

Journée nationale de la vérité et de la  
réconciliation

[UDEMNOUVELLES](#) | LE 27 SEPTEMBRE 2021 | CHRISTINE FORTIER



L'éducation à la  
réconciliation

Journée nationale de la vérité et de la réconciliation

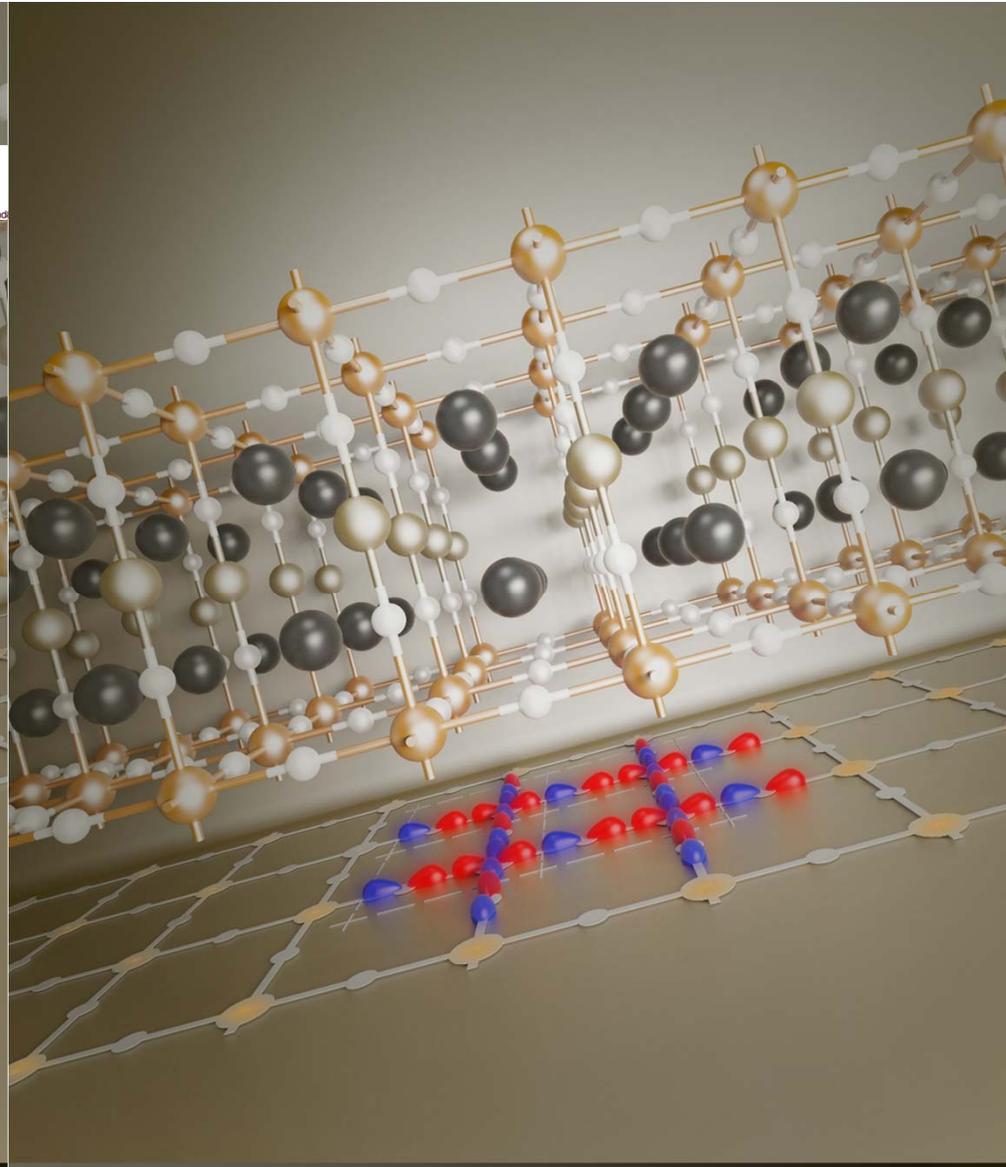


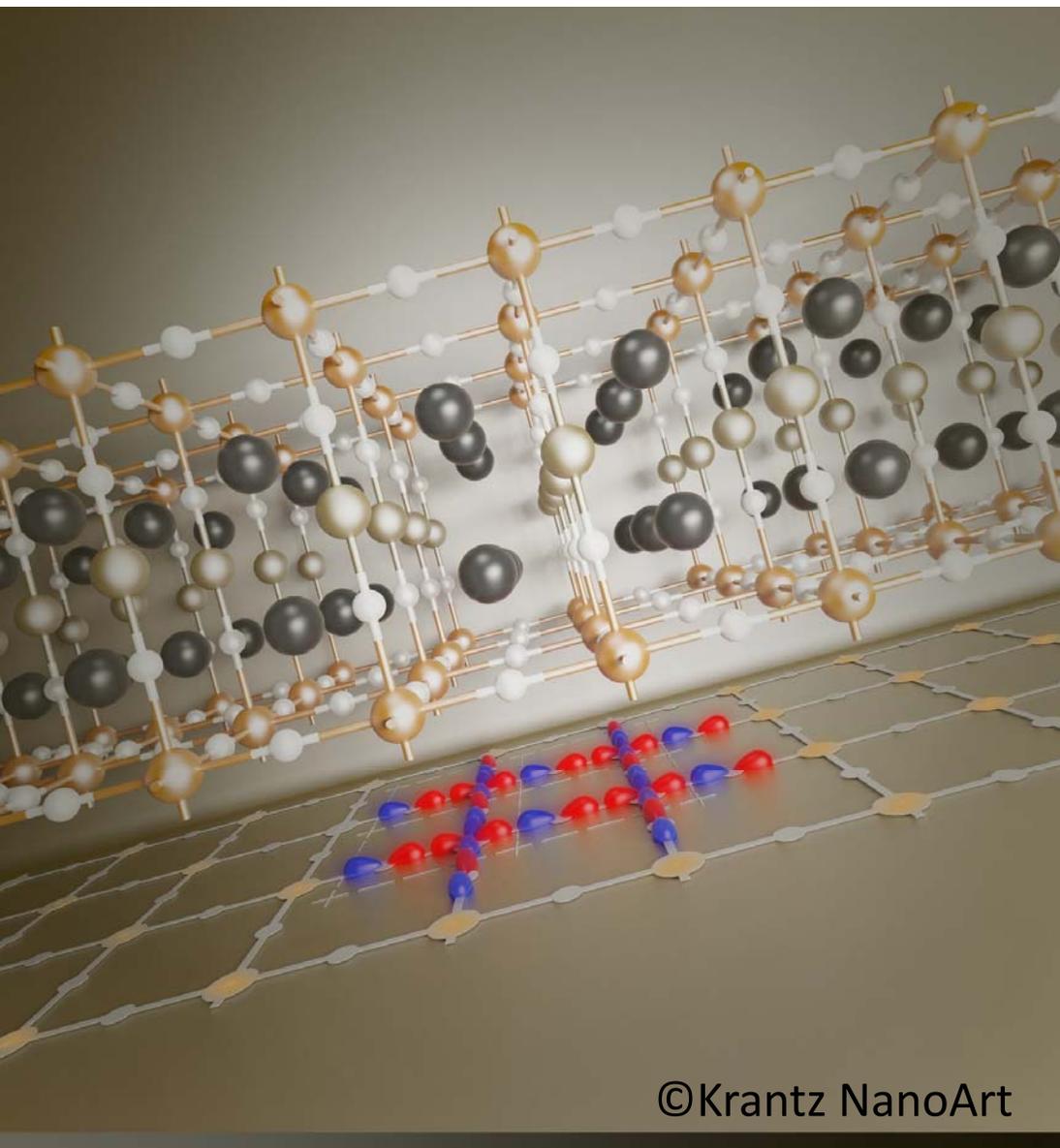


# Superconductivity in ultra-quantum matter: Part I

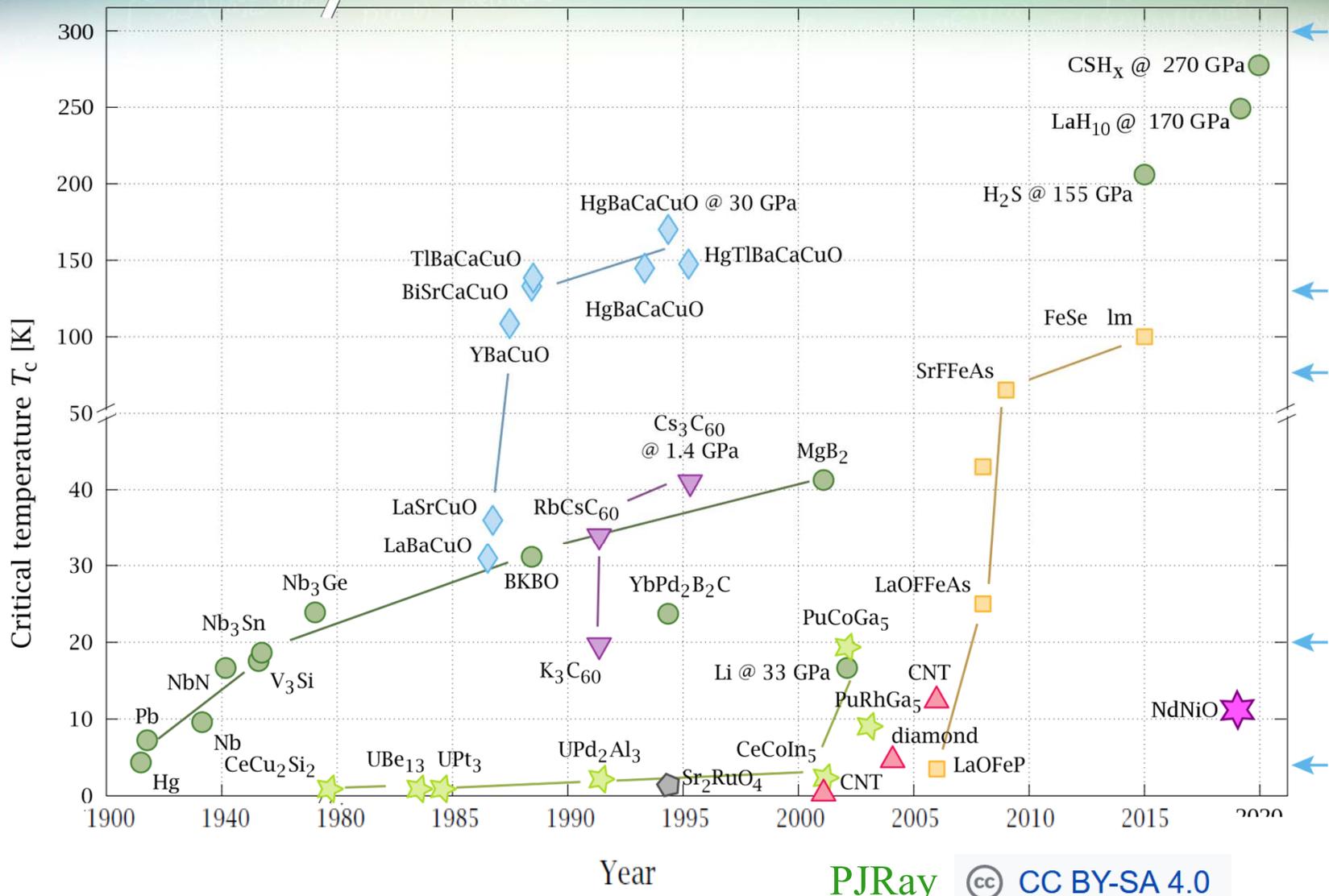
A.-M.S. Tremblay

RQMP  
30 September 2021  
10:30

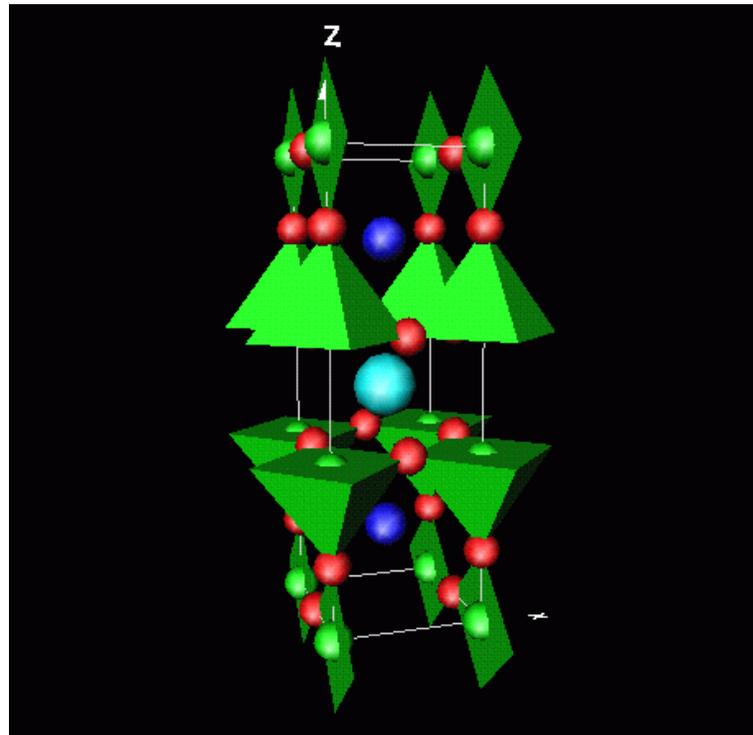
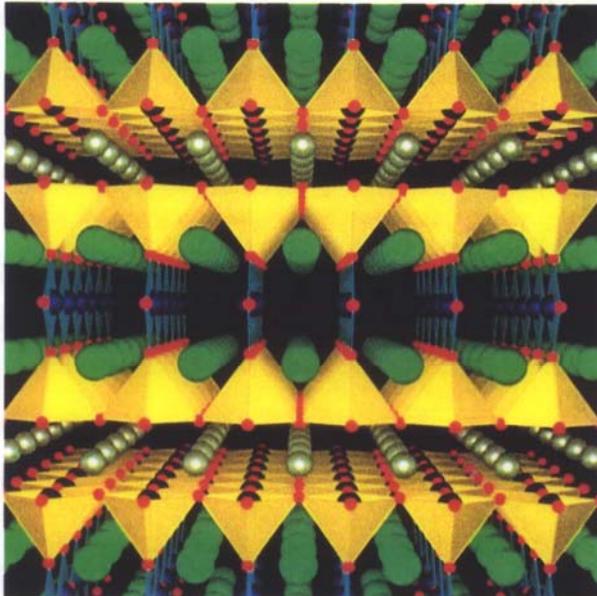




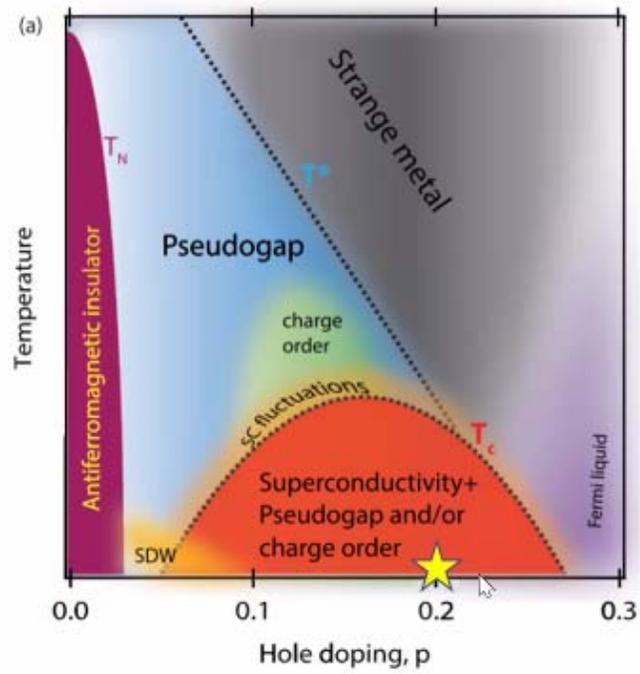
[USHERBROOKE.CA/IQ](http://USHERBROOKE.CA/IQ) 4



# Atomic structure



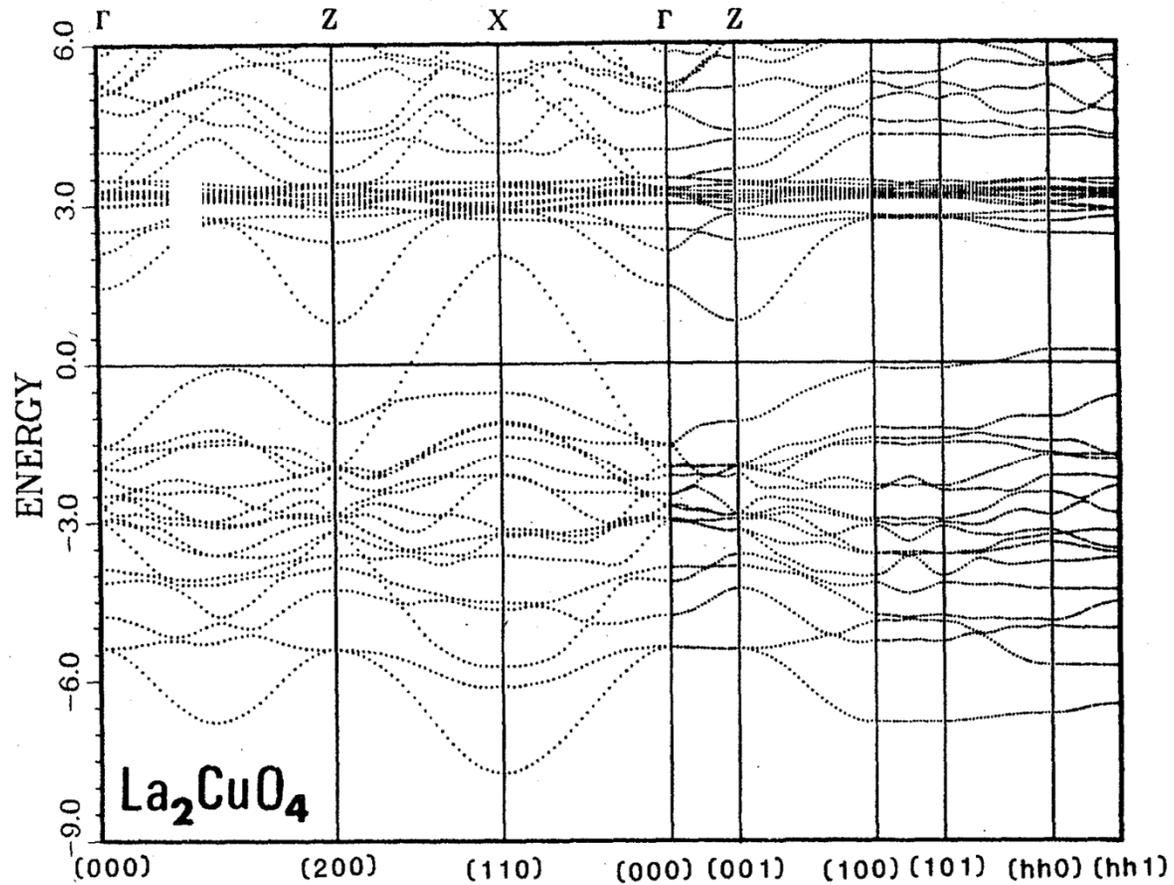
- Who ordered this?



Vishik, Rep. Prog. Phys. (2018)

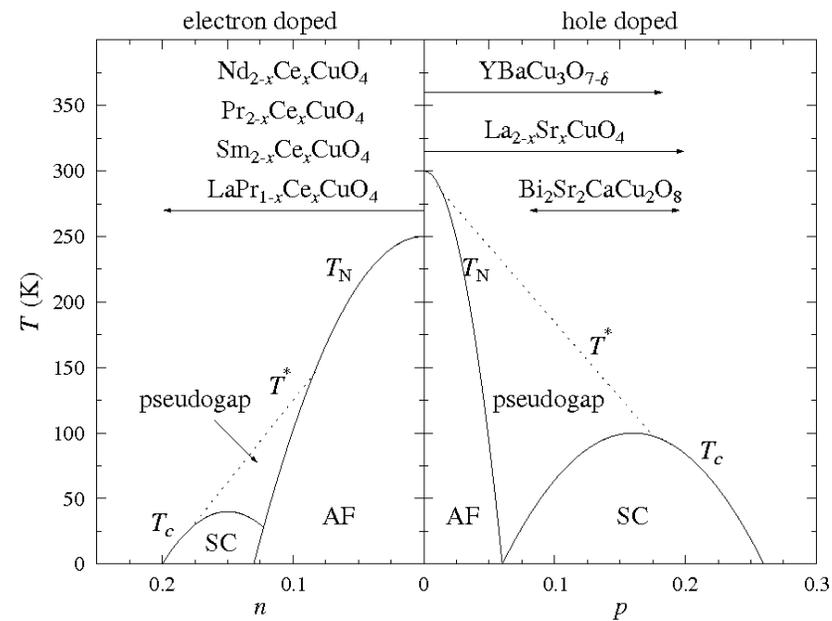


# Band structure for high Tc



W. Pickett, Rev. Mod. Phys. 1989

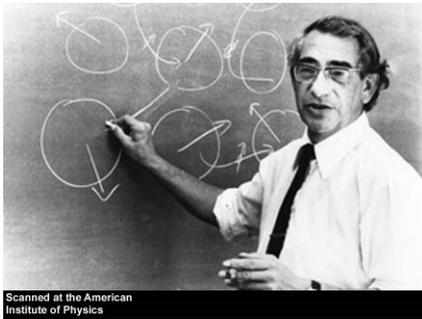
# "Universal" phase diagram



Public Domain,

<https://en.wikipedia.org/w/index.php?curid=21641300>

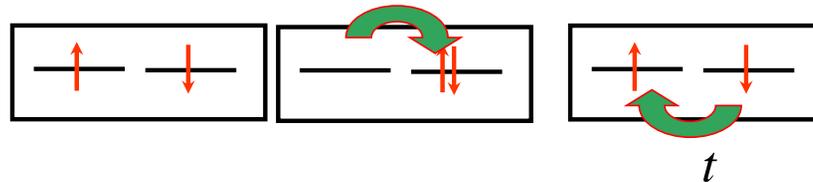
# Hubbard model



1931-1980

$$H = - \sum_{\langle ij \rangle \sigma} t_{i,j} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

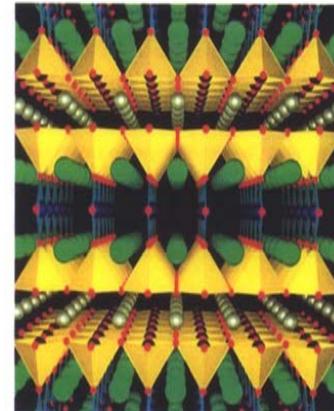
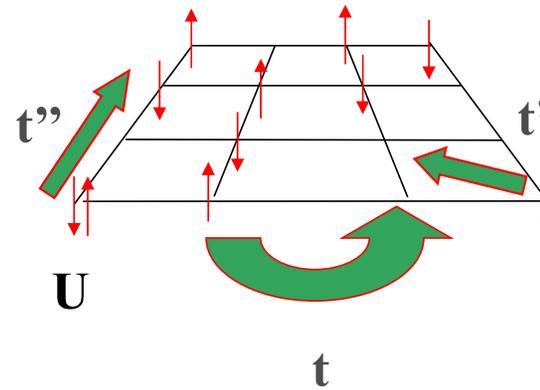
Spin 1/2



Effective model, Heisenberg:  $J = 4t^2 / U$

Attn: Charge transfer insulator

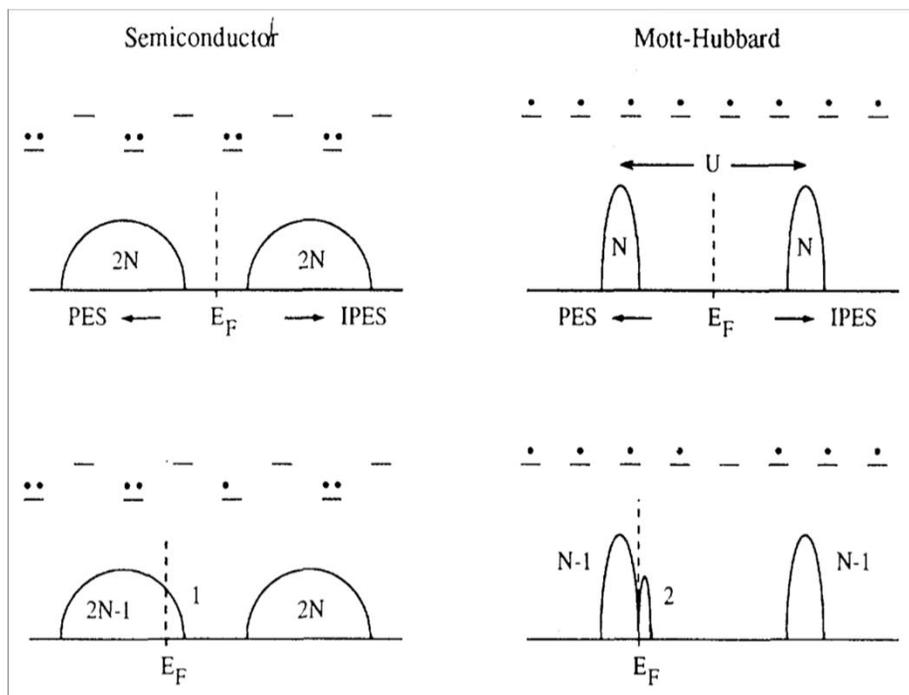
$\mu$



# Mott Insulator : X-Ray absorption

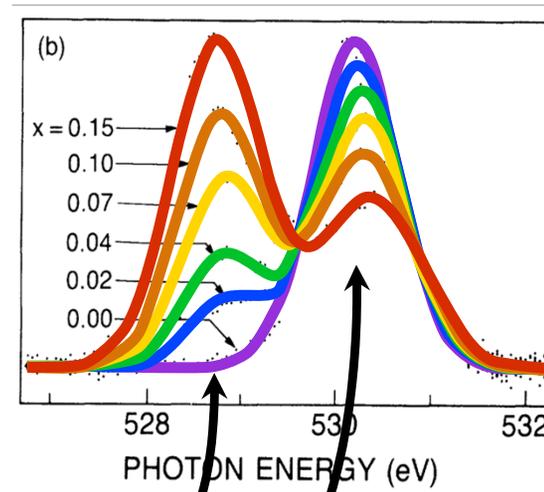
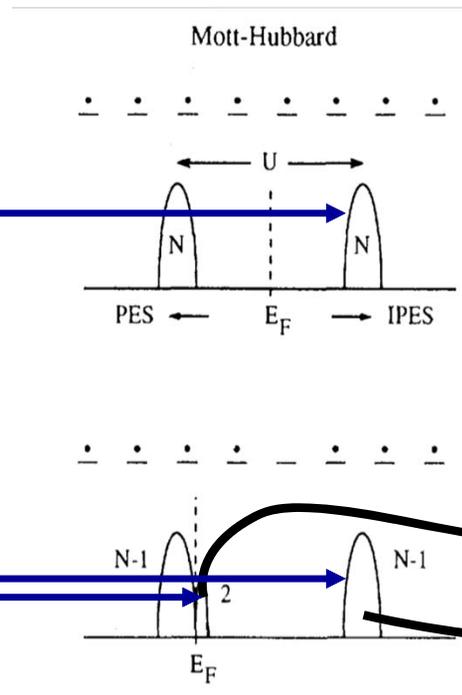


Meinders *et al.* PRB **48**, 3916 (1993)



# Mott Insulator : X-Ray absorption

Meinders *et al.* PRB **48**, 3916 (1993)



Chen *et al.* PRL **66**, 104 (1991)

## Take home messages

- Most of the main features of the phase diagram follow from the Hubbard model.
- This physics is continuously connected to the Mott transition at half-filling
- We need to look beyond traditional tools of solid state physics to work this out.

# Minuterie



- 10 Minutes

# Outline

- Method
- One-band Hubbard model
  - Phase diagram
  - Pseudogap
  - d-wave superconductivity
  - A phase transition at the heart of the phase diagram
- Three-band Hubbard model : oxygen can probe the details
  - Pseudogap
  - d-wave superconductivity

# Method :

## The precursors

Hohenberg-Kohn : Exchange correlation

Kohn-Sham : Basis set

Density Functional Theory

# Method

Metzner, Vollhardt PRL **62**, 324 (1989)

Georges, Kotliar, PRB **45**, 6479 (1992)

Jarrell PRL **69**, 168 (1992)

Review: Georges, Kotliar, Krauth, Rozenberg, RMP **68**, 13 (1996)

## Dynamical Mean-Field Theory : DMFT

# For additional physical intuition: Compare with more analytical approaches

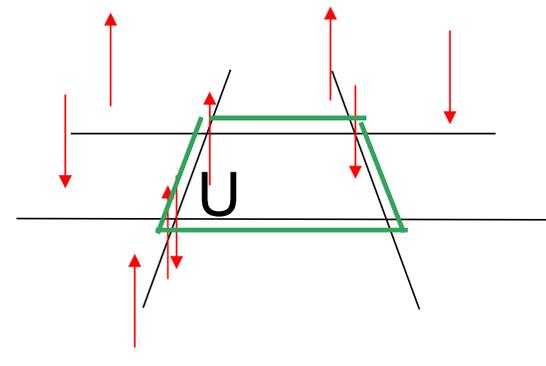
- Pseudogap

- Wei Wu, Scheurer, Chatterjee, Sachdev, Georges, Ferrero PRX 8, 021048 (2018)
- Scheurer, Chatterjee, Wu, Ferrero, Georges, Sachdev, PNAS **115**, E3665 (2018).

# Localized and delocalized pictures



## Localized



Lichtenstein *et al.*, PRB 2000  
Kotliar *et al.*, PRB 2000  
M. Potthoff, EJP 2003

## REVIEWS

Maier, Jarrell *et al.*, RMP. (2005)  
Kotliar *et al.* RMP (2006)  
AMST *et al.* LTP (2006)

$$(G^{-1})_{ij} = (G_0^{-1})_{ij} - \Sigma_{ij}$$

# Skeleton expansion for the Anderson impurity problem

$$\Sigma_{ij} = \text{Diagram 1} + \text{Diagram 2} + \dots$$

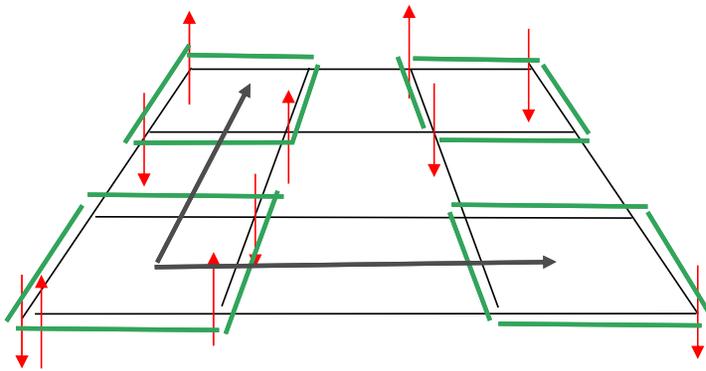
The diagrammatic expansion shows the self-energy  $\Sigma_{ij}$  as a sum of terms. The first term is a crossed diagram with two vertices connected by two lines. The second term is a diagram with two vertices connected by a horizontal line labeled  $G_{ij}$ , and a loop above it consisting of two lines labeled  $G_{ij}$  and  $G_{ji}$ .

$$(G^{-1})_{ij} = (G_0^{-1})_{ij} - \Sigma_{ij}$$

# Localized and delocalized pictures



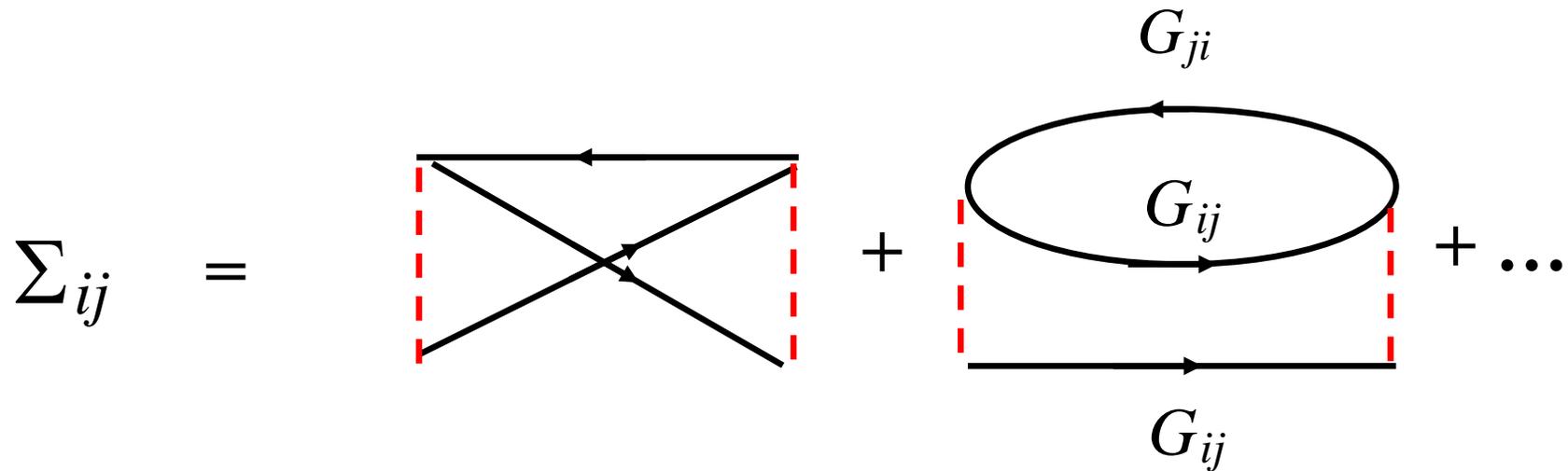
## Delocalized



$$\mathbf{R} \rightarrow \tilde{\mathbf{k}}$$

$$G_{ij}(\tilde{\mathbf{k}}) = \left( \frac{1}{(i\omega_n + \mu)I - \varepsilon(\tilde{\mathbf{k}}) - \Sigma} \right)_{ij}$$

# Skeleton expansion for the connected clusters

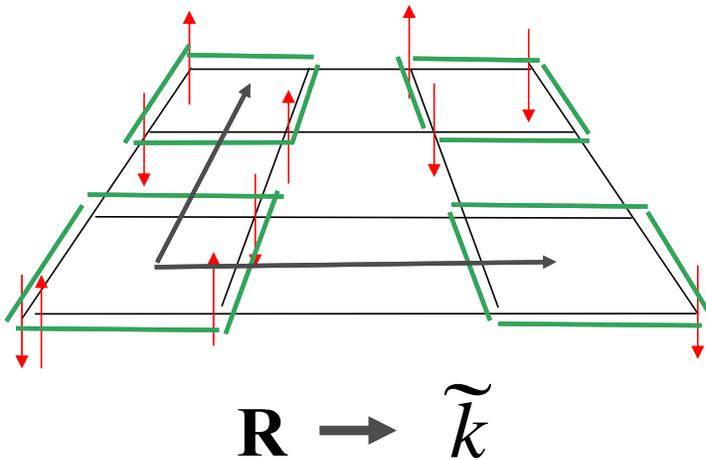


$$G_{ij}(\tilde{k}) = \left( \frac{1}{(i\omega_n + \mu)I - \varepsilon(\tilde{k}) - \Sigma} \right)_{ij} \longrightarrow G_{ij} = \int \frac{d^d \tilde{k}}{(2\pi)^d} G_{ij}(\tilde{k})$$

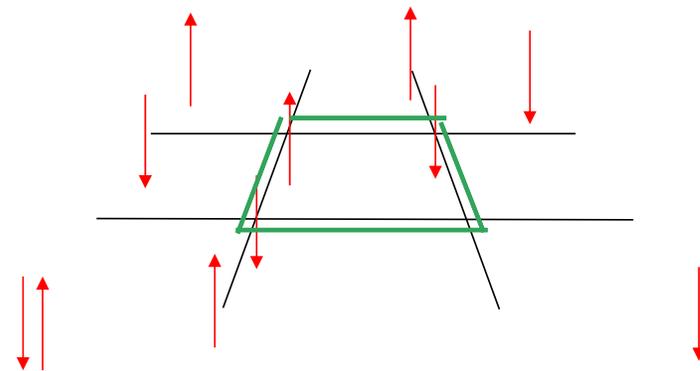
# Localized and delocalized pictures **C-DMFT**



## Delocalized



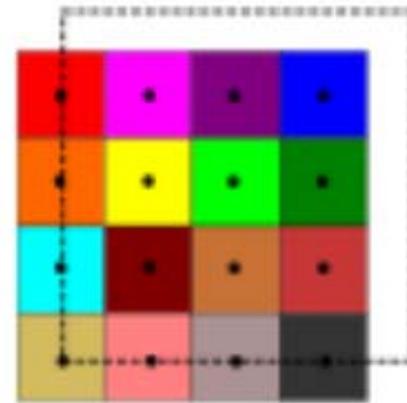
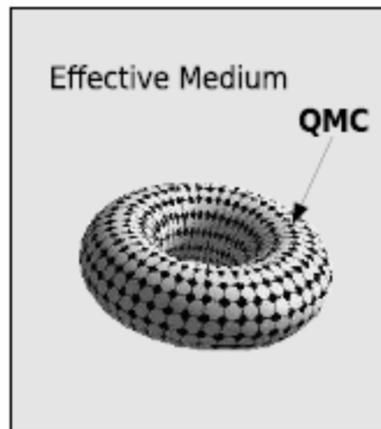
## Localized



$$G_{ij} = \int \frac{d^d \tilde{\mathbf{k}}}{(2\pi)^d} \left( \frac{1}{(i\omega_n + \mu)I - \varepsilon(\tilde{\mathbf{k}}) - \Sigma} \right)_{ij}$$

$$(G^{-1})_{ij} = (G_0^{-1})_{ij} - \Sigma_{ij}$$

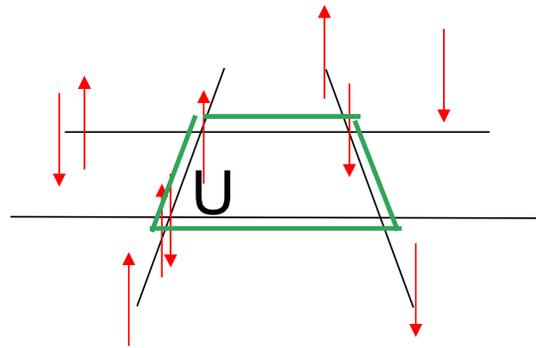
# Dynamical cluster approximation (DCA)



Hettler ...Jarrell...Krishnamurty PRB 58 (1998)

# Impurity solvers

# Impurity solver (Exact diagonalisation)



Caffarel, Krauth, PRL **72** 1545 (1994)

QCM David Sénéchal

# Impurity solver : continuous-time quantum Monte Carlo

$$Z = \int \mathcal{D}[\psi^\dagger, \psi] e^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{K}} \psi_{\mathbf{K}}^\dagger(\tau) \Delta_{\mathbf{K}}(\tau, \tau') \psi_{\mathbf{K}}(\tau')}$$

Hybridization expansion :

Werner Millis PRB **74**, 155107 (2006)

Werner Millis B **75**, 085108 (2007)

Haule, PRB **75**, 155113 (2007)

Sémon, Sordi, AMST PRB **89**, 165113 (2014)

Sémon, Yee, Haule, AMST PRB **90**, 075149 (2014)

triqs

ALPSCore / CT-HYB

iQIST

ComCTQMC

# Impurity solver : continuous-time quantum Monte Carlo

$$S = \int_0^\beta d\tau d\tau' \sum_{\sigma=\uparrow,\downarrow} \xi_\sigma^*(\tau) [g_{0\sigma}^{-1}(\tau - \tau')] \xi_\sigma(\tau') \\ + U \int_0^\beta d\tau \left( n_\uparrow(\tau)n_\downarrow(\tau) - \frac{n_\uparrow(\tau) + n_\downarrow(\tau)}{2} \right)$$

CT-AUX : [Gull, Werner, Parcollet, Troyer, 2008, Europhys. Lett. \*\*82\*\*, 57003 \(2008\)](#)

DCA++

Review of these methods

[Gull, Millis, Lichtenstein, Rubtsov, Troyer, Werner RMP \*\*83\*\*, 349 \(2011\)](#)

# Some groups using these methods for cuprates



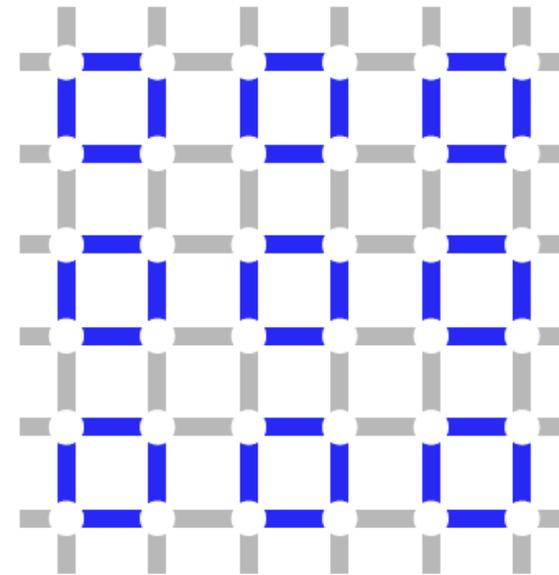
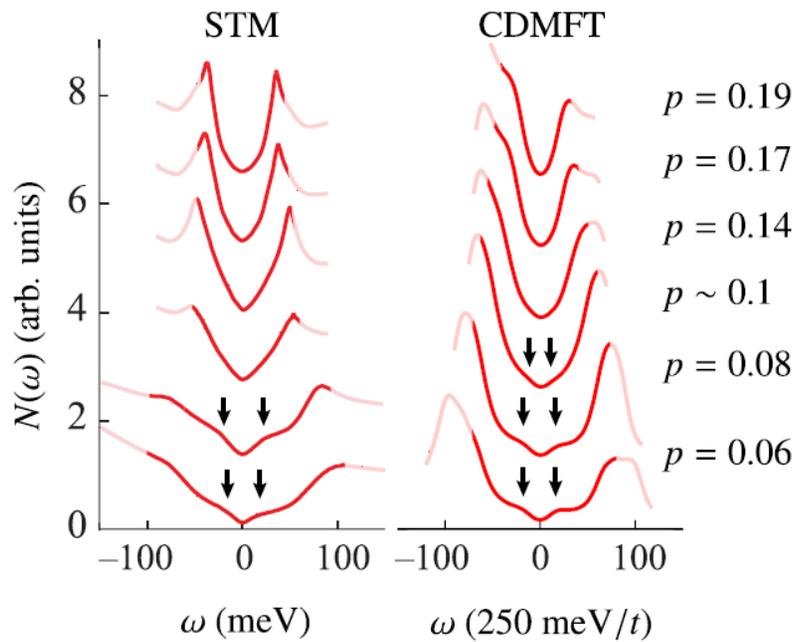
- Europe:
  - Georges, Parcollet, Ferrero, Civelli, Wu (Paris)
  - Lichtenstein, Potthoff, (Hamburg) Aichhorn (Graz), Liebsch (Jülich) de Medici (Grenoble) Capone (Italy)
- USA:
  - Gull (Michigan) Millis (Columbia)
  - Kotliar, Haule (Rutgers)
  - Jarrell (Louisiana)
  - Maier, Okamoto (Oakridge)
- Japan
  - Imada (Tokyo) Sakai, Tsunetsugu, Motome

# Critique

## + and -

- Long range order:
  - No mean-field factorization on the cluster
  - Symmetry breaking allowed in the bath
- Included exactly:
  - Short-range dynamical and spatial correlations
- Missing:
  - Long wavelength p-h and p-p fluctuations
  - Hence good when the corresponding correlation lengths are small

# Possible artefacts



Verret, Roy, Foley, Charlebois, Sénéchal, A.-M.S.T, RPB **100**, 224520 (2019)

STM Kohsaka, ... Davis, Nature (London) **454**, 1072 (2008).

# What to do

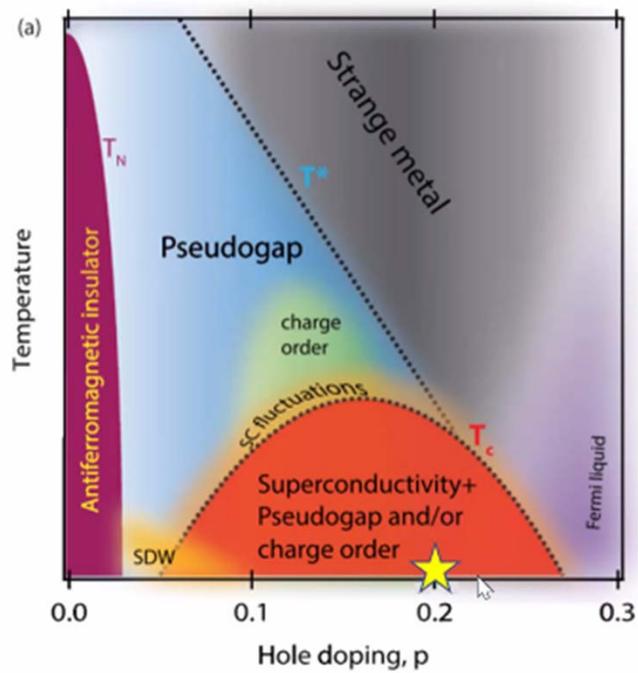
- Exact in the infinite size limit of the cluster
  - Compare different cluster sizes
  - Compare real-space (CDMFT) and momentum space (DCA) clusters

# Minuterie

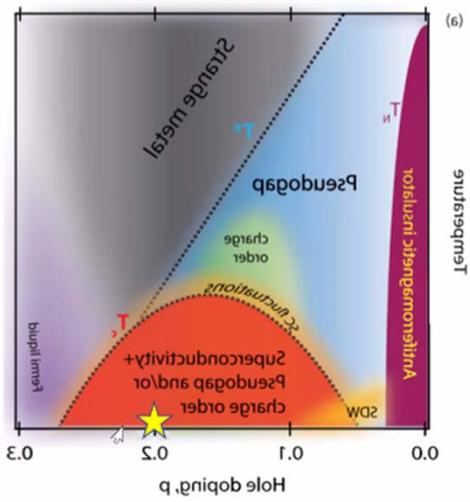


- 10 Minutes
- 6 minutes = 16 minutes

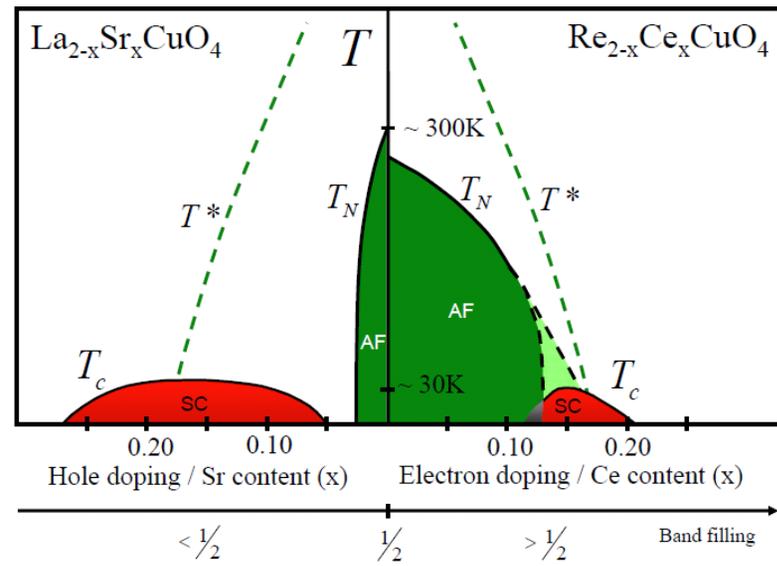
# Back to our problem: Phase diagram

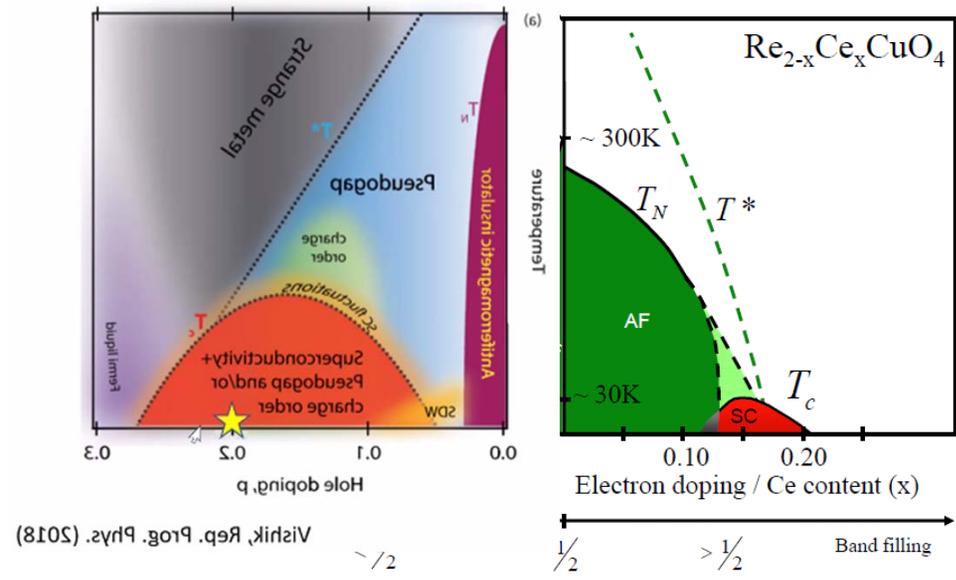


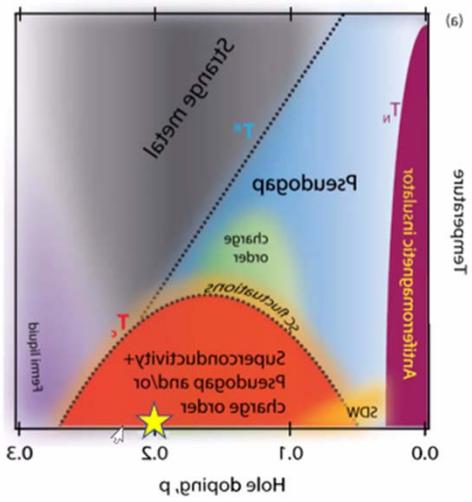
Vishik, Rep. Prog. Phys. (2018)



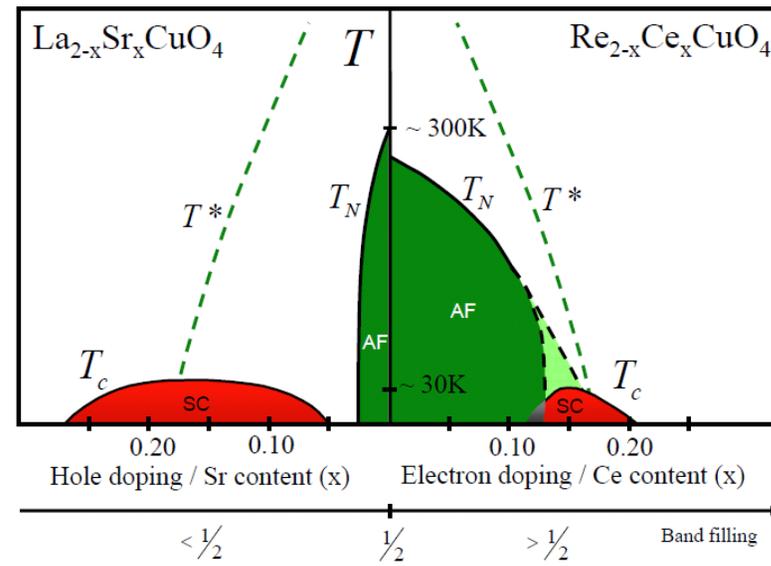
Vishik, Rep. Prog. Phys. (2018)





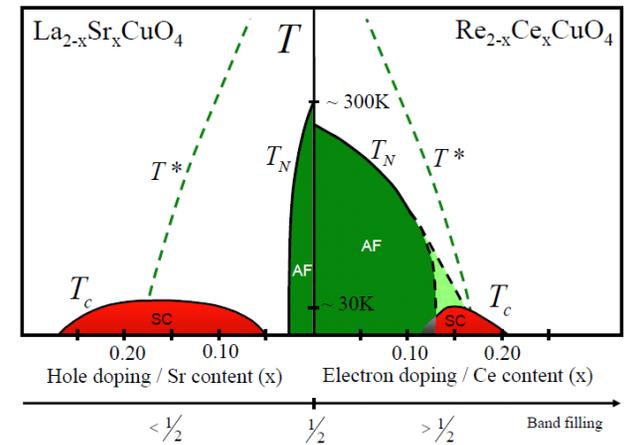
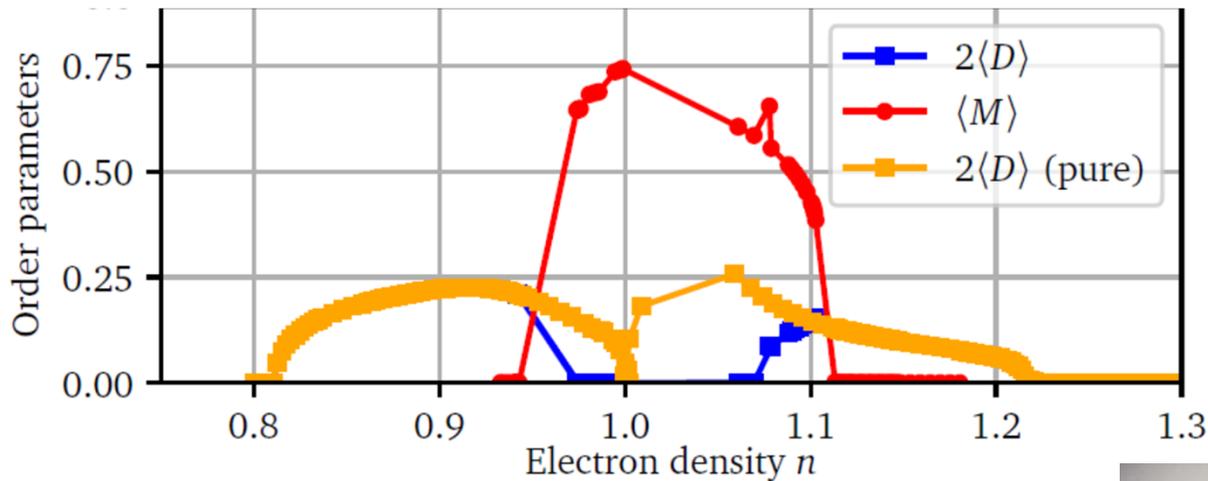


Vishik, Rep. Prog. Phys. (2018)

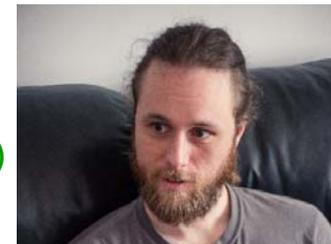


# A bird's eye overview of the $T = 0$ phase diagram

$$U = 8t, t' = -0.3t, t'' = 0.2t$$



- A. Foley *et al.* Phys. Rev. B **99**, 184510 (2019)
- S. S. Kancharla, *et al.* Phys. Rev. B **77**, 184516 (2008)
- D. Sénéchal, *et al.* Phys. Rev. Lett. **94**, (2005)
- M. Jarrell *et al.* EPL **56** 563, (2001)



A. Foley



S. Verret



D. Sénéchal

CDMFT 4 sites

# Competing ground states

PHYSICAL REVIEW X **10**, 031016 (2020)

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## Absence of Superconductivity in the Pure Two-Dimensional Hubbard Model

Mingpu Qin<sup>1,2,\*</sup>, Chia-Min Chung<sup>3,4,\*</sup>, Hao Shi,<sup>5</sup> Ettore Vitali,<sup>6,2</sup> Claudius Hubig<sup>7</sup>,  
Ulrich Schollwöck<sup>3,4</sup>, Steven R. White<sup>8</sup>, and Shiwei Zhang<sup>5,2</sup>

PRL **113**, 046402 (2014)

PHYSICAL REVIEW LETTERS

week ending  
25 JULY 2014

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## Competing States in the $t$ - $J$ Model: Uniform $d$ -Wave State versus Stripe State

Philippe Corboz,<sup>1,2</sup> T. M. Rice,<sup>1</sup> and Matthias Troyer<sup>1</sup>

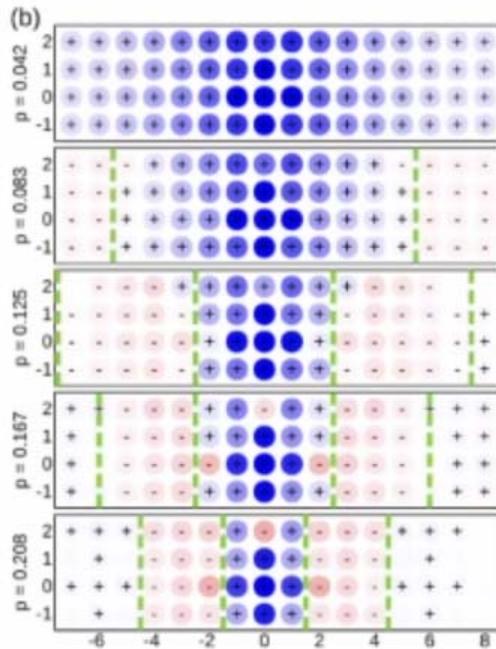
# Competing ground states



ARTICLE OPEN

## Stripe order from the perspective of the Hubbard model

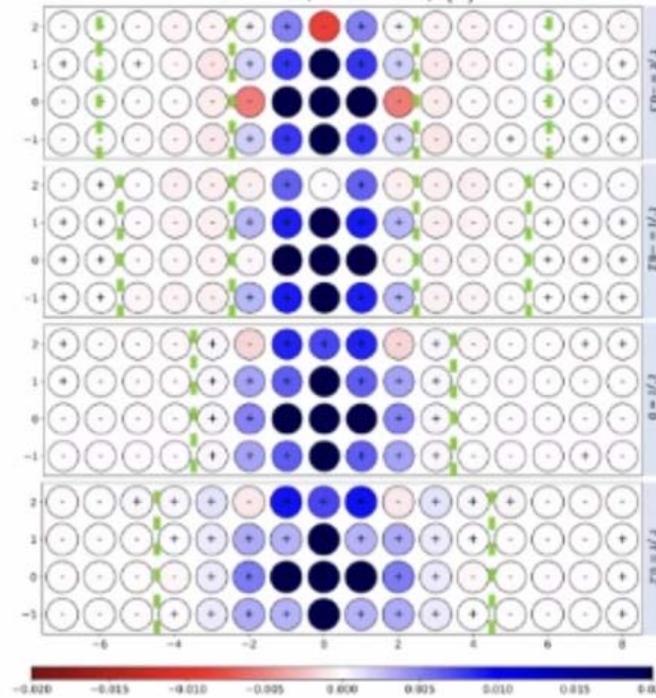
Edwin W. Huang<sup>1,†</sup>, Christian B. Mendl<sup>2</sup>, Hong-Chen Jeng<sup>2</sup>, Brian Moritz<sup>1,3</sup> and Thomas F. Devereaux<sup>4\*</sup>



DQMC on 16 x 4 Hubbard:  
 $U/t = 6, t'/t = -0.25, T/t = 0.22$

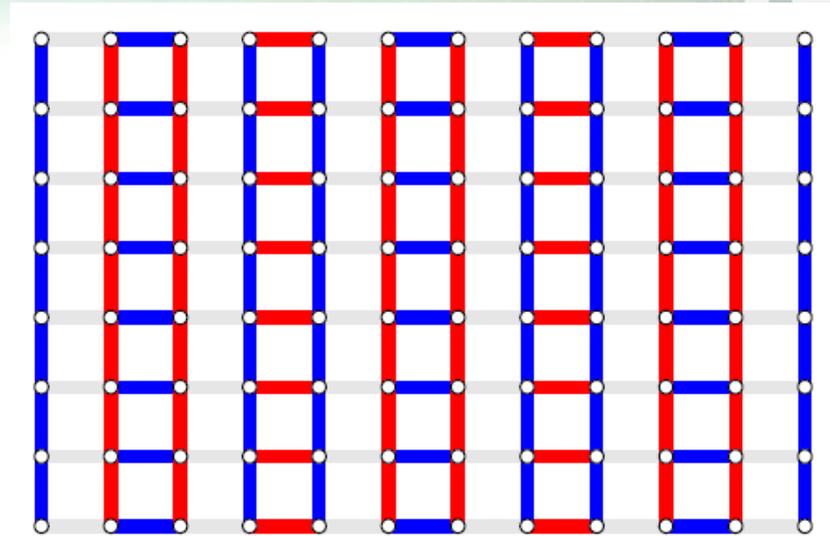
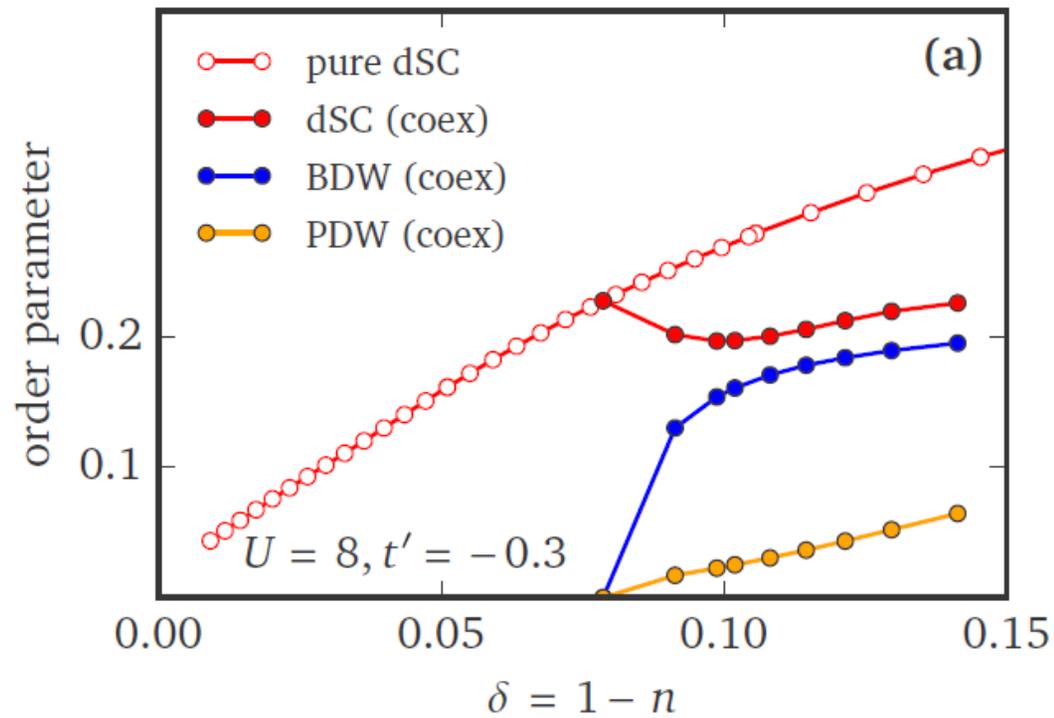
## DCA (16 x 4 cluster)

$U/t = 6, T/t = 0.2, \langle n \rangle = 0.8$



P. Mai, S. Karakuzu, S. Johnston & TAM, in preparation

# Competing states



**VCA**

Faye Sénéchal, PRB **95** (2017)



J.-P. Faye Latyr



D. Sénéchal

# Minuterie

- 10 minutes
- 6 minutes
- 3 minutes = 19



# Minuterie

- 10 minutes
- 6 minutes
- 3 minutes
- 4 minutes
- 5 minutes = 28



# Pseudogap



Alexis Reymbaut



Simon Bergeron



Maxime Charlebois

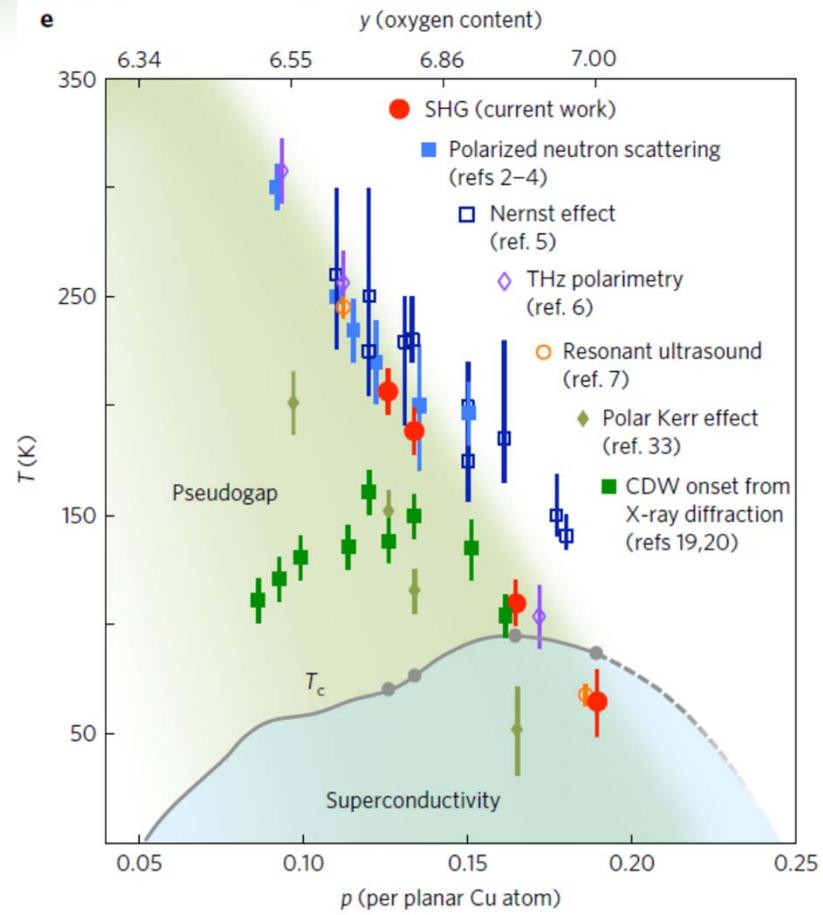


Patrick Sémon



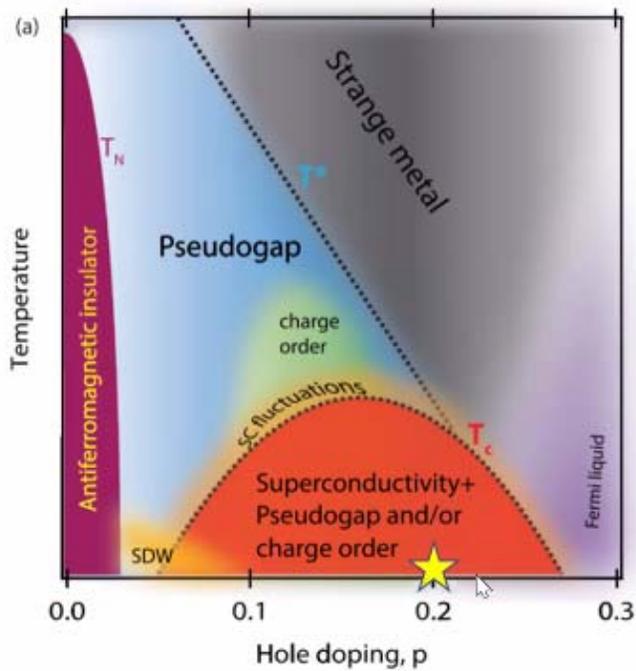
Marion Thénault

Reymbaut, *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

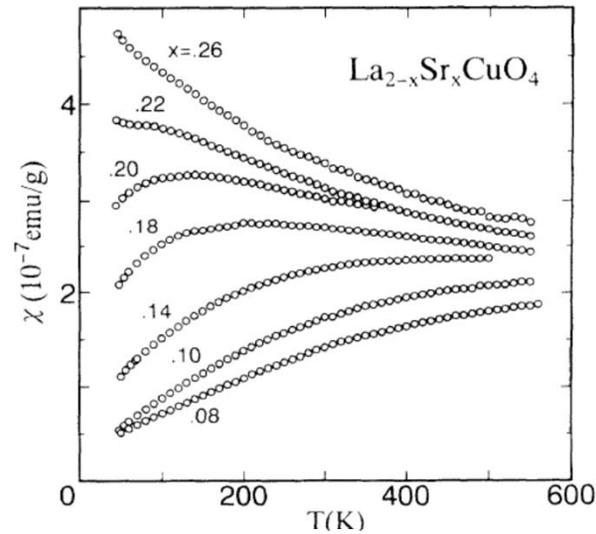


Zhao *et al.* Nat. Phys. 13, 250 (2017).

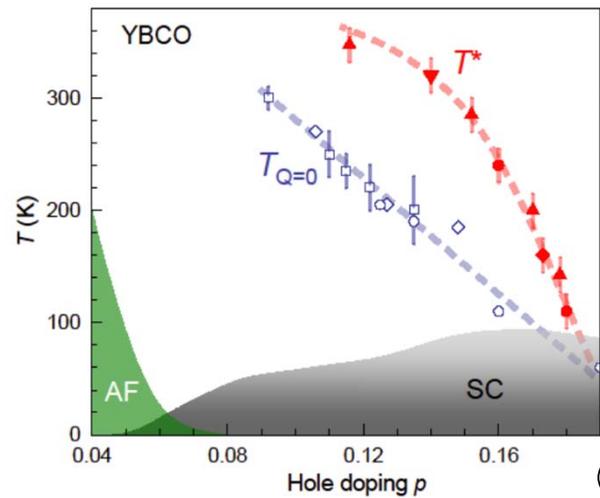
# Pseudogap



Vishik, Rep. Prog. Phys. (2018)



Nakano *et al.* Phys. Rev. B **49**, 16000 (1994)  
Alloul *et al.* (1989)

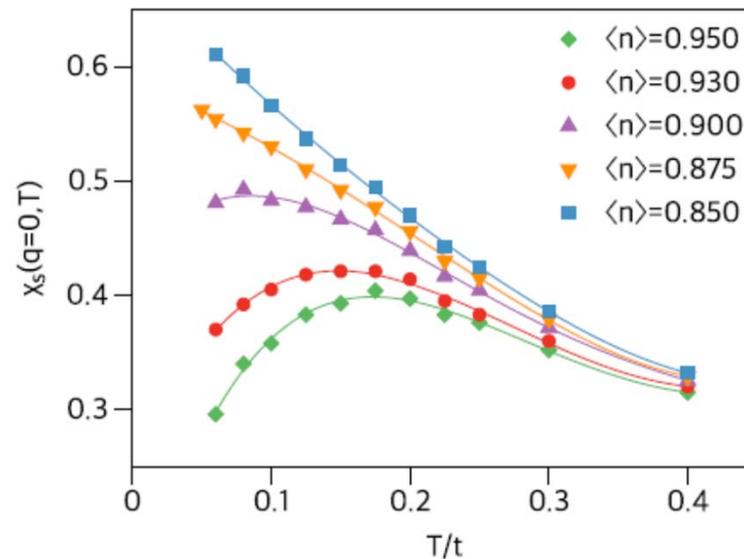


$T^*$  from NMR Knight shift  
Alloul *et al.*  
Curro *et al.*  
Berthier *et al.*  
Julien *et al.*

$T_{Q=0}$  from  
Neutron scattering (Bourges *et al.*)  
Torque (Matsuda *et al.*)  
SHG (Hsieh *et al.*)

© M-H-Julien

# Knight shift ( $Q=0$ spin susceptibility)

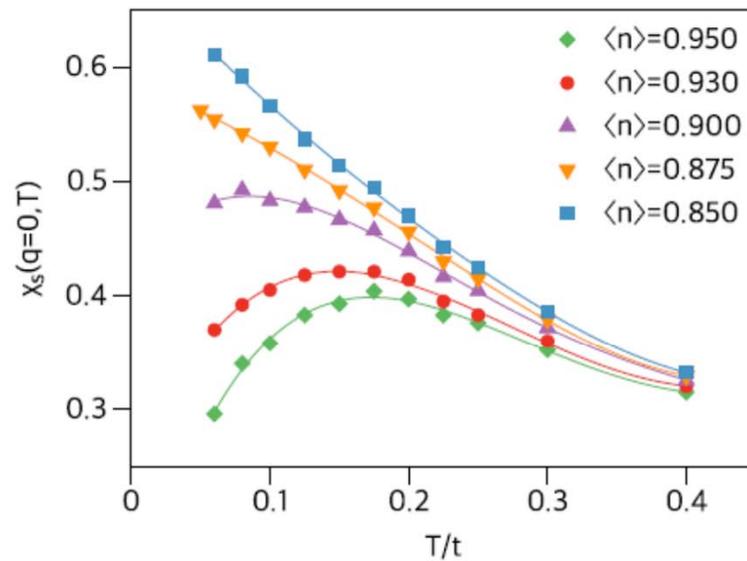
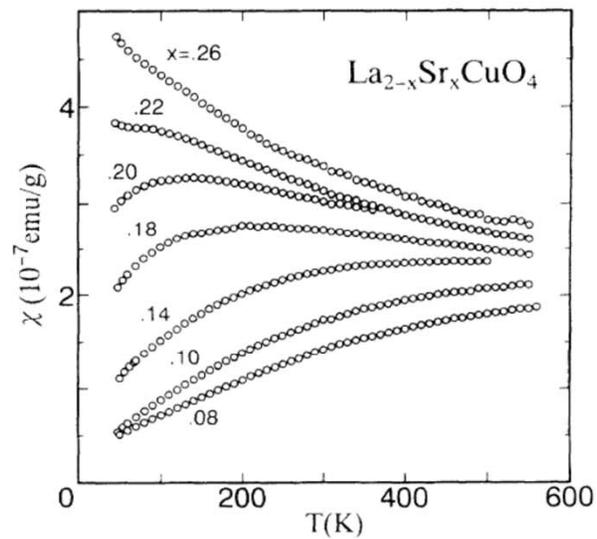


**Fig. 3** Temperature and doping dependence of the  $q=0$  spin susceptibility. At the smaller dopings (larger filling  $\langle n \rangle$ ),  $\chi_s(T)$  exhibits a peak in the temperature dependence indicating the opening of a PG

DCA 12 sites,  $t'=0$ ,  $U = 7$

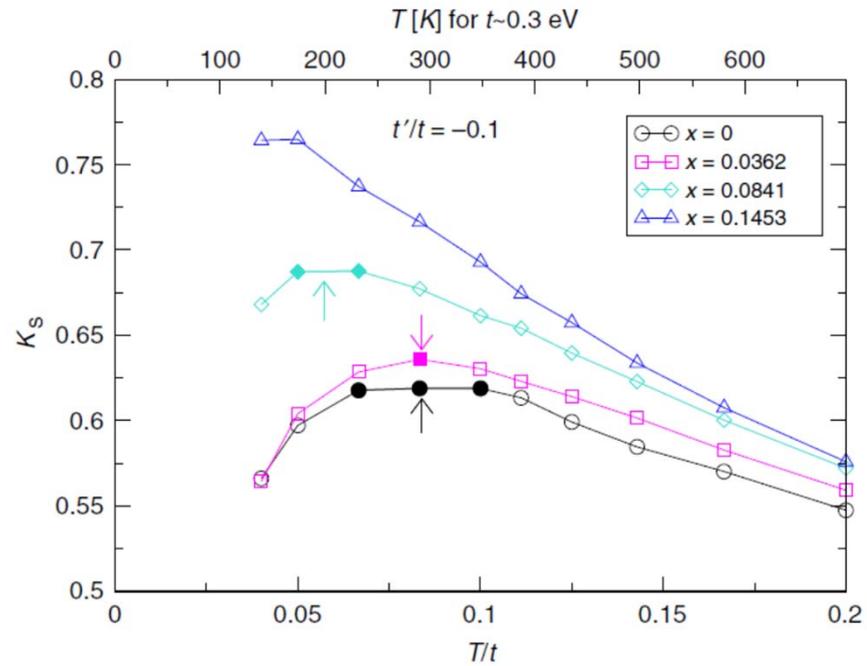
T.A. Maier, D.J. Scalapino, npj Quantum Materials (2019)

# Comparison



**Fig. 3** Temperature and doping dependence of the  $q=0$  spin susceptibility. At the smaller dopings (larger filling  $\langle n \rangle$ ),  $\chi_s(T)$  exhibits a peak in the temperature dependence indicating the opening of a PG

# Knight shift

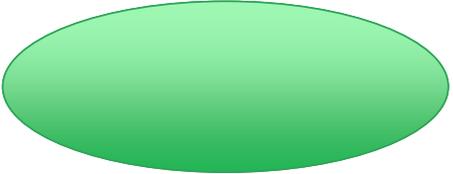


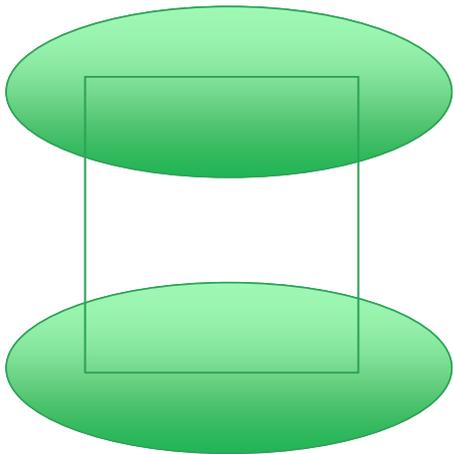
DCA 8 sites,  $U = 6$ ,  $t' = -0.1t$

Chen, LeBlanc, Gull, Nature Com. Apr. 2017

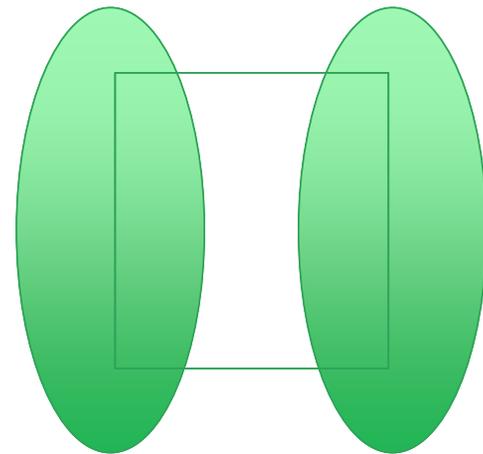
See also Jarrell *et al.* 2001, 2002

# Physical origin of the pseudogap

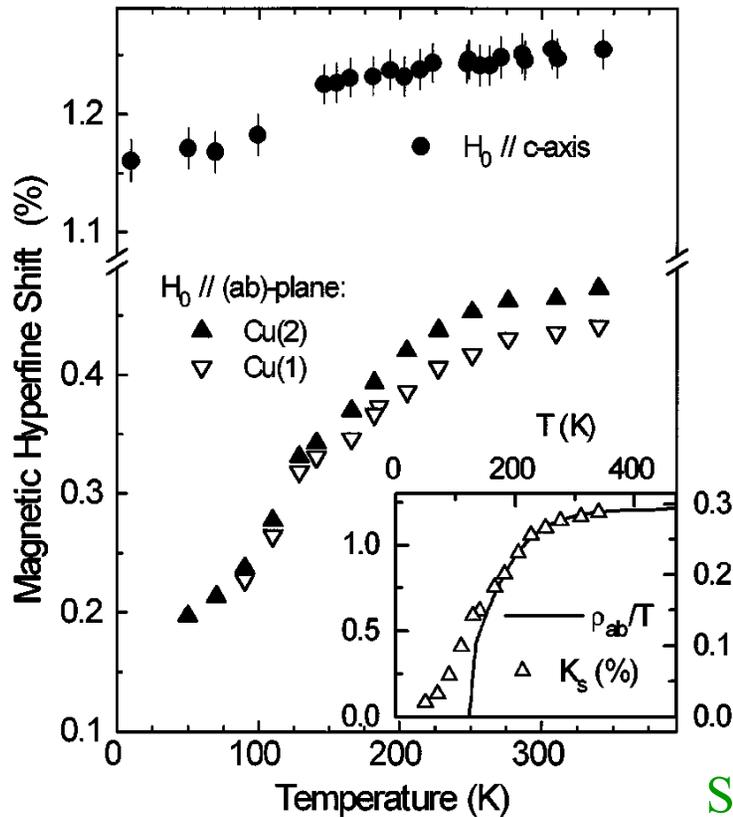

$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



-

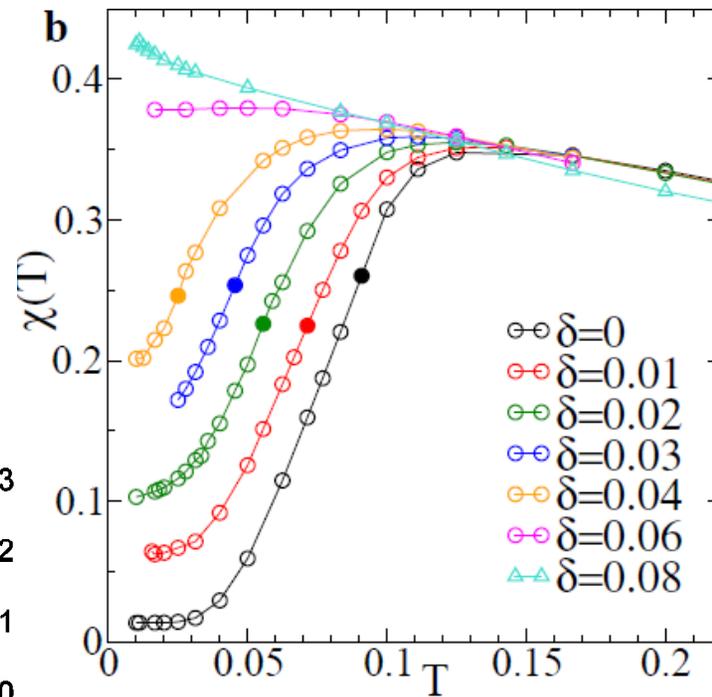


# Spin susceptibility



Underdoped Hg1223

Julien et al. PRL 76, 4238 (1996)



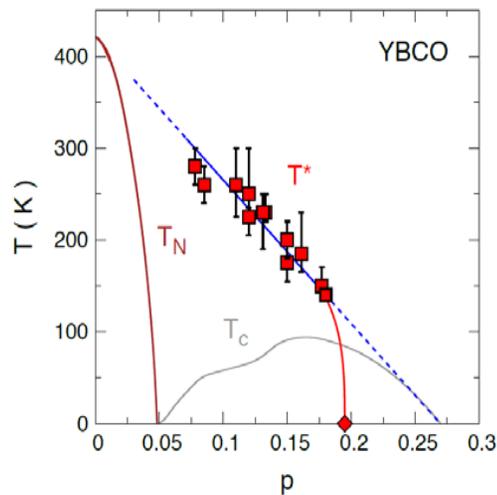
Sordi et al., Sci. Rep. 2 547 (2012);

$$U = 6.2, t' = 0$$

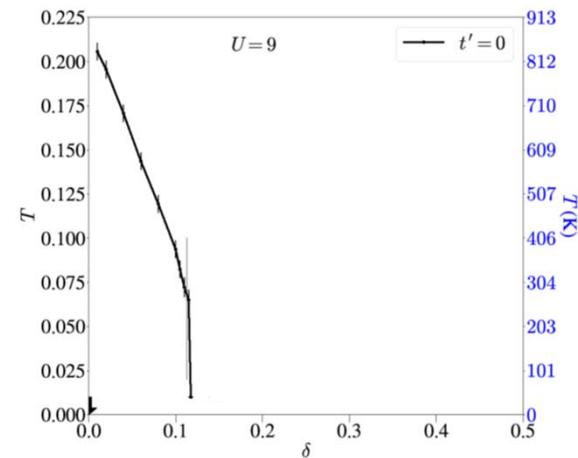
# Experiments and $T^*$



G.Sordi et al. Phys. Rev. B **87**,  
041101(R) (2013)



Cyr-Choinières et al. Phys. Rev. B **97**, 064502



$p$

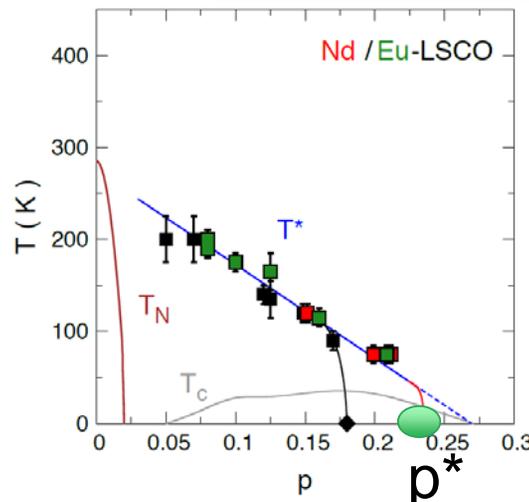
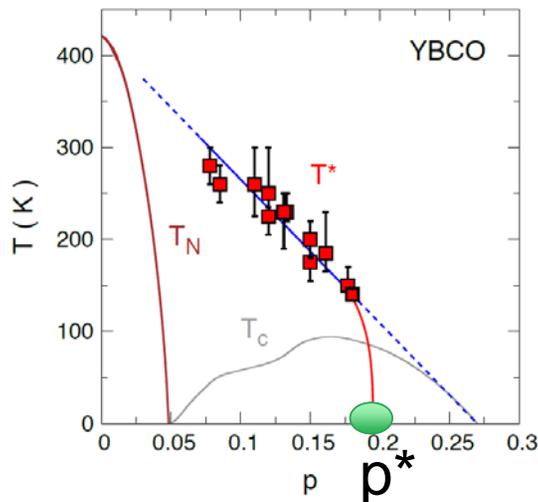
A. Reymbaut, M. Thénault, L. Fratino, G. Sordi,  
P. Sémon, AMT, Phys. Rev. Research **1**, 023015 (2019)

W Wu, A Georges, M Ferrero Phys. Rev. X **8**, 021048 (2018).

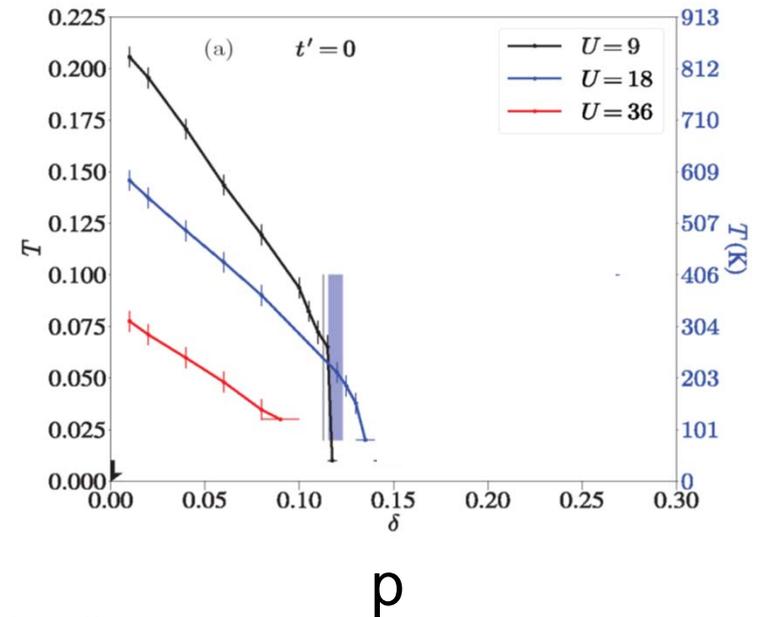
Bragança, Sakai, Aguiar, Civelli, PRL **120**, 067002 (2018)

# Experiments and $T^*$

$$k_B T^* \sim J$$



$$J = 4t^2/U$$



Cyr-Choinières et al. Phys. Rev. B **97**, 064502

A. Reymbaut, M. Thénault, L. Fratino, G. Sordi,  
P. Sémon, AMT, Phys. Rev. Research **1**, 023015 (2019)

W Wu, A Georges, M Ferrero Phys. Rev. X **8**, 021048 (2018).

Bragança, Sakai, Aguiar, Civelli, PRL **120**, 067002 (2018)

# Entropy maximum

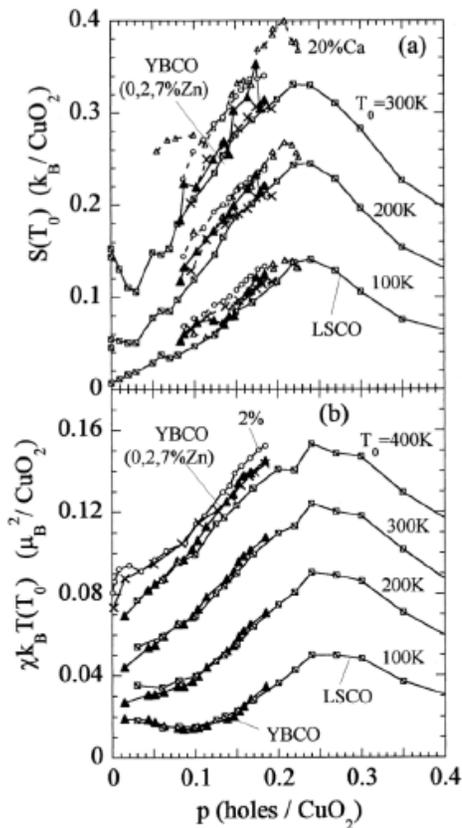


Fig. 14. (a) Entropy  $S(T_0)$  (in  $k_B/\text{CuO}_2$ ) at fixed  $T = T_0$  vs  $p$ , reflecting spectral weight within fixed energy windows  $\sim E_F \pm 2k_B T_0$  for LSCO and YBCO (0, 2, 7% Zn and 20% Ca). (b)  $\chi k_B T(T_0)$  (in  $\mu_B^2/\text{CuO}_2$ ) at fixed  $T = T_0$  vs  $p$  for LSCO and YBCO(0, 2, 7% Zn).

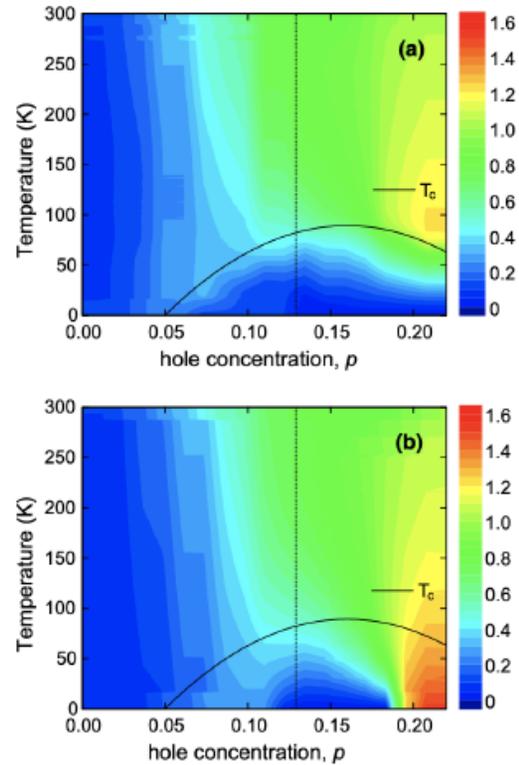
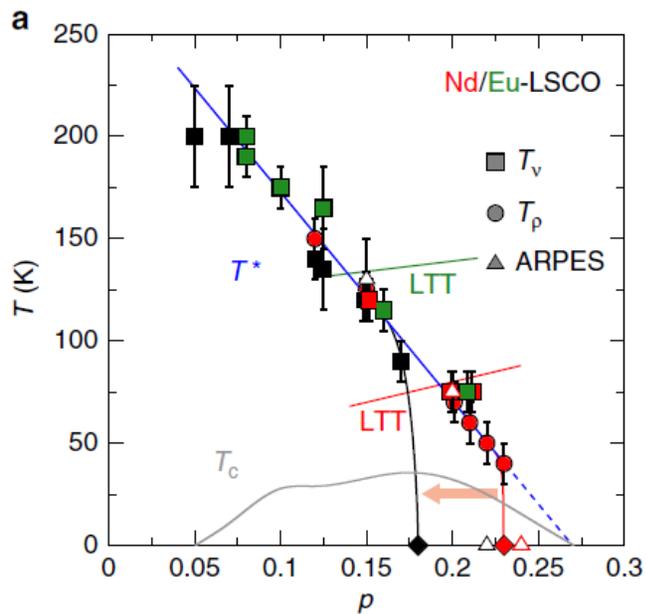


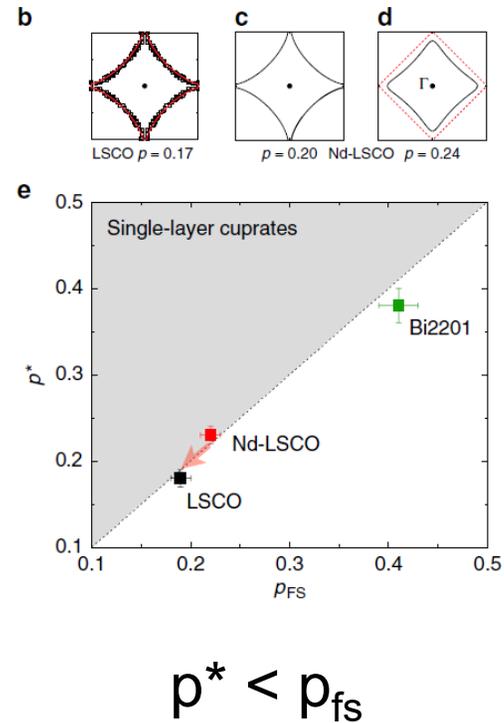
Fig. 2. False color contour plot of  $S(T)/T$  for (a) the real superconducting HTS system and (b) for the normal-state extrapolated to  $T = 0$ . The plots are a composite of Bi-2212 data for  $p > 0.13$  and Y-123 data for  $p < 0.13$ . The crossover is marked by the vertical dashed line.

Tallon, Loram (2001)  
 Physica C: Supercon 349(1):53–68.  
 Tallon, *et al.* (2004)  
 Physica C: Supercon. 415(1):9–14.

# Results : effect of $t'$ on $T^*$

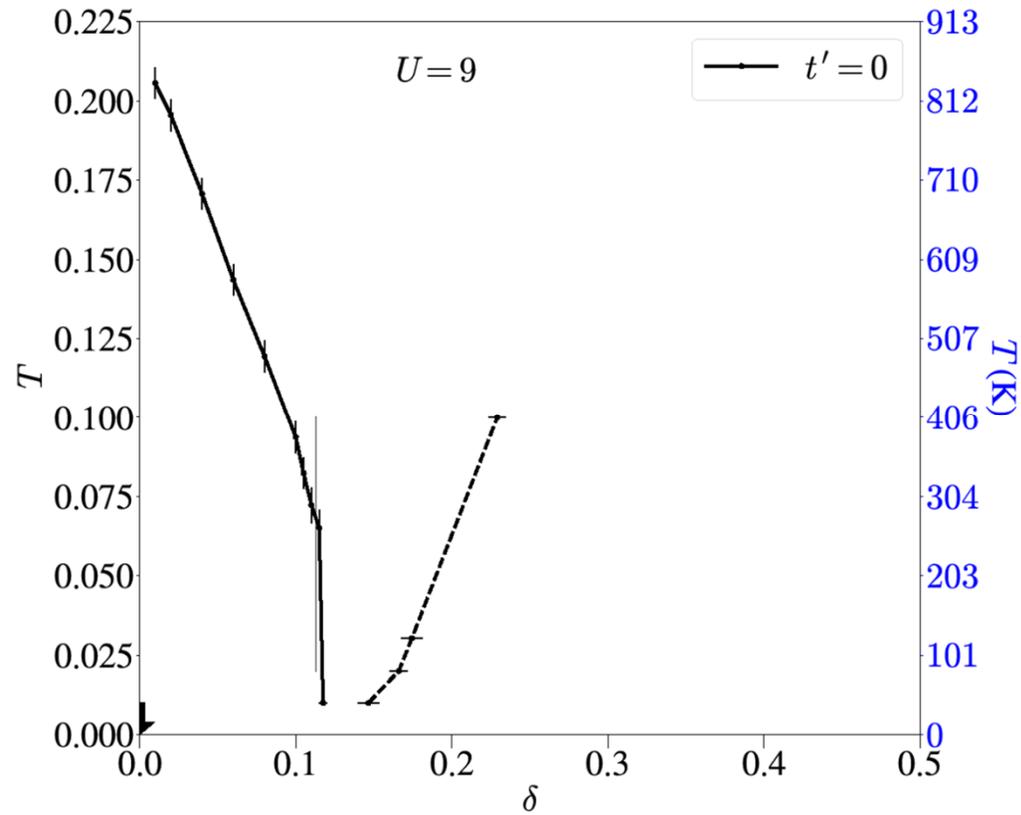


Doiron-Leyraud *et al.*  
Nature Comm. **8** 2044



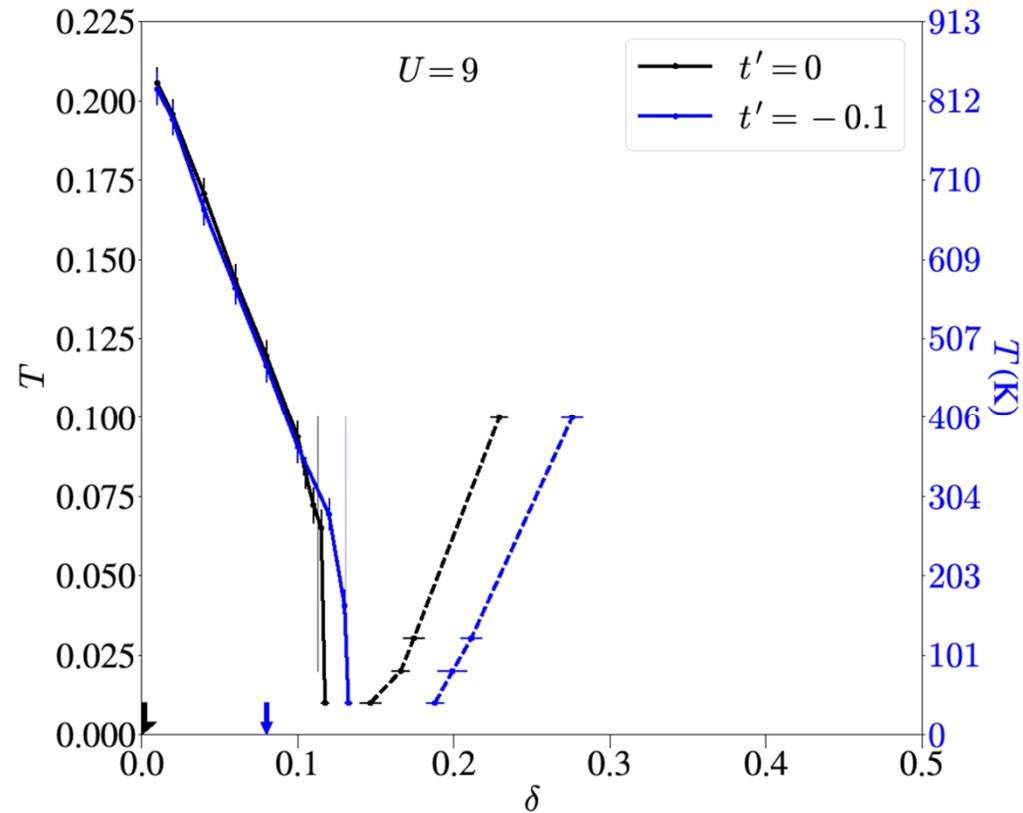
Doiron-Leyraud *et al.*  
Nature Comm. **8** 2044

# Effect of $t'$



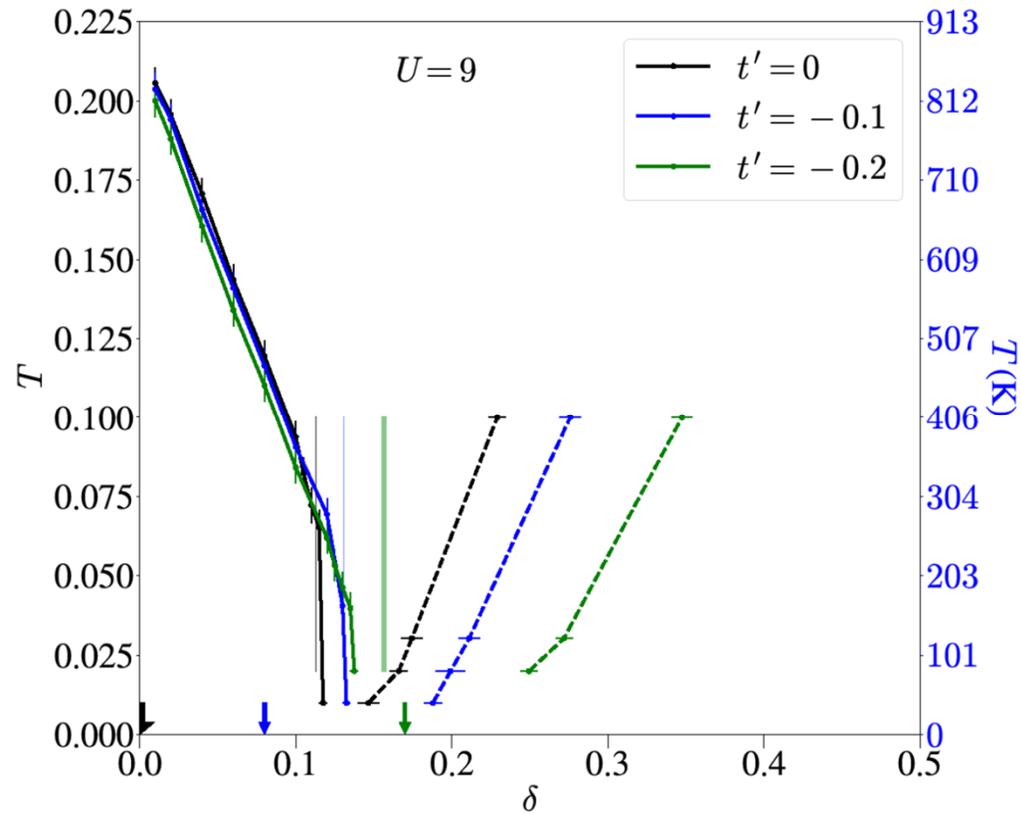
A. Reymbaut *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

# Effect of $t'$



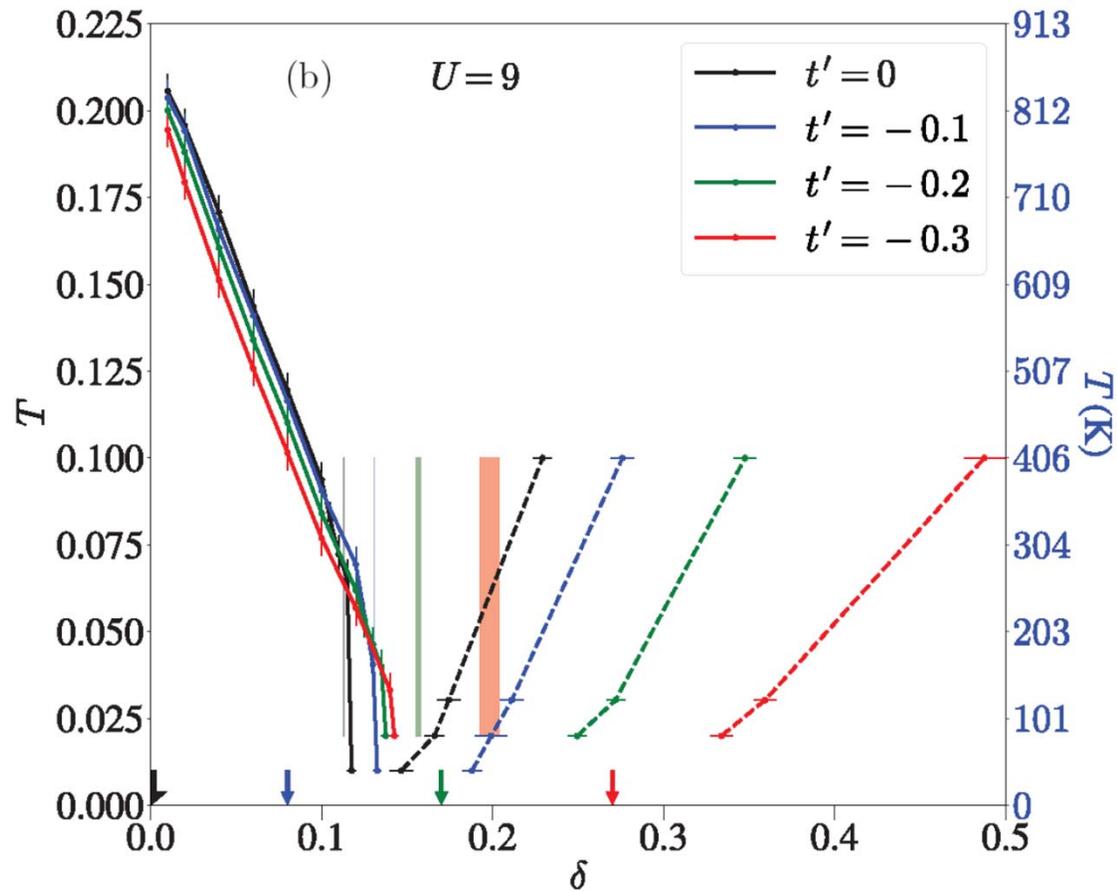
A. Reymbaut *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

# Effect of $t'$



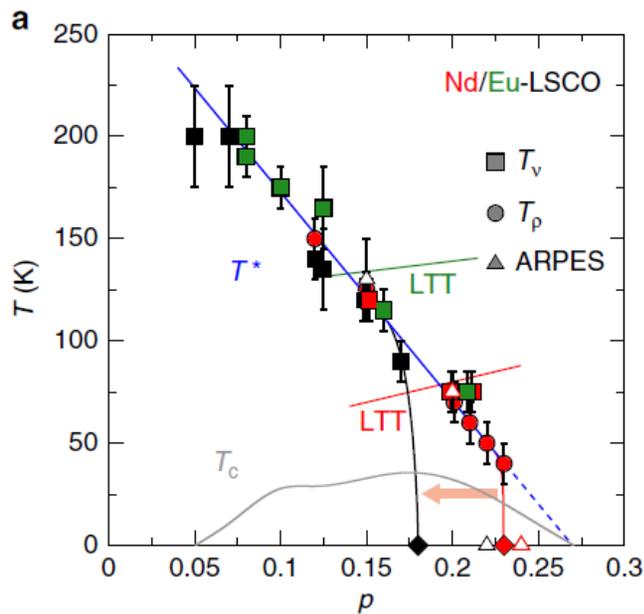
A. Reymbaut *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

# Effect of $t'$

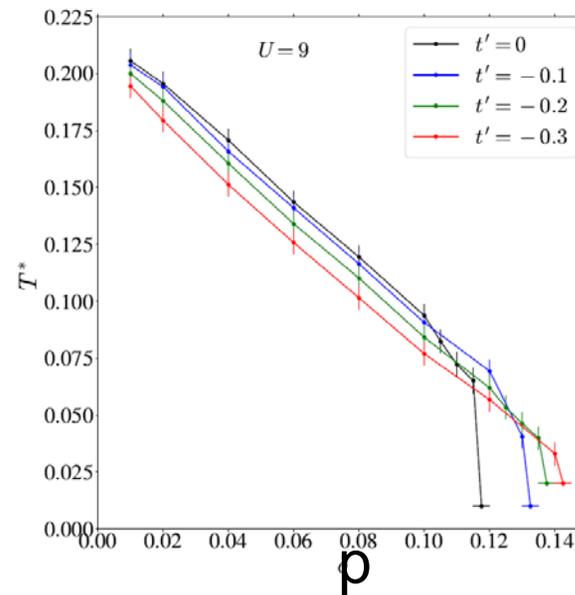


A. Reymbaut *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

# Results : effect of $t'$ on $T^*$

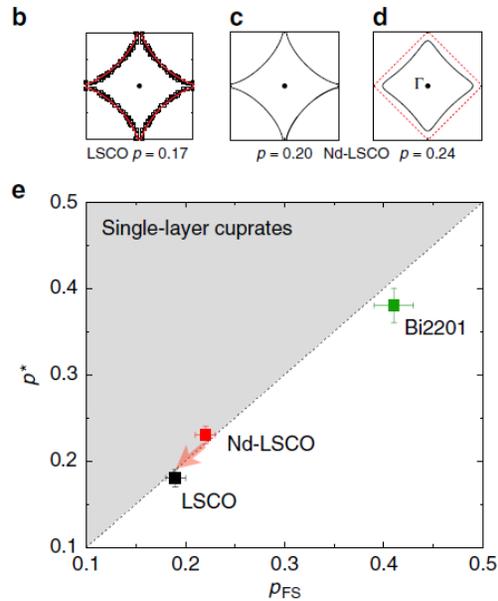


Doiron-Leyraud *et al.*  
Nature Comm. **8** 2044



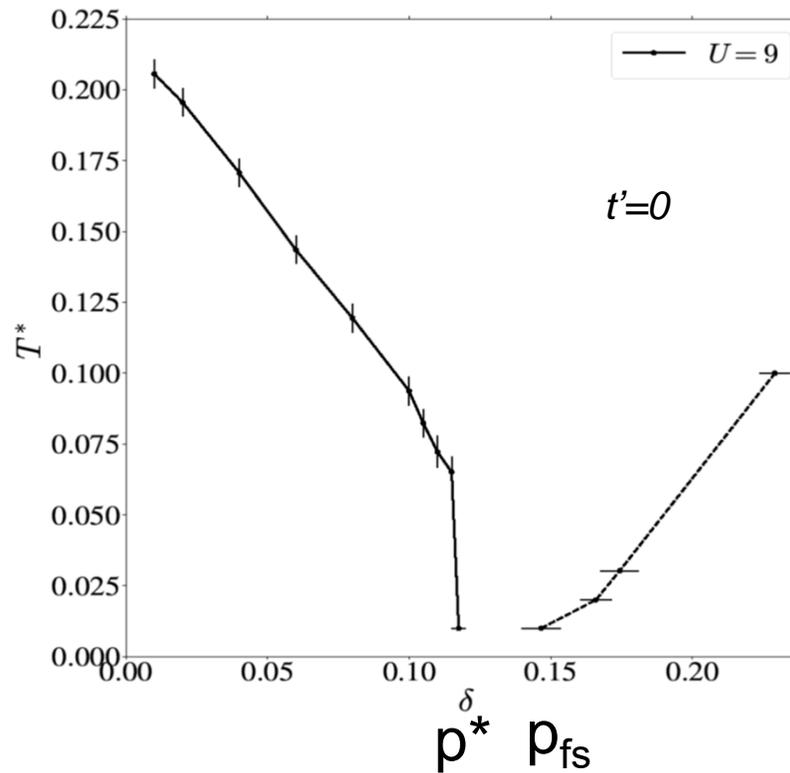
A.Reymbaut, *et al.*  
Phys. Rev. Research **1**, 023015 (2019)

# Results: van Hove singularity



$$p^* < p_{fs}$$

Doiron-Leyraud *et al.*  
 Nature Comm. **8** 2044



A.Reymbaut, *et al.*

Phys. Rev. Research **1**, 023015 (2019)

W Wu, A Georges, M Ferrero Phys. Rev. X **8**, 021048 (2018).

# Minuterie

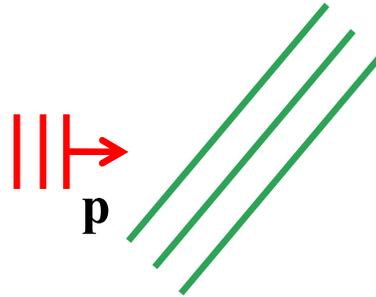


- 10 minutes
- 6 minutes
- 3 minutes
- 4 minutes = 23

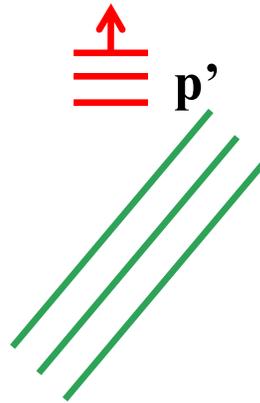
# d-wave superconductivity

# Superconductivity

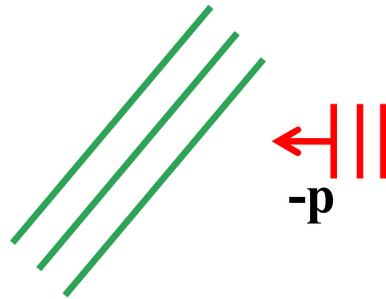
# Attraction mechanism in the metallic state



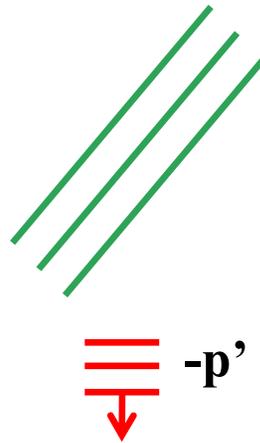
# Attraction mechanism in the metallic state



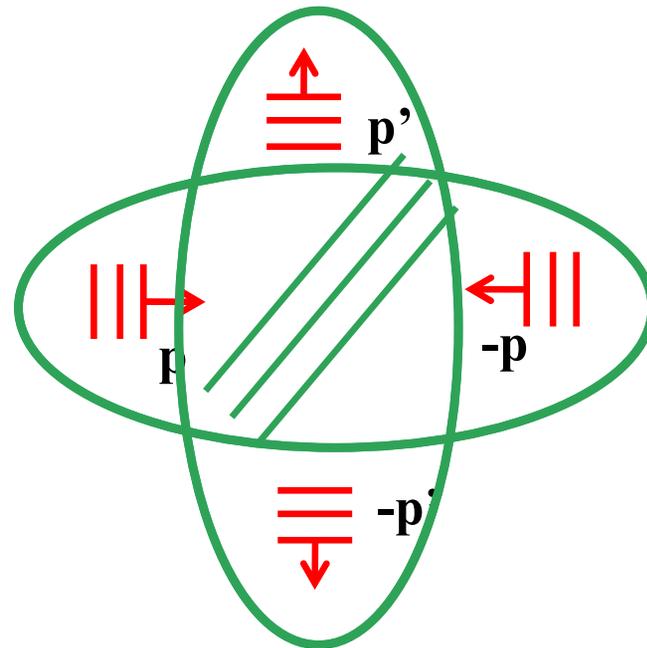
# Attraction mechanism in the metallic state



# Attraction mechanism in the metallic state



# Attraction mechanism in the metallic state



# #1 Cooper pair, #2 Phase coherence



$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow}$$

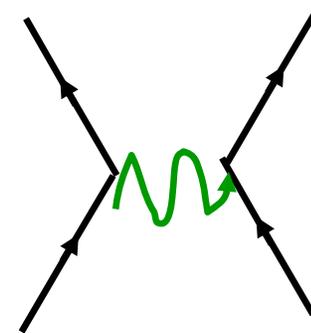
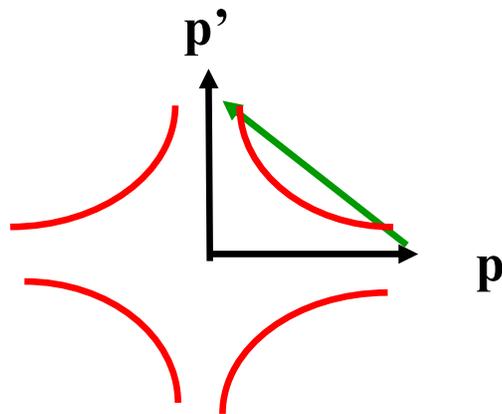
$$E_P = \sum_{\mathbf{p}, \mathbf{p}'} U_{\mathbf{p}-\mathbf{p}'} \langle \psi_{\mathbf{p}'\uparrow, -\mathbf{p}'\downarrow}^* \rangle \langle \psi_{\mathbf{p}\uparrow, -\mathbf{p}\downarrow} \rangle$$

$$|\text{BCS}(\theta)\rangle = \dots + e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$

# Cartoon « BCS » weak-coupling picture



$$\Delta_{\mathbf{p}} = -\frac{1}{2V} \sum_{\mathbf{p}'} U(\mathbf{p} - \mathbf{p}') \frac{\Delta_{\mathbf{p}'}}{E_{\mathbf{p}'}} (1 - 2n(E_{\mathbf{p}'}))$$



Exchange of spin waves?  
Kohn-Luttinger  
 $T_c$  with pressure

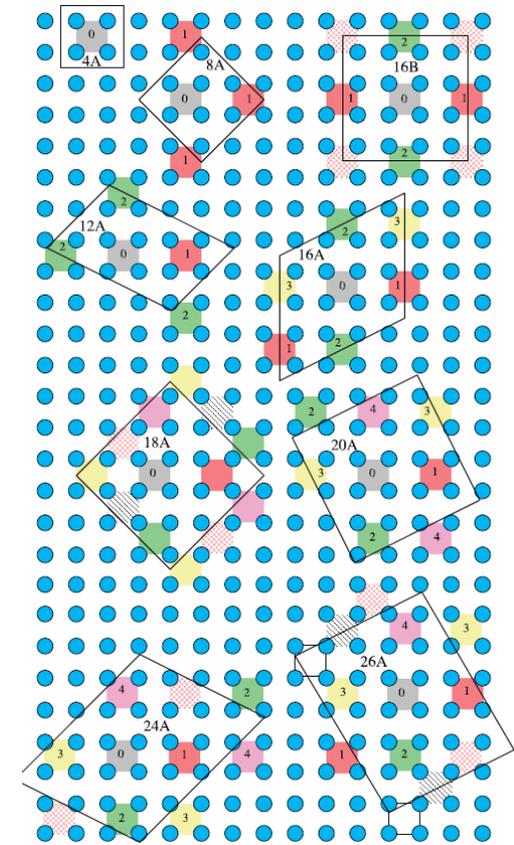
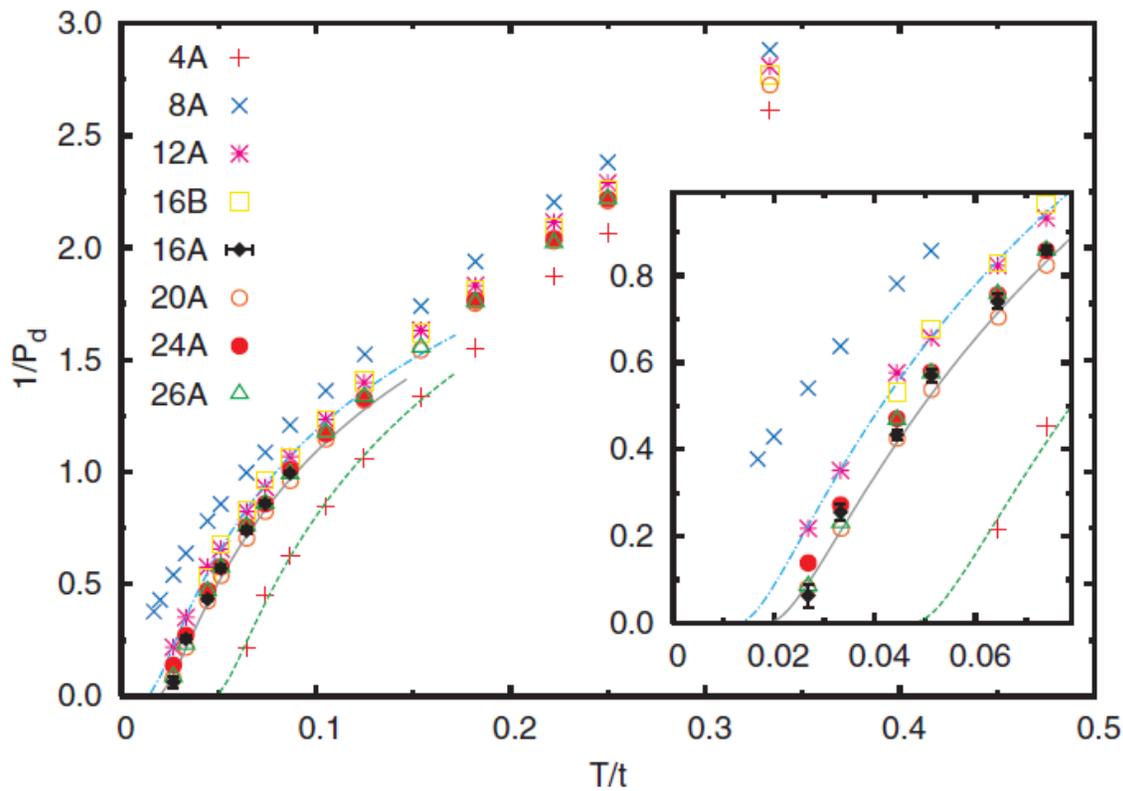
Béal–Monod, Bourbonnais, Emery  
P.R. B. **34**, 7716 (1986).

D. J. Scalapino, E. Loh, Jr., and J. E. Hirsch  
P.R. B **34**, 8190-8192 (1986).

Kohn, Luttinger, P.R.L. **15**, 524 (1965).

P.W. Anderson Science 317, 1705 (2007)

# Exchange of spin waves, $U = 4t$ doping 10%



Maier, Jarrell, Schulthess, Kent, White PRL **95**, 237001 (2005)

# A cartoon strong correlation picture



$$J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = J \sum_{\langle i,j \rangle} \left( \frac{1}{2} c_i^\dagger \vec{\sigma} c_i \right) \cdot \left( \frac{1}{2} c_j^\dagger \vec{\sigma} c_j \right)$$

$$d = \langle \hat{d} \rangle = 1/N \sum_{\vec{k}} (\cos k_x - \cos k_y) \langle c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow} \rangle$$

$$H_{MF} = \sum_{\vec{k},\sigma} \varepsilon(\vec{k}) c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} - 4Jm\hat{m} - Jd(\hat{d} + \hat{d}^\dagger) + F_0$$

Pitaevskii Brückner:

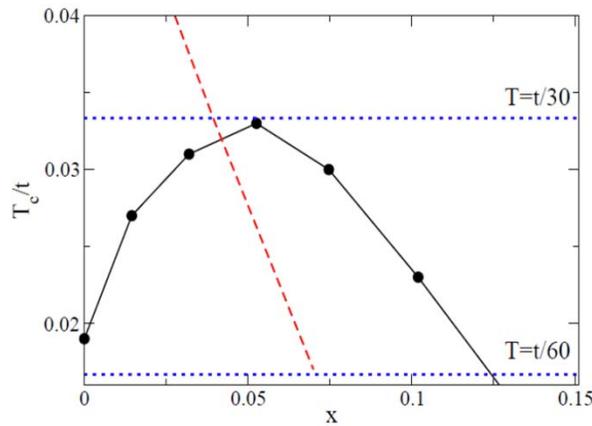
Pair state orthogonal to repulsive core of Coulomb interaction

P.W. Anderson *Science*  
317, 1705 (2007)

Miyake, Schmitt-Rink, and Varma  
*P.R. B* 34, 6554-6556 (1986)

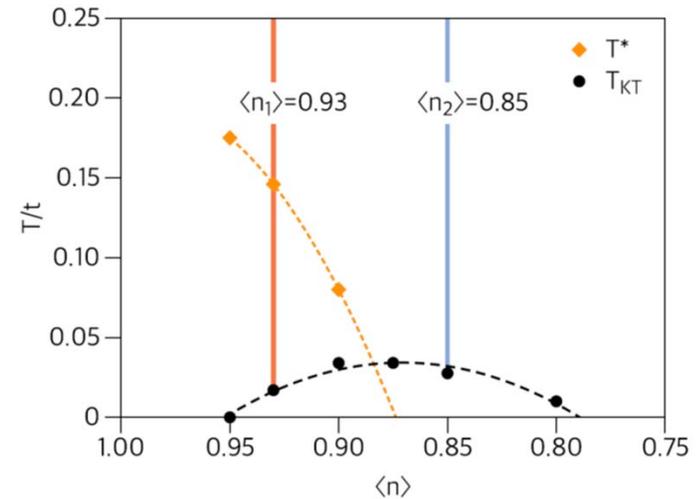
More sophisticated Slave Boson: Kotliar Liu *PRB* 1988

# Superconducting transition temperature



E. Gull and A. J. Millis  
Phys. Rev. B 88, 075127

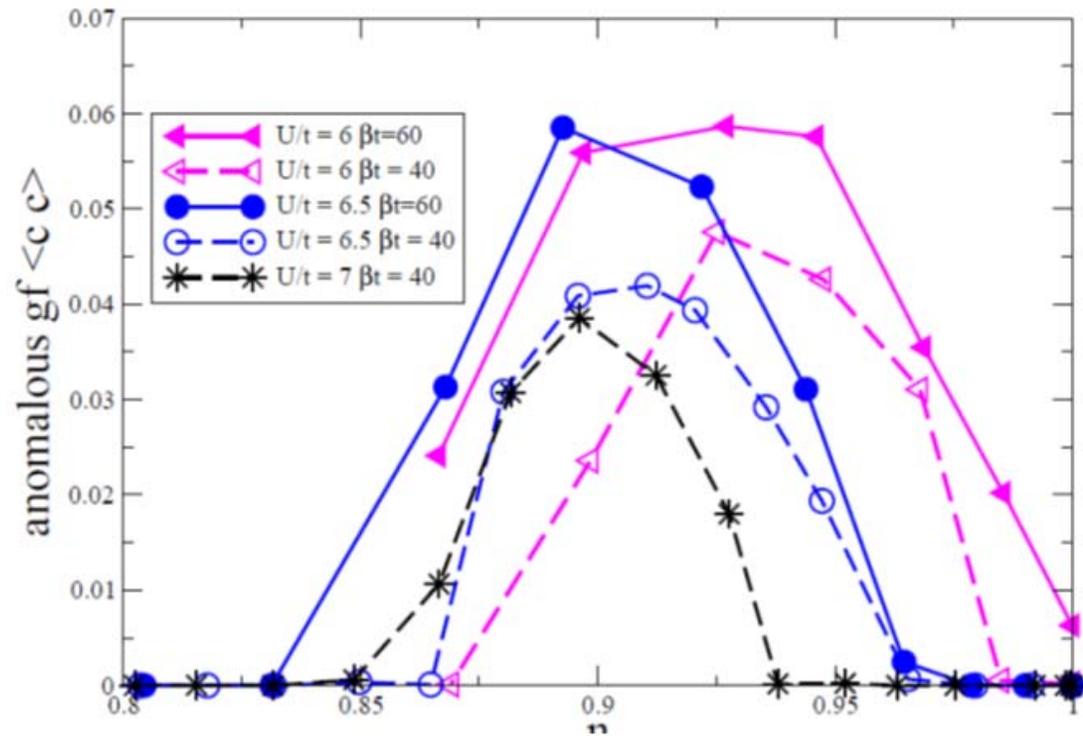
DCA, 8 sites,  $U/t = 6$  and  $t'=0$



T.A. Maier, D.J. Scalapino, npj Quantum Materials (2019)

DCA, 12 sites,  $U/t = 7$  and  $t'/t = -0.15$

$t' = 0$  DCA, 8 site



Gull *et al.* *Phys. Rev. Lett.* **110**, 216405 (2013)

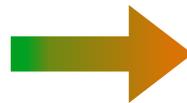
# d-wave in mean-field

$$\hat{\mathcal{H}}_{\text{modèle } t-J} = -t \sum_{\langle i,j \rangle \sigma} \hat{P} \left( \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + c.h \right) \hat{P} + J \sum_{\langle i,j \rangle} \left( \hat{S}_i \cdot \hat{S}_j - \frac{1}{4} \hat{n}_i \hat{n}_j \right)$$

$$\begin{aligned} J \hat{S}_i^z \hat{S}_j^z &= J (\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow}) (\hat{n}_{j\uparrow} - \hat{n}_{j\downarrow}) \\ &= J (\hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} - \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow}) (\hat{c}_{j\uparrow}^\dagger \hat{c}_{j\uparrow} - \hat{c}_{j\downarrow}^\dagger \hat{c}_{j\downarrow}) \\ &= -J (\hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow} \hat{c}_{j\uparrow}^\dagger \hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} \hat{c}_{j\downarrow}^\dagger \hat{c}_{j\downarrow}) + \dots \\ &= -J (\hat{c}_{j\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow} \hat{c}_{j\uparrow} + \hat{c}_{i\uparrow}^\dagger \hat{c}_{j\downarrow}^\dagger \hat{c}_{j\downarrow} \hat{c}_{i\uparrow}) + \dots \end{aligned}$$

Hartree-Fock :

$$d^* = \langle \hat{c}_{j\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger \rangle \mathcal{H}_{\text{modèle } t-J}$$

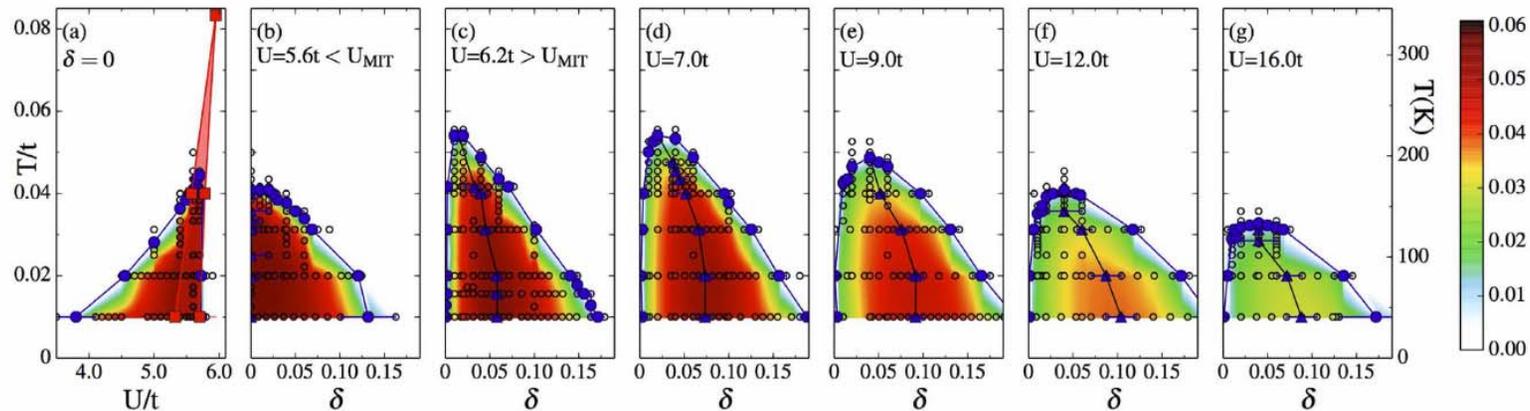


$$\langle J \hat{S}_i^z \hat{S}_j^z \rangle = -2J d^* d + \dots$$

Miyake, Schmitt-Rink et Varma, [PRB 34, 6554-6556](#) (1986)

Anderson, Baskaran, Zou et Hsu, [PRL 58, 26](#) (1987)

# $T_c$ controlled by $J$ , CDMFT 2x2



Fratino et al.  
Sci. Rep. 6, 22715

Some experiments that suggest  $T_c < T_{\text{pair}} < T^*$

T. Kondo *et al.* PRL **111** (2013)

Kondo, Takeshi, et al. Kaminski Nature Physics 2011, **7**, 21-25

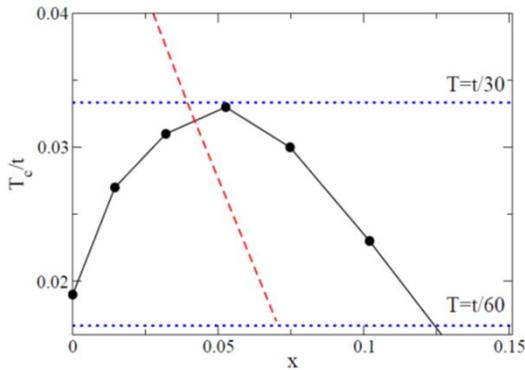
A. Pushp, Parker, ... A. Yazdani, Science **364**, 1689 (2009)

Lee ... Tajima (Osaka) <https://arxiv.org/pdf/1612.08830>

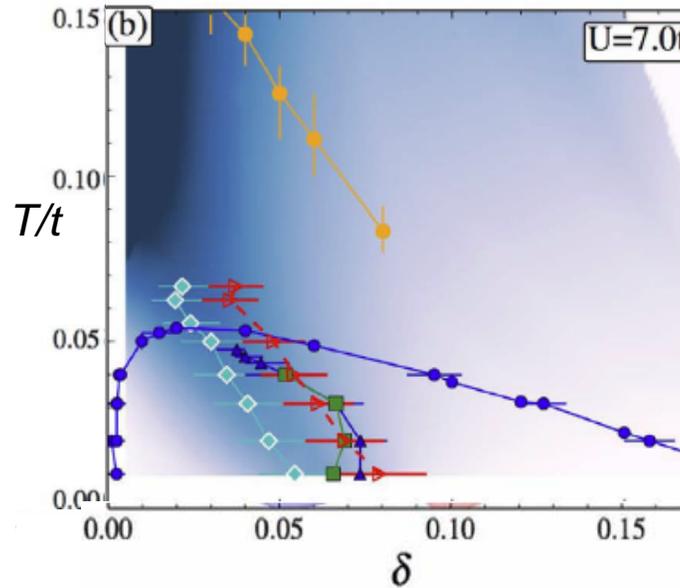
Patrick M. Rourke, et al. Hussey Nature Physics **7**, 455–458 (2011)

Lee et al. J. Phys. Soc. Jpn. **86**, 023701 (2017)

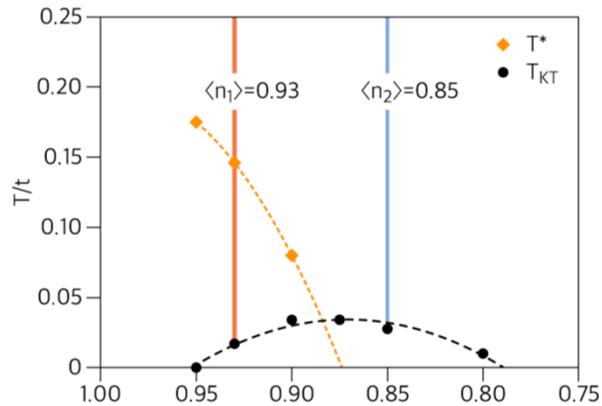
# Phase diagram



E. Gull and A. J. Millis  
 Phys. Rev. B 88, 075127

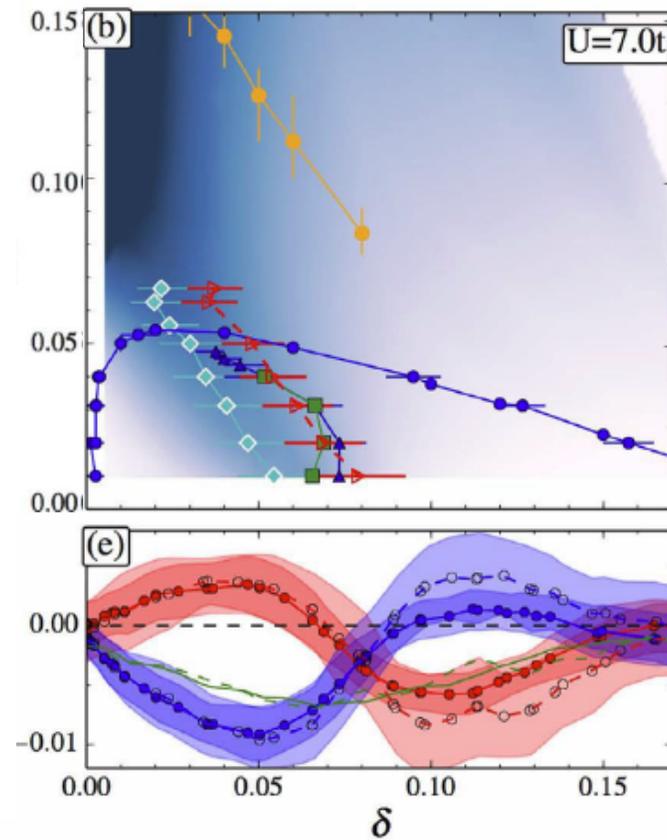


Fratino et al.  
 Sci. Rep. 6, 22715



T.A. Maier, D.J. Scalapino, npj Quantum Materials (2019)

# Condensation energy



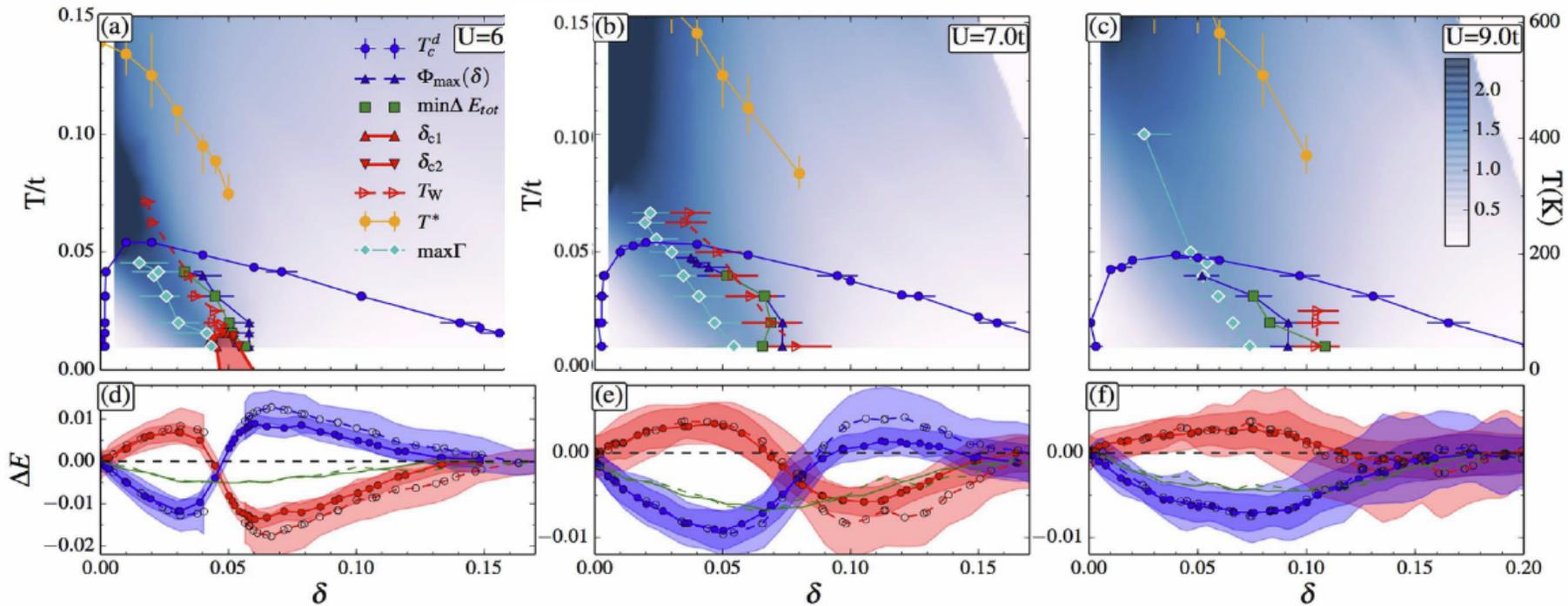
Fratino et al.  
*Sci. Rep.* **6**, 22715

Theory, see also  
Jarrel PRL  
(2004), Gull  
Millis PRB  
(2014)

Experiments:  
Bontemps,  
Santander-Syro  
Van der Marel ...

# Condensation energy

Fratino et al.  
Sci. Rep. 6, 22715



Theory, see also Jarrel PRL (2004), Gull  
Millis PRB (2014)

Experiments: Bontemps, Santander-Syro  
Van der Marel ...

# Minuterie

- 10 minutes
- 6 minutes
- 3 minutes
- 4 minutes
- 5 minutes
- 9 minutes = 37





D. Sénéchal



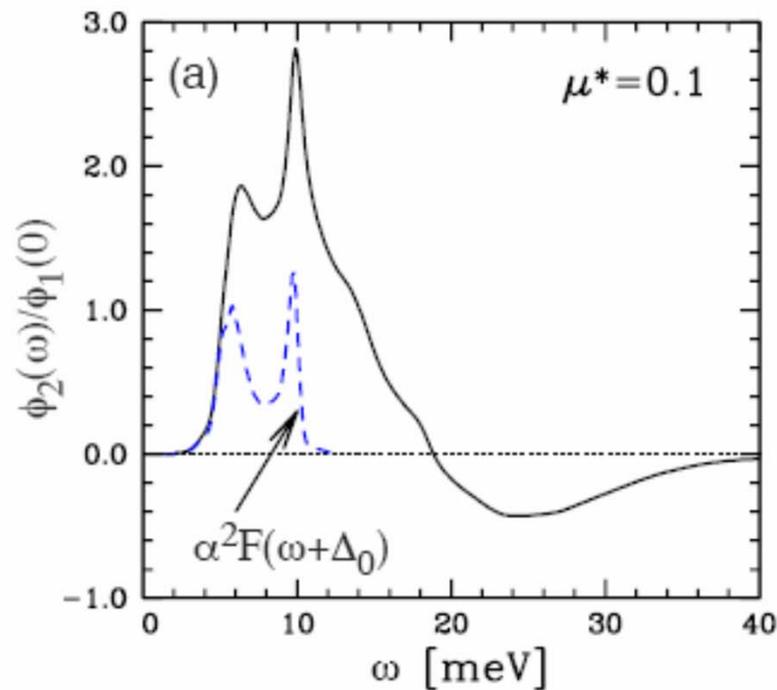
Bumsoo Kyung

## The glue

Kyung, Sénéchal, Tremblay, Phys. Rev. B **80**, 205109 (2009)  
Sénéchal, Day, Bouliane, AMST, Phys. Rev. B **87**, 075123 (2013)  
A. Reymbaut *et al.* PRB **94** 155146 (2016)

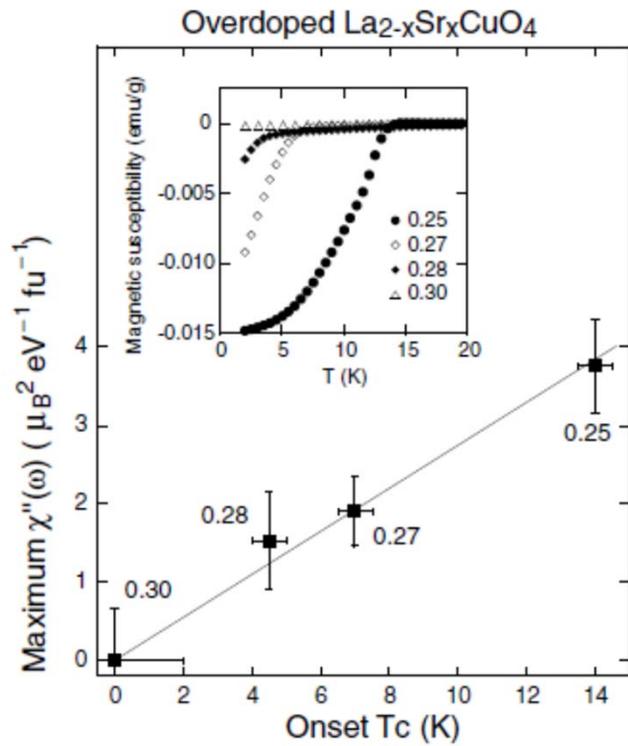
# $\text{Im } \Sigma_{an}$ and electron-phonon in Pb

Maier, Poilblanc, Scalapino, PRL (2008)

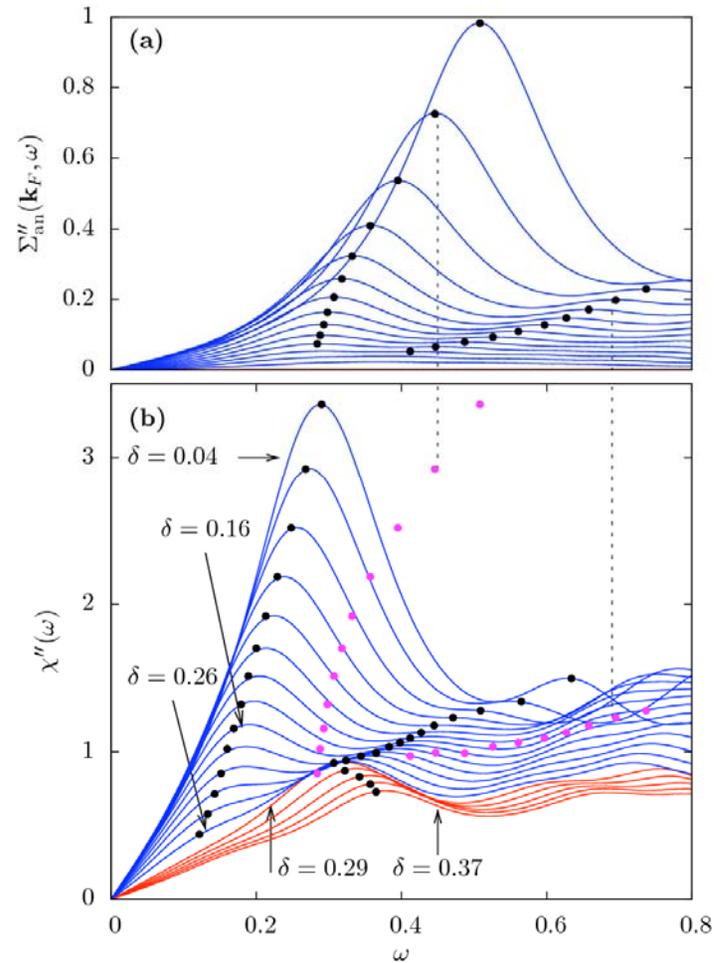


# The glue CDMFT 2x2, T=0

Kyung, Sénéchal, Tremblay, Phys. Rev. B  
**80**, 205109 (2009)



Wakimoto ... Birgeneau  
 PRL (2004)



# The glue and neutrons

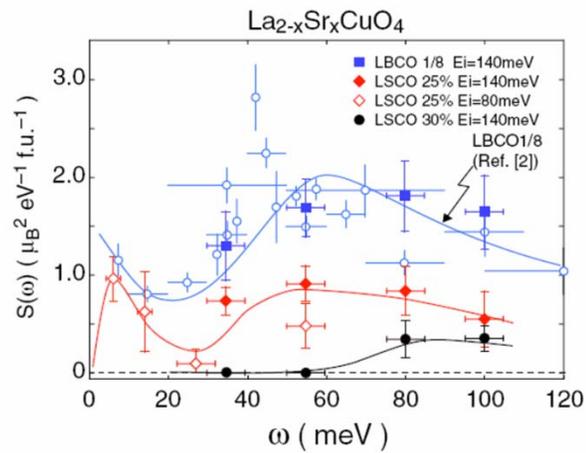
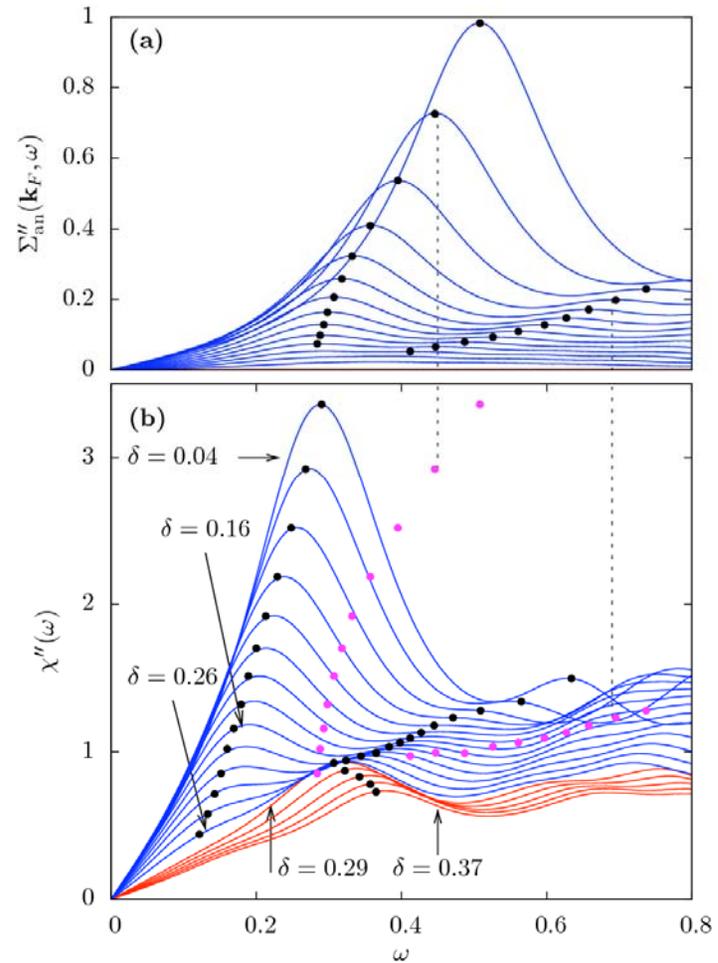


FIG. 3 (color online).  $\mathbf{Q}$ -integrated dynamic structure factor  $S(\omega)$  which is derived from the wide- $H$  integrated profiles for LBCO 1/8 (squares), LSCO  $x = 0.25$  (diamonds; filled for  $E_i = 140$  meV, open for  $E_i = 80$  meV), and  $x = 0.30$  (filled circles) plotted over  $S(\omega)$  for LBCO 1/8 (open circles) from [2]. The solid lines following data of LSCO  $x = 0.25$  and  $0.30$  are guides to the eyes.

Wakimoto ... Birgeneau PRL (2007);  
PRL (2004)



# Minuterie

- 10 minutes
- 6 minutes
- 3 minutes
- 4 minutes
- 5 minutes
- 9 minutes
- 3 minutes = 40





Giovanni Sordi



Patrick Sémon

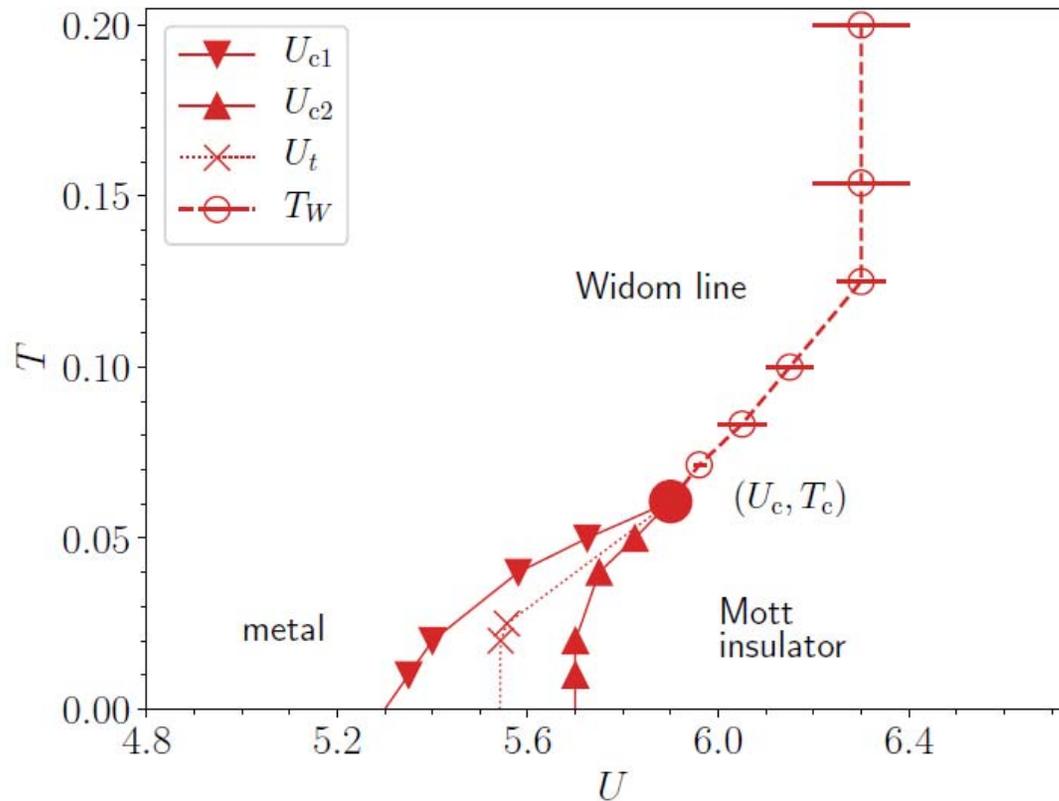


Kristjan Haule

# A phase transition at the heart of the phase diagram (and its relation to Mott)

G. Sordi, *et al.* *Scientific Reports* 2, 547 (2012)

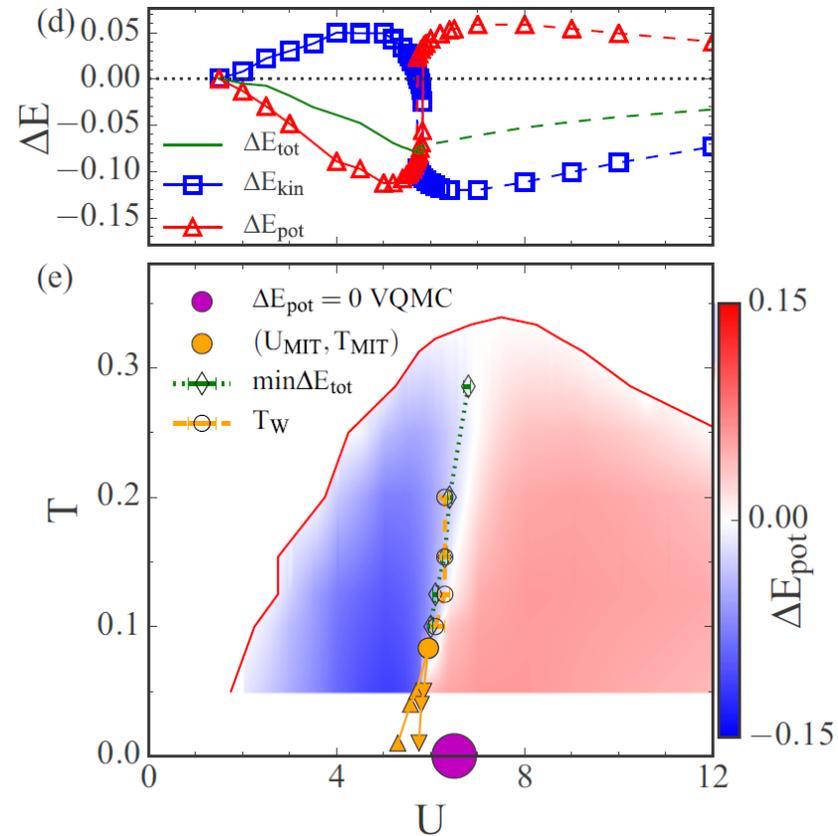
# Mott transition at half-filling, CDMFT 2 x 2



Walsh, Sémon, Poulin, G. Sordi, AMST, PRB **99**, 075122 (2019)

Park, Haule, Kotliar, Phys. Rev. Lett. **101**, 186403 (2008)

# Change in potential energy due to large $\xi$

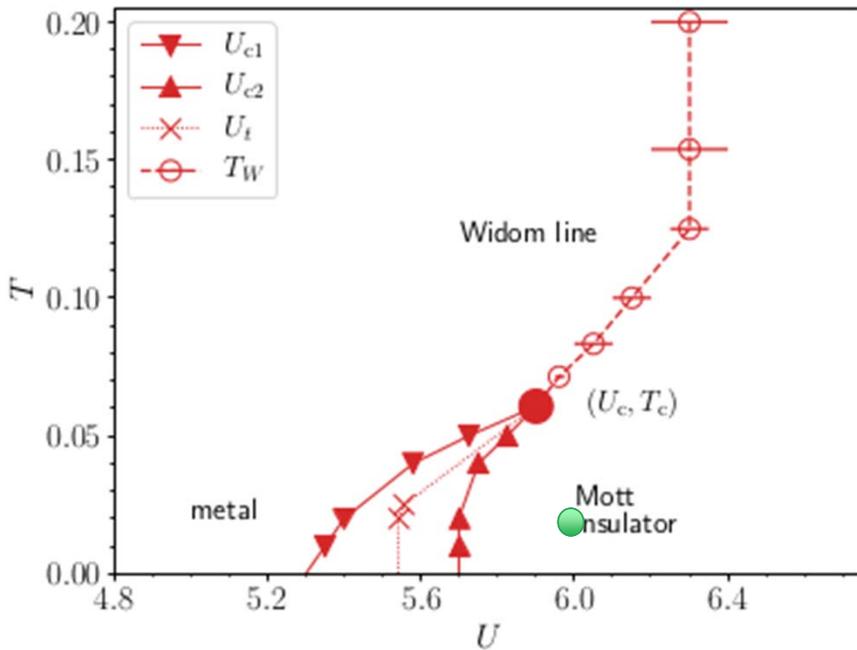


Fratino, Sémon, Charlebois, Sordi, AMST, PRB **95**, 235109 (2017)

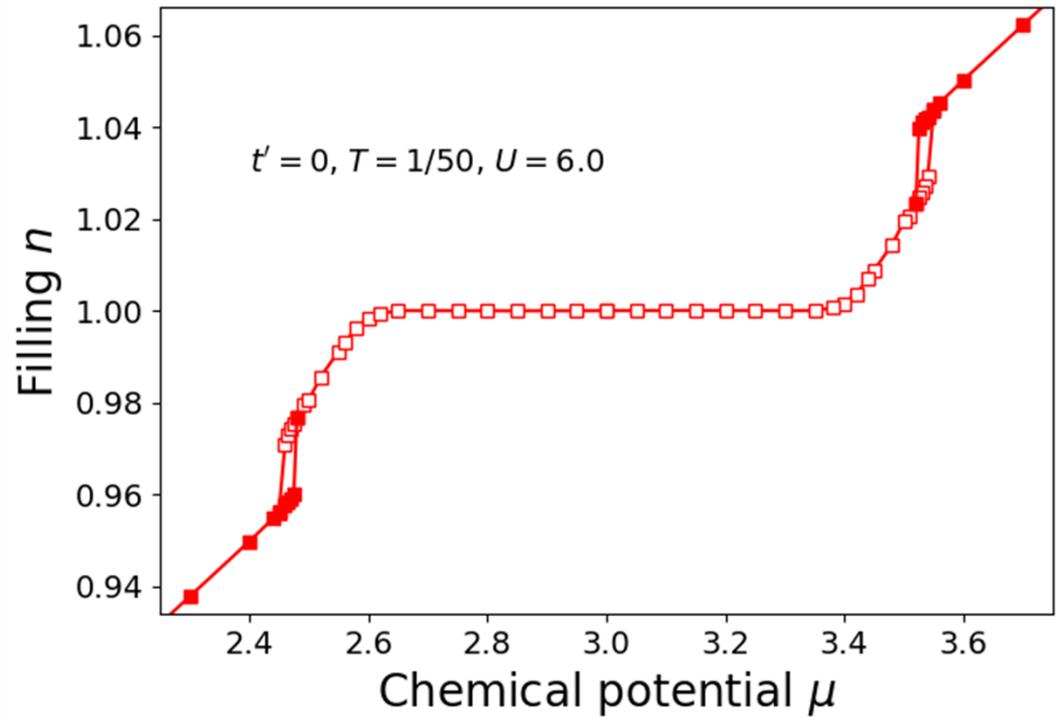
# Mott and Sordi transition: CDMFT 2 x 2



CDMFT

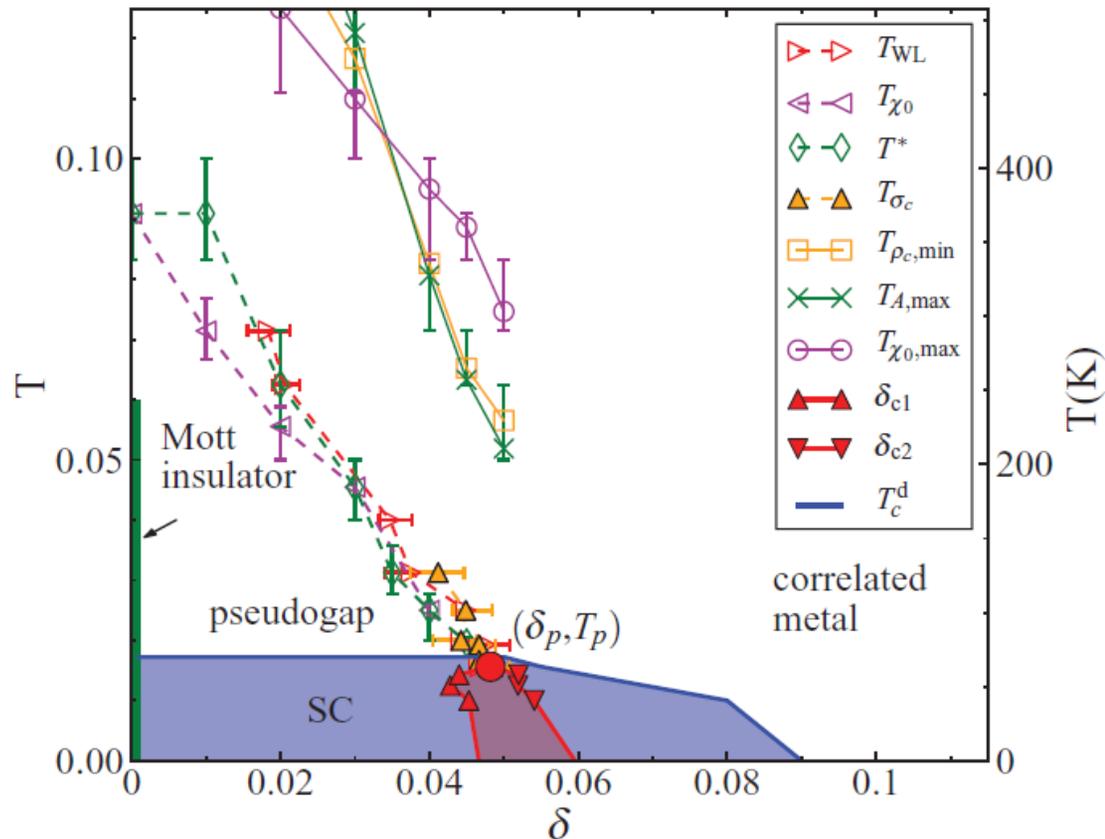


C. Walsh, Phys. Rev. Lett., 122 : 067203 – Feb 2019



G. Sordi, PhysRevB.84.075161 – August 2011

# Widom line and its precursor



Not in  $d=1$

J. P. L. Faye and D. Sénéchal  
Phys. Rev. B **96**, 195114

Sordi, Sémon, Haule, AMST, PRB **87**, 041101(R) (2013)

Sordi, Haule, AMST, PRB **84**, 075161 (2011)

Sordi, Haule, AMST, PRL **104**, 226402 (2010)

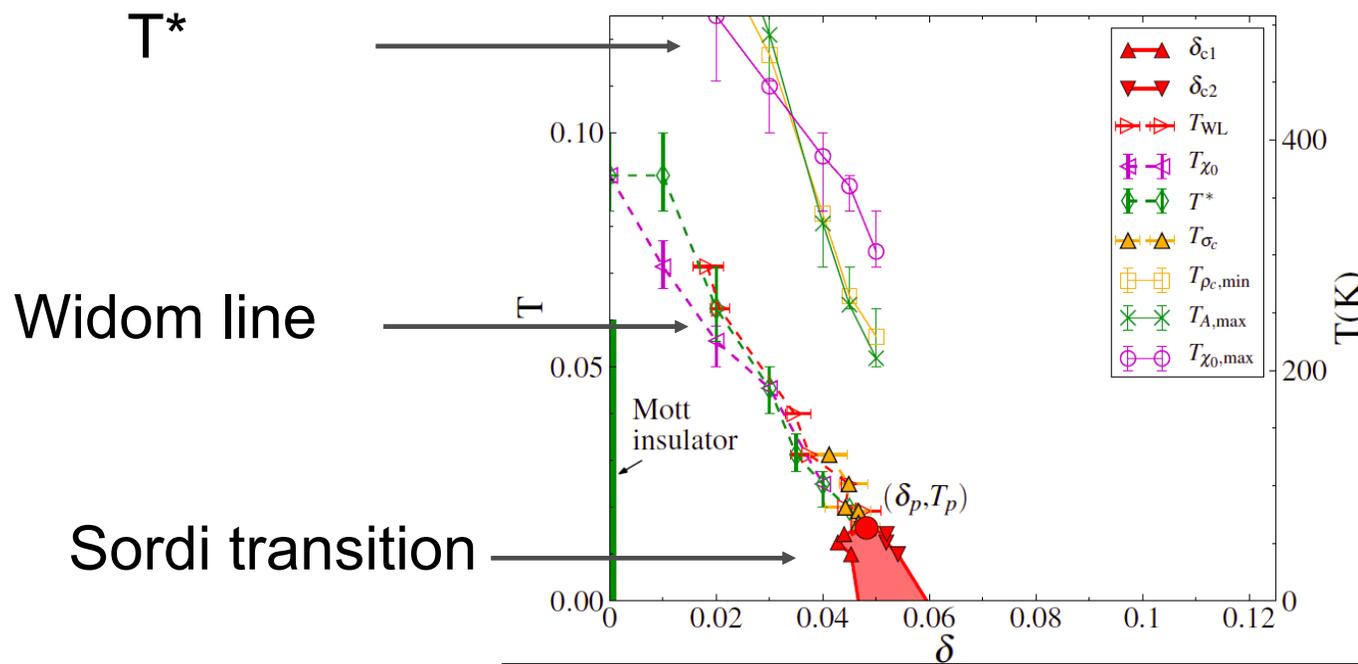
# Two crossover lines



Giovanni Sordi



Patrick Sémon



Not in  $d=1$

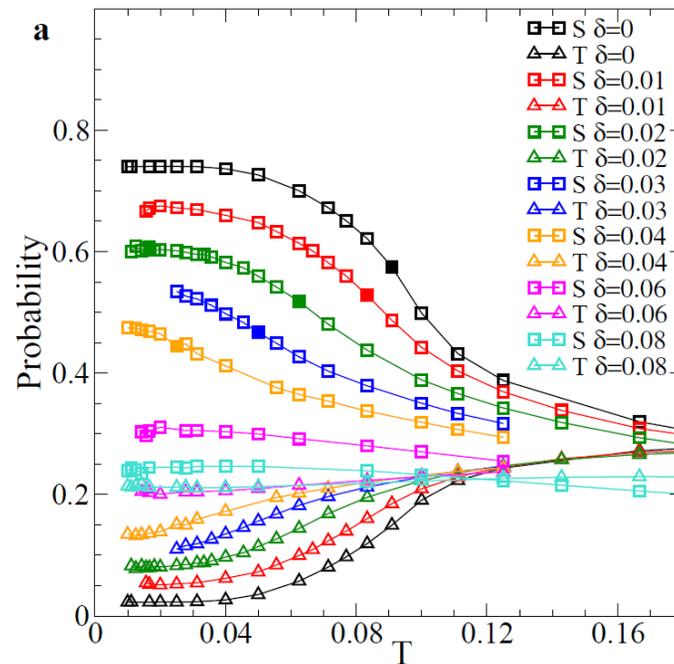
J. P. L. Faye and D. Sénéchal  
Phys. Rev. B **96**, 195114

G. Sordi et al. Phys. Rev. Lett. 108, 216401/1-6 (2012)

G.Sordi et al. Phys. Rev. B 87, 041101(R)/1-5 (2013)

P. Sémon, G. Sordi, et al., Phys. Rev. B **89**, 165113/1-6 (2014)

# Physics: Plaquette eigenstates



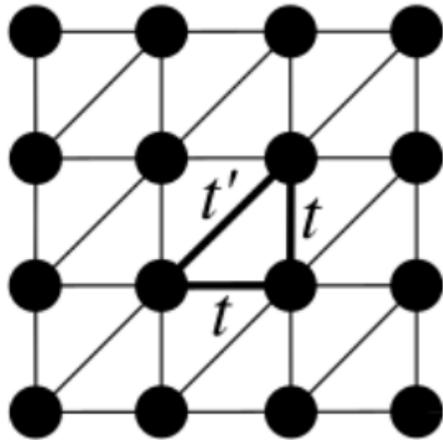
$$U = 6.2; t' = 0$$

Sordi et al., Sci. Rep. 2 547 (2012);

See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges  
PRB **80**, 064501 (2009)

# Anisotropic triangular lattice



Pierre-Olivier  
Downey



Maxime Charlebois

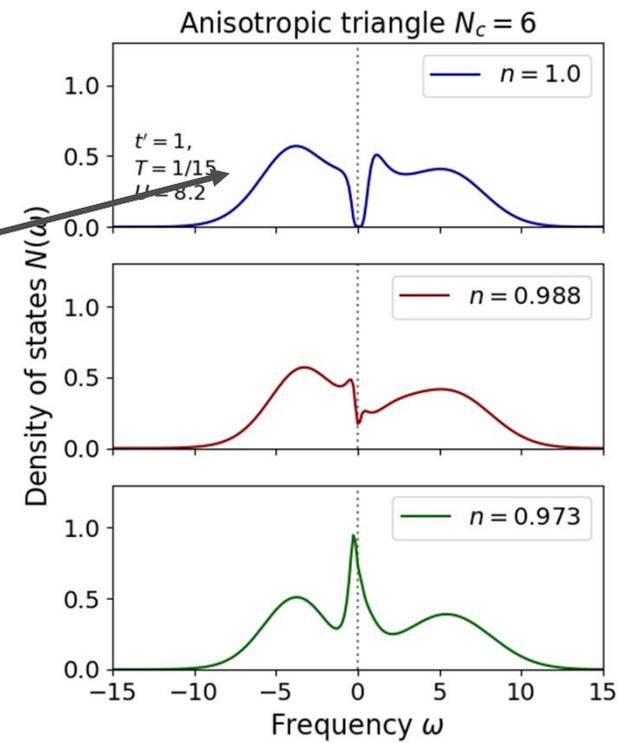
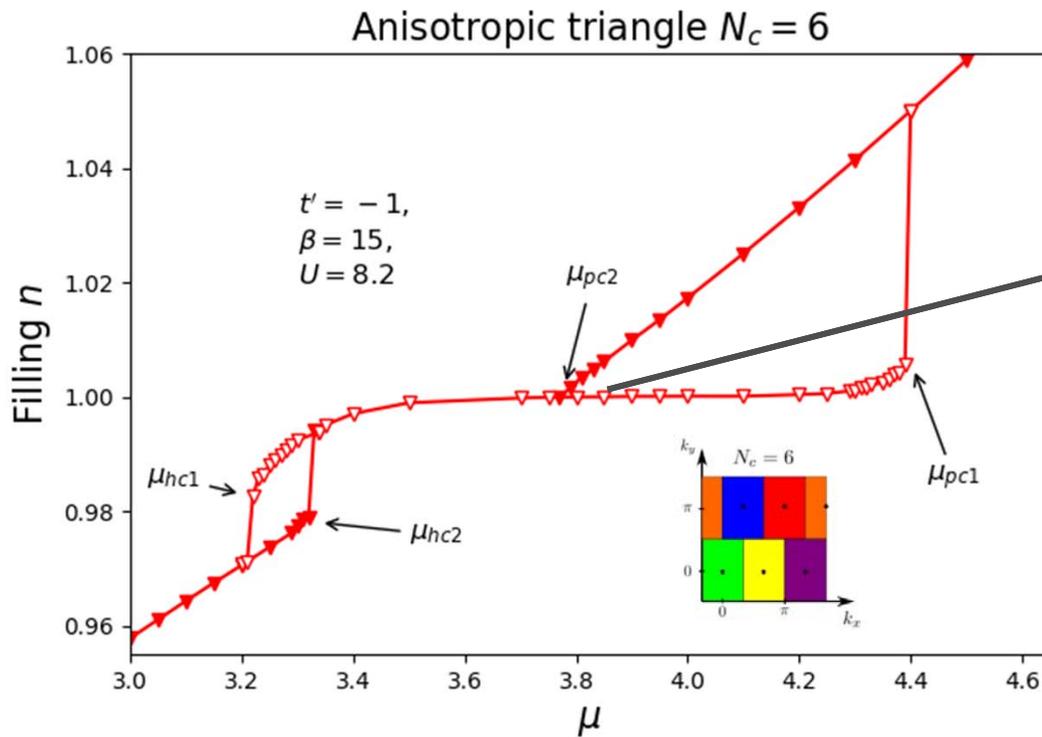


Charles-David Hébert

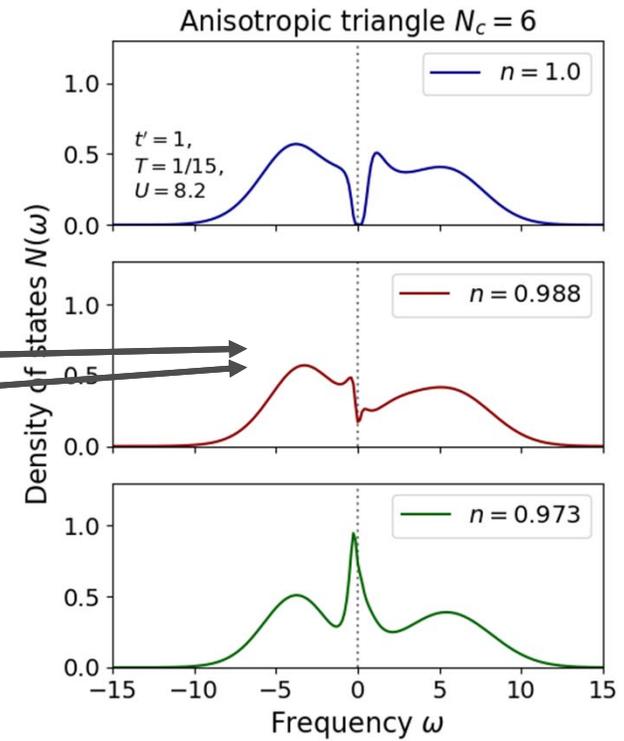
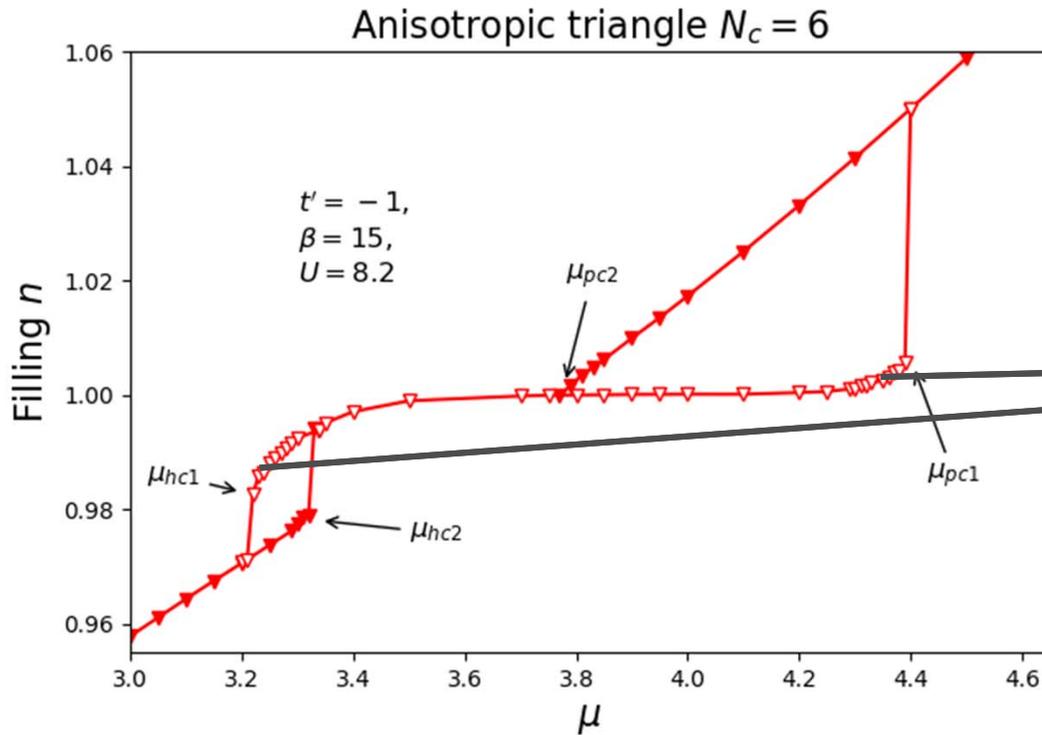


Olivier Gingras

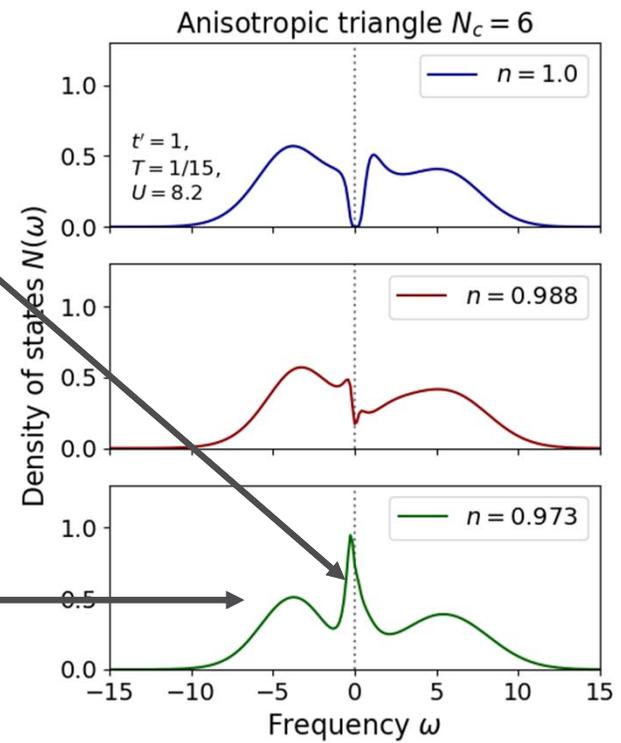
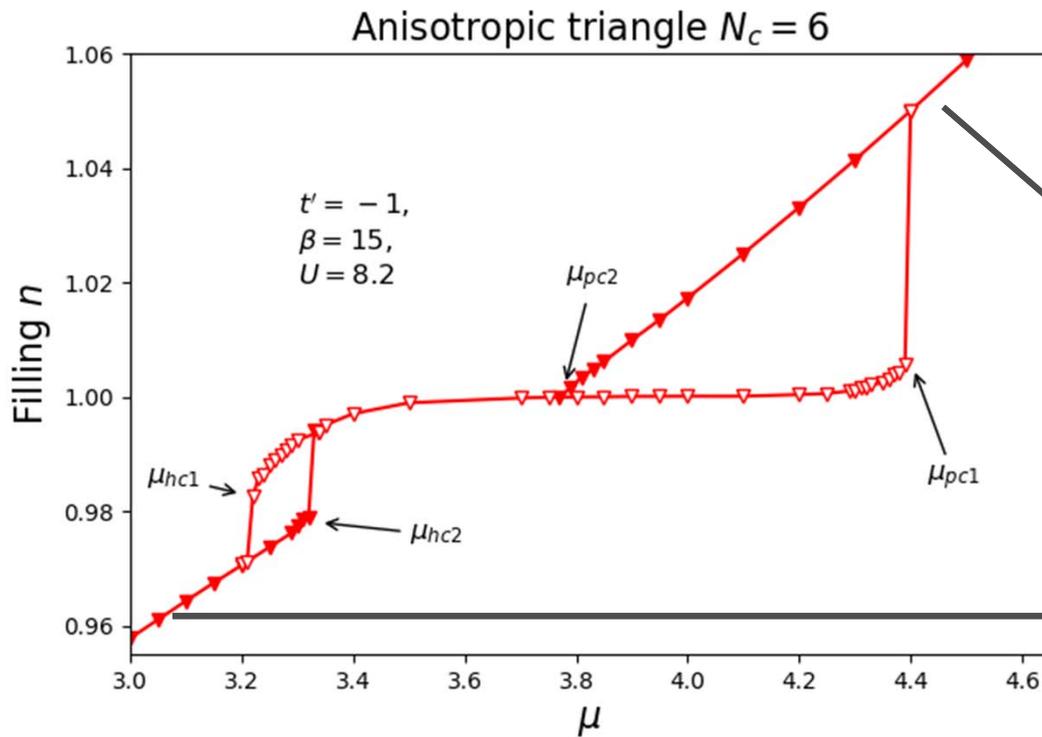
# Same Physics on the triangular lattice



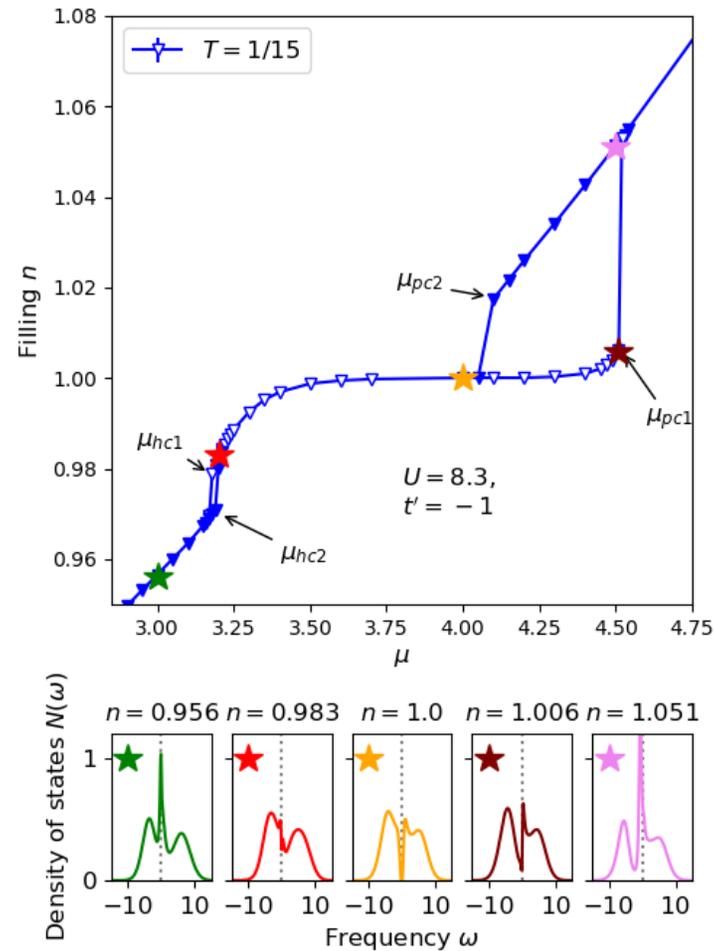
# Same Physics on the triangular lattice



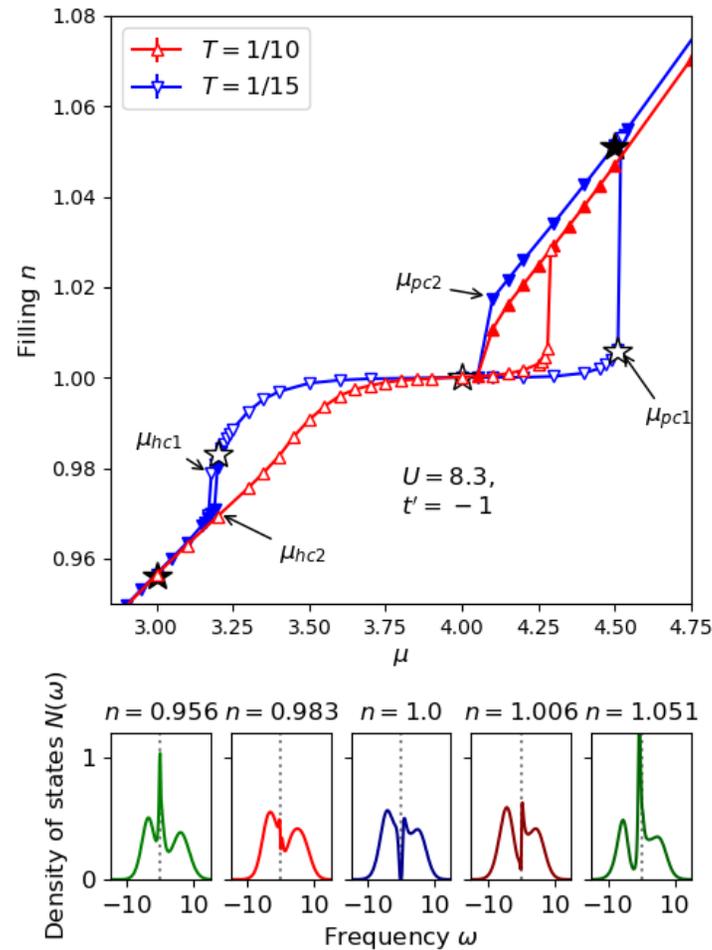
# Same Physics on the triangular lattice



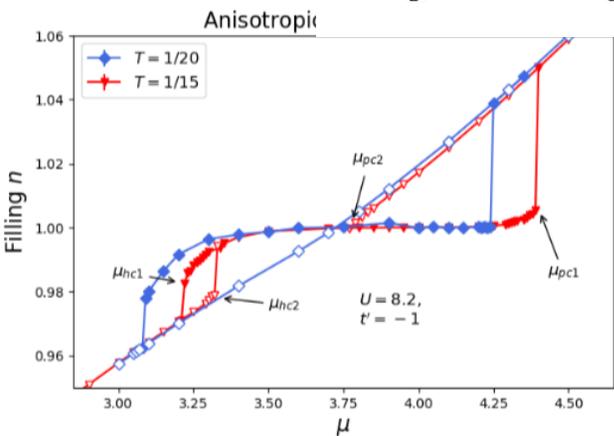
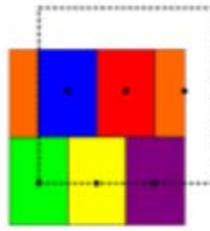
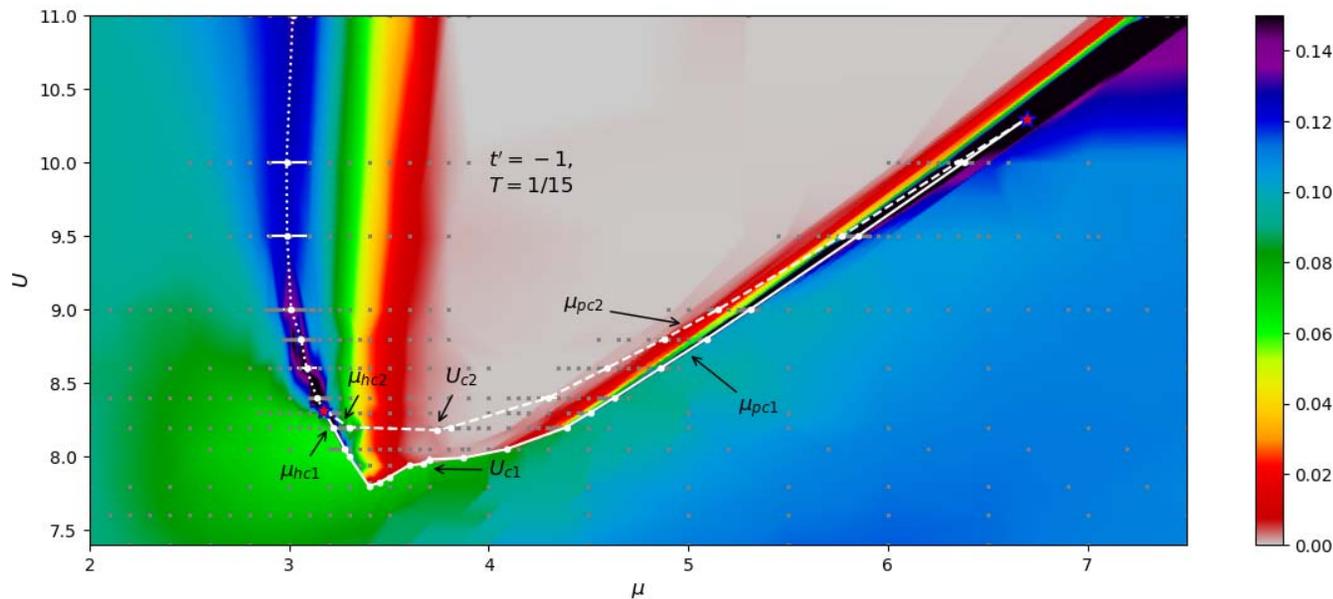
# Triangular lattice with DCA, 6 patches



# Some Physics on the triangular lattice



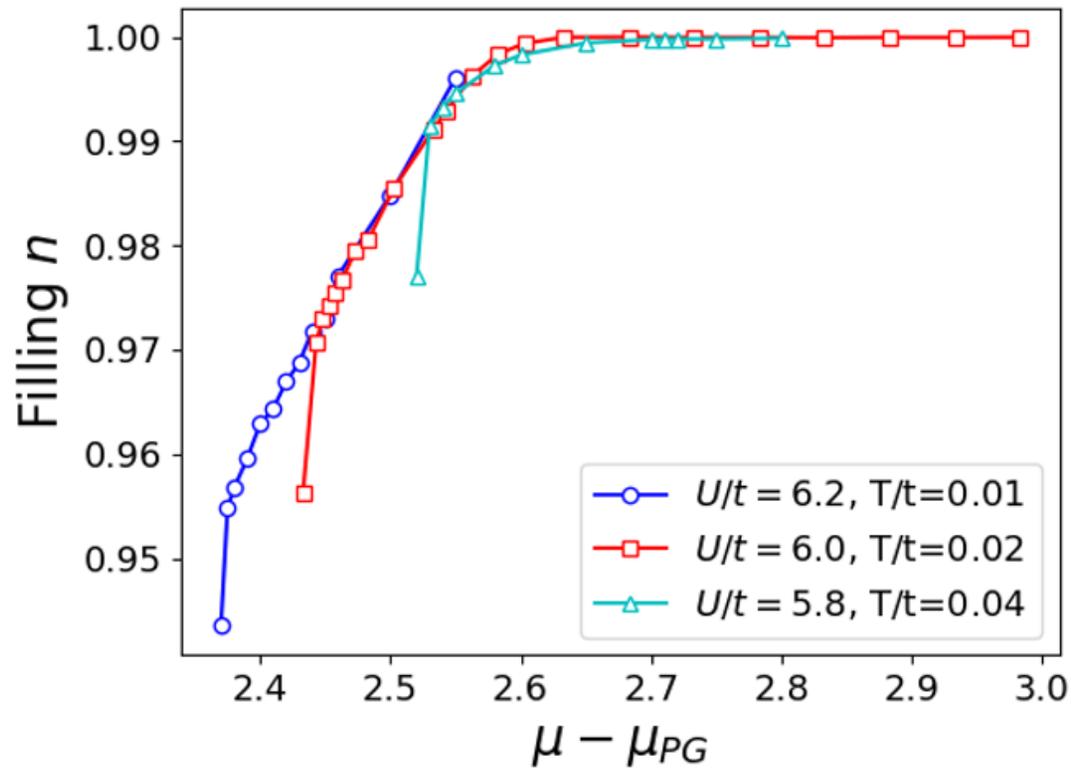
# Mott and Sordani transition on the triangular lattice DCA, $N=6$



# (Topological) stability

Depends on

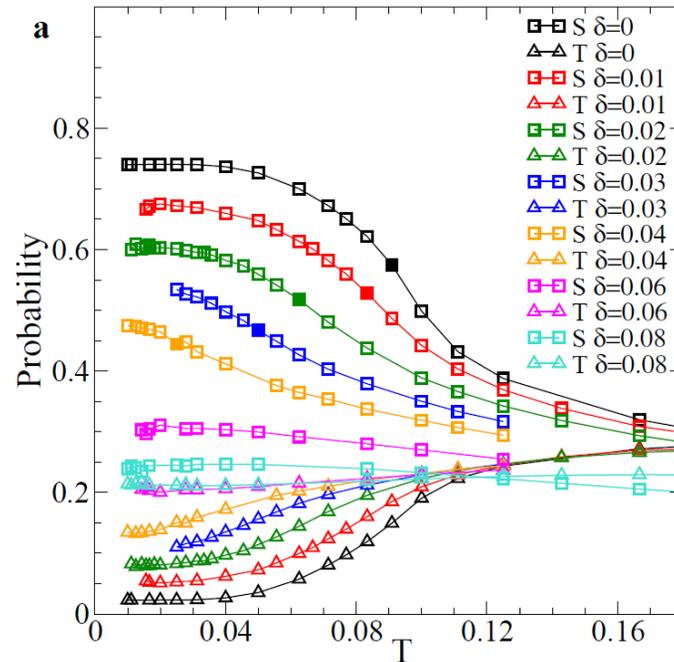
- Cluster
- Large changes in  $t'$



# Another *Fermi Surface Reconstruction without Symmetry Breaking*

- Gazit, Assaad, Sachdev Phys. Rev. X 10, 041057

# Physics: Plaquette eigenstates



$$U = 6.2; t' = 0$$

Sordi et al., Sci. Rep. 2 547 (2012);

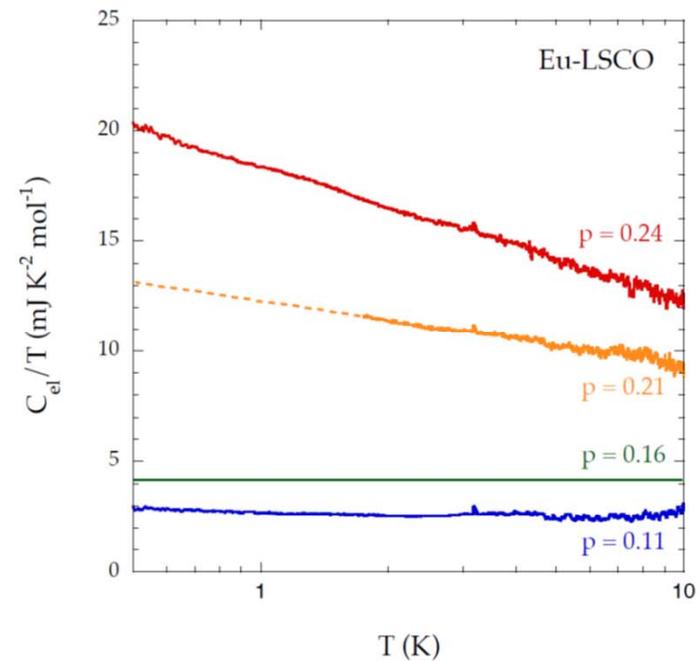
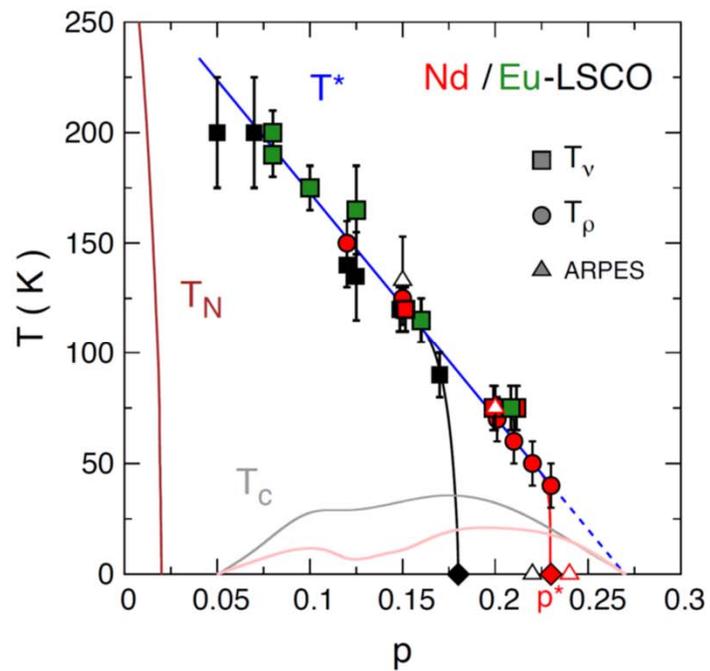
See also:

Michel Ferrero, P. S. Cornaglia, L. De Leo, O. Parcollet, G. Kotliar, A. Georges  
PRB **80**, 064501 (2009)

# Quantum Critical point Back to square lattice

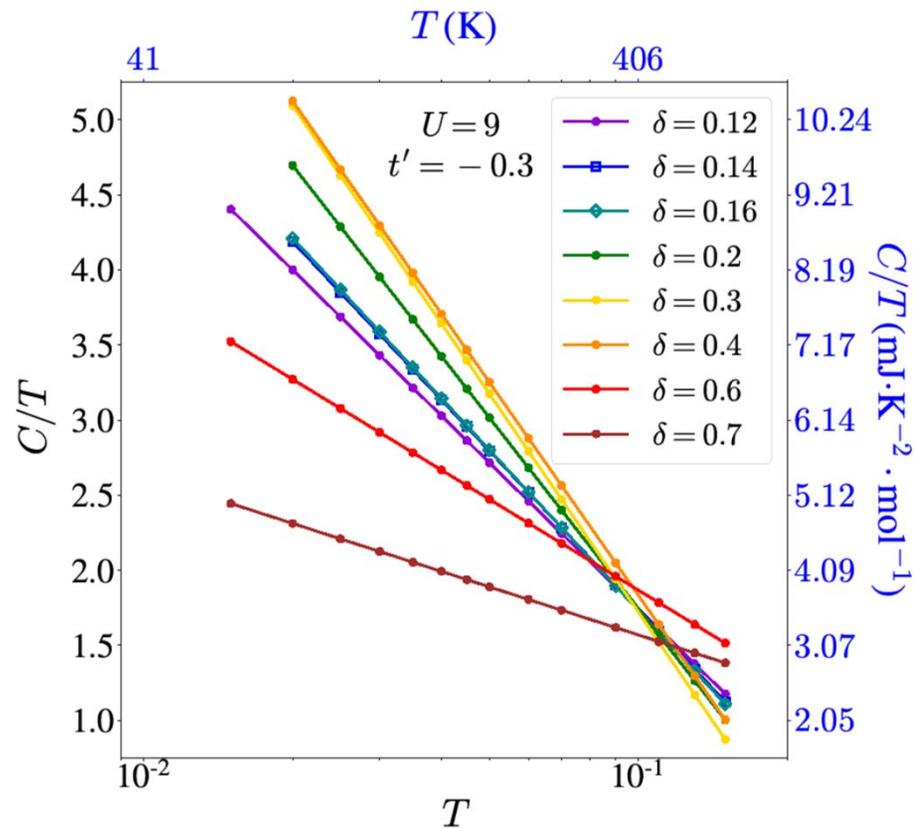
Yang, ... Zaanen, and Jarrell PRL **106**, 047004 (2011)

# Specific heat in the strange metal phase



B. Michon, C. Girod, Taillefer, Klein, Nature 567, 218 (2019)

# Specific heat in the strange metal phase



A.Reymbaut, *et al.* Phys. Rev. Research **1**, 023015 (2019)

See also for  $C_v$  Maximum: Sordi, Walsh, Sémon, and A.-M.S.T, PRB **100**, 121105(R) (2019)

# Summary Conclusion

# Summary

- Intrinsic to the doped Mott insulator
  - Pseudogap
  - First-order transition – QCP
  - d-wave superconductivity
    - Short-range spin fluctuations ( $J$ )
    - Role of charge-transfer gap and of oxygen-hole doping
- Other effects that have not been discussed  $V \gg J$ 
  - Reymbaut, Charlebois, Fellous Asiani, Fratino, Sémon, Sordi A.-M.S.T. PRB **94**, 155146 (2016)
- Other experiment consistent with doped Mott picture
  - Frachet, ... Leboeuf, Julien Nat. Phys. 10.1038/s41567-020-0950-5

# Entanglement properties

- Sharp variation in the entanglement-related properties and not broken symmetry phases characterizes the onset of the pseudogap phase at finite temperature.
  - Walsh, Sémon, Poulin, Sordi, A.-M.S.T. PRX QUANTUM **1**, 020310 (2020)

# Mammoth



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**Merci**

**10**