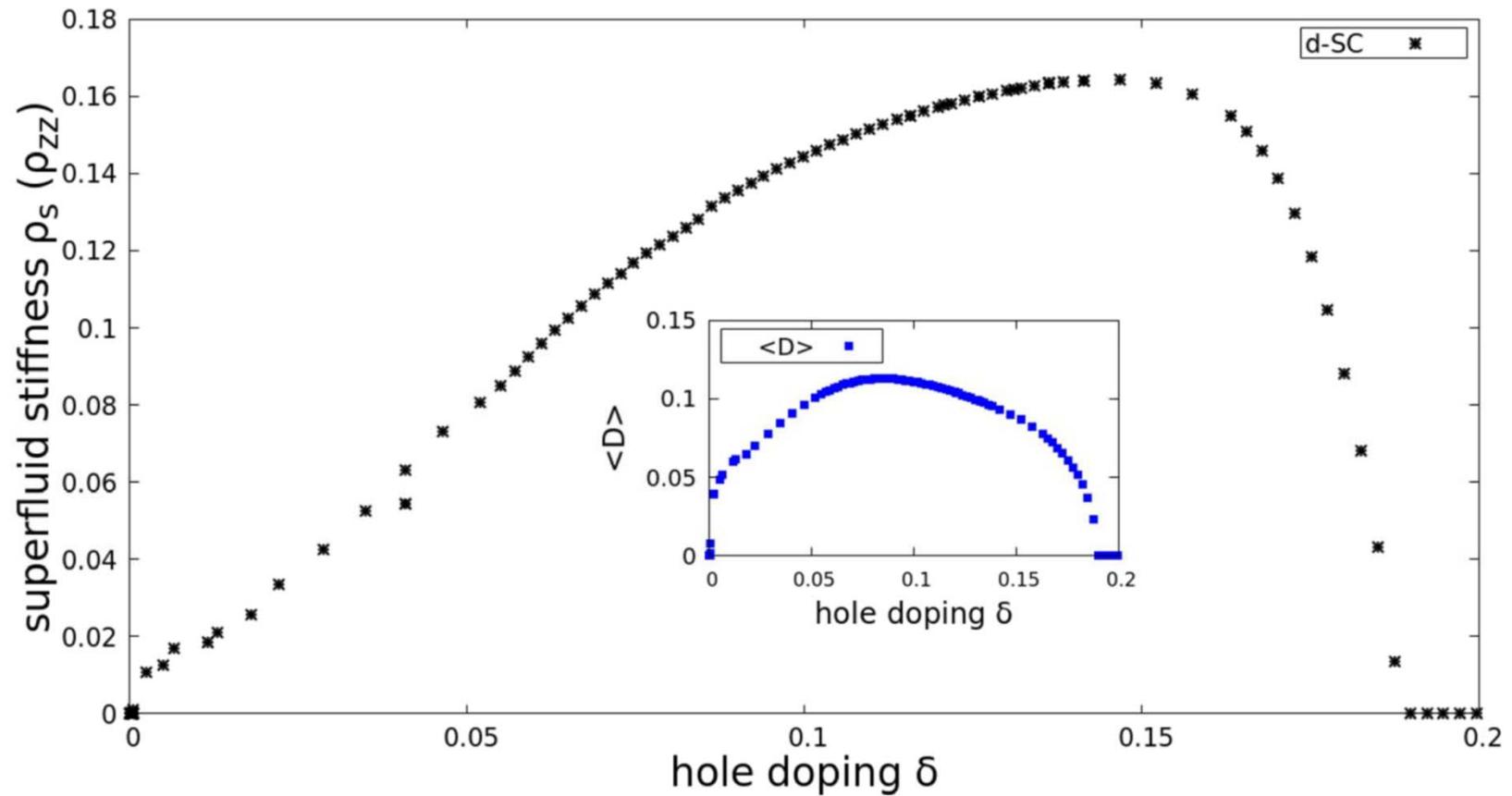
## What causes $T_c$ to drop near $n = 1$ ?

O. Simard, C.-D. Hébert, A. Foley, A.-M.S. Tremblay, D. Sénéchal, unpublished

# Superfluid stiffness $T=0$

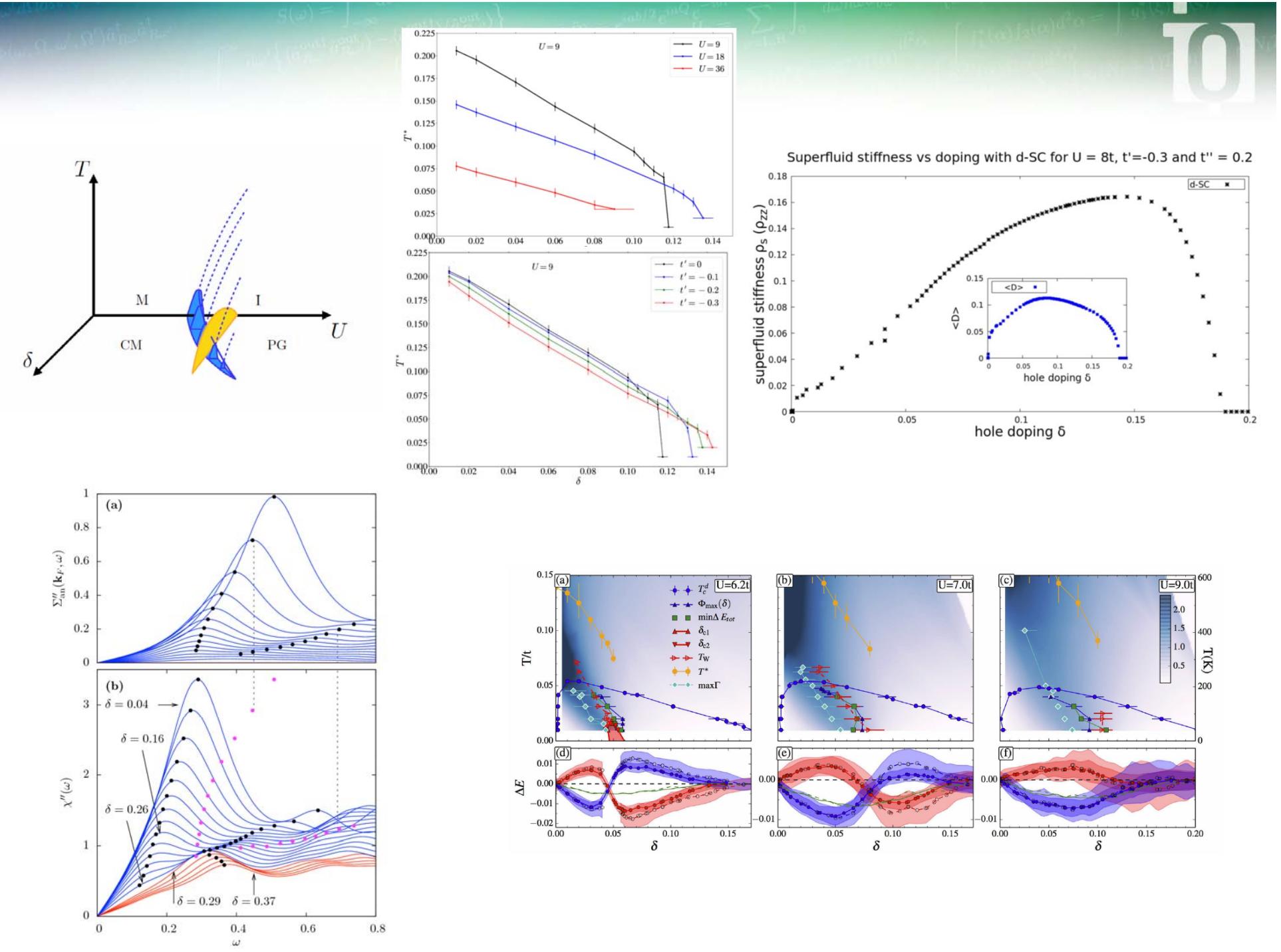
$$\rho_{ab} = \langle \hat{J}_a (\mathbf{r}, \tau) \hat{J}_b (\mathbf{r}', \tau') \rangle = \frac{-\beta}{V} \frac{\delta^2 \mathcal{F}}{\delta A_a (\mathbf{r}, \tau) \delta A_b (\mathbf{r}', \tau')} \Big|_{A=0}$$

Superfluid stiffness vs doping with d-SC for  $U = 8t$ ,  $t'=-0.3$  and  $t'' = 0.2$



# Conclusion





# Mammouth



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**Le calcul de haute performance**  
CRÉER LE SAVOIR  
ALIMENTER L'INNOVATION  
BÂTIR L'ÉCONOMIE NUMÉRIQUE

  
**Calcul Québec**

Merci  
Thank you



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