



Bad metals and more

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**Quantum Matter:
Computation Meets Experiments**

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Bad metals



Reza Nourafkan



Alexis Reymbaut



Charles-David Hébert



Simon Bergeron

Peter T. Brown, *et al.*

Bad metallic transport in a cold atom Fermi-Hubbard system

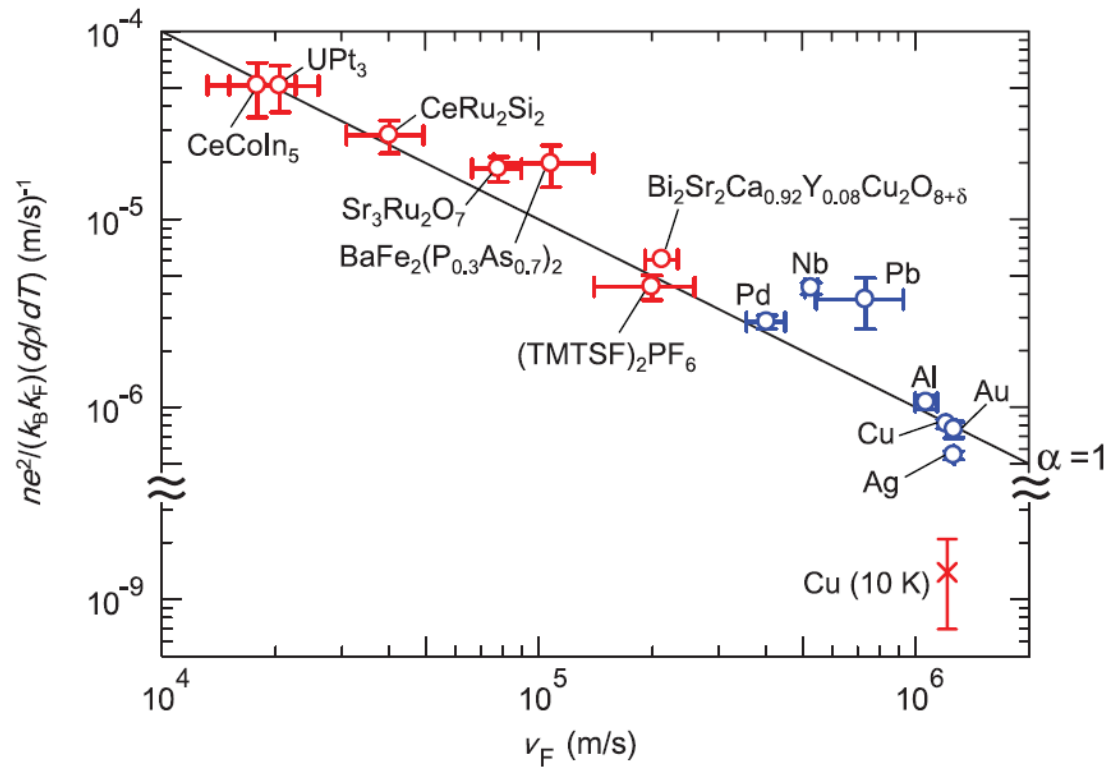
Science [10.1126/science.aat4134](https://doi.org/10.1126/science.aat4134) (2018).

Motivation

$$\rho = (m^*/ne^2) (1/\tau)$$

$$\hbar/\tau = k_B T$$

$$(T\tau)^{-1} \sim \alpha k_B/\hbar$$



J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie SCIENCE 339, 804 (2013)

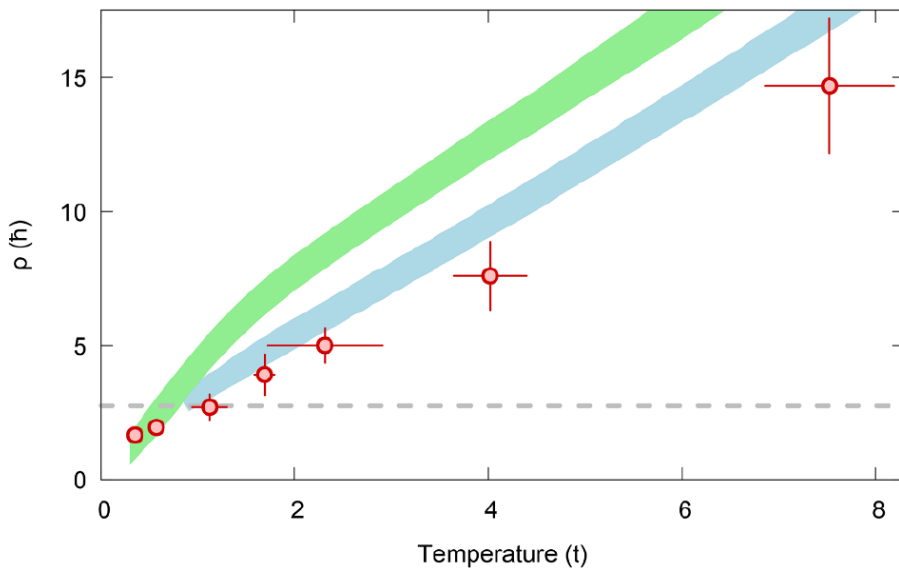
Cold atoms, high temperature

Linear resistivity at high temperature

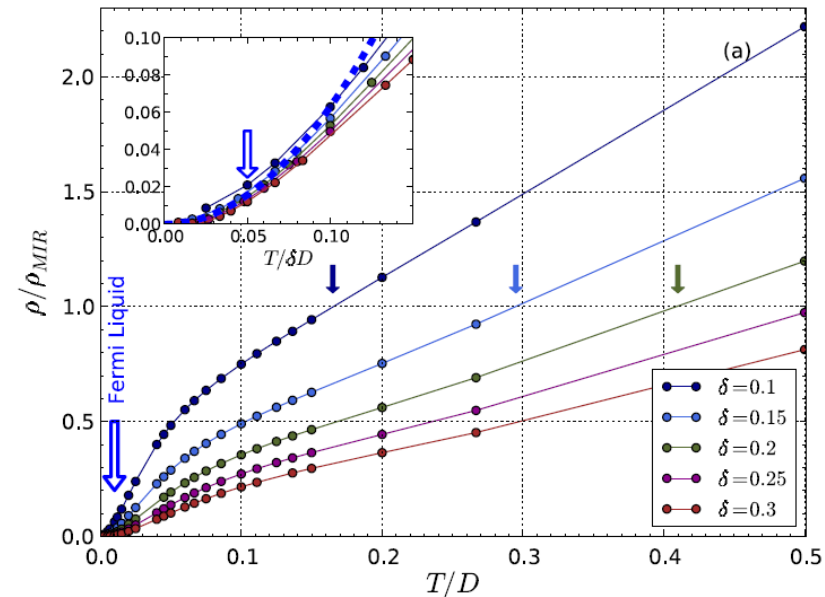


Deng *et al.* PRL **110**, 086401 (2013)

DMFT (Green), FTLM (Blue)



$$U = 7.5t, \quad n = 0.825, \quad t' = 0$$



$$U = 4D = 16t$$

H. Terletska, *et al.* Phys. Rev. Lett. **107**, 026401 (2011).

X. Deng, *et al.* Phys. Rev. Lett. **110**, 086401 (2013).

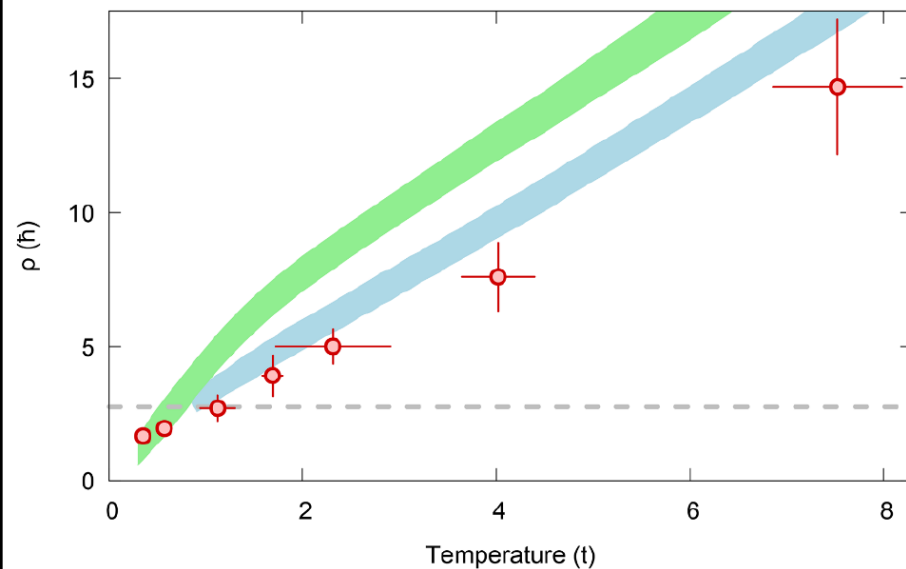
W. Xu, *et al.* Phys. Rev. Lett. **111**, 036401 (2013).

J. Vučičević, Phys. Rev. Lett. **114**, 246402 (2015).

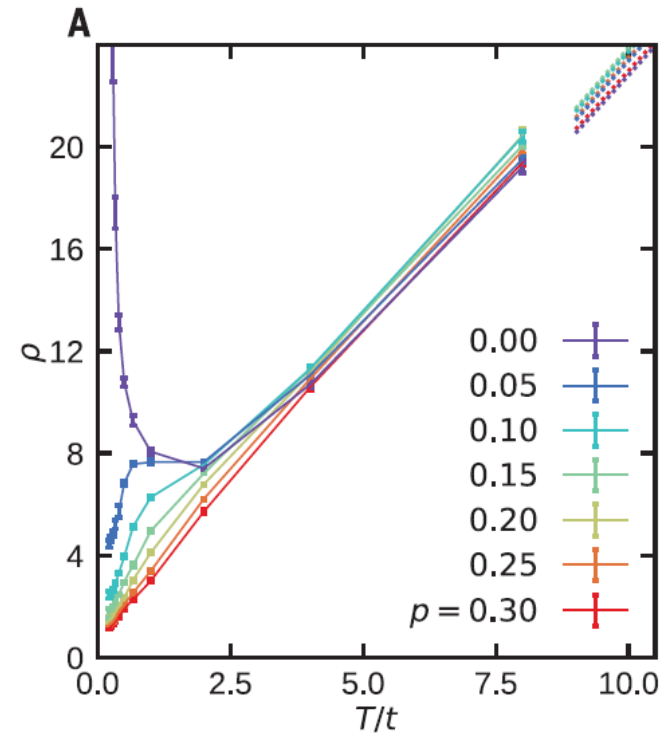
Linear resistivity at high temperature



DMFT (Green), FTLM (Blue)



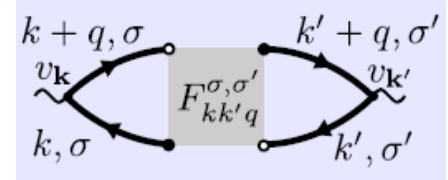
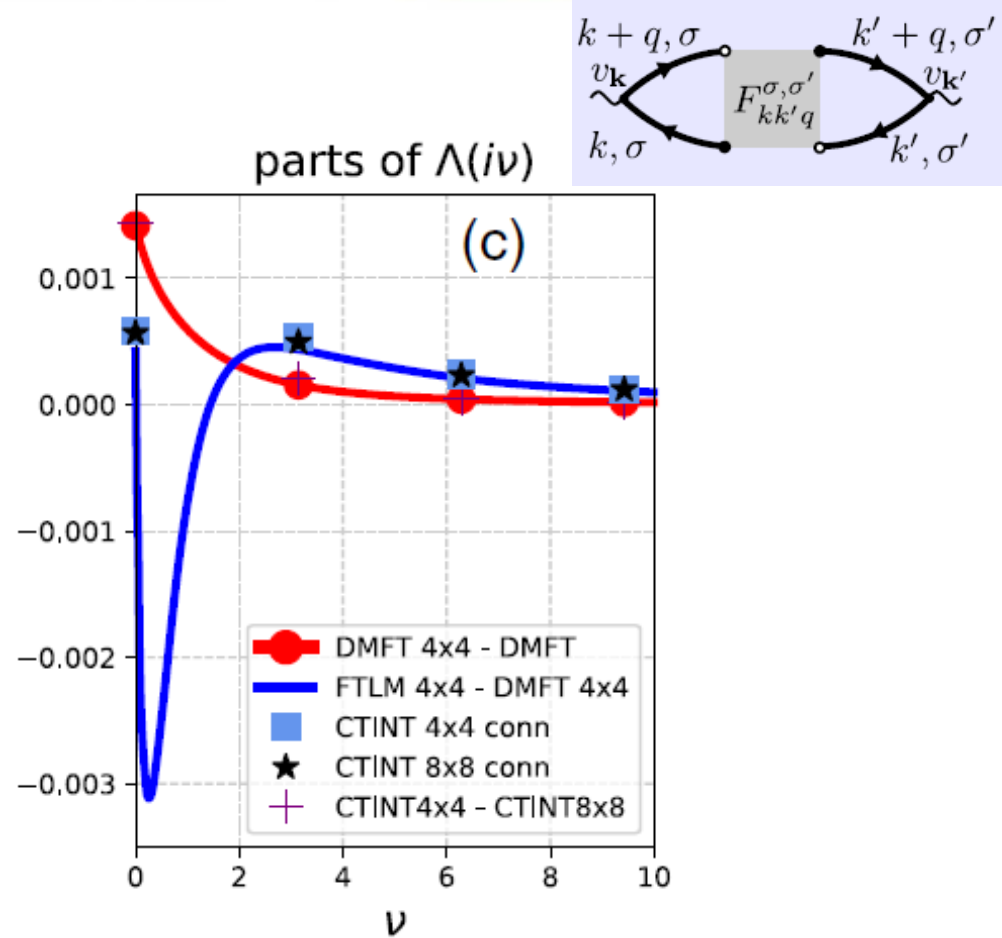
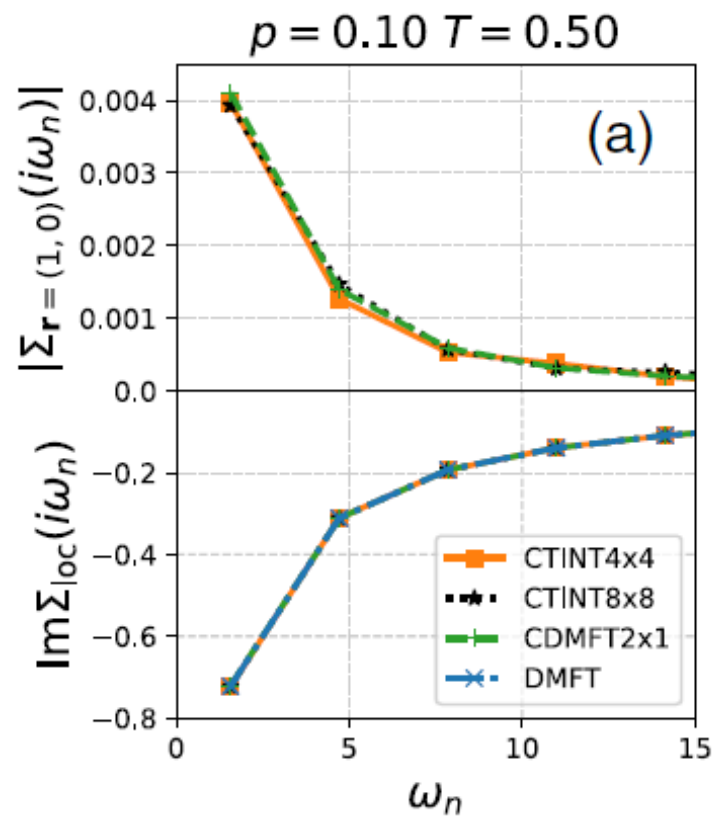
$$U = 7.5t, \quad n = 0.825, \quad t' = 0$$



Huang *et al.* Science 366, 987 (2019)

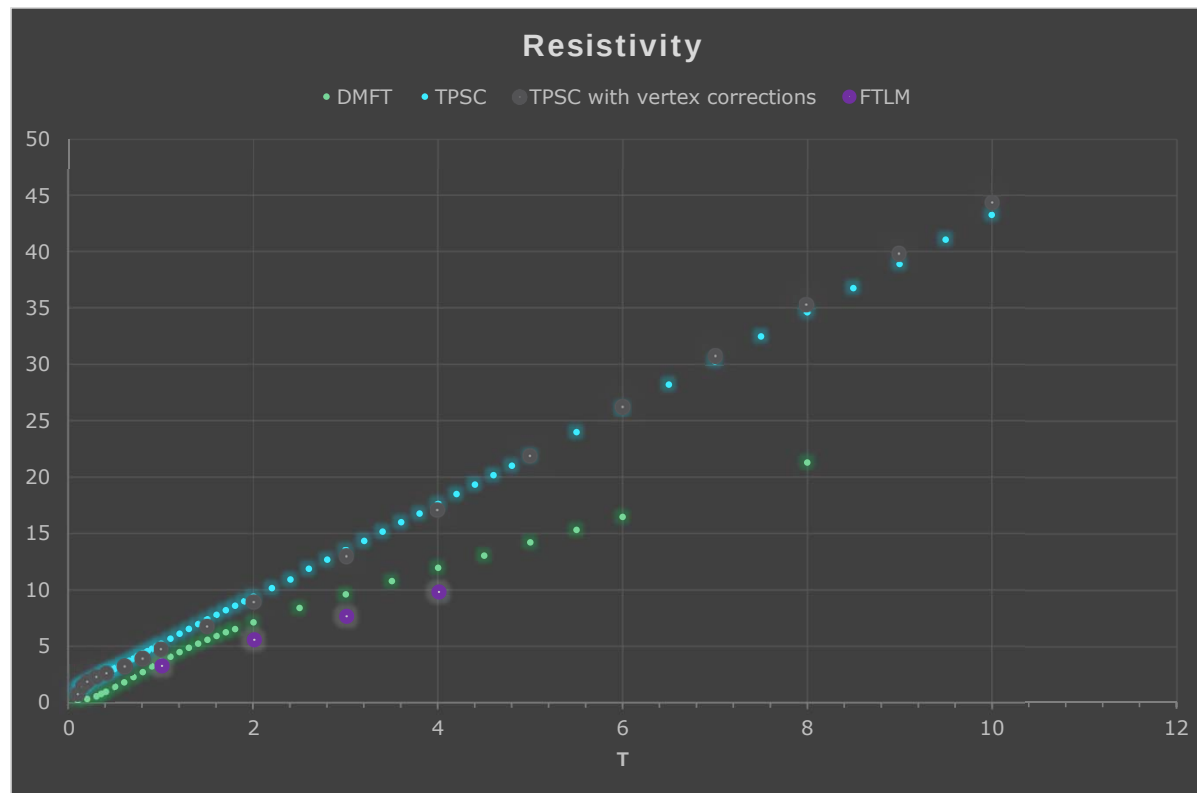
$$U = 6t, \quad t' = -0.25$$

Linear resistivity at high temperature : vertex corrections



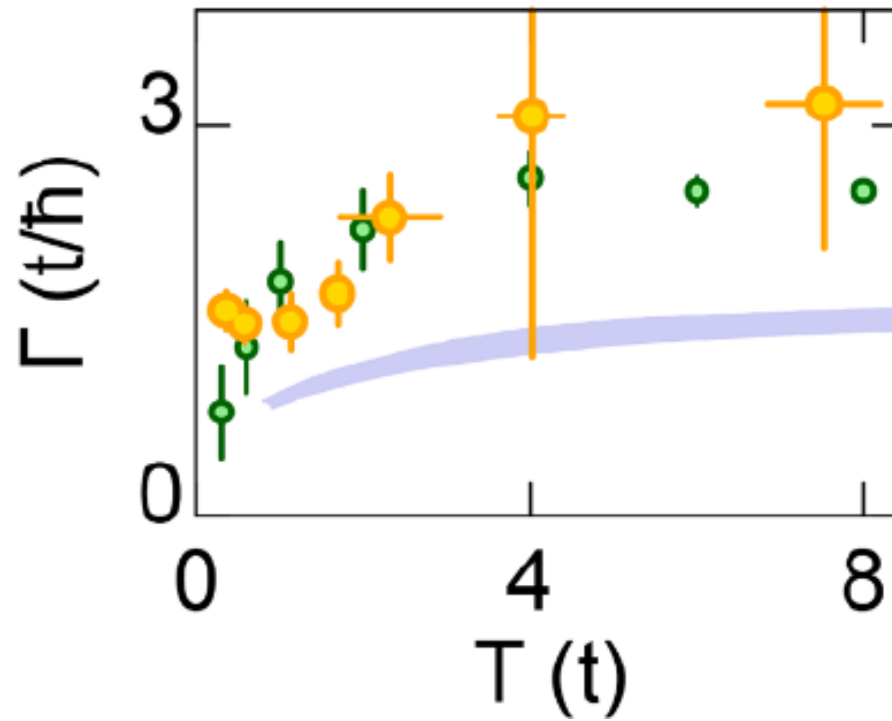
J. Vučičević *et al.* PRL **123**, 036601 (2019)

Another view of vertex corrections (TPSC)



Dominic Bergeron

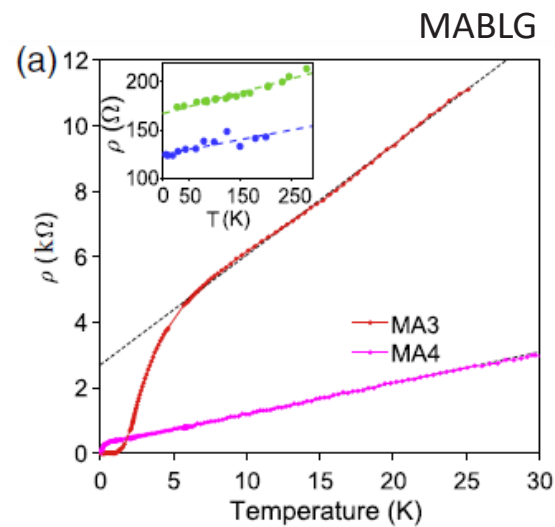
Unexplained : Width of the Drude Peak



Perspective

$$\rho = (m^*/ne^2) (1/\tau)$$

$$\hbar/\tau = k_B T$$

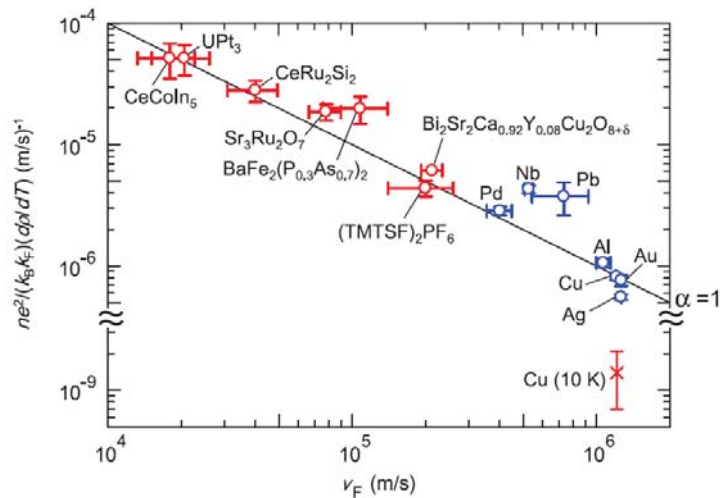


Cao *et al.* PRL **124**, 076801 (2020)

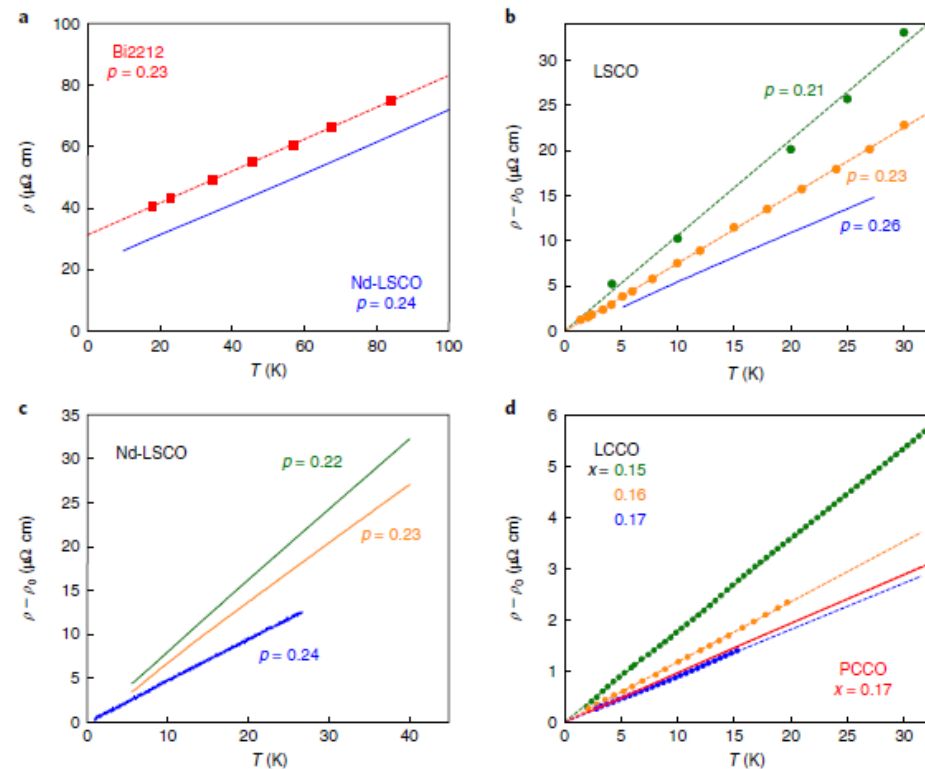
Low temperature ($k_B T \ll E_F$) experiments



Cuprates



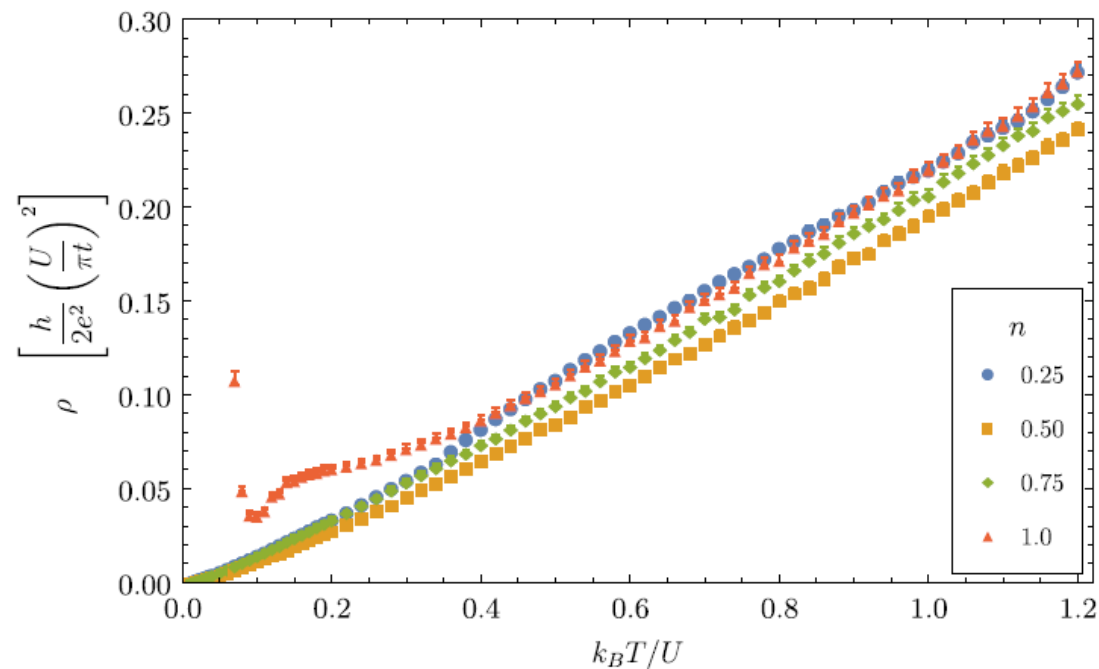
J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie *SCIENCE* **339**, 804 (2013)



A. Legros *et al.* *Nature Physics* (2018)

Modified Hubbard model

$$H = t \sum_{\langle ij \rangle, s} c_{is}^\dagger c_{js} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{V}{2} \sum_{i \neq j} e^{-|\vec{x}_i - \vec{x}_j|/\ell} n_i n_j.$$



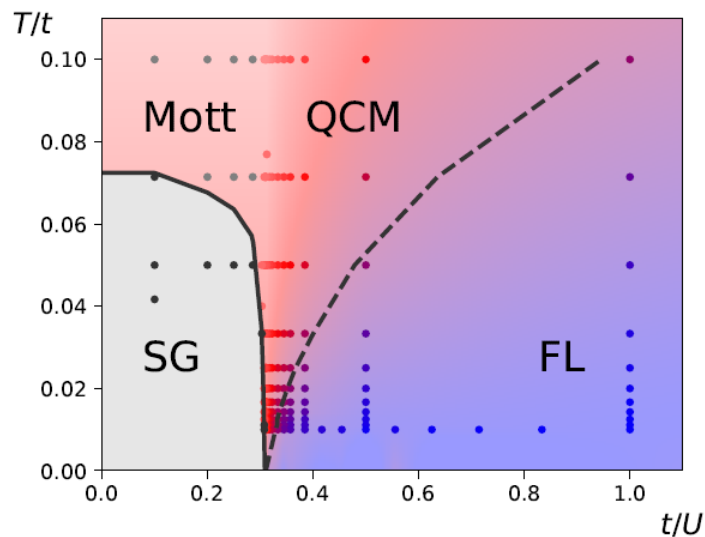
Mousatov *et al.* PRL **122**, 186601 (2019)

$$\begin{aligned} \tilde{V} &= 0.1U \\ t &\ll k_B T \lesssim U \end{aligned}$$

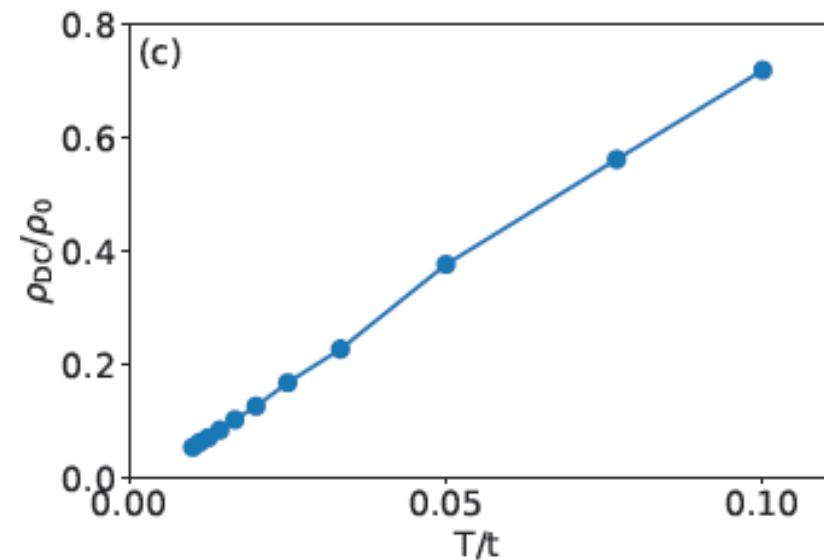
SYK like models (similar to DMFT at high T)



$$H = - \sum_{\langle ij \rangle, s=\uparrow, \downarrow} t_{ij} c_{is}^\dagger c_{js} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i < j} \frac{J_{ij}}{\sqrt{N}} \vec{S}_i \cdot \vec{S}_j$$



P. Cha *et al.* arXiv:2002.07181



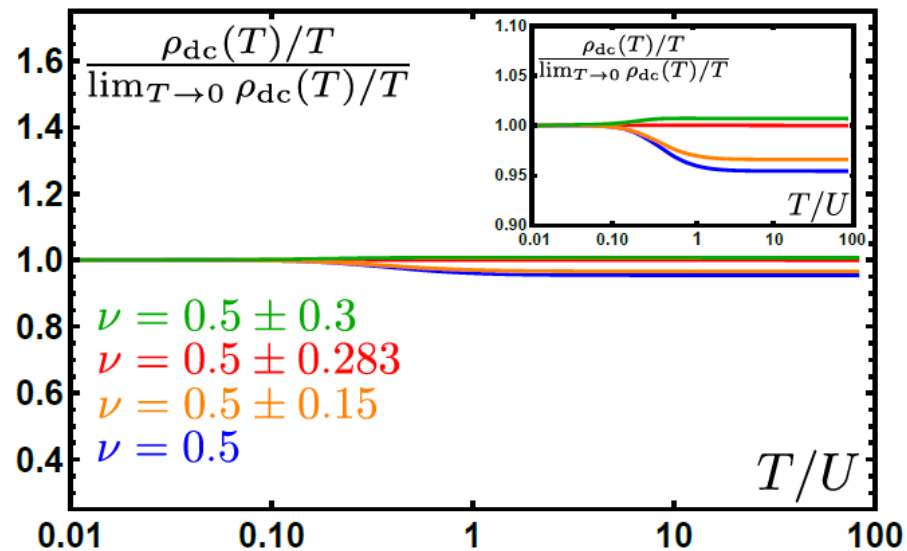
P. Cha *et al.* arXiv:2002.07181

- X.-Y. Song, *et al.* Phys. Rev. Lett. **119**, 216601 (2017).
- A. A. Patel, *et al.* Phys. Rev. X **8**, 021049 (2018).
- D. Chowdhury, *et al.* Phys. Rev. X **8**, 031024 (2018).
- S. A. Hartnoll, A. Lucas, and S. Sachdev, *Holographic Quantum Matter* (MIT Press, Cambridge, MA, 2018).

SYK like models (similar to DMFT at high T)



« Slope invariance »

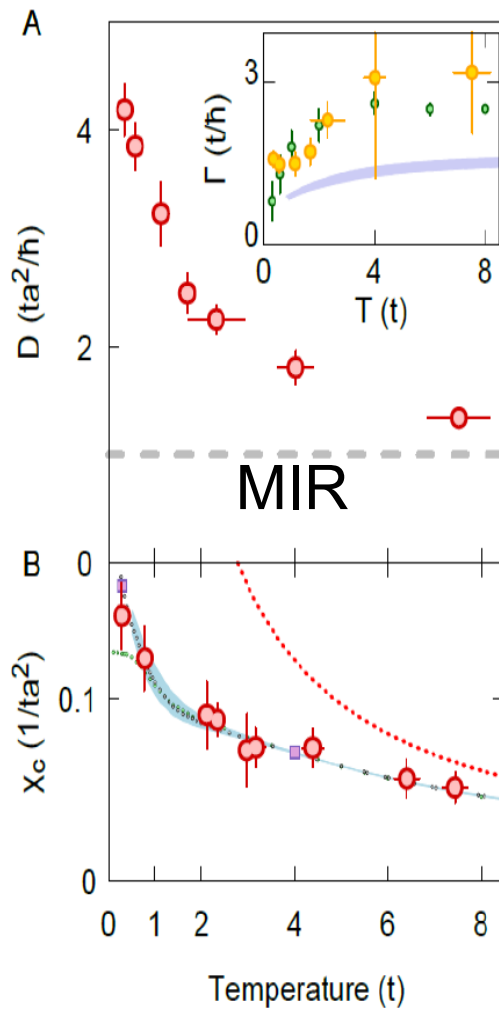


P. Cha *et al.* arXiv:1910.07530

- X.-Y. Song, *et al.* Phys. Rev. Lett. **119**, 216601 (2017).
- A. A. Patel, *et al.* Phys. Rev. X **8**, 021049 (2018).
- D. Chowdhury, *et al.* Phys. Rev. X **8**, 031024 (2018).

Back to high temperature :
Why linear?

Linear at high temperature, but not because of scattering



$$\sigma = \chi_c D$$

J. Kokalj, Phys. Rev. B 95, 041110(R) (2017)

$$D \gtrsim ta^2 / \hbar$$

$$\chi_c \sim n(1 - n/2)/T$$

E. Perepelitsky, *et al.* Phys. Rev. B **94**, 235115 (2016).

General Case

$$\sigma_{xx}(0) = \frac{\hbar}{k_B T} \sum_{n,m} \frac{e^{-\beta E_n}}{Z} \langle n | j_x | m \rangle \langle m | j_x | n \rangle \pi \delta(E_m - E_n)$$

Mukerjee *et al.* PRB **73**, 035113 (2006)

P. Cha *et al.* arXiv:1910.07530

E. Perepelitsky, *et al.* Phys. Rev. B **94**, 235115 (2016).

Entanglement entropy and mutual information near the Mott transition



Caitlin Walsh



Patrick Sémon



David Poulin



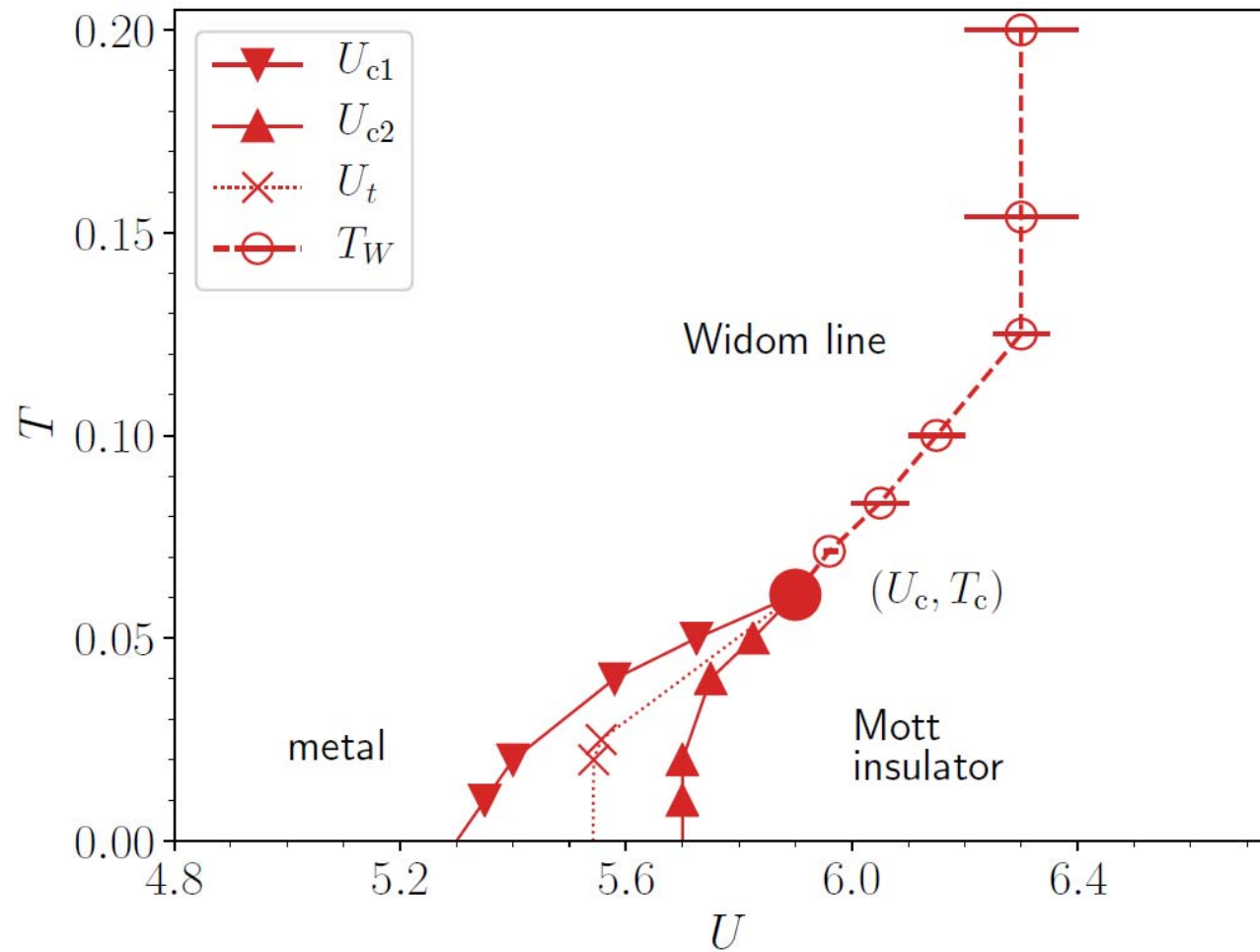
Giovanni Sordi

C. Walsh, *et al.*
Phys. Rev. Lett. **122**, 067203 (2019)
Phys. Rev. B **99**, 075122 (2019)

The Mott transition at half-filling

C. Walsh, *et al.* PRB **99**, 075122 (2019)

H. Park, *et al.* PRL **107**, 137007 (2011).



Motivation



PHYSICAL REVIEW X **7**, 031025 (2017)

Measuring Entropy and Short-Range Correlations in the Two-Dimensional Hubbard Model

E. Cocchi,^{1,2} L. A. Miller,^{1,2} J. H. Drewes,¹ C. F. Chan,¹ D. Pertot,¹ F. Brennecke,¹ and M. Köhl¹

First-order nature of the transition,
universality class of the end point,
crossovers emanating from the end point.

For quantum critical or finite temperature critical points

- A. Anfossi *et al.* Phys. Rev. Lett. **95**, 056402 (2005).
- L. Amico *et al.* Europhys. Lett. **77**, 17001 (2007).
- L. Amico *et al.* Rev. Mod. Phys. **80**, 517 (2008).
- D. Larsson *et al.* Phys. Rev. A **73**, 042320 (2006).
- D. Larsson *et al.* Phys. Rev. Lett. **95**, 196406 (2005).

Single-site entanglement entropy

What is measured (Using CDMFT CT-HYB on plaquette)

- Single site entanglement entropy for fermions [1]

$$\rho_A = \text{Tr}_B[\rho_{AB}] \quad s_A = -\text{Tr}_A[\rho_A \ln \rho_A]$$

$$\rho = \text{diag}(p_0, p_{\uparrow}, p_{\downarrow}, p_{\uparrow\downarrow}) \quad s_1 = -\sum_i p_i \ln(p_i)$$

$$p_{\uparrow\downarrow} = \langle n_{i\uparrow} n_{i\downarrow} \rangle \quad p_{\uparrow} = p_{\downarrow} = \langle n_{i\uparrow} - n_{i\uparrow} n_{i\downarrow} \rangle \quad p_0 = 1 - 2p_{\uparrow} - p_{\uparrow\downarrow}$$

[1] P. Zanardi *et al.* Phys. Rev. A **65**, 042101 (2002).

What is measured

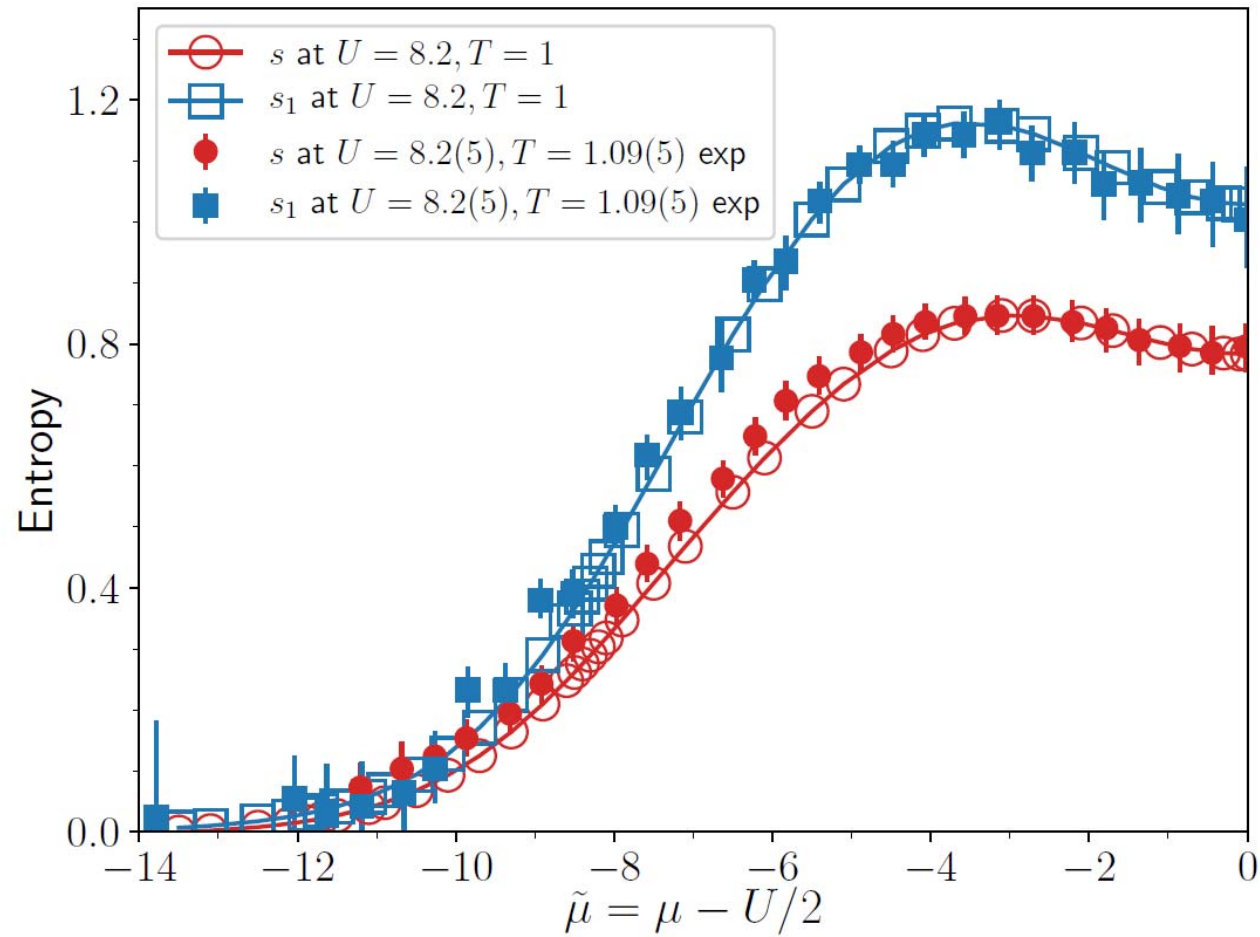
- Entropy

$$sdT - adP + nd\mu = 0.$$

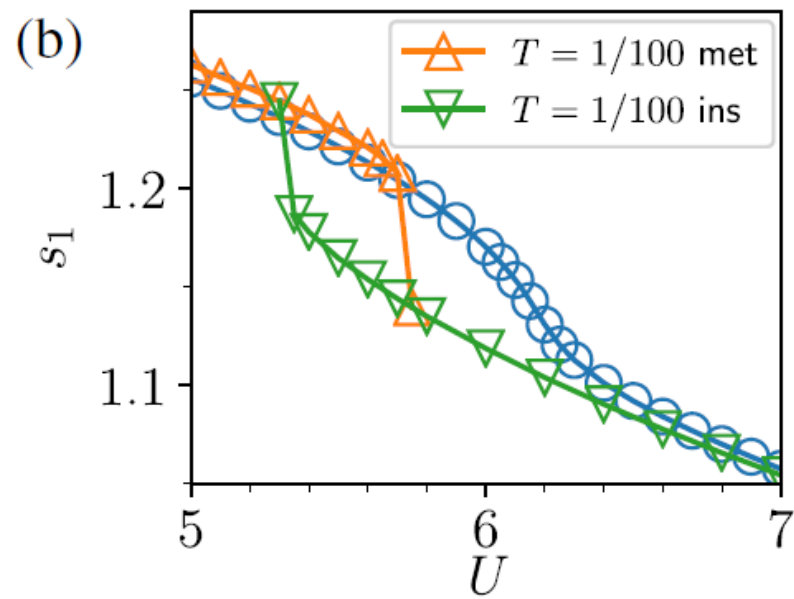
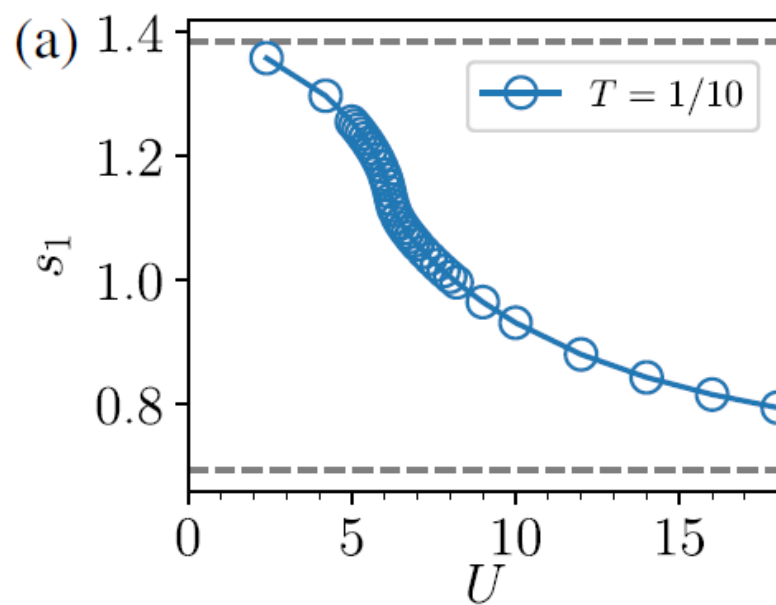
$$P(T)_U = \frac{1}{a} \int_{-\infty}^{U/2} n(\mu, T) d\mu$$

$$s = a(dP/dT)_\mu$$

Agreement with experiment

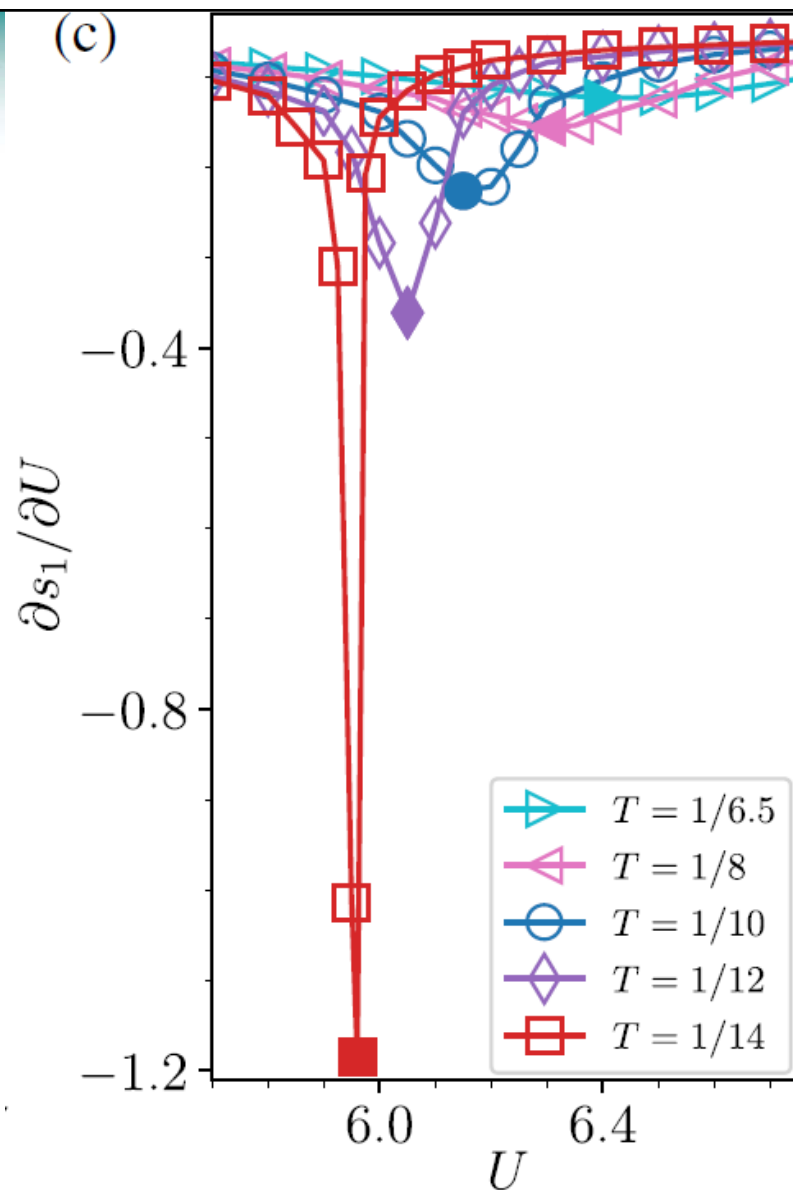


Results



$$\partial s_1 / \partial T < 0$$

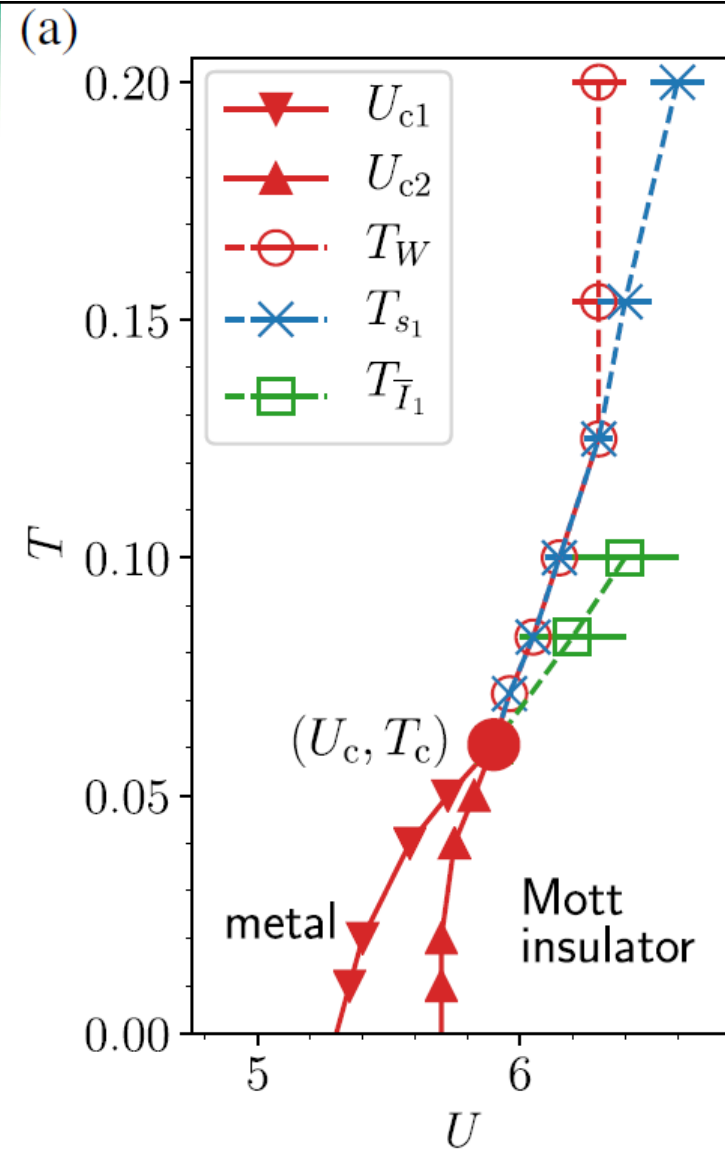
Results



$$(\partial s_1 / \partial D)(\partial D / \partial U)$$

$$\partial D / \partial U \sim -|U - U_c|^{-1+1/\delta}$$

Transition and crossovers



From single-site entanglement entropy



- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
 - Without knowledge of the order parameter of the transition

Mutual information

Mutual information

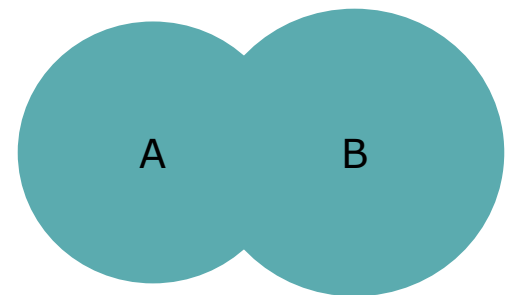
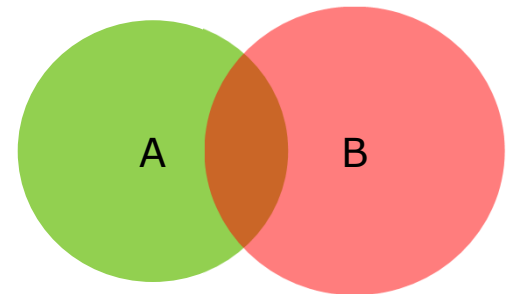


$$I(A:B) = s_A + s_B - s_{AB}$$

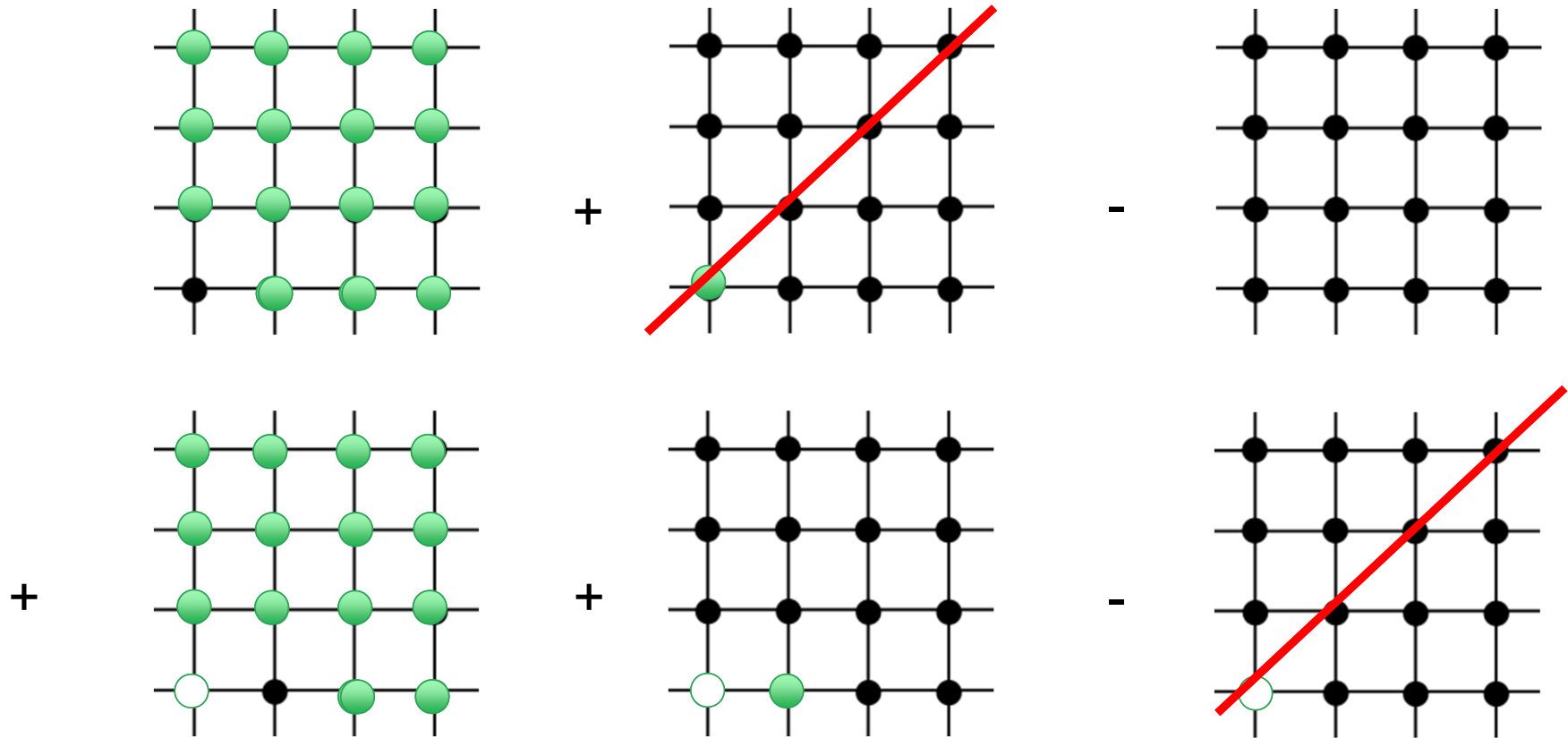
Here we are *not* looking at the area law

What is measured experimentally

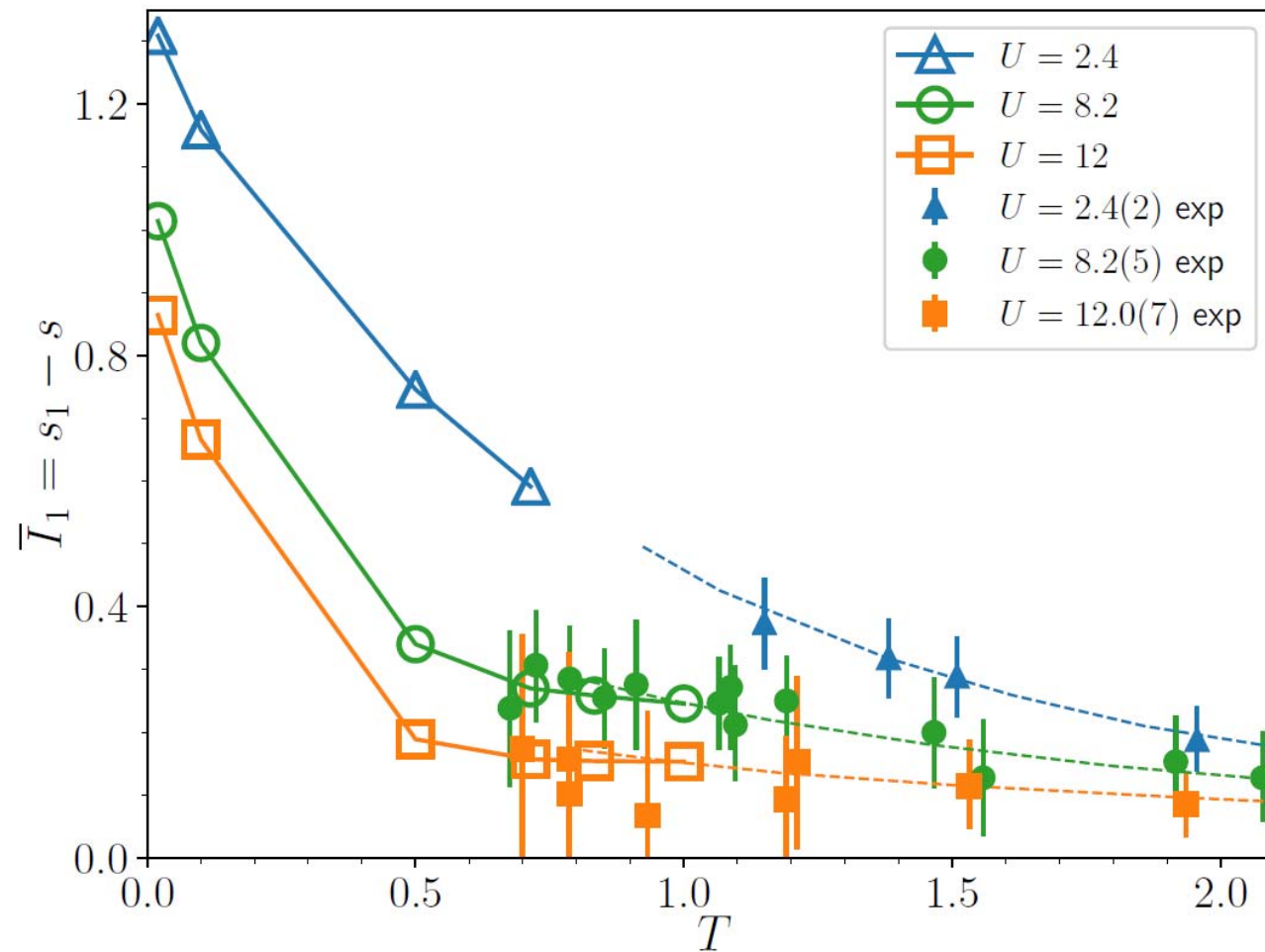
$$\bar{I}_1 = s_1 - s$$



Total mutual information

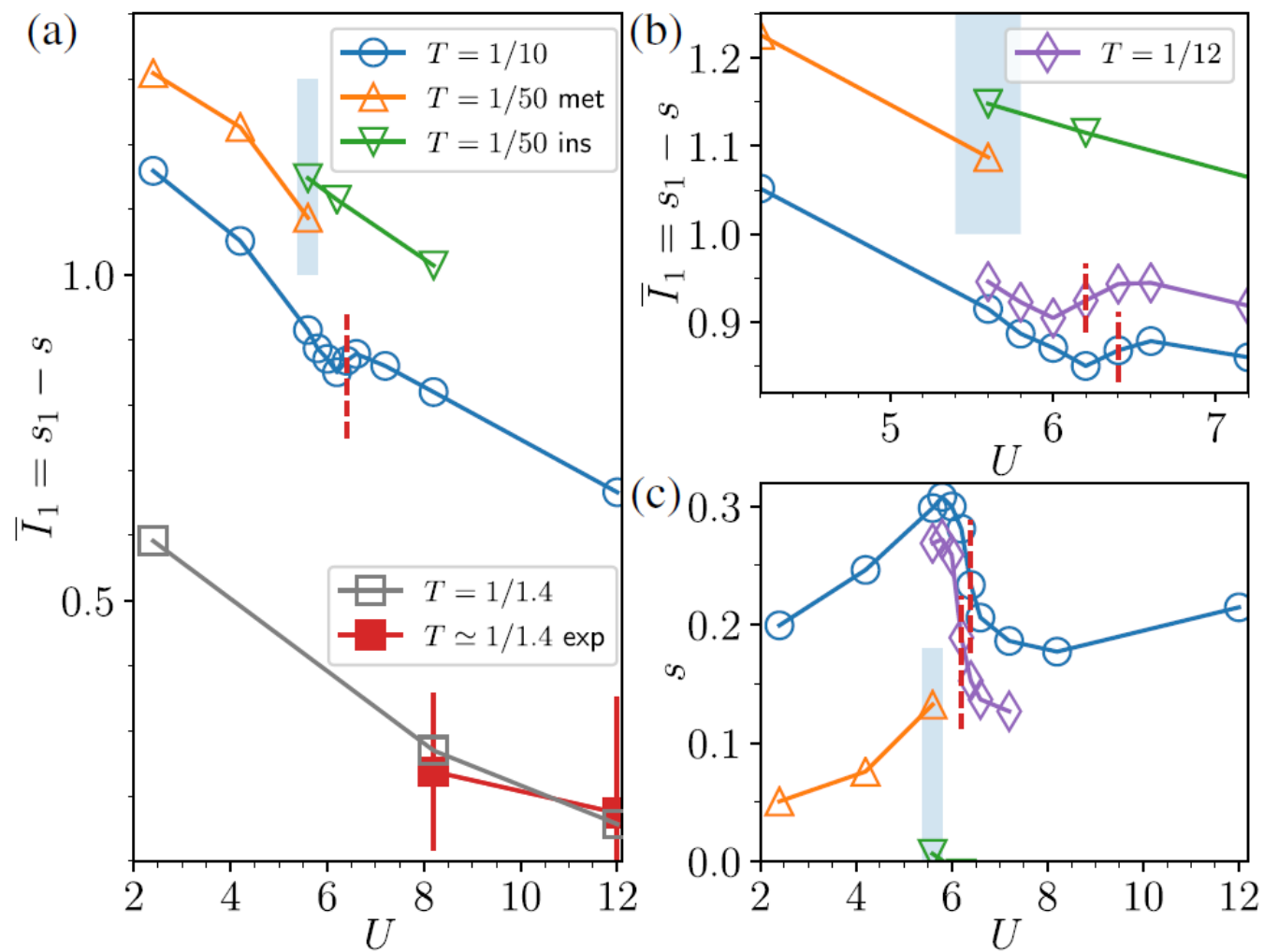


Agreement with experiment

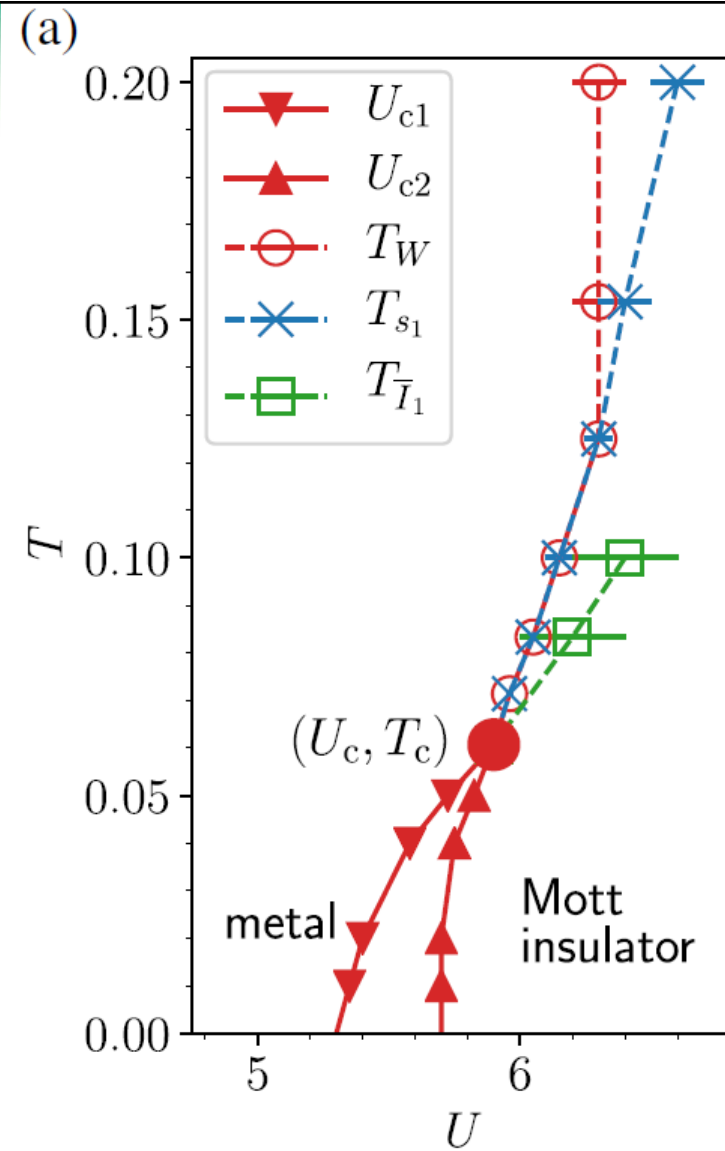


Results

$$\bar{I}_1 \sim \text{sgn}(U - U_c) |U - U_c|^{1/\delta}$$



Transition and crossovers



From average mutual information



- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
 - Without knowledge of the order parameter of the transition

Merci



Alexis Reymbaut



Charles-David Hébert



Reza Nourafkan



Simon Bergeron



Caitlin Walsh



Patrick Sémon



David Poulin



Giovanni Sordi