



Pseudogap and superconductivity in cuprates, a dynamical mean-field perspective

A.-M.S. Tremblay,

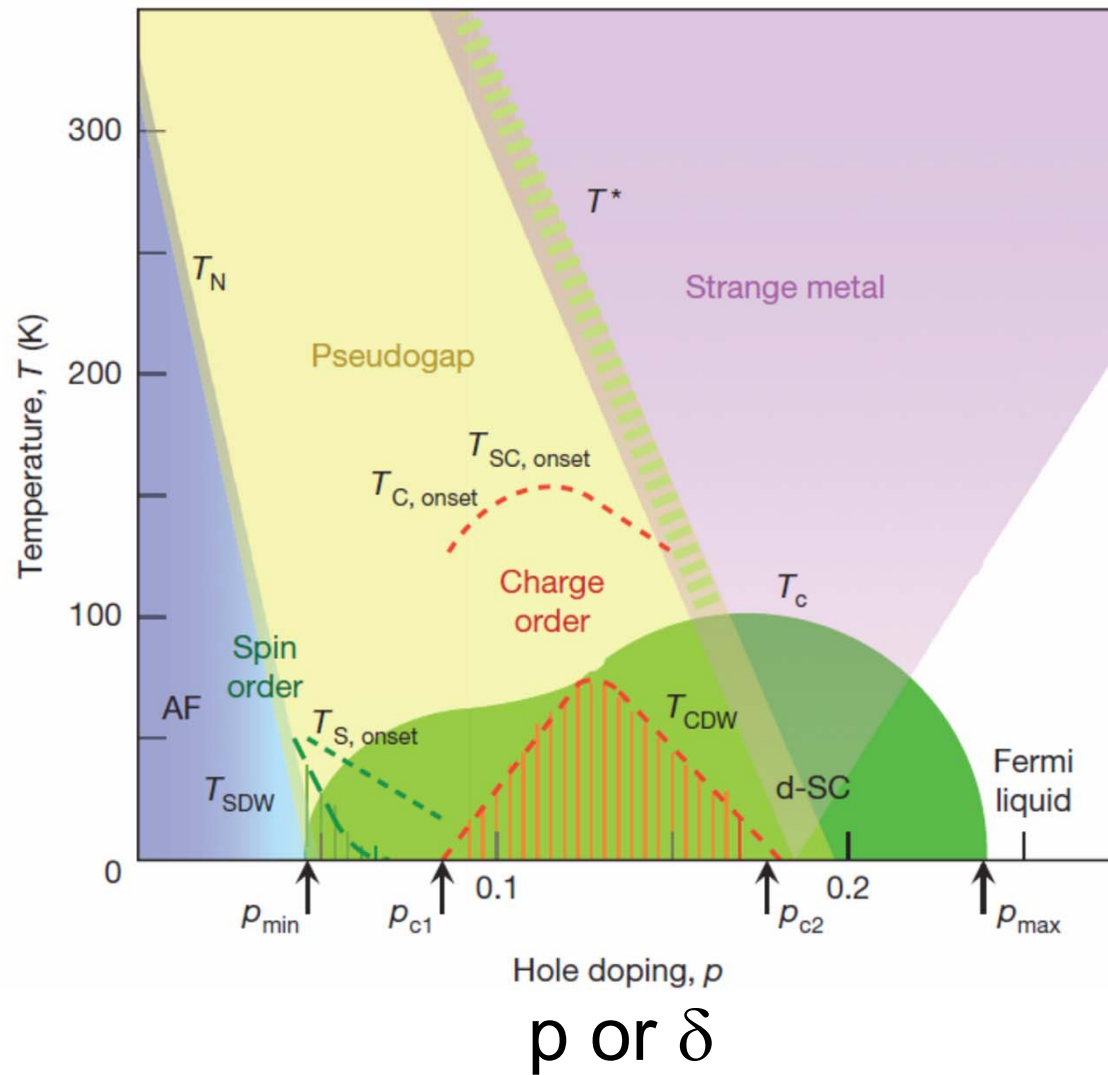
S. Bergeron, Maxime Charlebois, L. Fratino,

A. Foley, Charles-David Hébert, A. Reymbaut, D. Sénéchal

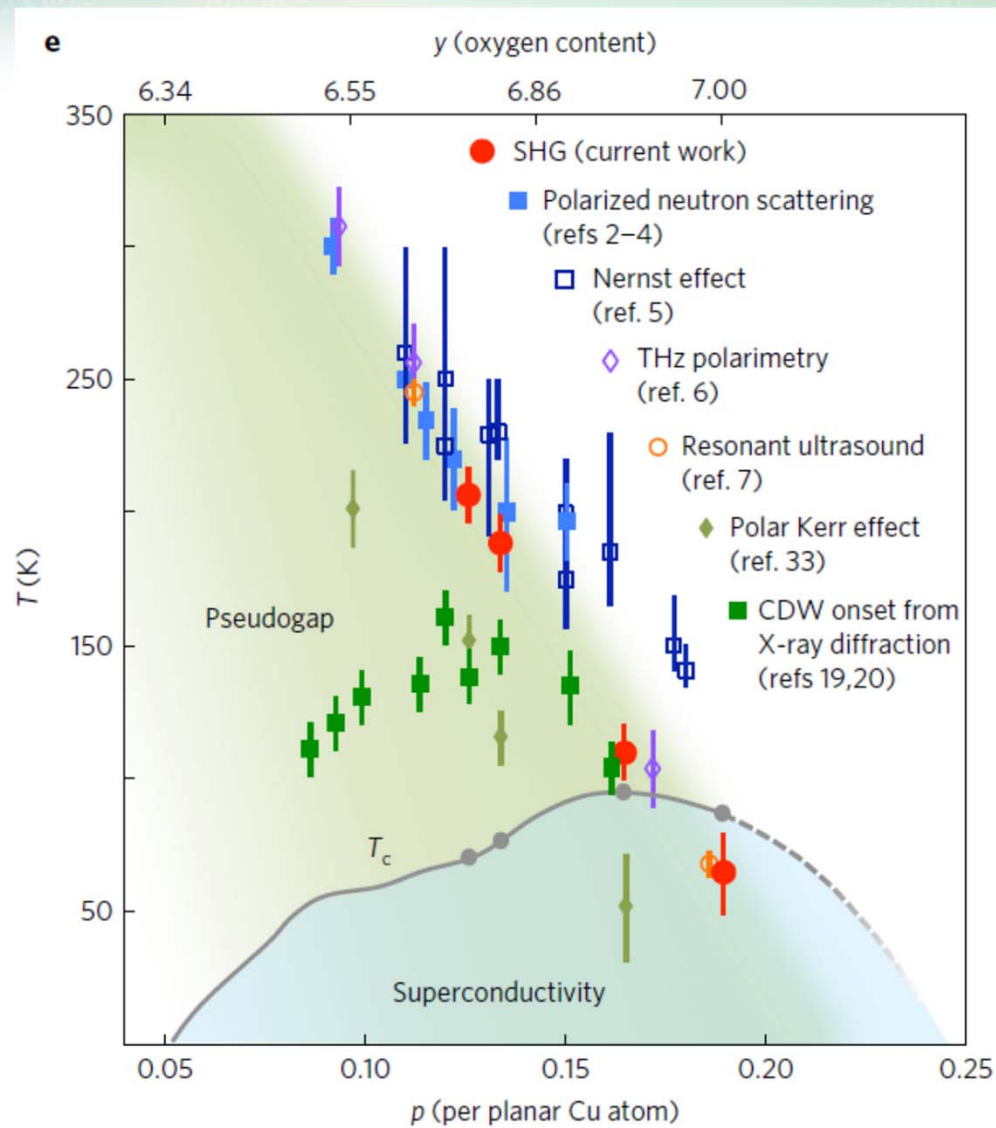
O. Simard, G. Sordi, Patrick Sémon, M. Thénault

Ringberg symposium on
Unconventional Superconductivity and Spin Liquids,
14-18 Oct. 2019

Phase diagram $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$

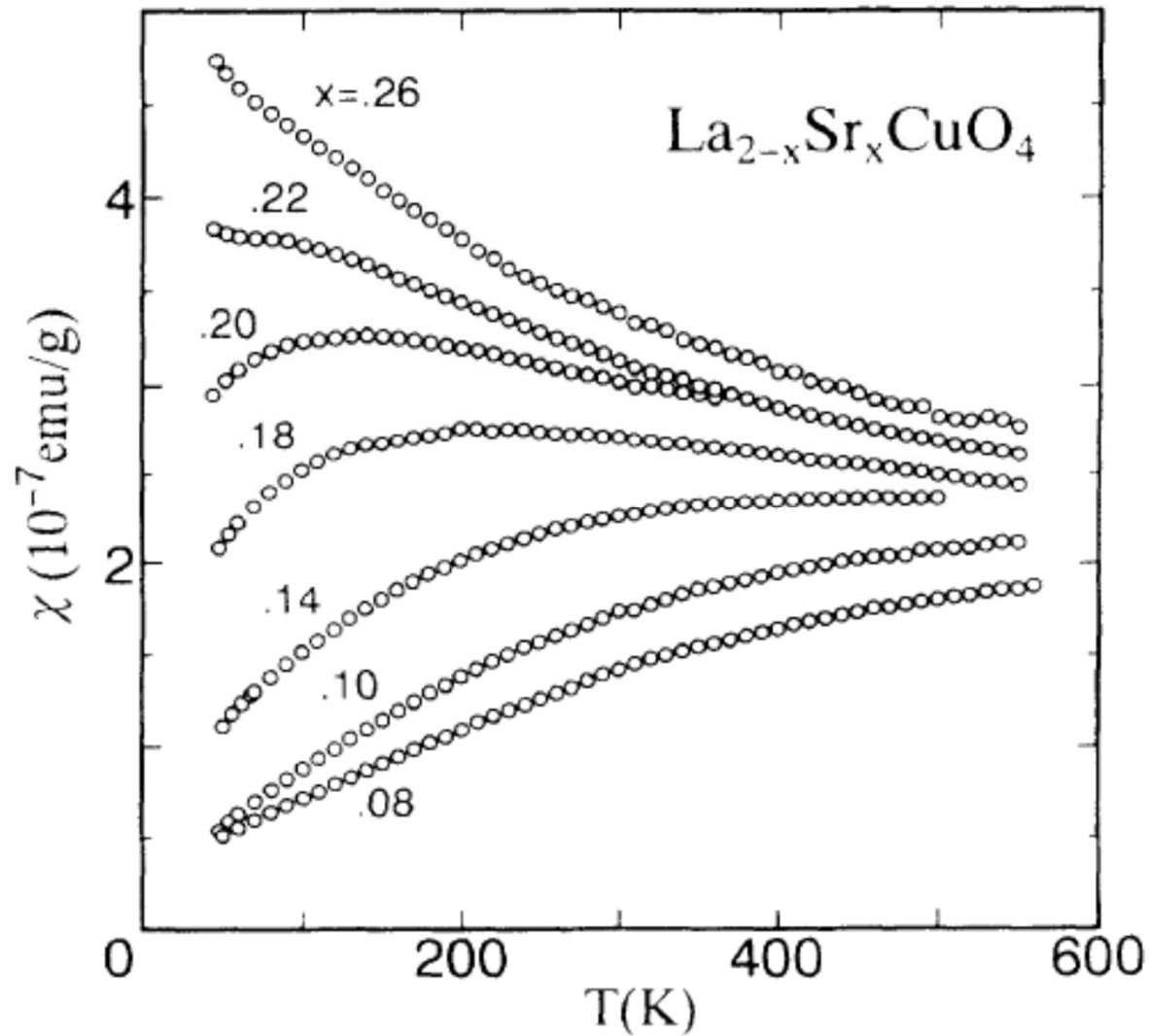


Keimer et al., Nature 518, 179 (2015)



Zhao *et al.* Nat. Phys. 13, 250 (2017).

Knight shift (Spin susceptibility)



Nakano *et al.* Phys. Rev. B **49**, 16000 (1994)
Alloul *et al.* (1989)

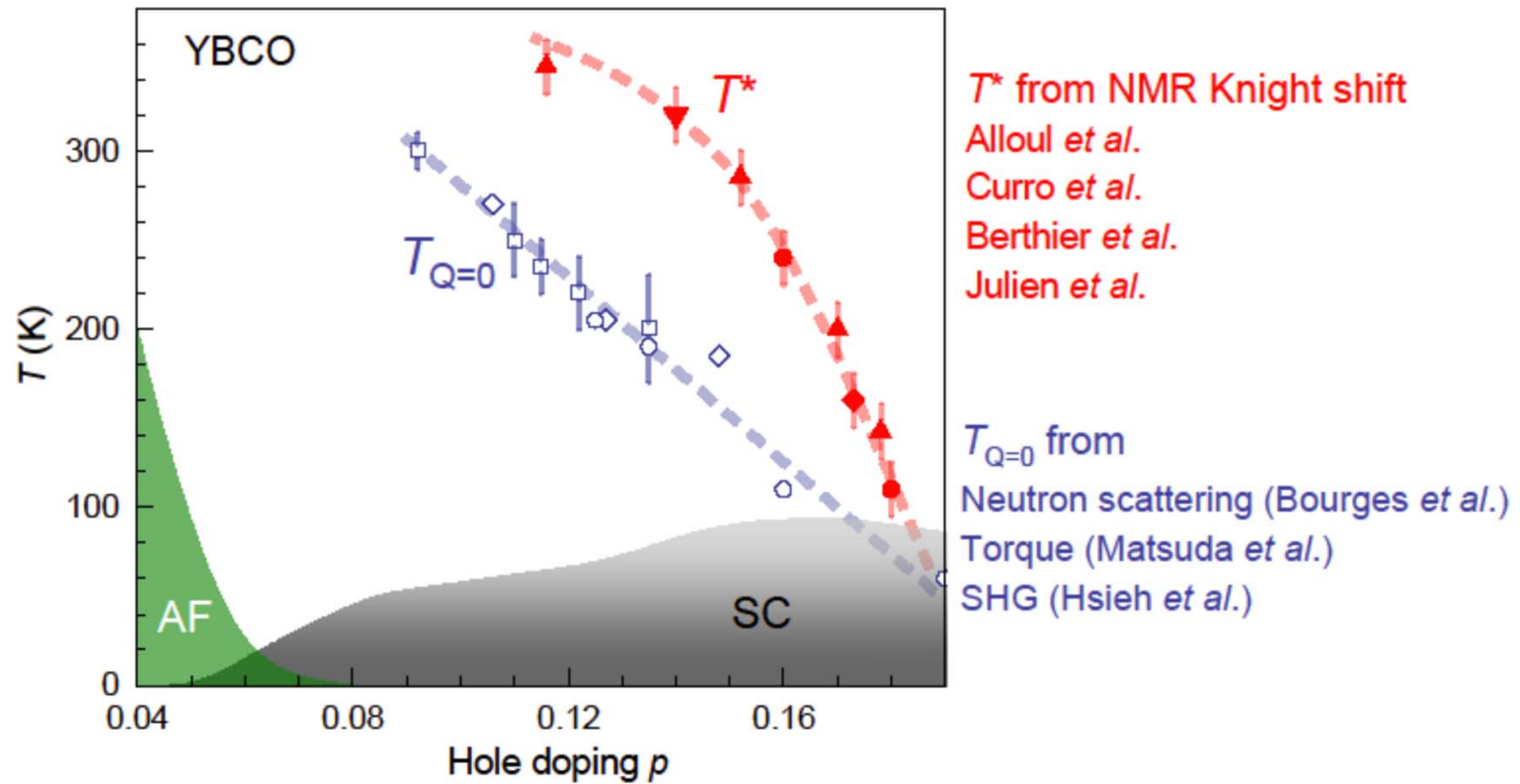
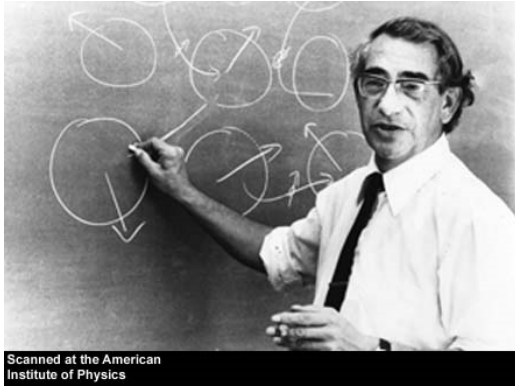


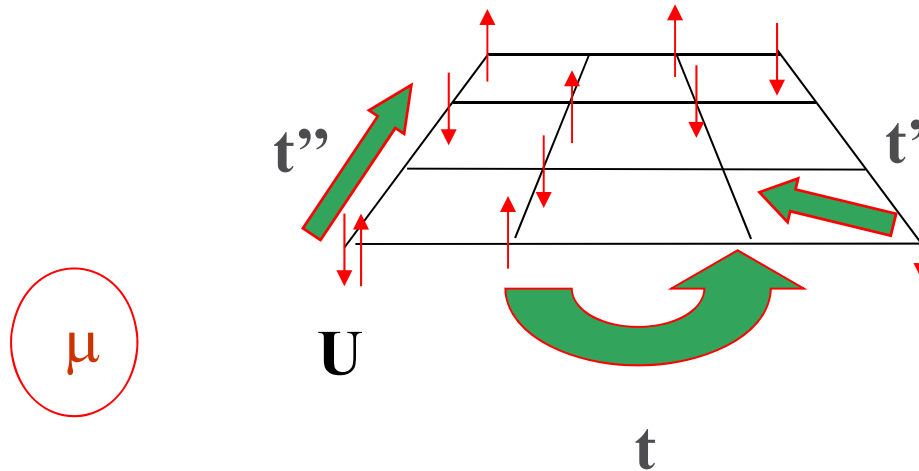
Figure from: Marc-Henri Julien

Model

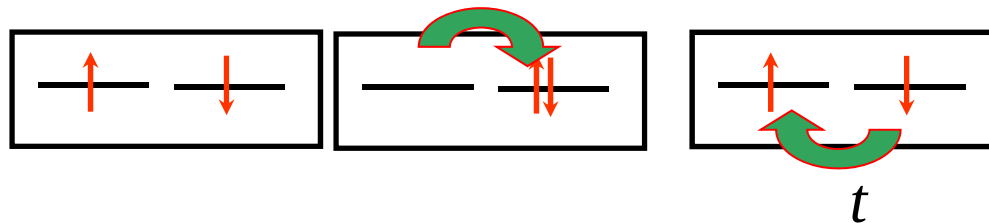
Hubbard model



1931-1980



$$H = -\sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Effective model, Heisenberg: $J = 4t^2 / U$

Attn: Charge transfer insulator

Method

- Dynamical Mean Field Theory
 - clusters
- Concept: atomic-like localized correlations consistent with delocalized aspect

REVIEWS

Maier, Jarrell *et al.*, RMP. (2005)

Kotliar *et al.* RMP (2006)

AMST *et al.* LTP (2006)

Hettler *et al.*, PRB 1998

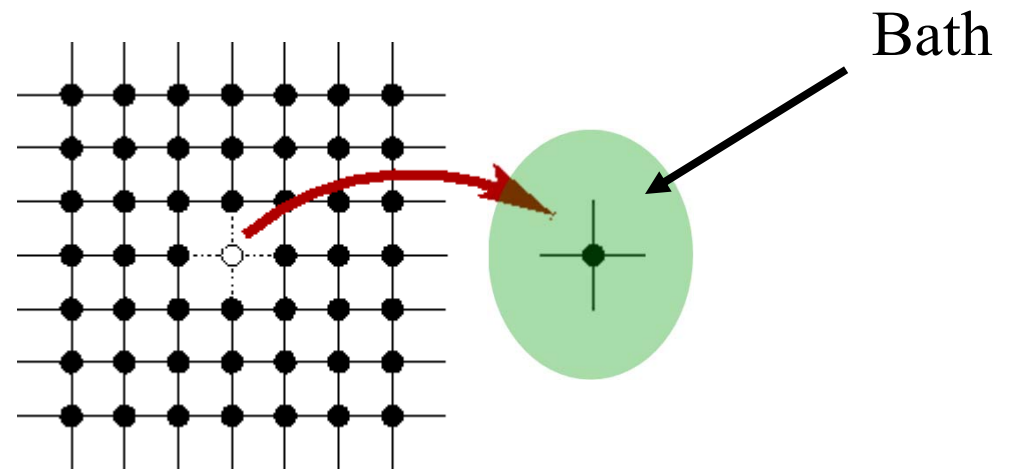
Lichtenstein *et al.*, PRB 2000

Kotliar *et al.*, PRB 2000

M. Potthoff, EJP 2003

Dynamical Mean-Field Theory.

The beginnings in $d = \textit{infinity}$



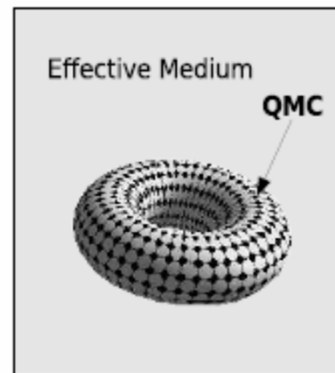
W. Metzner and D. Vollhardt, PRL (1989)

A. Georges and G. Kotliar, PRB (1992)

M. Jarrell PRB (1992)

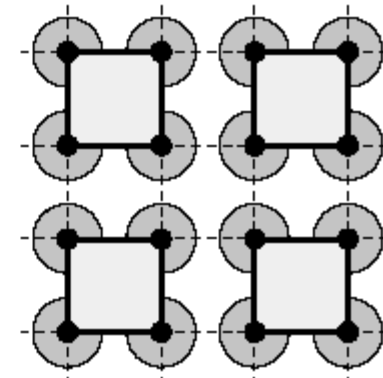
DMFT, ($d = 3$)

2d Hubbard: Quantum cluster method

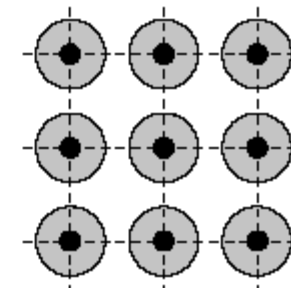


DCA

C-DMFT



DMFT



Hettler ...Jarrell...Krishnamurty PRB **58** (1998)

Kotliar et al. PRL **87** (2001)

M. Potthoff et al. PRL **91**, 206402 (2003).

REVIEWS

Maier, Jarrell et al., RMP. (2005)

Kotliar et al. RMP (2006)

AMST et al. LTP (2006)

+ and -

- Long range order:
 - No mean-field factorization on the cluster
 - Symmetry breaking allowed in the bath (mean-field)
- Included exactly:
 - Short-range dynamical and spatial correlations
- Missing:
 - Long wavelength p-h and p-p fluctuations
 - Hence good when the corresponding correlation lengths are small

Some groups using these methods for cuprates

- Europe:
 - Georges, Parcollet, Ferrero, Civelli, Wu (Paris)
 - Lichtenstein, Potthoff, (Hamburg) Aichhorn (Graz), Liebsch (Jülich) de Medici (Grenoble) Capone (Italy)
- USA:
 - Gull (Michigan) Millis (Columbia)
 - Kotliar, Haule (Rutgers)
 - Jarrell (Louisiana)
 - Maier, Okamoto (Oakridge)
- Japan
 - Imada (Tokyo) Sakai, Tsunetsugu, Motome

Outline

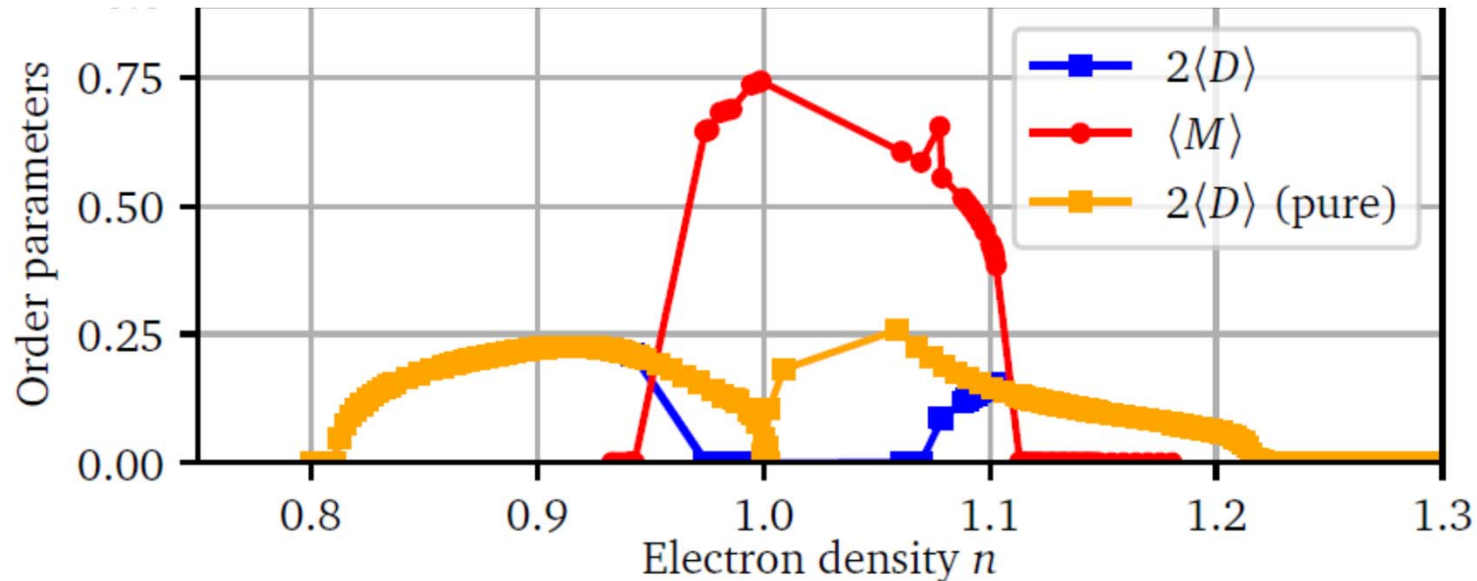
- The model
- The method
- Part I: $T = 0$ phase diagram
- Part II: The pseudogap from Knight shift
- Part III: Specific heat in the strange metal
- Part IV: Strongly correlated superconductivity
- Part V: Perspective

Part I

$T = 0$ phase diagram

$T = 0$ phase diagram

$$U = 8t, t' = -0.3t, t'' = 0.2t$$

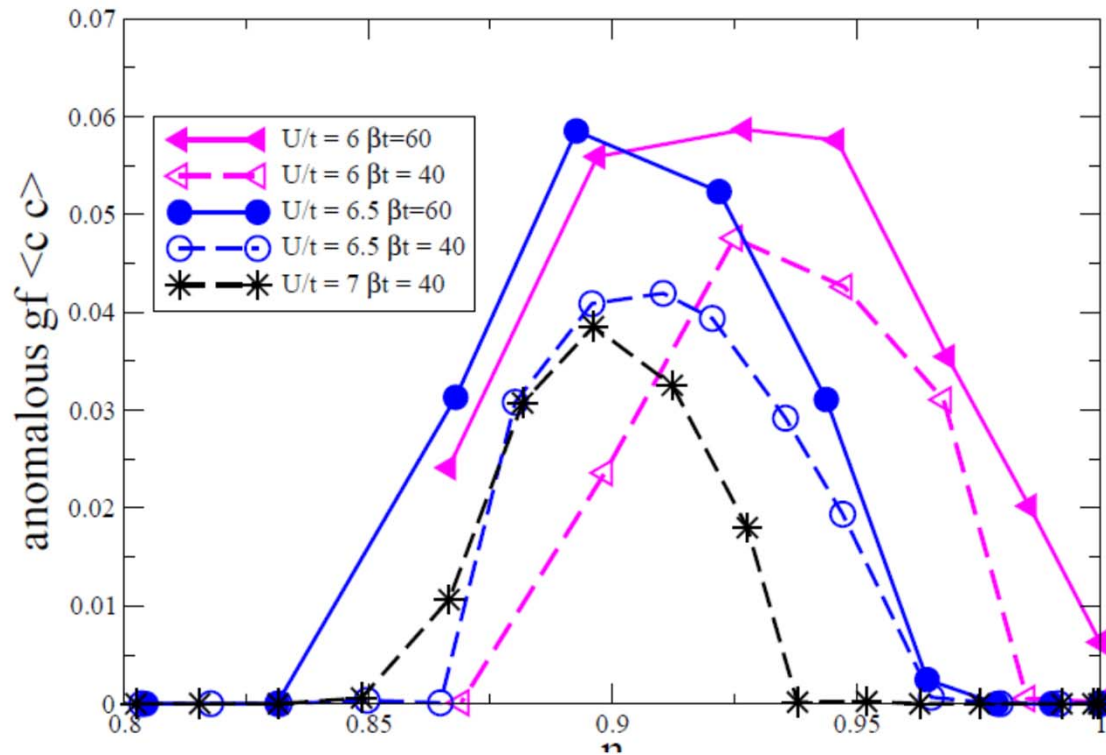


- A. Foley *et al.* Phys. Rev. B **99**, 184510 (2019)
S. S. Kancharla, *et al.* Phys. Rev. B **77**, 184516 (2008)
D. Sénéchal, *et al.* Phys. Rev. Lett. **94**, (2005)
M. Jarrell *et al.* EPL **56** 563, (2001)

CDMFT 4 sites

Fall at half-filling without AFM

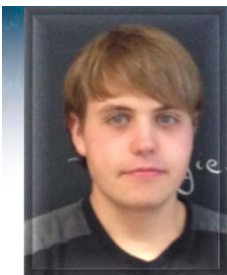
$t' = 0$ DCA, 8 site



Gull *et al.* Phys. Rev. Lett. **110**, 216405 (2013)

Part II:

The pseudogap



Simon Bergeron



Maxime Charlebois



Patrick Sémon



Alexis Reymbaut

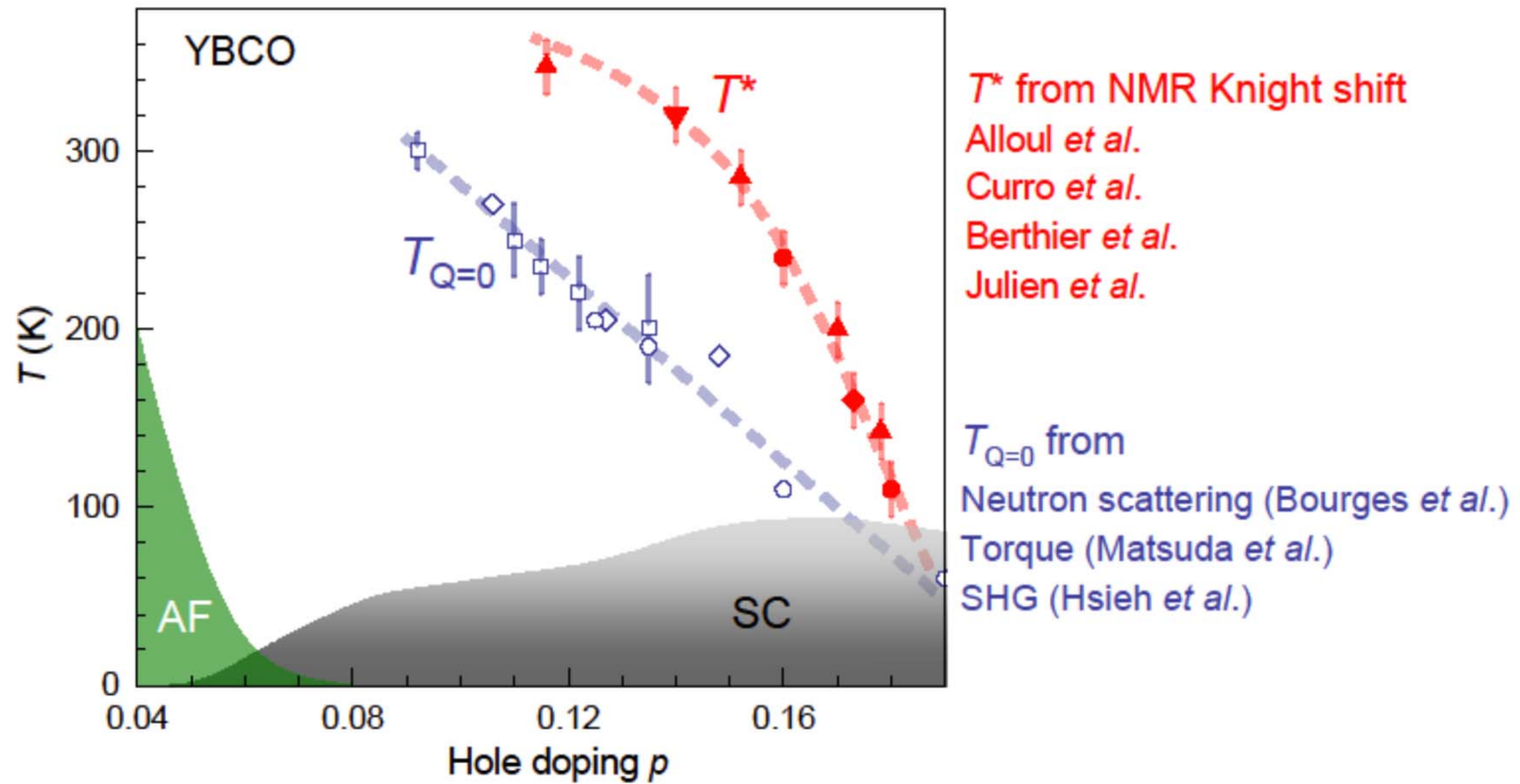
R. Garioud

The pseudogap from Knight shift

A. Reymbaut, *et al.* Phys. Rev. Research **1**, 023015 (2019)



Marion Thénault



Thanks: Marc-Henri Julien

Knight shift ($Q=0$ spin susceptibility)

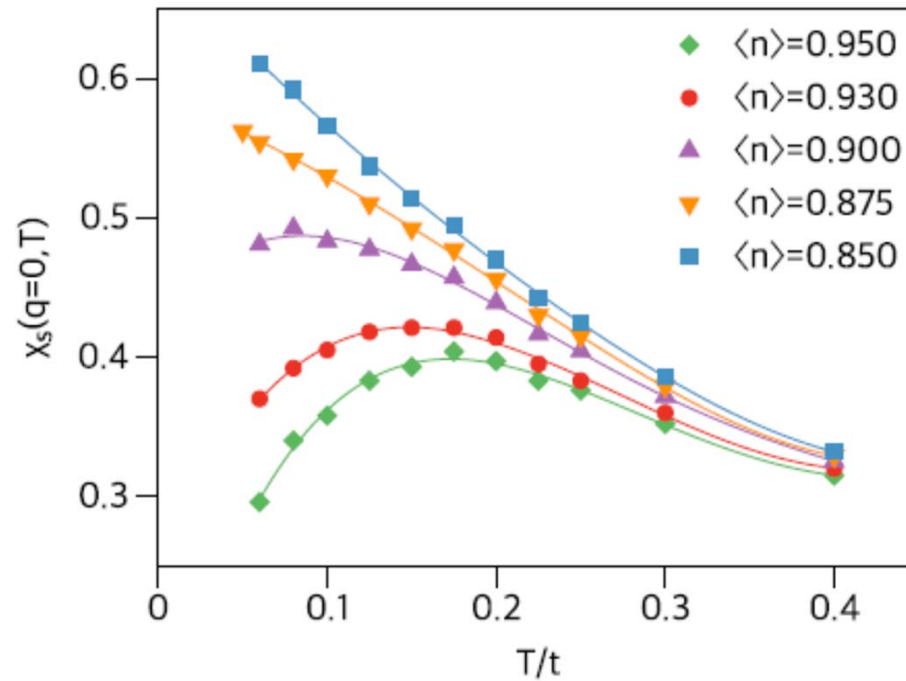


Fig. 3 Temperature and doping dependence of the $q=0$ spin susceptibility. At the smaller dopings (larger filling $\langle n \rangle$), $\chi_s(T)$ exhibits a peak in the temperature dependence indicating the opening of a PG

DCA 12 sites, $t'=0$, $U = 7$

T.A. Maier, D.J. Scalapino, npj Quantum Materials (2019)

Comparison

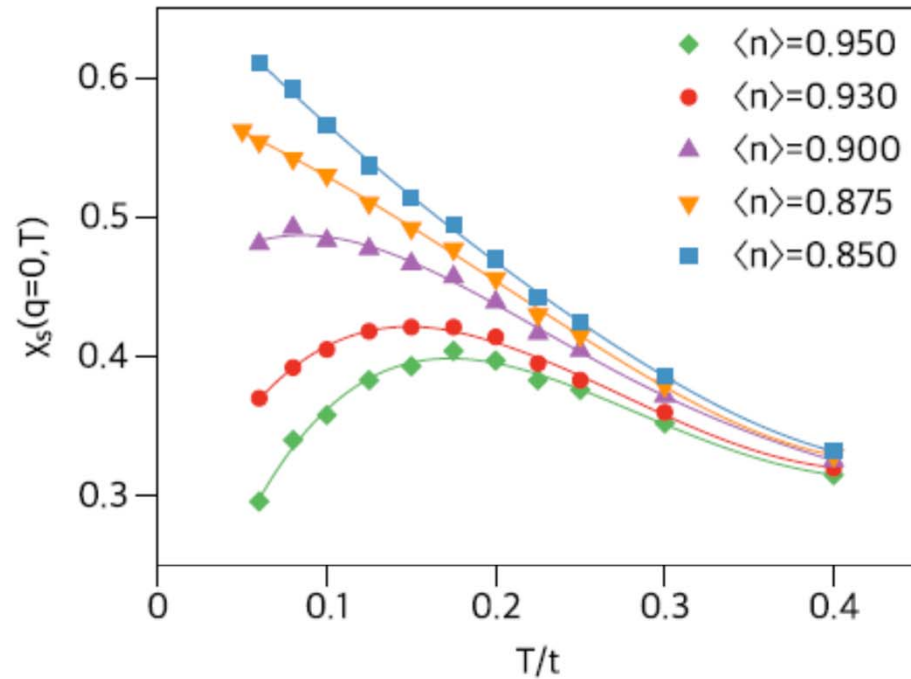
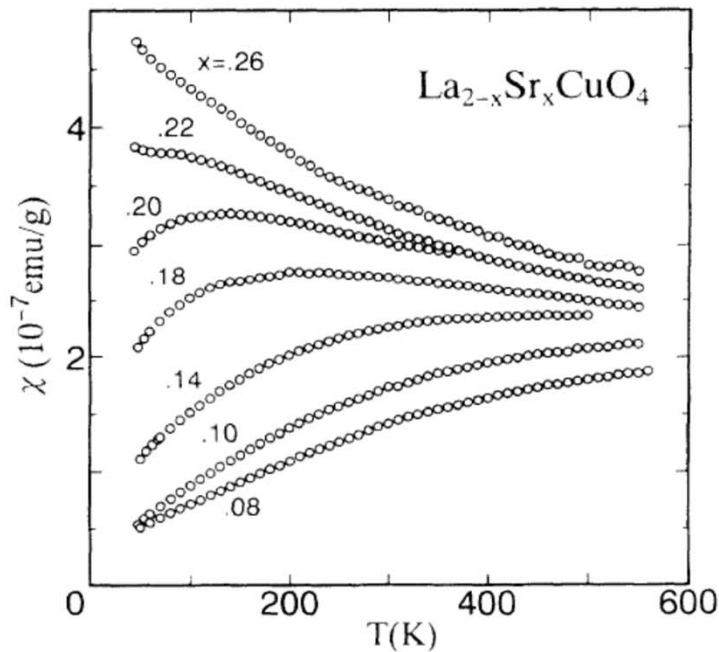
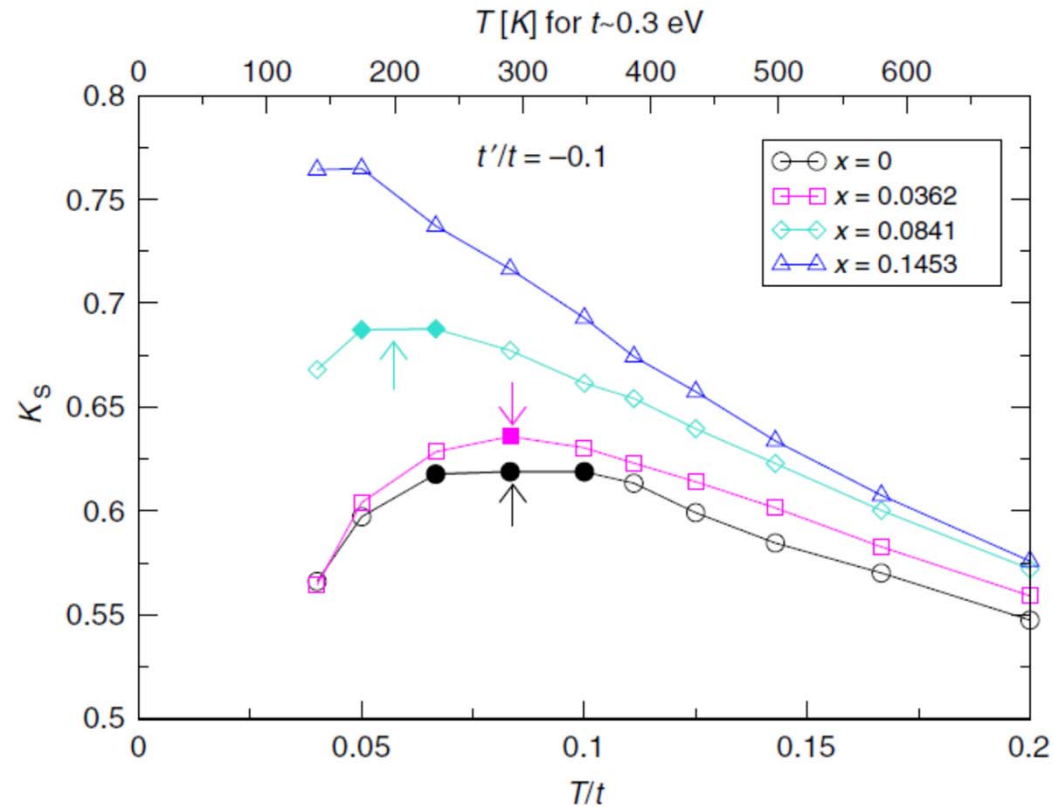


Fig. 3 Temperature and doping dependence of the $q=0$ spin susceptibility. At the smaller dopings (larger filling $\langle n \rangle$), $\chi_s(T)$ exhibits a peak in the temperature dependence indicating the opening of a PG

Knight shift

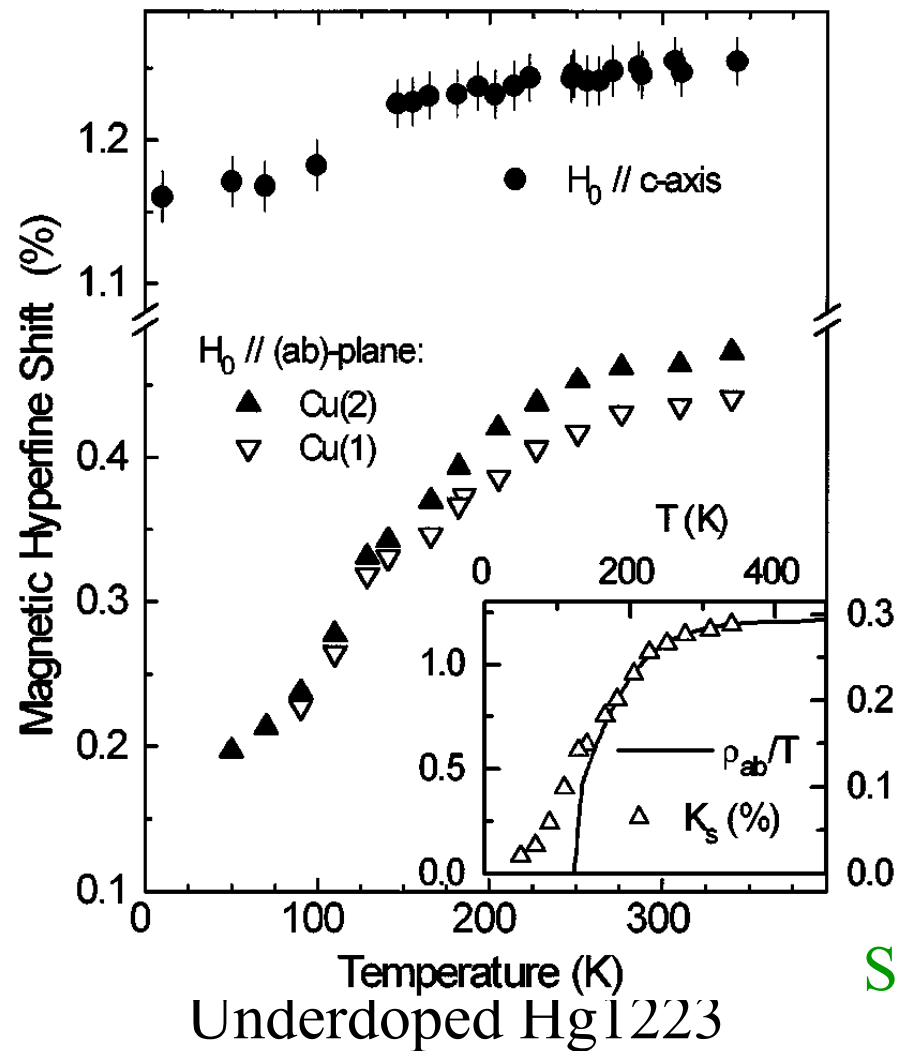


DCA 8 sites, $U = 6$, $t' = -0.1t$

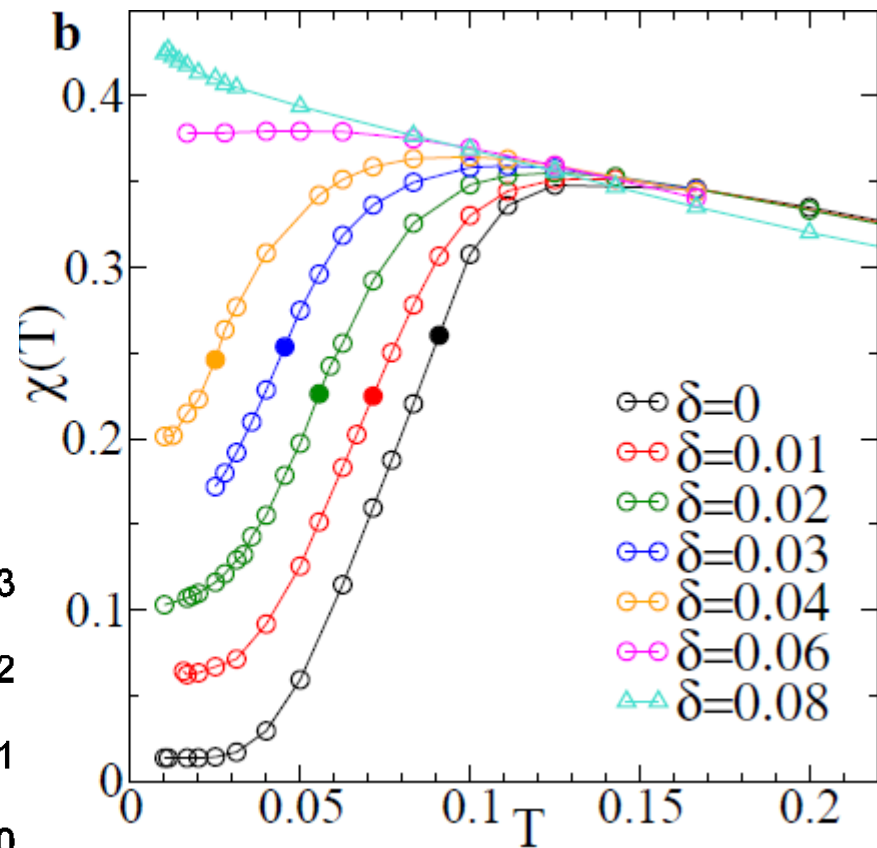
Chen, LeBlanc, Gull, Nature Com. Apr. 2017

See also Jarrell *et al.* 2001, 2002

Spin susceptibility



Julien et al. PRL **76**, 4238 (1996)

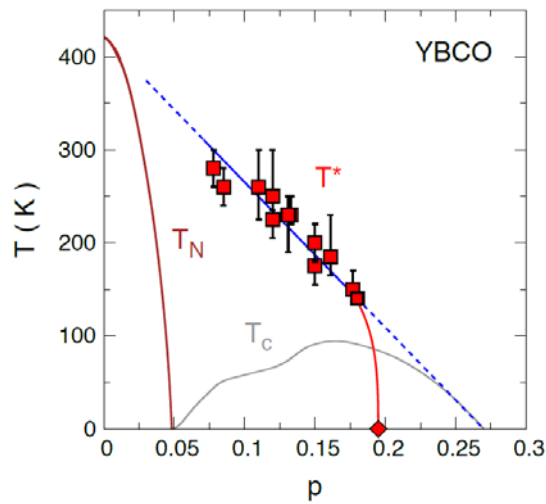


Sordi et al., Sci. Rep. 2 547 (2012);

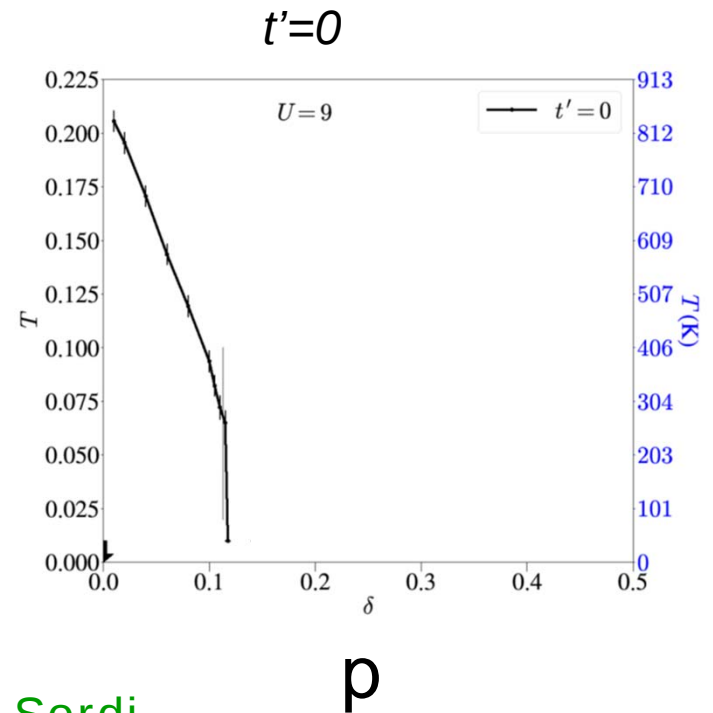
$$U = 6.2, t'=0$$

Results T^*

G.Sordi et al. Phys. Rev. B **87**,
041101(R) (2013)



Cyr-Choinières et al. Phys. Rev. B **97**, 064502



A. Reymbaut, M. Thénault, L. Fratino, G. Sordi,
P. Sémon, AMT, Phys. Rev. Research **1**, 023015 (2019)

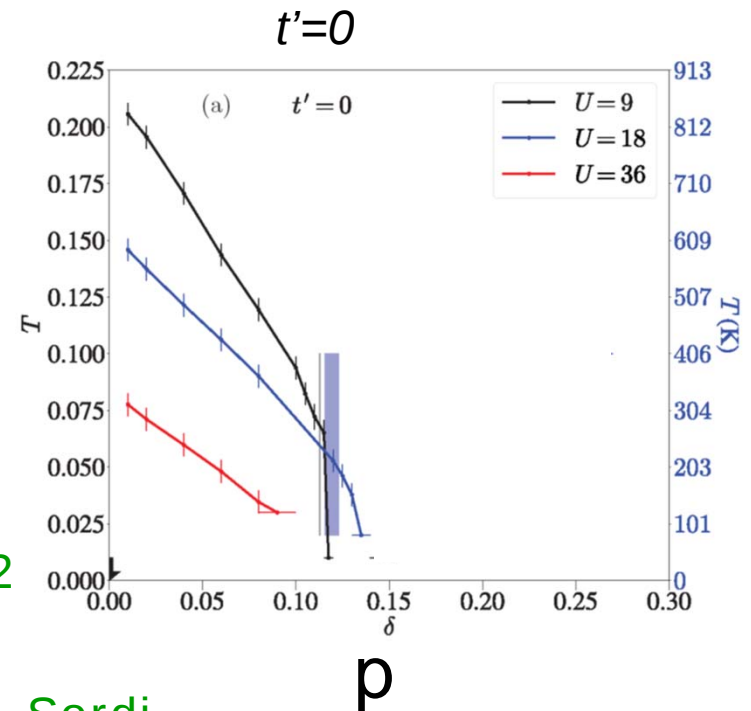
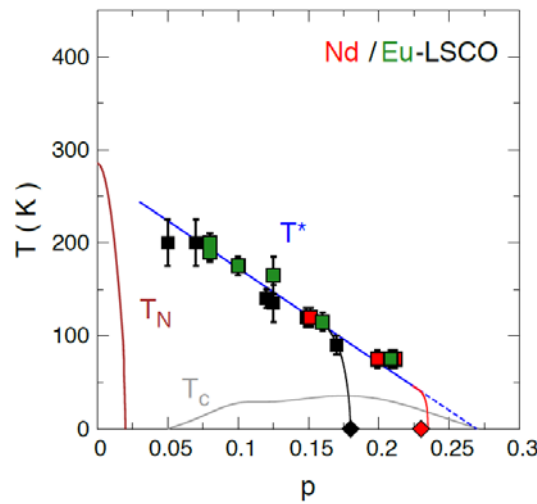
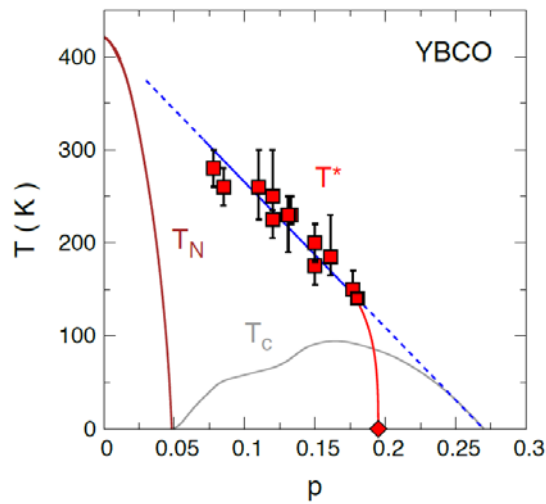
W Wu, A Georges, M Ferrero Phys. Rev. X **8**, 021048 (2018).

Bragança, Sakai, Aguiar, Civelli, PRL **120**, 067002 (2018)

Results T^*

$$k_B T^* \sim J$$

$$J = 4t^2/U$$



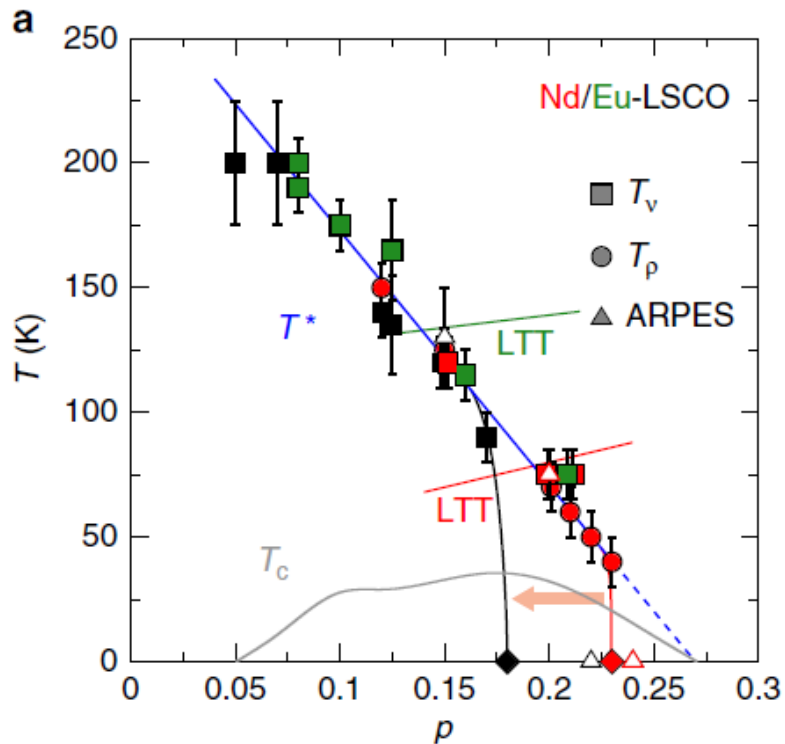
Cyr-Choinières et al. Phys. Rev. B **97**, 064502

A. Reymbaut, M. Thénault, L. Fratino, G. Sordi,
P. Sémon, AMT, Phys. Rev. Research **1**, 023015 (2019)

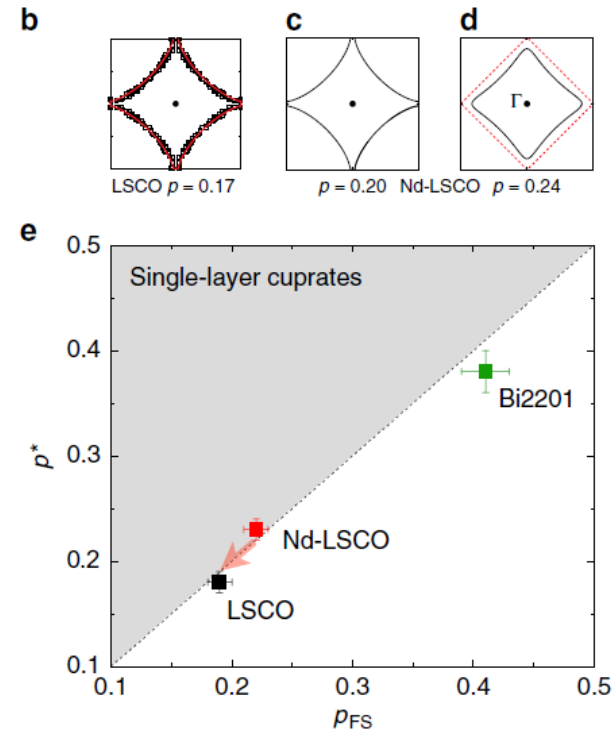
W Wu, A Georges, M Ferrero Phys. Rev. X **8**, 021048 (2018).

Bragança, Sakai, Aguiar, Civelli, PRL **120**, 067002 (2018)

Results : effect of t' on T^*



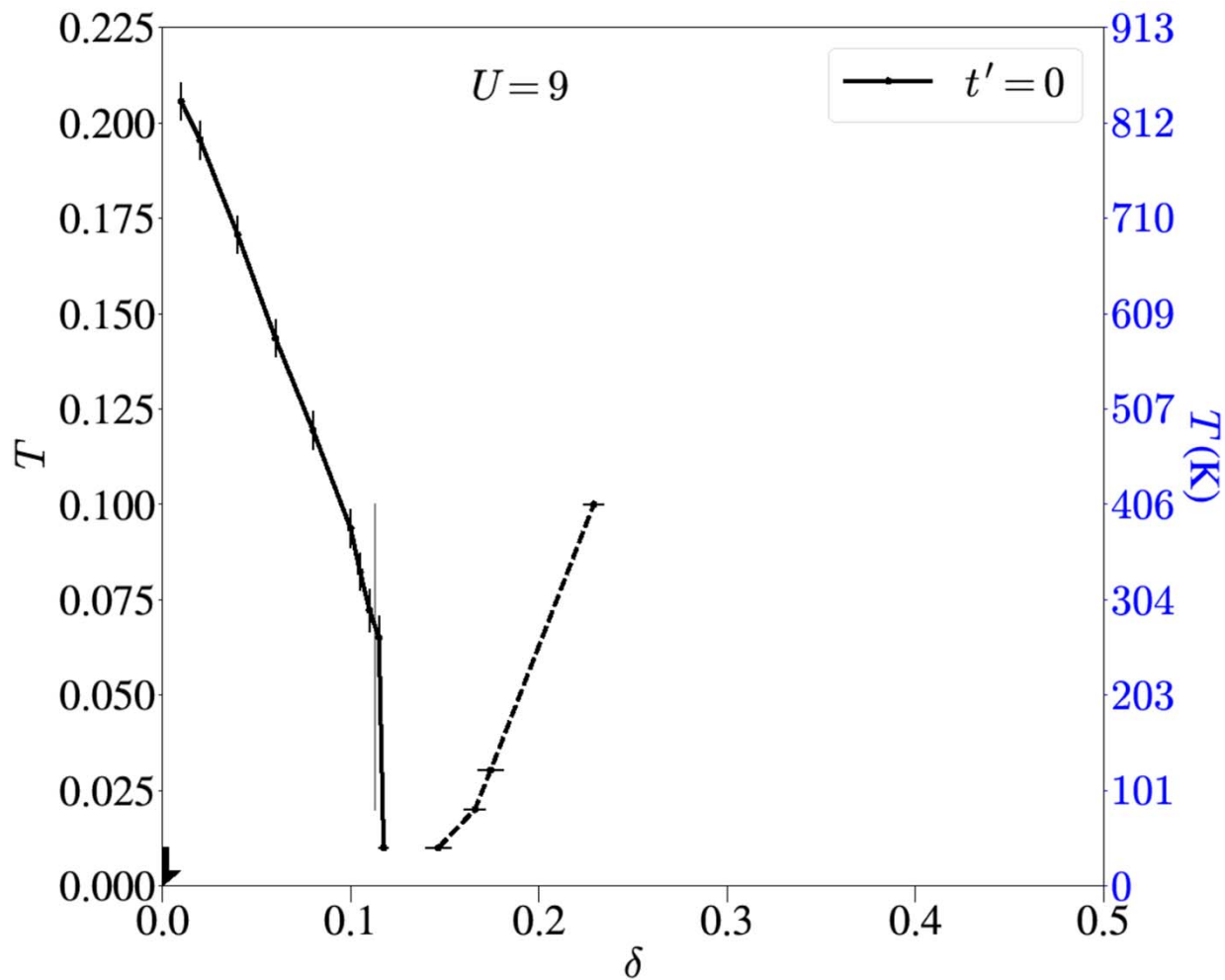
Doiron-Leyraud *et al.*
Nature Comm. **8** 2044



$$p^* < p_{fs}$$

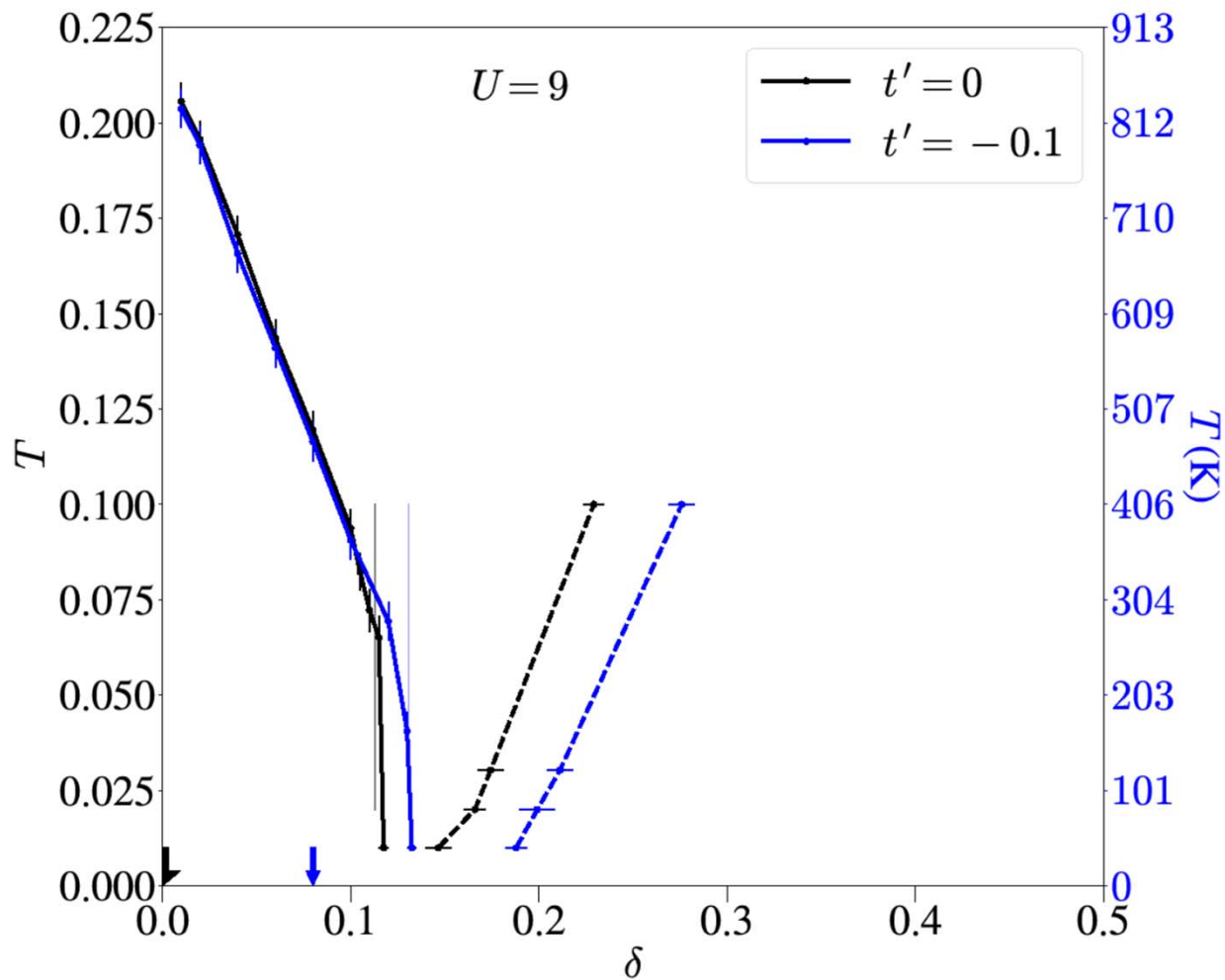
Doiron-Leyraud *et al.*
Nature Comm. **8** 2044

Effect of t'



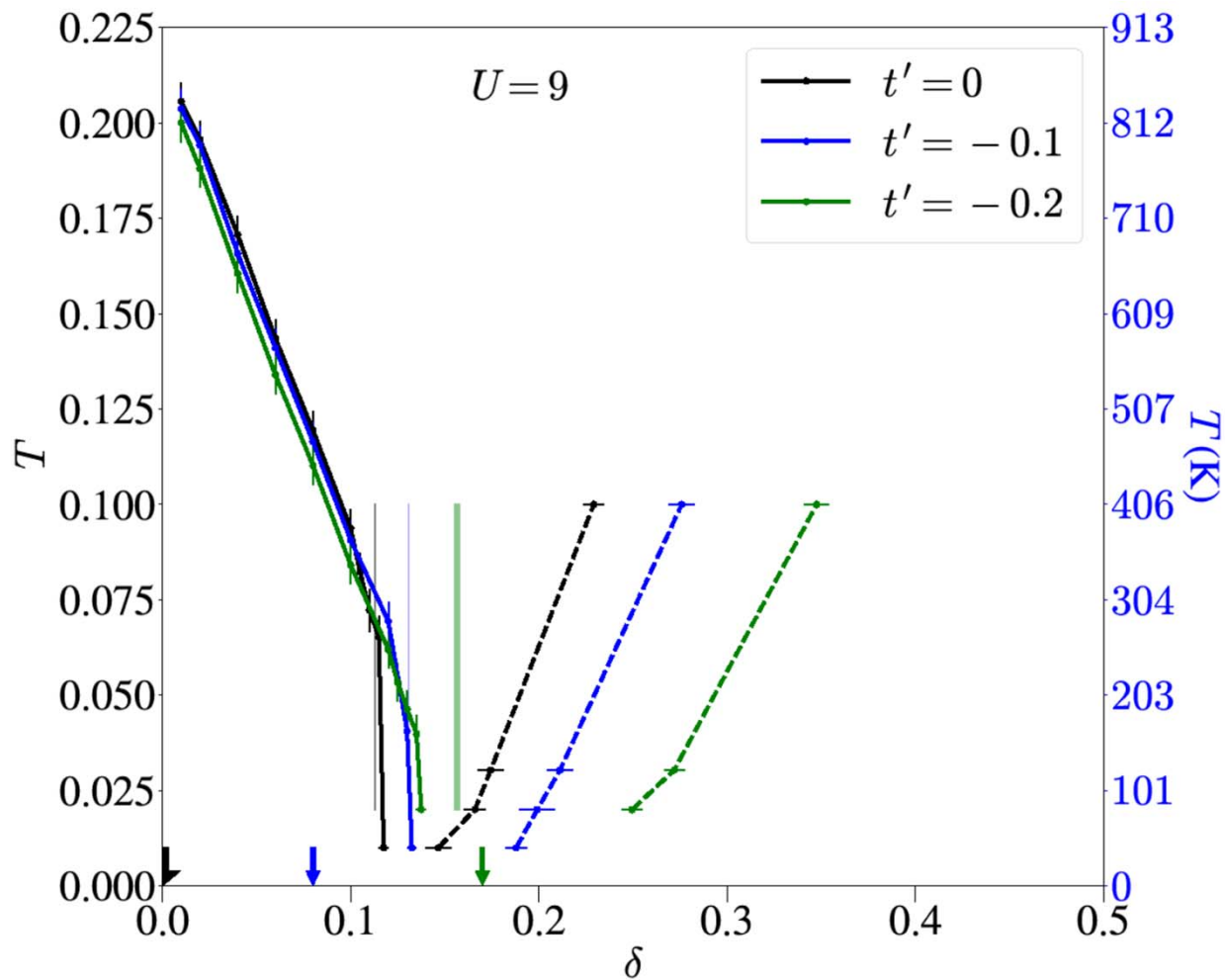
A. Reymbaut *et al.*
Phys. Rev. Research **1**, 023015 (2019)

Effect of t'



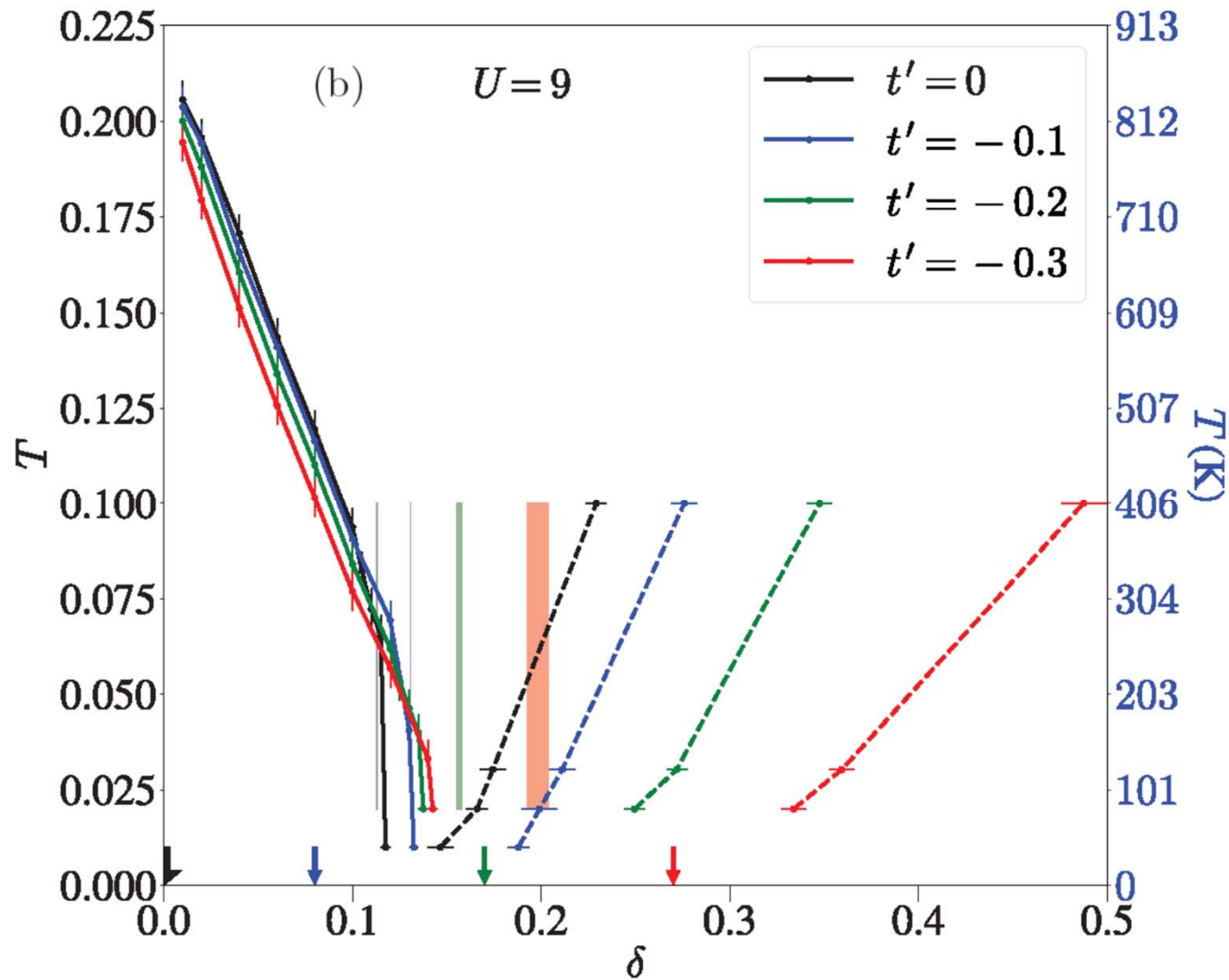
A. Reymbaut *et al.*
Phys. Rev. Research **1**, 023015 (2019)

Effect of t'



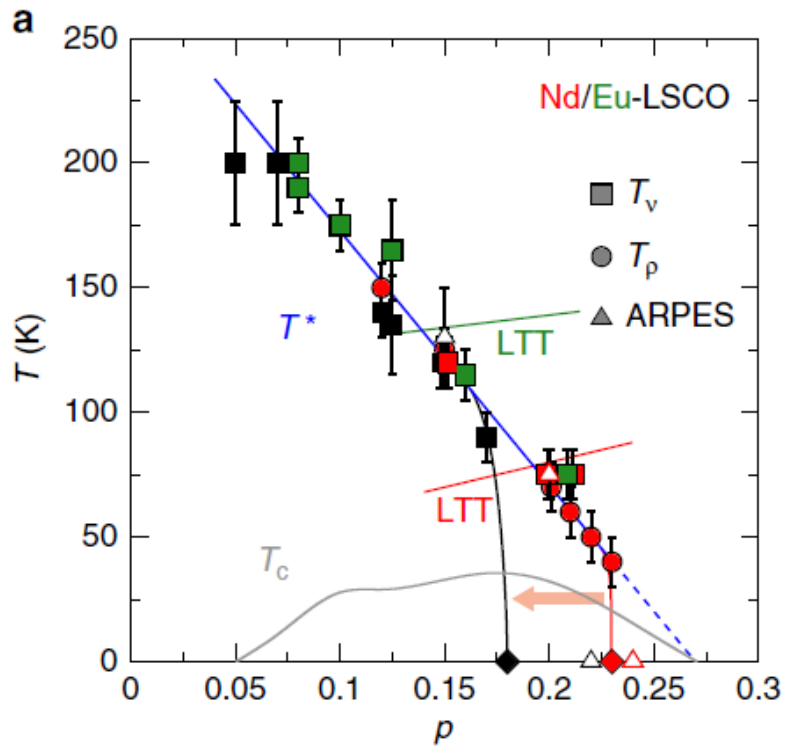
A. Reymbaut *et al.*
Phys. Rev. Research **1**, 023015 (2019)

Effect of t'

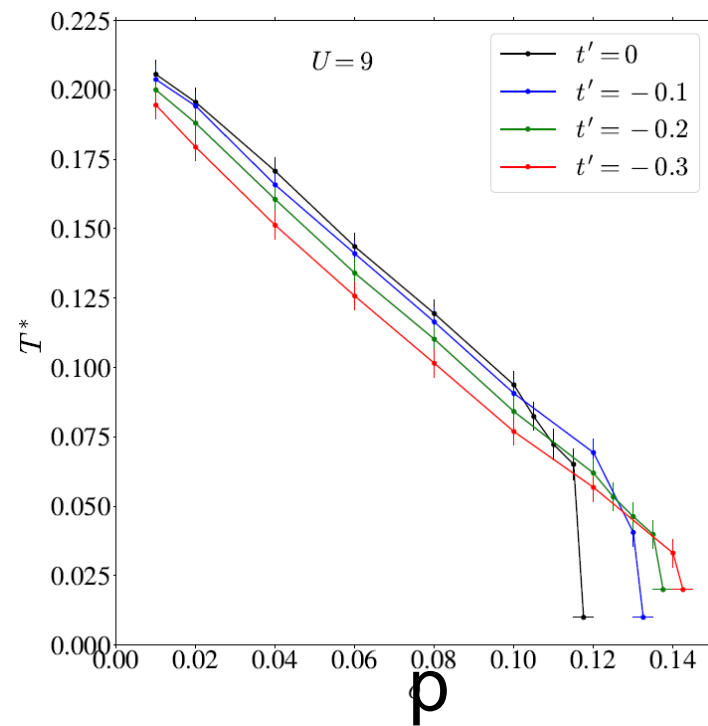


A. Reymbaut et al.
 Phys. Rev. Research **1**, 023015 (2019)

Results : effect of t' on T^*

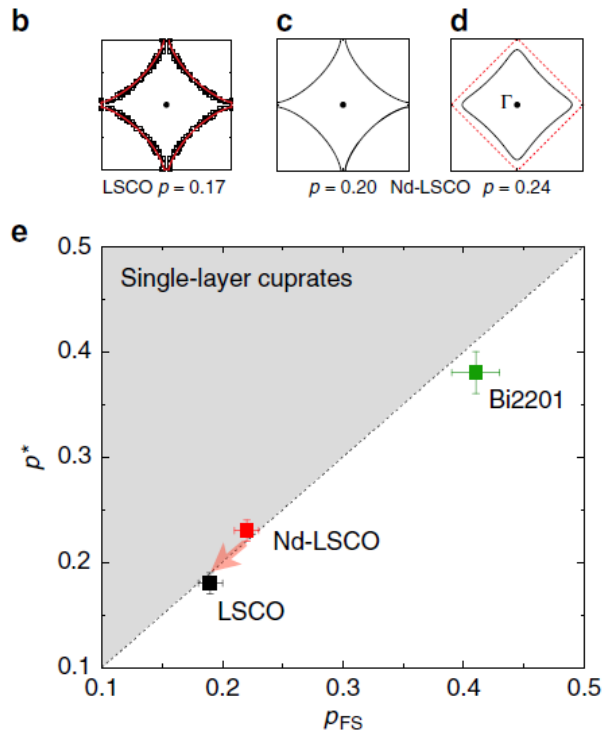


Doiron-Leyraud *et al.*
Nature Comm. **8** 2044



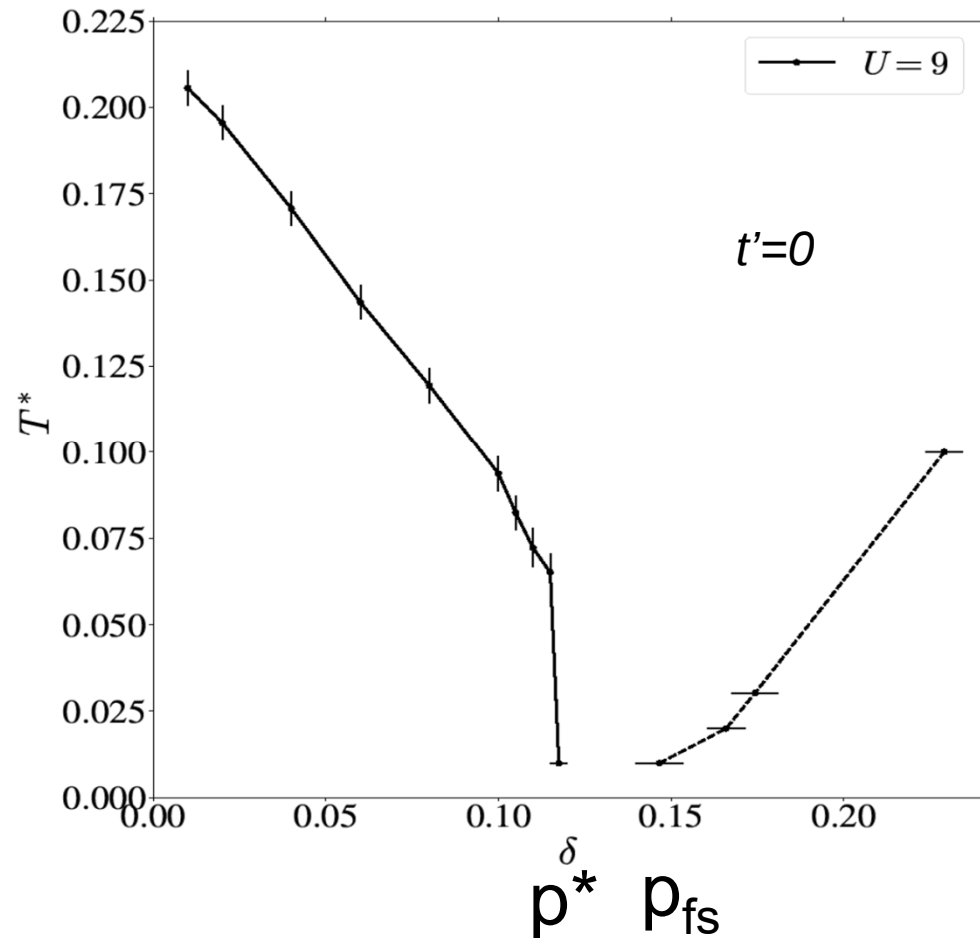
A.Reymbaut, *et al.*
Phys. Rev. Research **1**, 023015 (2019)

Results: van Hove singularity



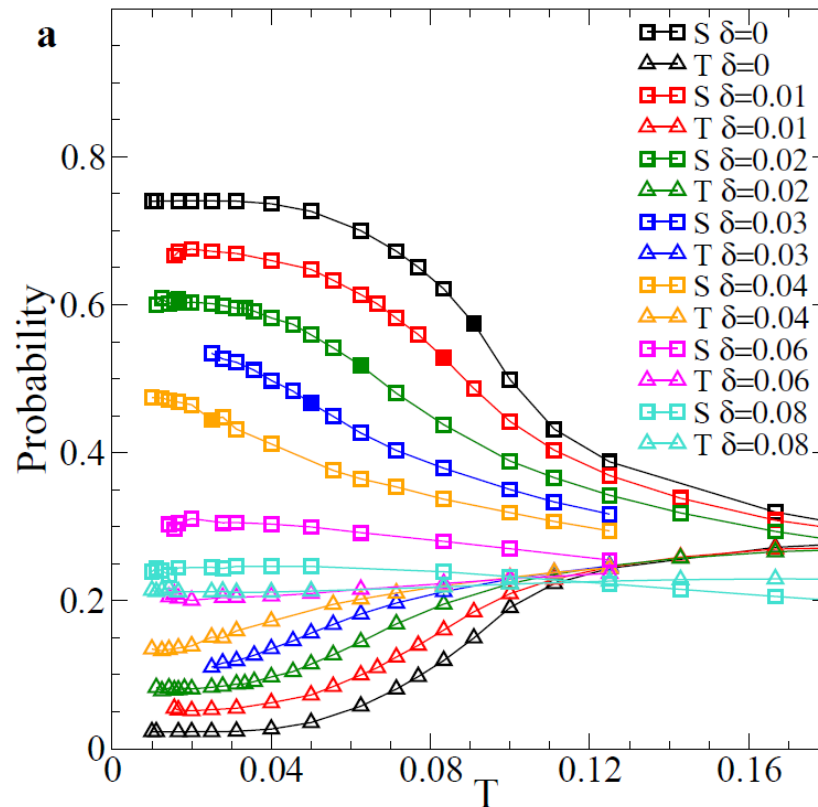
$$p^* < p_{fs}$$

Doiron-Leyraud *et al.*
Nature Comm. **8** 2044



A.Reymbaut, *et al.*
Phys. Rev. Research **1**, 023015 (2019)
Sordi *et al.*, Sci. Rep. **2** 547 (2012);

Physics: Plaquette eigenstates



$$U = 6.2; t' = 0$$

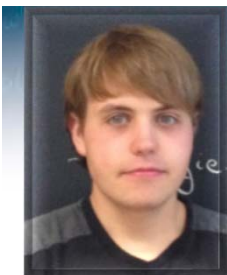
Sordi et al., Sci. Rep. 2 547 (2012);

See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges
PRB **80**, 064501 (2009)

Part III:

Specific heat in the strange metal phase



Simon Bergeron



Maxime Charlebois



Patrick Sémon



Alexis Reymbaut

Specific heat in the strange metal phase

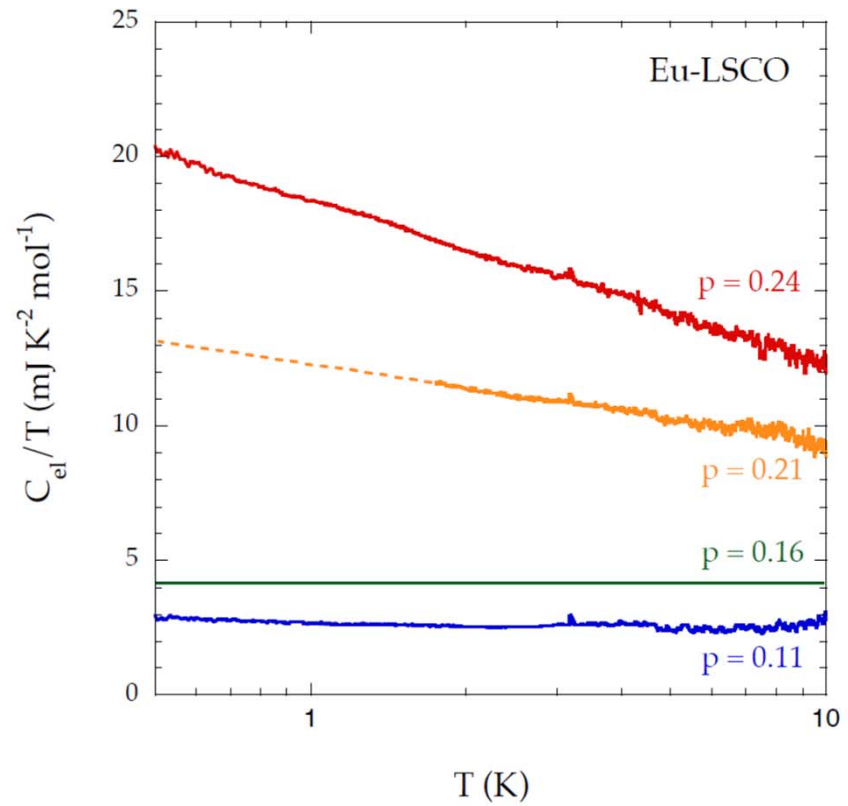
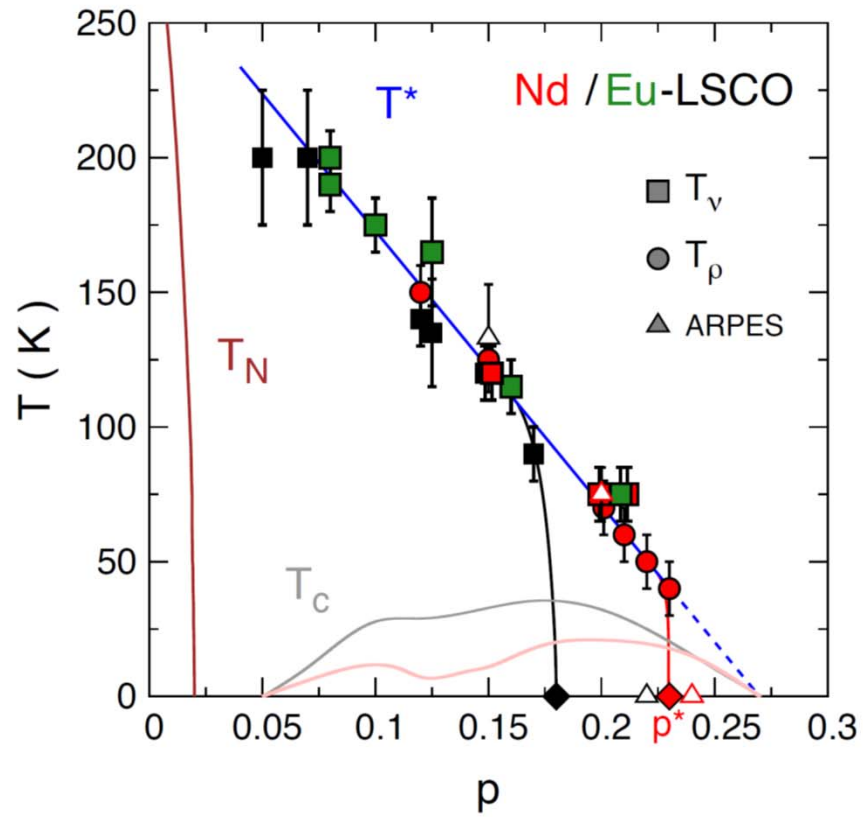
A. Reymbaut, *et al.* Phys. Rev. Research **1**, 023015 (2019)



Marion Thénault

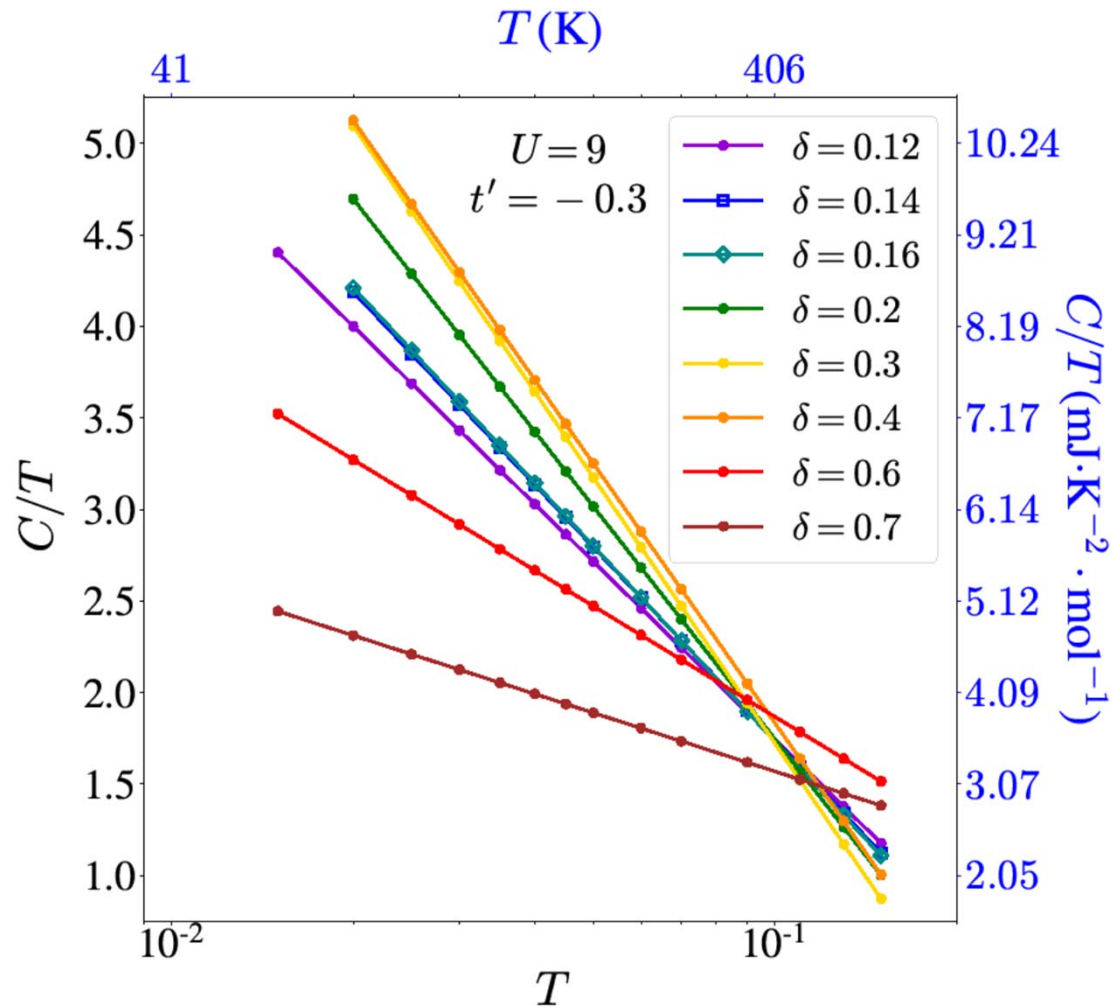
R. Garioud

Specific heat in the strange metal phase



B. Michon, C. Girod, Taillefer, Klein, Nature **567**, 218 (2019)

Specific heat in the strange metal phase



Part IV:

Strongly correlated superconductivity



Giovanni Sordi



Patrick Sémon



Lorenzo Fratino

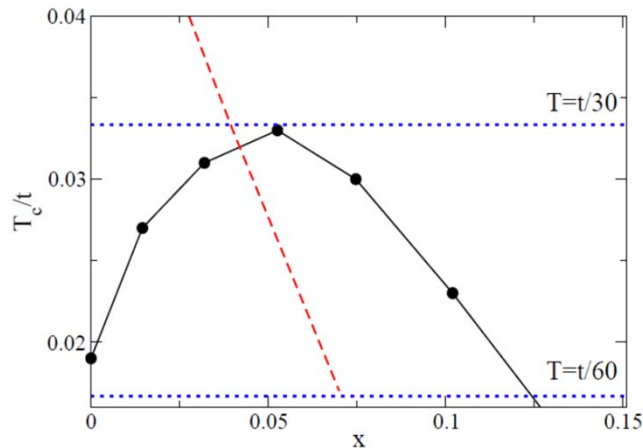
Superconductivity for large U

Sordi et al. PRL **108**, 216401 (2012)

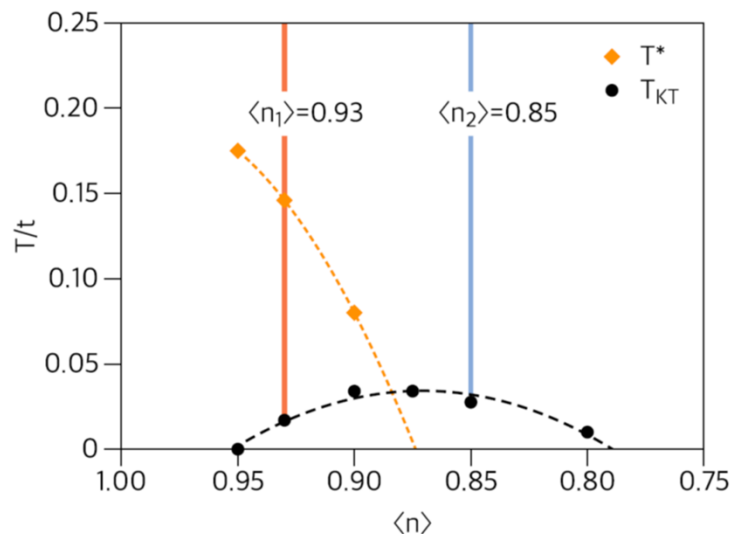
Fratino et al.

Sci. Rep. **6**, 22715 (2016)

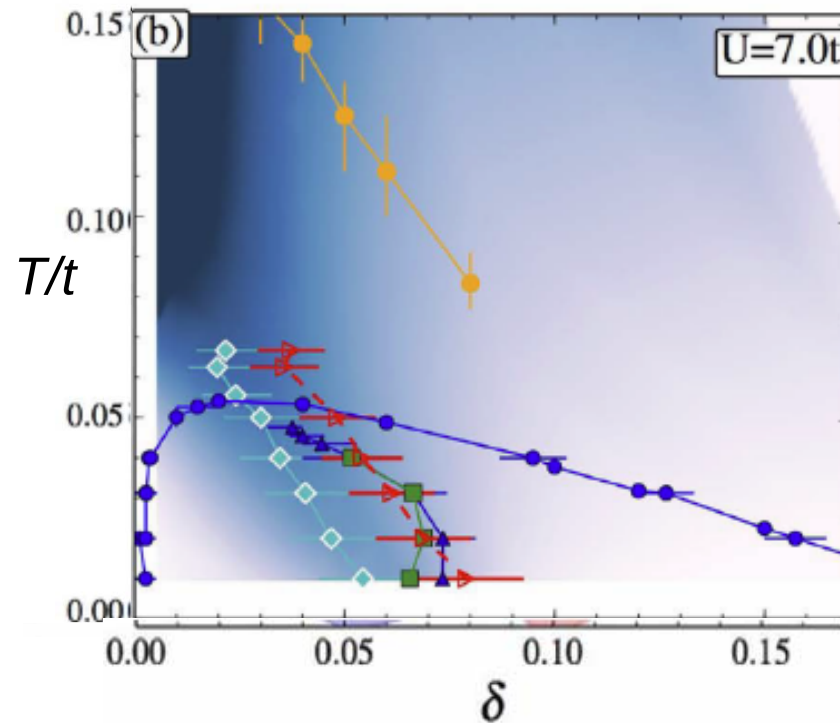
Superconducting transition temperature



E. Gull and A. J. Millis
Phys. Rev. B 88, 075127



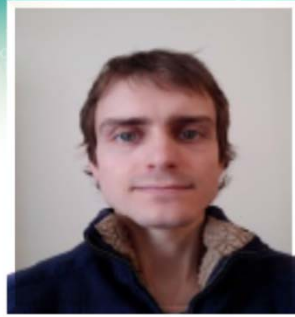
T.A. Maier, D.J. Scalapino, npj Quantum Materials (2019)



Fratino et al.
Sci. Rep. 6, 22715



Olivier Simard



Charles-David Hébert



Alexandre Foley



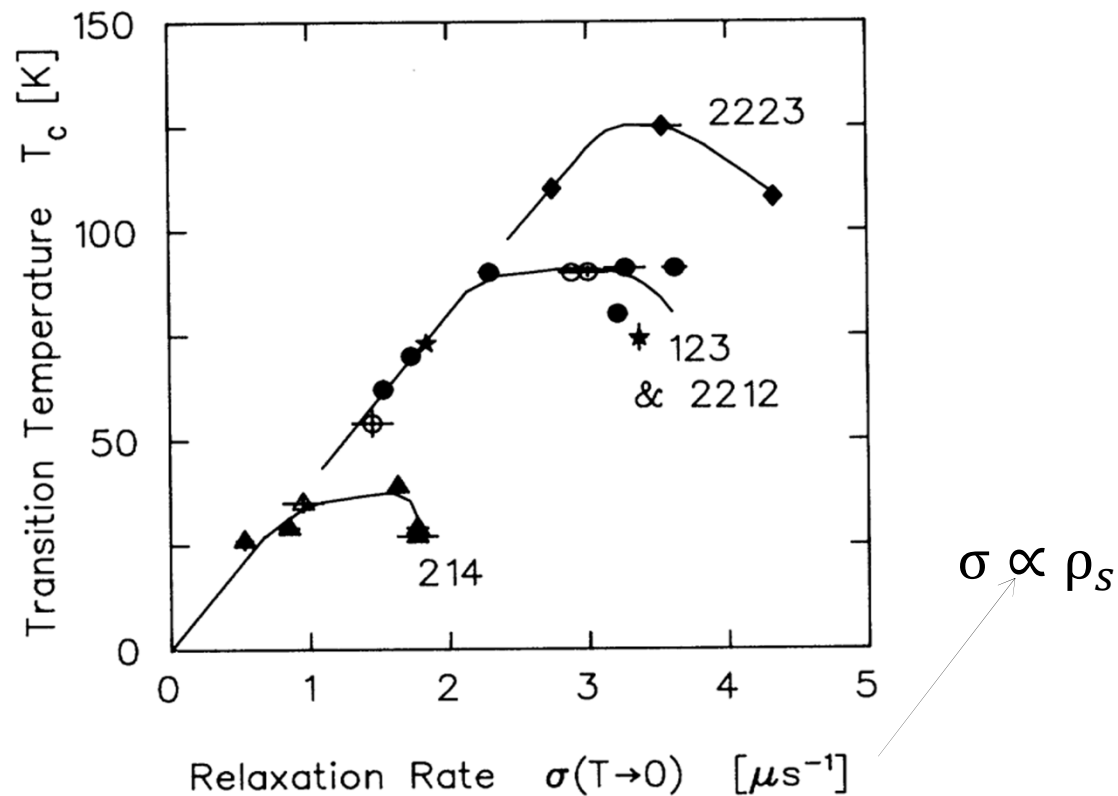
David Sénéchal

What causes T_c to drop near $n = 1$?

O. Simard, C.-D. Hébert, A. Foley, A.-M.S. Tremblay, D. Sénéchal, Phys. Rev. B **100**, 094506 (2019)

What causes T_c to drop?

Phase fluctuations? Emery Kivelson Nature **374** (1995)



Uemura, Y.J. *et al.*, PRL vol.62, (1989)
Tallon *et al.*, PRB **68**, 180501(R) (2003)

Superfluid stiffness $T=0$

$$j = -\rho_s A$$

$$d = 2$$

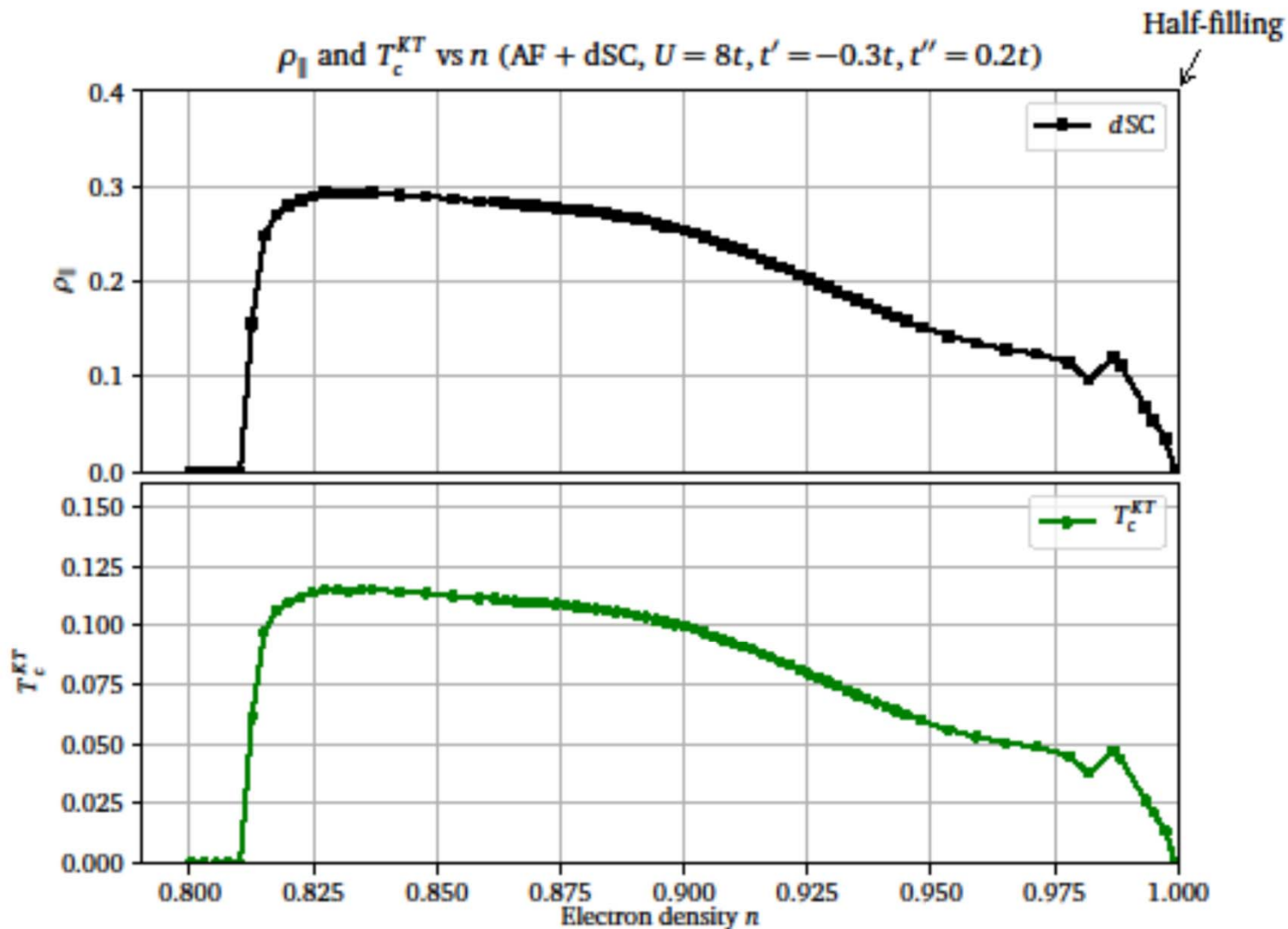
$$\frac{1}{\lambda^2} = \rho_s \mu_0$$

$$T_c^{KT} = \frac{\pi}{8e^2} \rho_s(T_c^{KT})$$

$$T_c^{KT} < \frac{\pi}{8e^2} \rho_s(T_c^{KT} = 0)$$

$$\hbar = 1 ; k_B = 1$$

Emery Kivelson, Nature **374**, 434 (1995)
Metzner, Yamase, Phys. Rev. B **100**, 014504 (2019)
Hazra, Verma, and Randeria, Phys. Rev. X **9**, 031049 (2019)



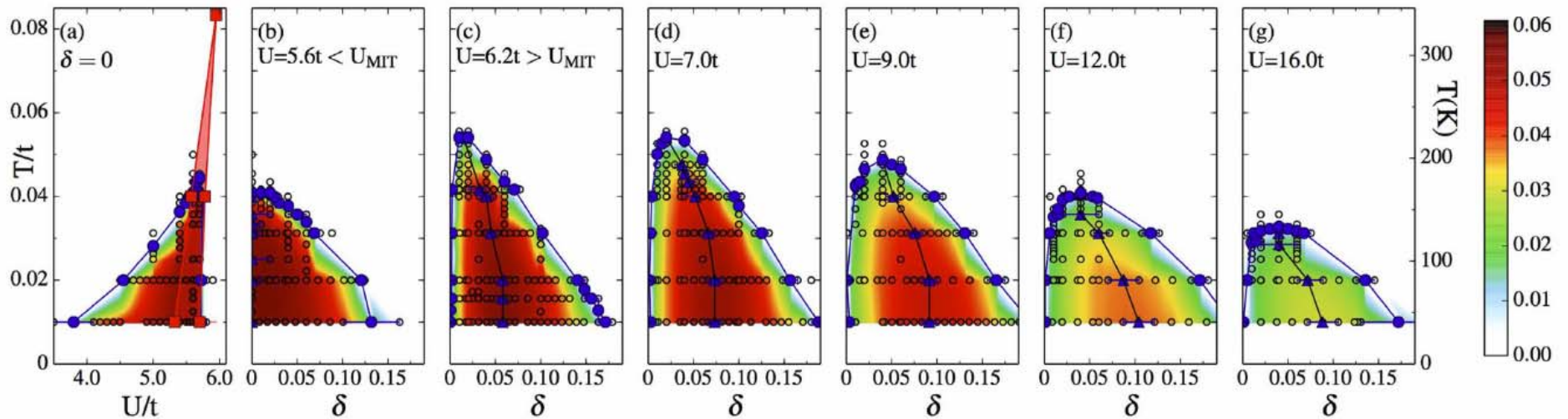
O. Simard, *et al.* Phys. Rev. B **100**, 094506 (2019)

See also E. Gull, A.J. Millis, Phys. Rev. B **88**, 075127 (2013)



What energy scale controls T_c ?

T_c controlled by J



Fratino et al.
Sci. Rep. **6**, 22715

Some experiments that suggest $T_c < T_{\text{pair}} < T^*$

T. Kondo *et al.* PRL **111** (2013)

Kondo, Takeshi, et al. Kaminski Nature Physics 2011, **7**, 21-25

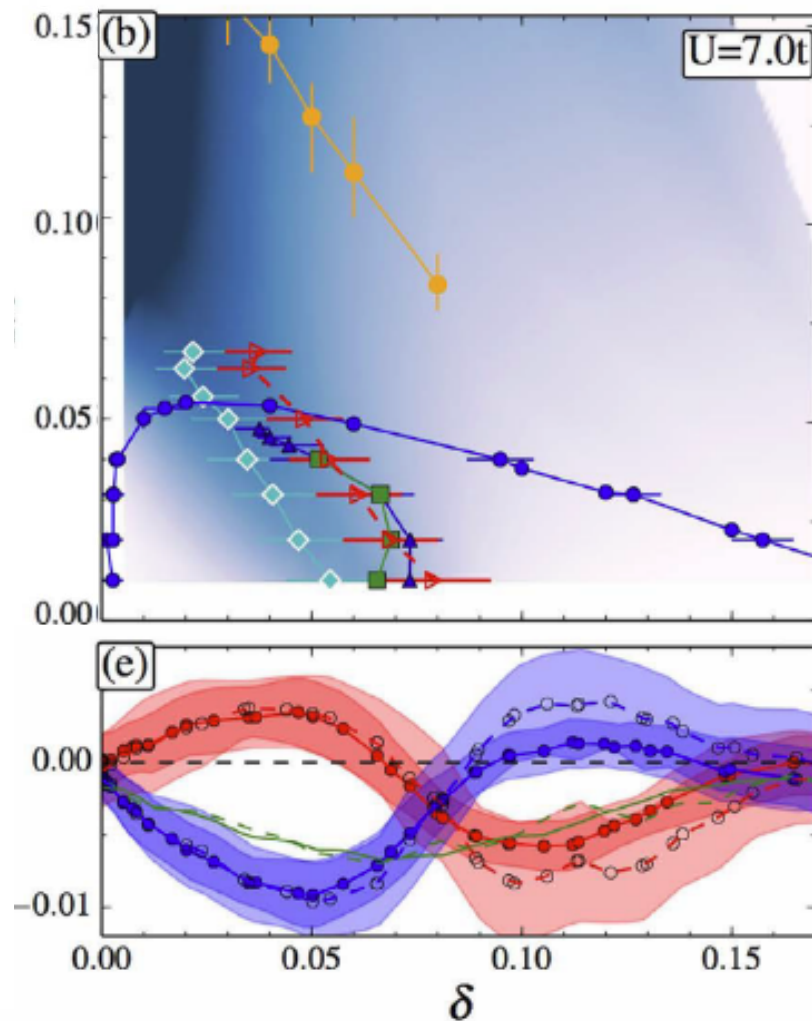
A. Pushp, Parker, ... A. Yazdani, Science **364**, 1689 (2009)

Lee ...Tajima (Osaka) <https://arxiv.org/pdf/1612.08830>

Patrick M. Rourke, et al. Hussey Nature Physics **7**, 455–458 (2011)

Lee et al. J. Phys. Soc. Jpn. **86**, 023701 (2017)

Condensation energy



Fratino et al.
Sci. Rep. 6, 22715

Theory, see also
Jarrel PRL
(2004), Gull
Millis PRB
(2014)

Experiments:
Bontemps,
Santander-Syro
Van der Marel ...

Part V:

Perspective



Giovanni Sordi

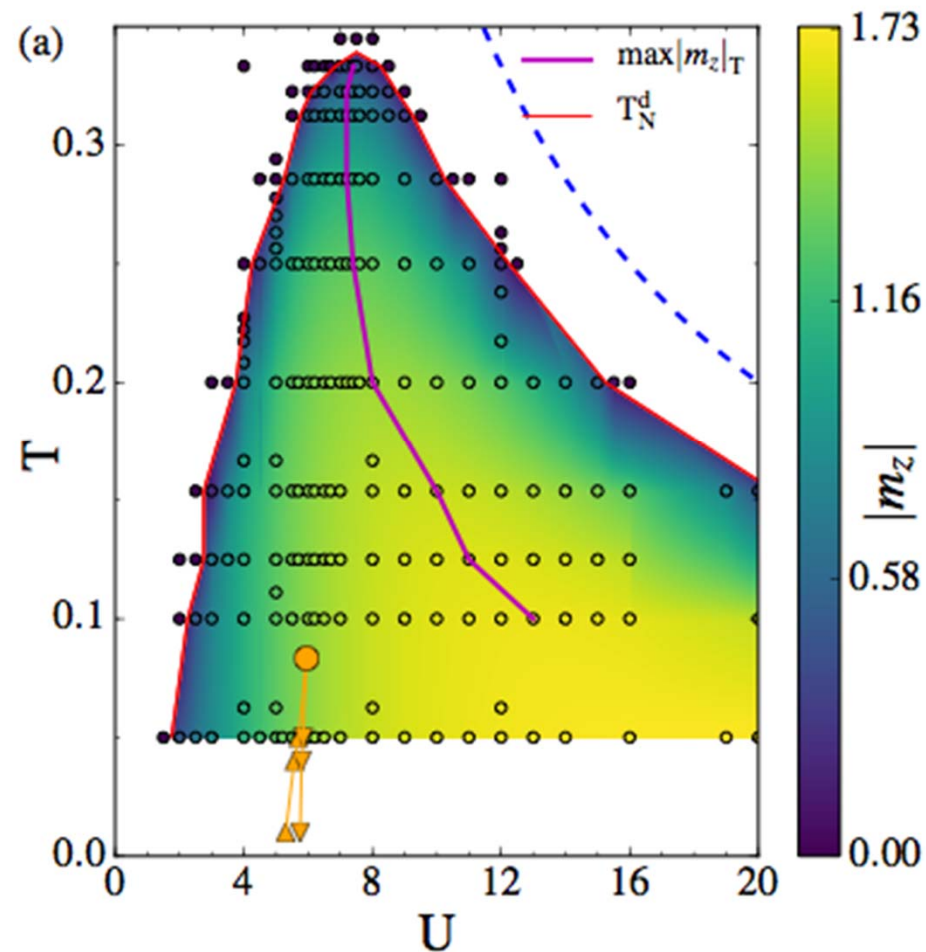


Kristjan Haule

Influence of the Mott transition away from half-filling

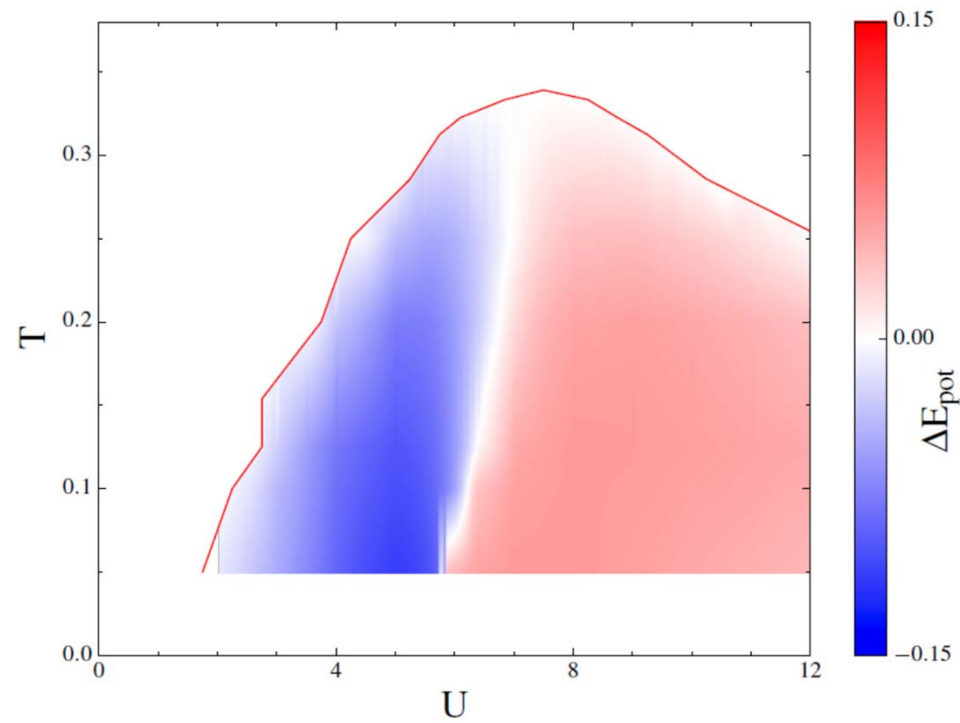
- Sordi et al., PRL 104, 226402 (2010)
- Sordi et al., PRB 84, 075161 (2011)
- Fratino et al., PRB 93, 245147 (2016) [Emery model]
- Sordi et al., Sci. Rep. 2 547 (2012);
- Sordi et al., PRB 87, 041101(R) (2013)

AFM phase diagram $d=2$, $t'=0$



L. Fratino, P. Sémon, M. Charlebois, G. Sordi, AMT Phys. Rev. B **95**, 235109 (2017)

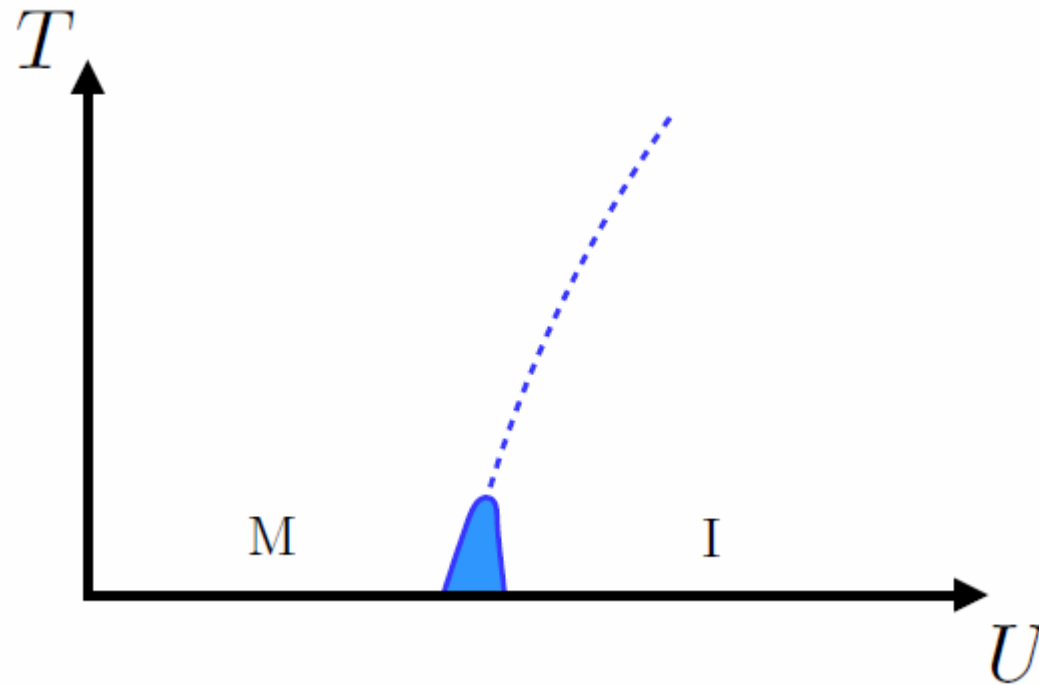
Change in potential energy due to large ξ



L. Fratino,¹ P. Sémon,² M. Charlebois,² G. Sordi,¹ and A.-M. S. Tremblay^{2,3}
[arXiv:1702.01821](https://arxiv.org/abs/1702.01821)

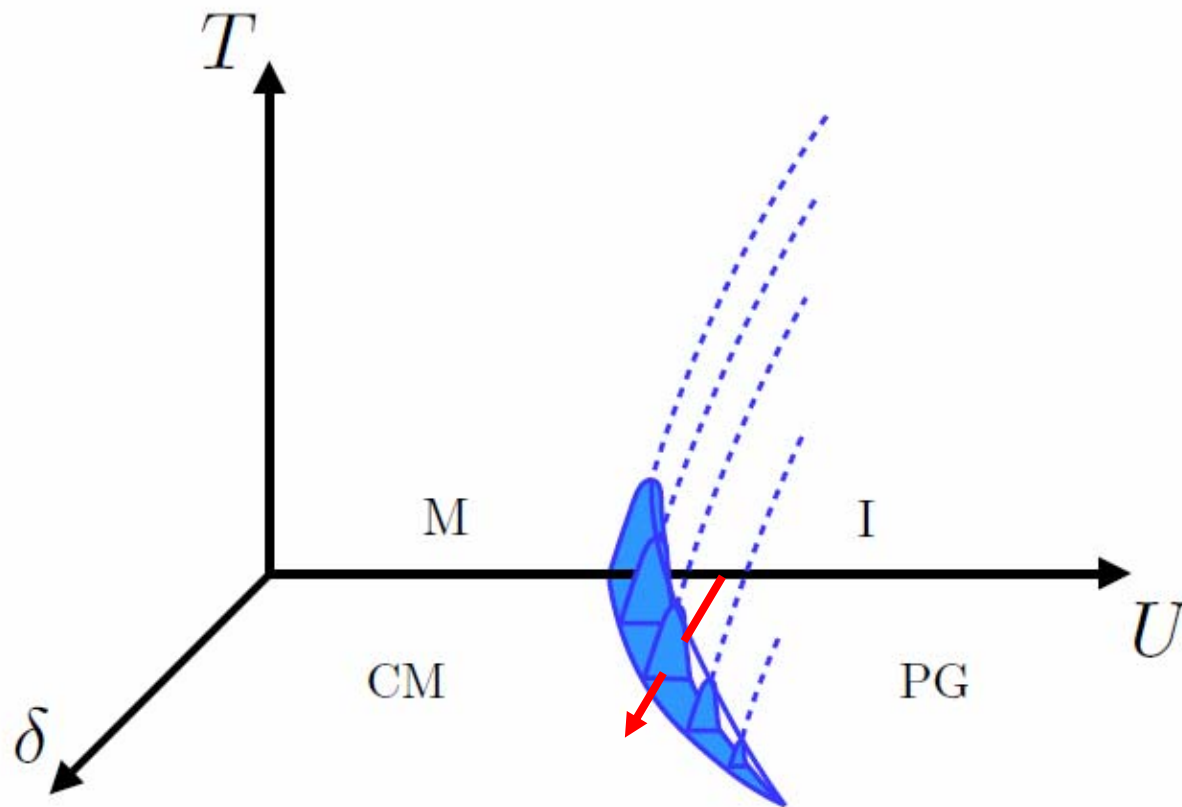
Influence of Mott transition away from half-filling

$n = 1, d = 2$ square lattice



Influence of Mott transition away from half-filling

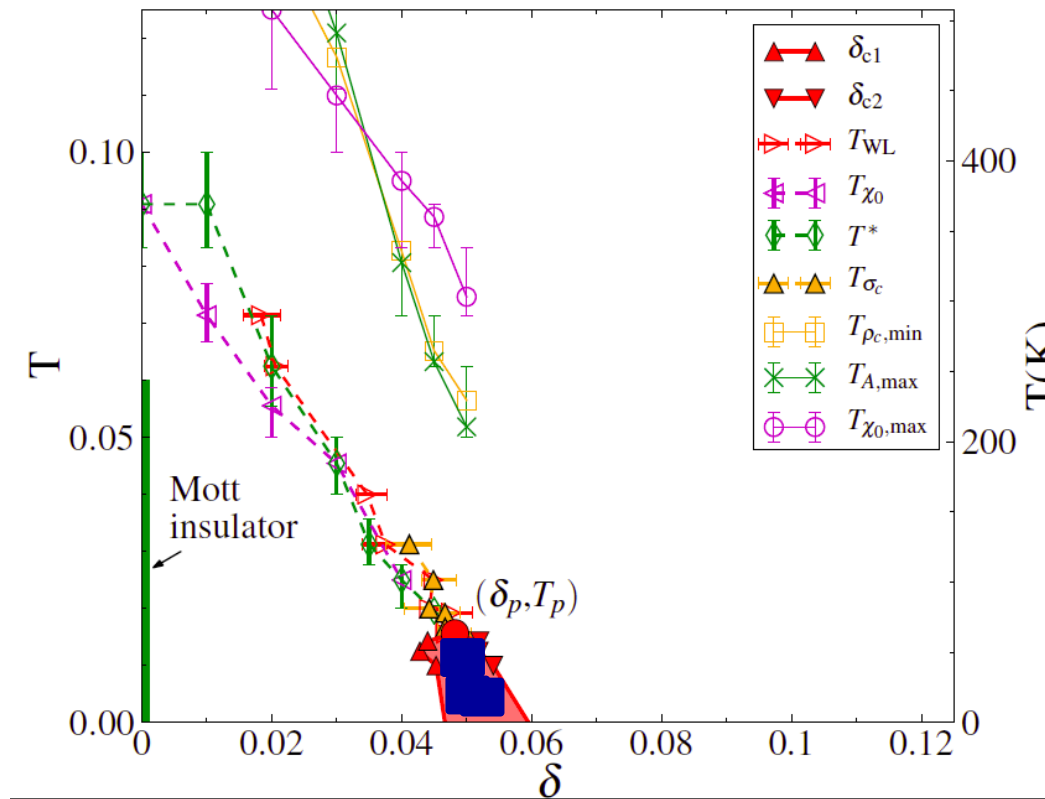
$n = 1, d = 2$ square lattice



Two crossover lines



Giovanni Sordi



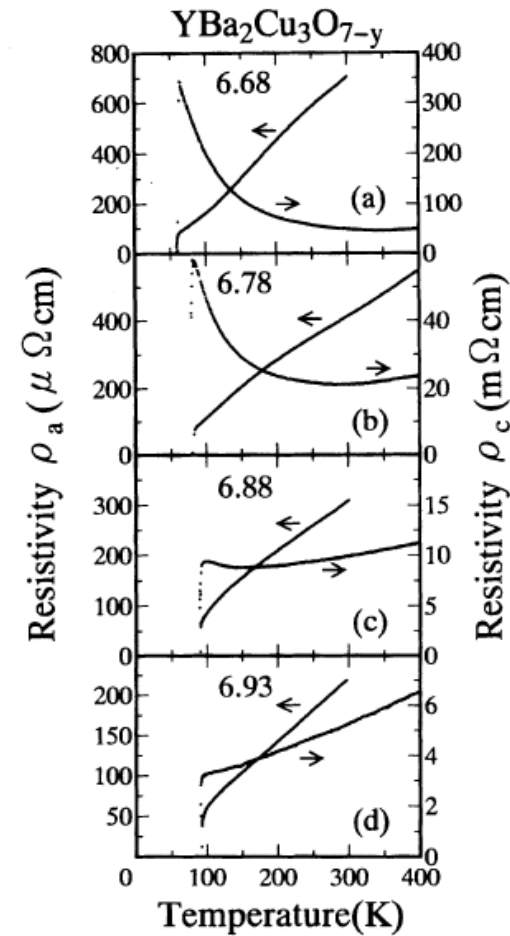
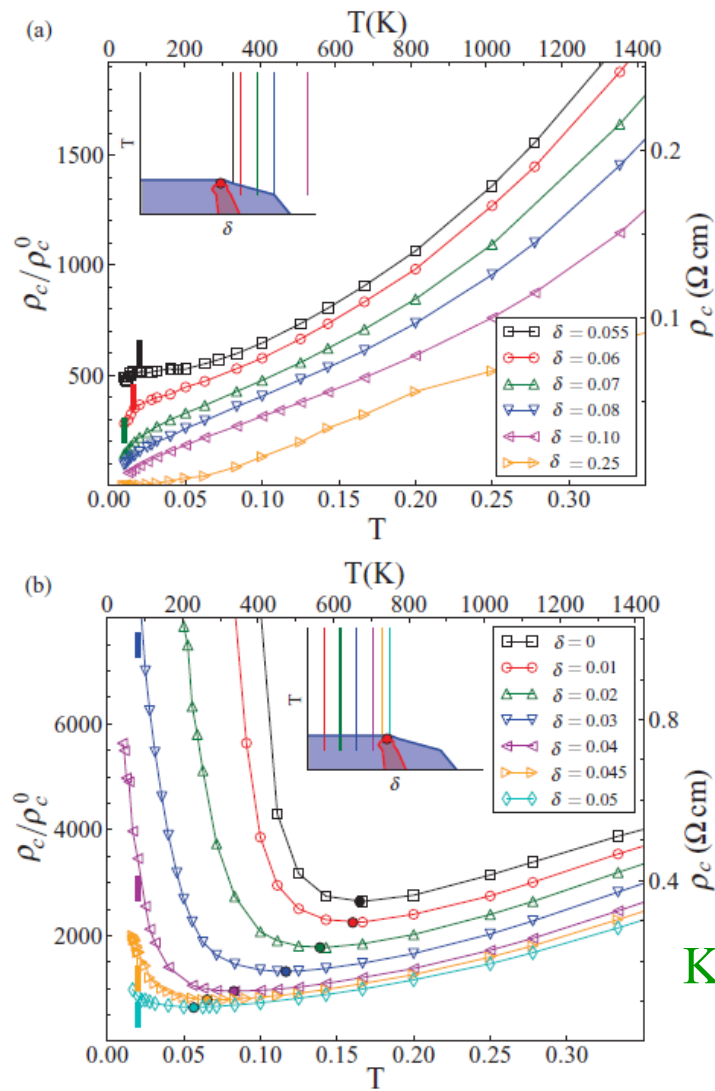
Patrick Sémon

G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

G. Sordi et al. Phys. Rev. B 87, 041101(R)/1-5 (2013)

P. Sémon, G. Sordi, et al., Phys. Rev. B **89**, 165113/1-6 (2014)

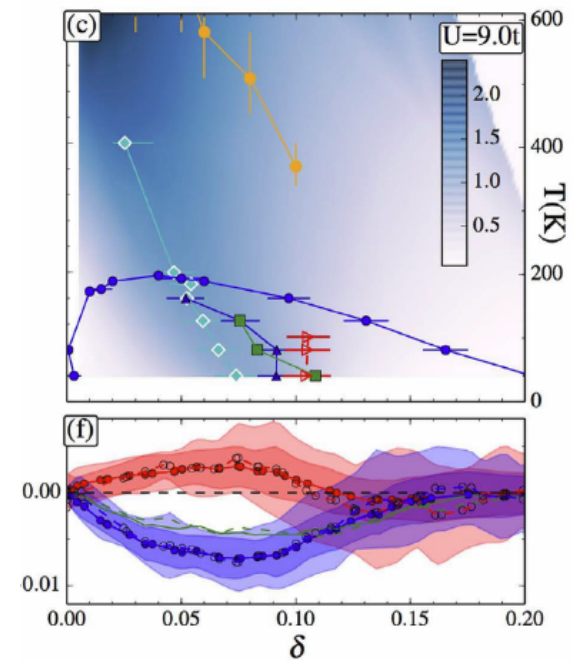
c-axis resistivity



K. Takenaka, K. Mizuhashi, H. Takagi, and S. Uchida,
Phys. Rev.B 50, 6534 (1994).

G.Sordi et al. Phys. Rev. B 87, 041101(R)/1-5 (2013)

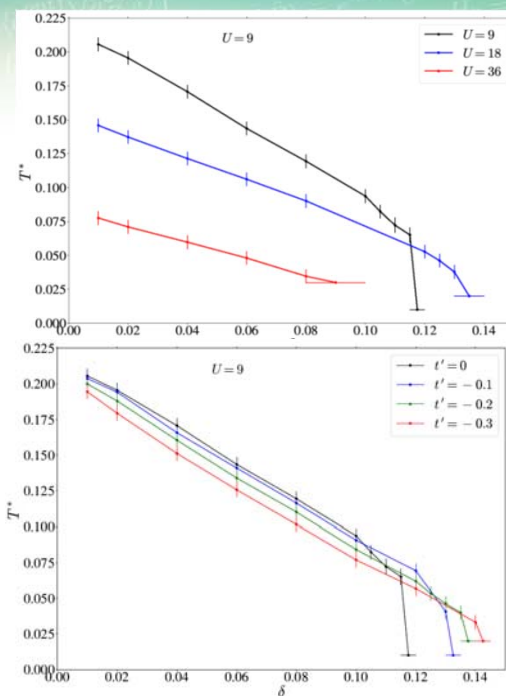
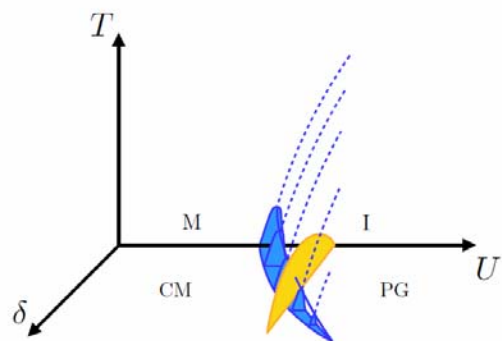
Connecting the finite doping behavior to the Mott transition at half-filling



Conclusion

p^* in Hubbard is the end
of Mott physics

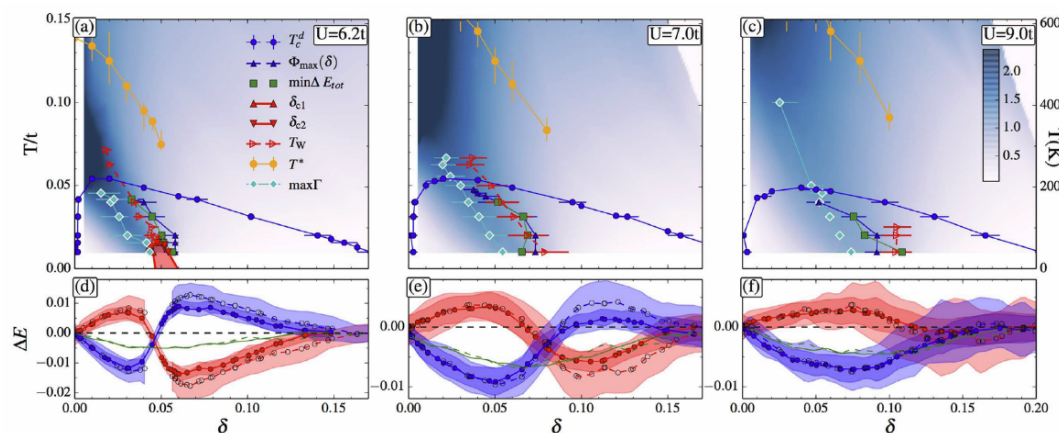
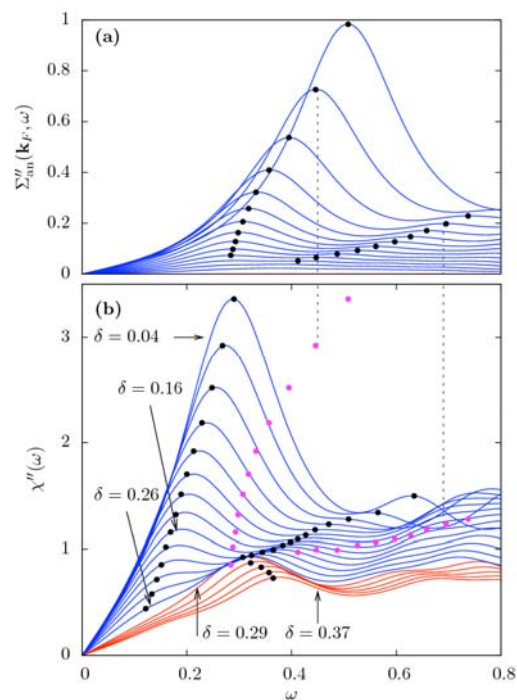
Mott transition and its
finite doping extension is
the organizing principle



$$T^* \sim J$$

$$p^* < p_{fs}$$

Pressure dependence
of p^*



$$T_c \sim J$$

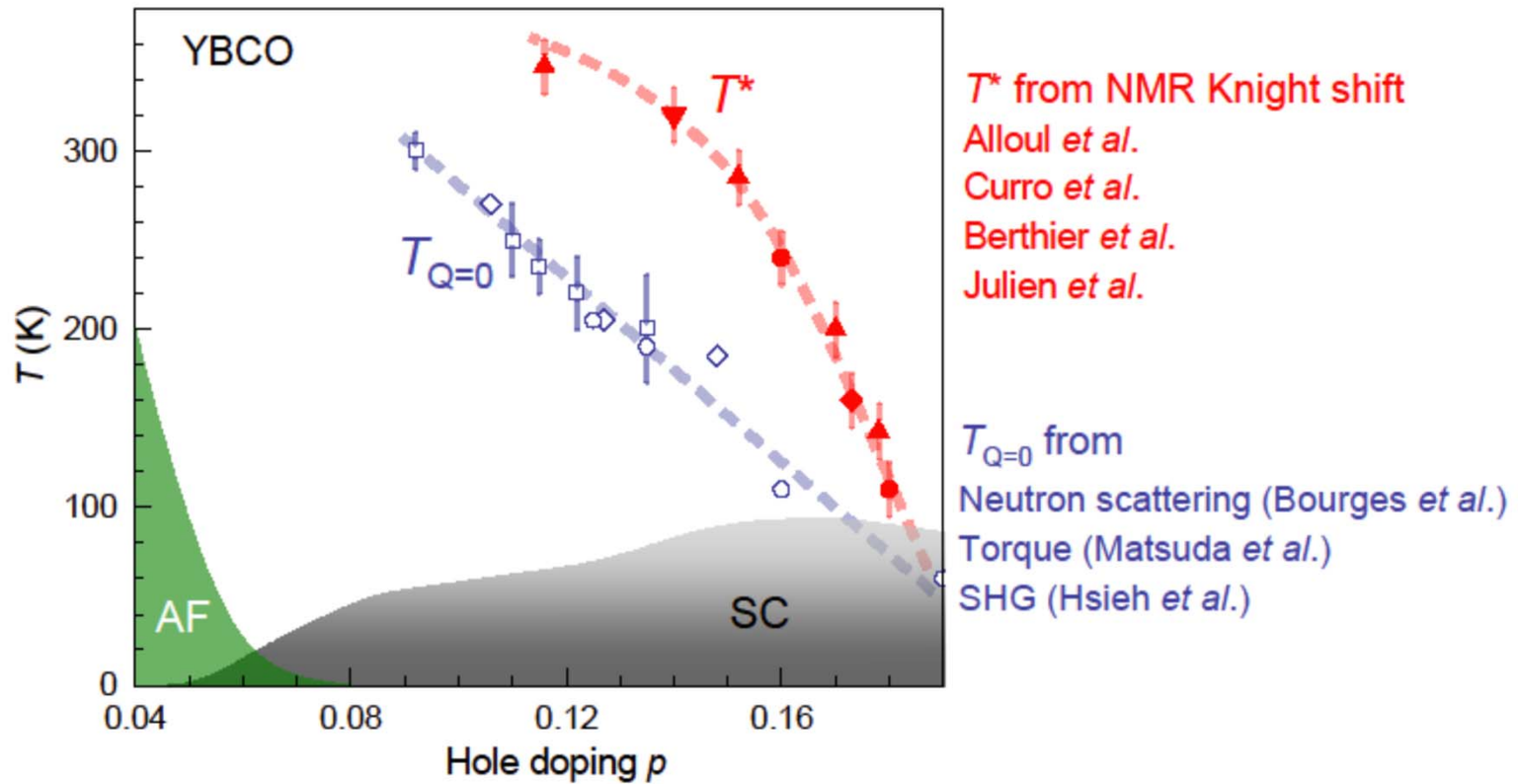


Figure from: Marc-Henri Julien

Mammoth



Éducation,
Loisir et Sport
Québec



Canada Foundation for Innovation
Fondation canadienne pour l'innovation



compute + calcul
CANADA

High Performance Computing

CREATING KNOWLEDGE
DRIVING INNOVATION
BUILDING THE DIGITAL ECONOMY

Le calcul de haute performance

CRÉER LE SAVOIR
ALIMENTER L'INNOVATION
BÂTIR L'ÉCONOMIE NUMÉRIQUE


Calcul Québec

Merci
Thank you



D. Sénéchal



Bumsoo Kyung

The glue

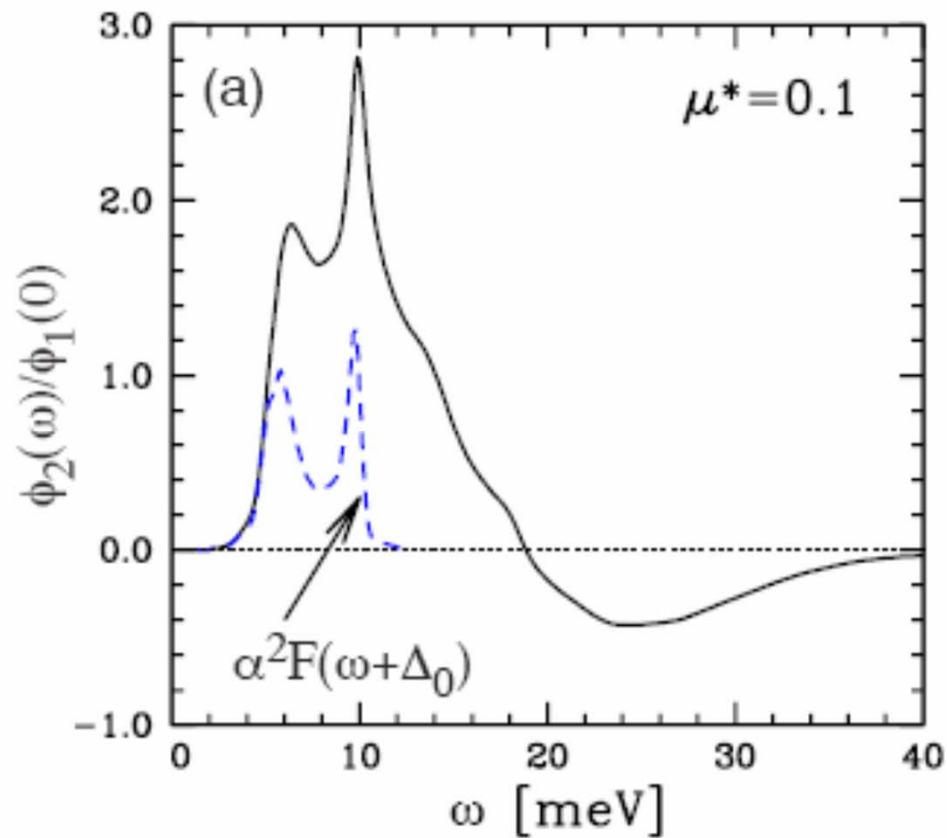
Kyung, Sénéchal, Tremblay, Phys. Rev. B **80**, 205109 (2009)

Sénéchal, Day, Bouliane, AMST, Phys. Rev. B **87**, 075123 (2013)

A. Reymbaut *et al.* PRB **94** 155146 (2016)

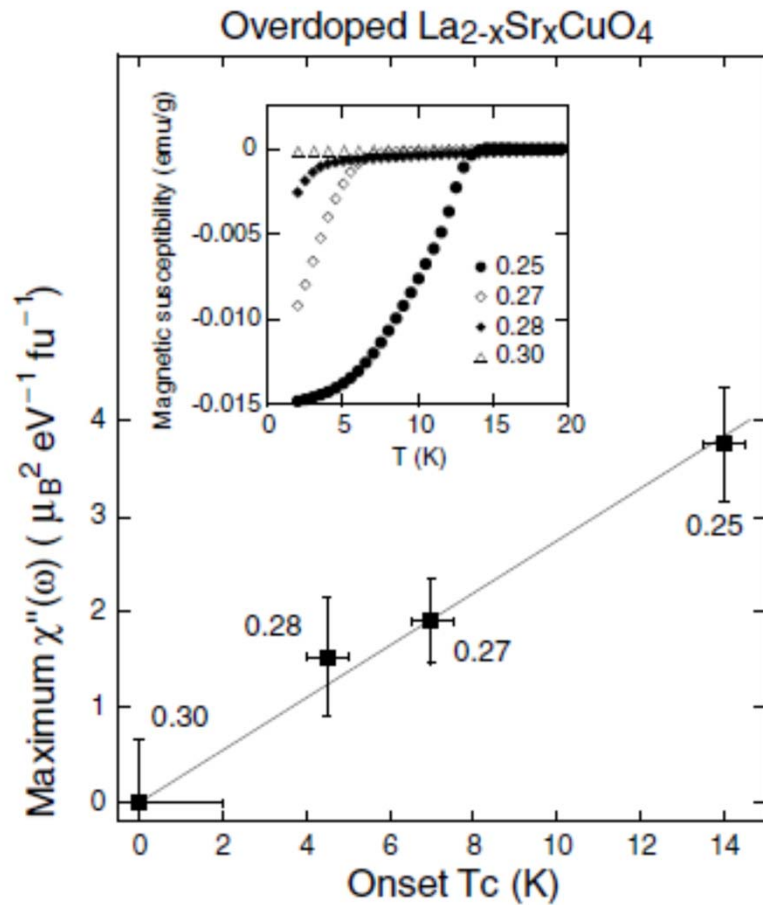
$\text{Im } \Sigma_{\text{an}}$ and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

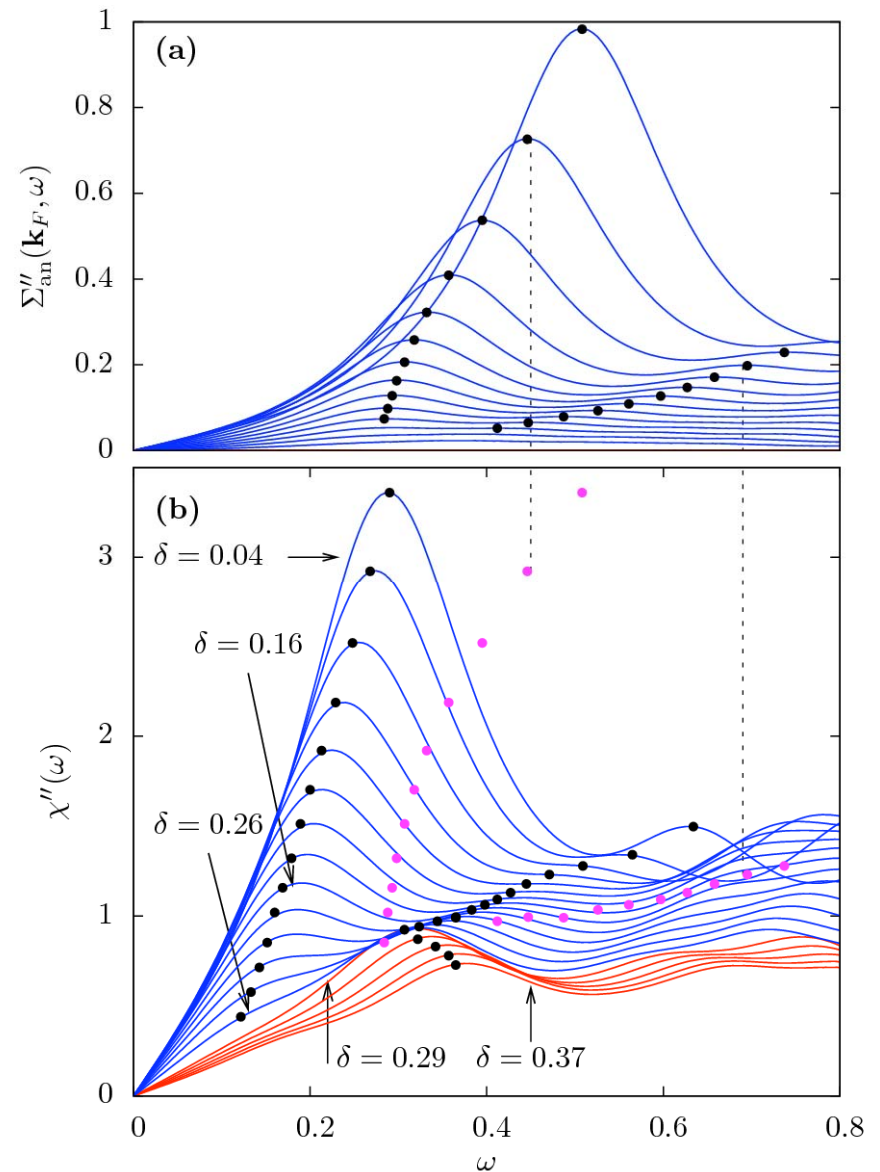


The glue

Kyung, S  n  chal, Tremblay, Phys. Rev. B
80, 205109 (2009)



Wakimoto ... Birgeneau
 PRL (2004)



The glue and neutrons

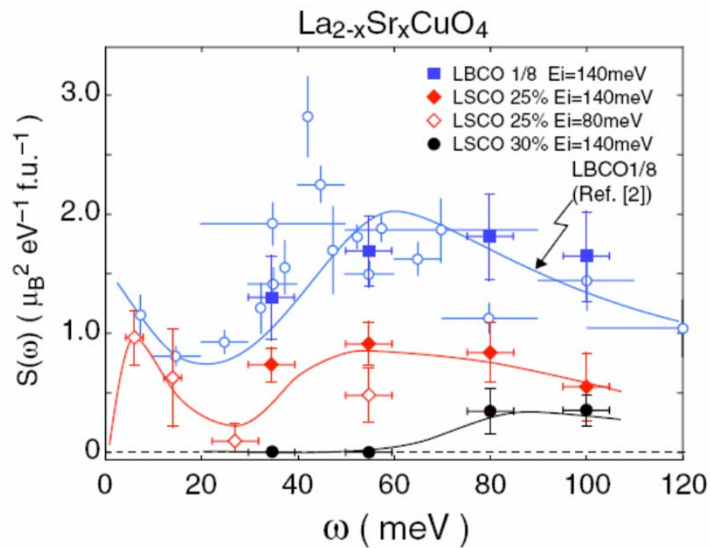


FIG. 3 (color online). $\mathbf{Q}$ -integrated dynamic structure factor $S(\omega)$ which is derived from the wide- H integrated profiles for LBCO 1/8 (squares), LSCO $x = 0.25$ (diamonds; filled for $E_i = 140$ meV, open for $E_i = 80$ meV), and $x = 0.30$ (filled circles) plotted over $S(\omega)$ for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO $x = 0.25$ and 0.30 are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007);
PRL (2004)

