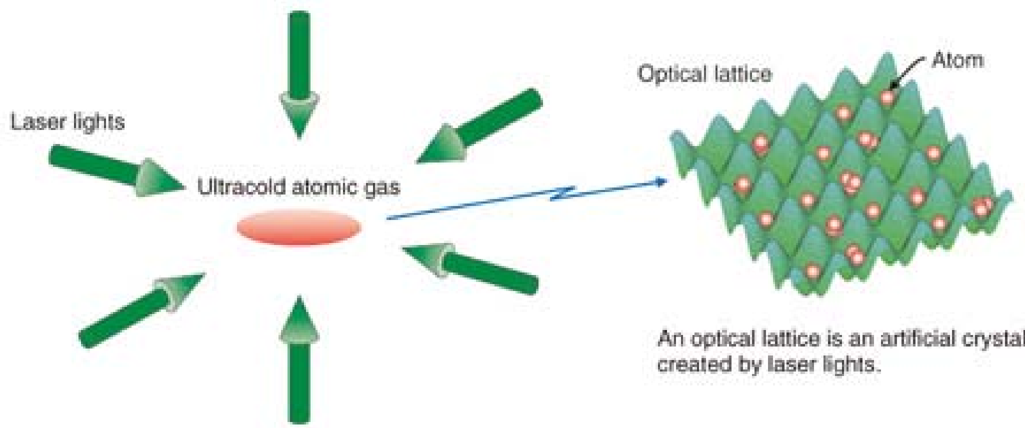
# Analog simulators for the Hubbard model

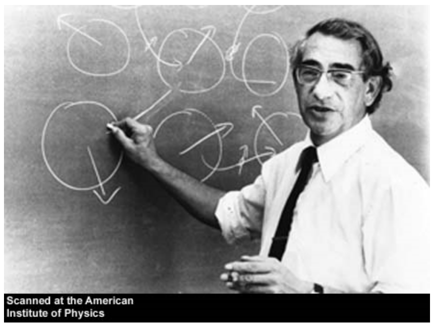



$\boldsymbol{P} = \alpha \boldsymbol{E}$

$-\boldsymbol{E} \cdot \boldsymbol{P} = -\alpha \boldsymbol{E}^2$

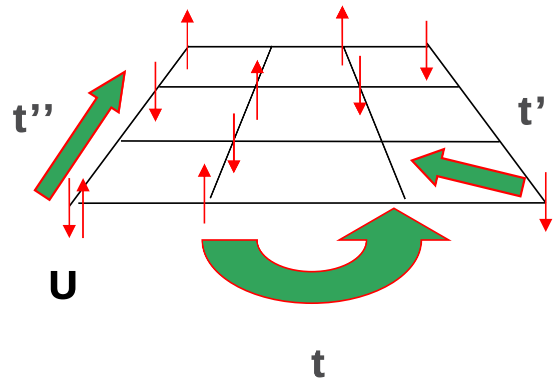
https://www.ntt-review.jp/archive/ntttechnical.php?contents=ntr201209fa5_s.html

# Hubbard Model




1931-1980

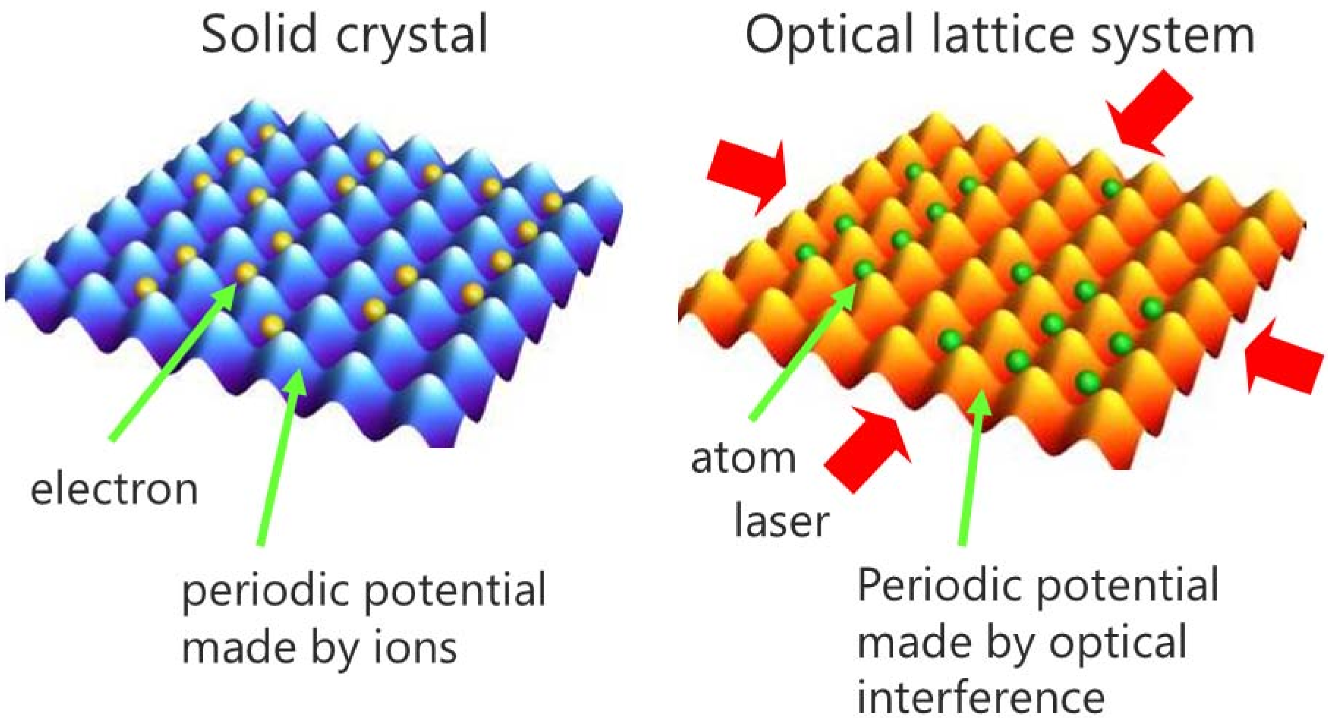
μ




$$H = -\sum_{<ij>\sigma} t_{i,j}\left(c_{i\sigma}^{\dagger}c_{j\sigma} + c_{j\sigma}^{\dagger}c_{i\sigma}\right) + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

$t = 1, \ k_B = 1, \ \hbar = 1$

# The analogy

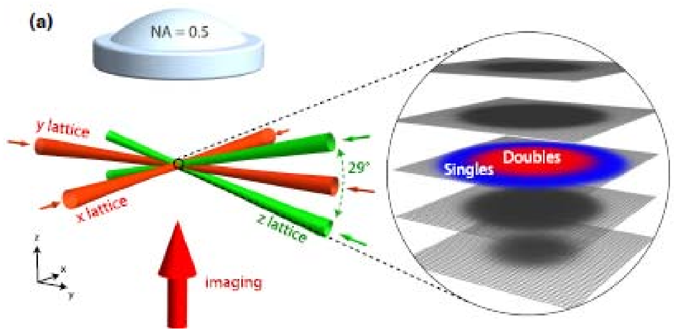


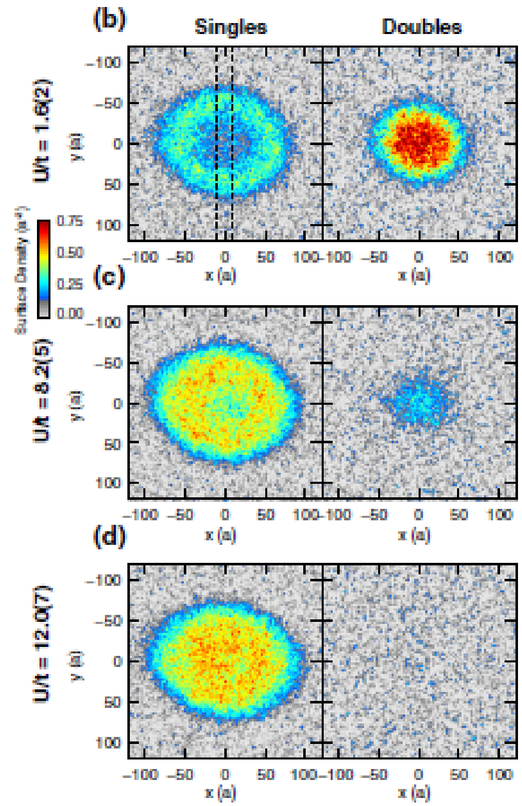

http://www.kozuma-eng.sci.titech.ac.jp/research_category/entry17.html

Selected for a Viewpoint in *Physics*
PRL **116,** 175301 (2016) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2016

# Equation of State of the Two-Dimensional Hubbard Model

Eugenio Cocchi,[1,2] Luke A. Miller,[1,2] Jan H. Drewes,[1] Marco Koschorreck,[1] Daniel Pertot,[1]
Ferdinand Brennecke,[1] and Michael Köhl[1,*]




*T* = 0.3 *t*

# Entanglement entropy and mutual information near the Mott transition


Caitlin Walsh


Patrick Sémon


David Poulin


Giovanni Sordi

C. Walsh *et al.*
PRX Quantum 1, 020310 (2020)
Phys. Rev. Lett. **122**, 067203 (2019)
Phys. Rev. B **99**, 075122 (2019)
See also later today

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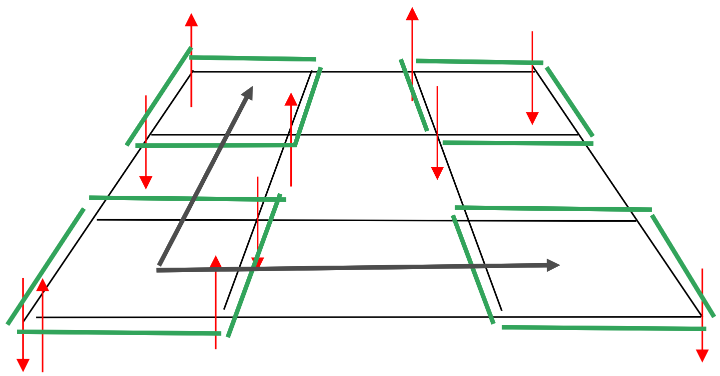
# Localized and delocalized pictures C-DMFT

## Delocalized

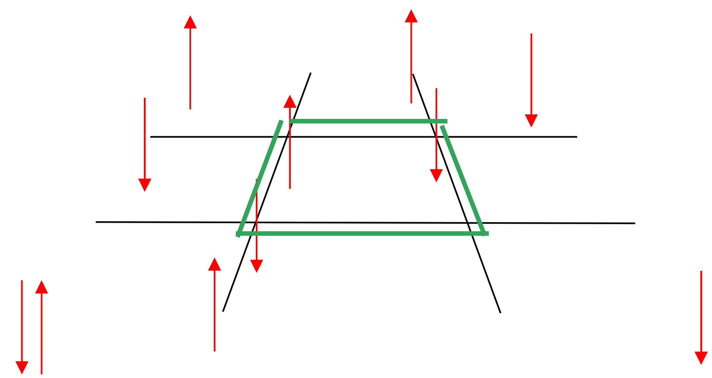


$$\mathbf{R} \rightarrow \tilde{k}$$

$$G_{ij} = \int \frac{d^d\tilde{k}}{(2\pi)^d} \left( \frac{1}{(i\omega_n+\mu)I-\varepsilon\left(\tilde{k}\right)-\Sigma} \right)_{ij}$$

REVIEWS
Maier, Jarrell et al., RMP. (2005)
Kotliar *et al.* RMP (2006)
AMST *et al.* LTP (2006)

## Localized

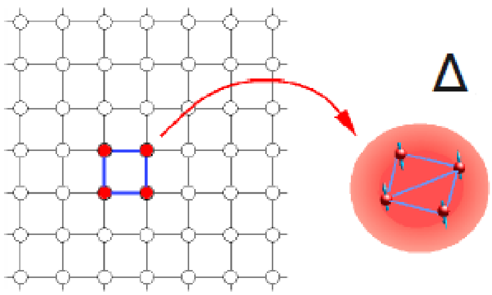


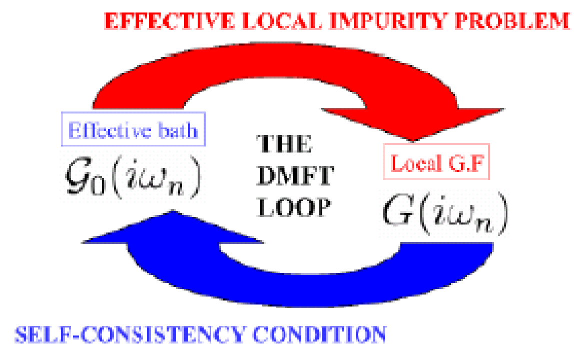
$$\left(G^{-1}\right)_{ij} = \left(G_0^{-1}\right)_{ij} - \Sigma_{ij}$$

Lichtenstein *et al.*,PRB 2000
Kotliar *et al.*, PRB 2000
M. Potthoff, EJP 2003

# Cellular Dynamical Mean-Field Theory: Impurity solver

$$Z = \int \mathcal{D}[\psi^\dagger, \psi]\, \mathrm{e}^{-S_c - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\mathbf{K}} \psi^\dagger_{\mathbf{K}}(\tau) \Delta_{\mathbf{K}}(\tau,\tau') \psi_{\mathbf{K}}(\tau')}$$




Mean-field is not a trivial problem! Many impurity solvers.

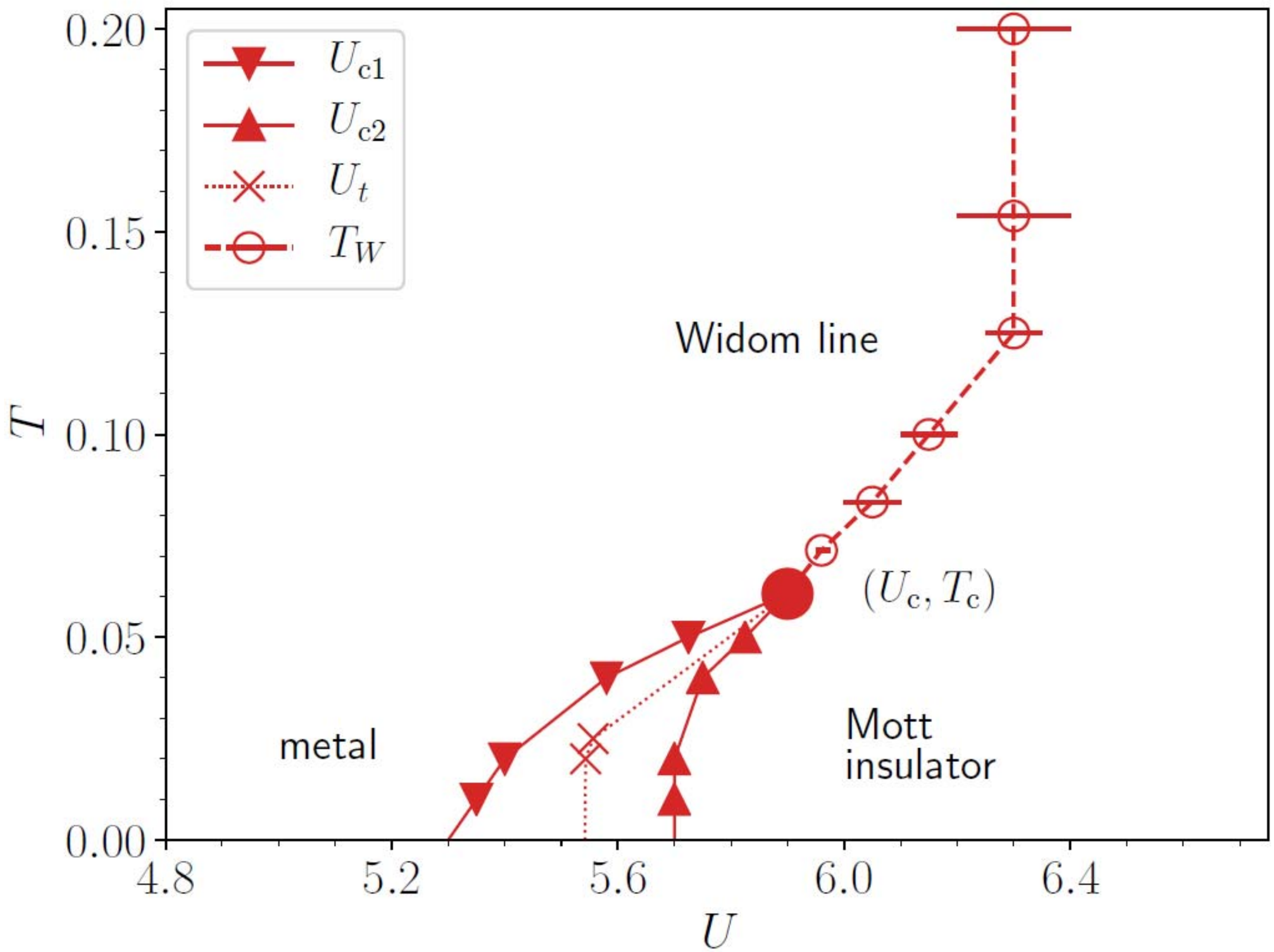
Here: continuous time QMC

P. Werner, PRL 2006
P. Werner, PRB 2007
K. Haule, PRB 2007

$$\Delta(i\omega_n) = i\omega_n + \mu - \Sigma_c(i\omega_n) - \left[\sum_{\tilde{k}} \frac{1}{i\omega_n + \mu - t_c(\tilde{k}) - \Sigma_c(i\omega_n)}\right]^{-1}$$

# The Mott transition at half-filling

C. Walsh, *et al.* PRB **99**, 075122 (2019)
H. Park, *et al.* PRL **107**, 137007 (2011).



# Single-site entanglement entropy

Schrödinger: I would not call [entanglement] *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

Proceedings of the Cambridge Philosophical Society **31**, 555 (1935); **32**, 446 (1936).

# Motivation

PHYSICAL REVIEW X **7**, 031025 (2017)

**Measuring Entropy and Short-Range Correlations in the Two-Dimensional Hubbard Model**

E. Cocchi,[1,2] L. A. Miller,[1,2] J. H. Drewes,[1] C. F. Chan,[1] D. Pertot,[1] F. Brennecke,[1] and M. Köhl[1]

First-order nature of the transition,
universality class of the end point,
crossovers emanating from the end point.

For quantum critical or finite temperature critical points

A. Anfossi *et al.* Phys. Rev. Lett. **95**, 056402 (2005).
L. Amico *et al.* Europhys. Lett. **77**, 17001 (2007).
L. Amico *et al.* Rev. Mod. Phys. **80**, 517 (2008).
D. Larsson *et al.* Phys. Rev. A **73**,042320 (2006).
D. Larsson *et al.* Phys. Rev. Lett. **95**, 196406 (2005).

# What is measured (Using CDMFT CT-HYB on plaquette)

- Single site entanglement entropy for fermions [1]

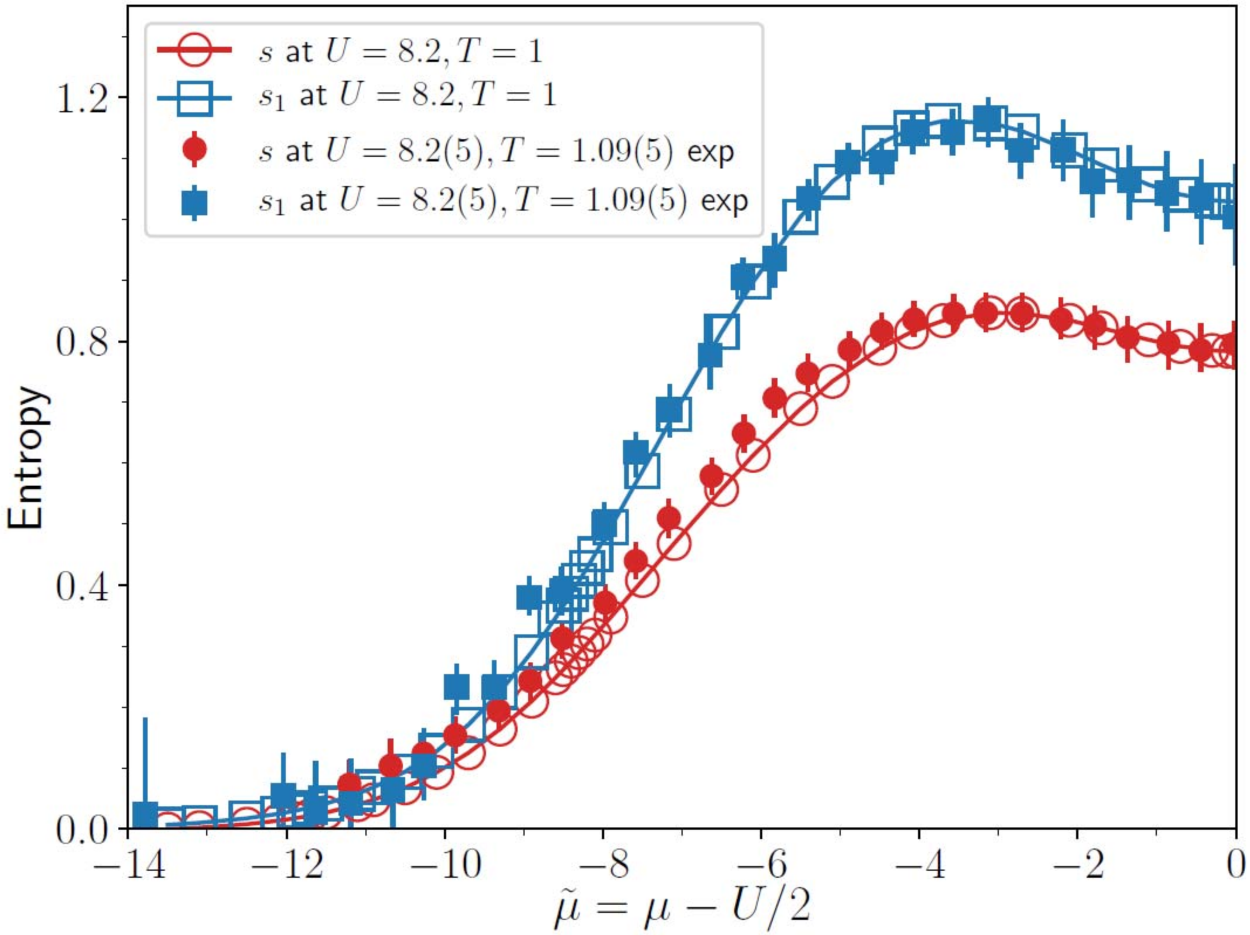
$$\rho_A = \mathrm{Tr}_B[\rho_{AB}] \qquad s_A = -\mathrm{Tr}_A[\rho_A \ln \rho_A]$$

$$\rho = \mathrm{diag}(p_0, p_\uparrow, p_\downarrow, p_{\uparrow\downarrow}) \qquad s_1 = -\sum_i p_i \ln(p_i)$$

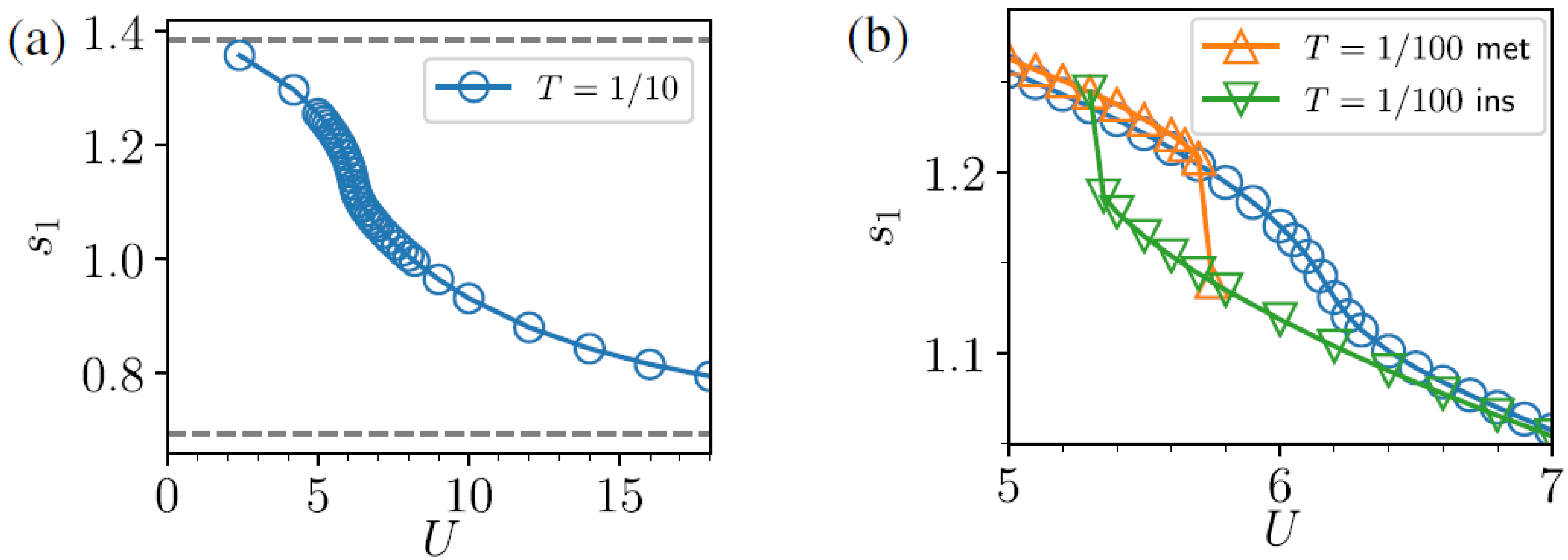
$$p_{\uparrow\downarrow} = \langle n_{i\uparrow} n_{i\downarrow} \rangle \qquad p_\uparrow = p_\downarrow = \langle n_{i\uparrow} - n_{i\uparrow} n_{i\downarrow} \rangle \qquad p_0 = 1 - 2p_\uparrow - p_{\uparrow\downarrow}$$

[1] P. Zanardi *et al.* Phys. Rev. A **65**, 042101 (2002).

# Agreement with experiment

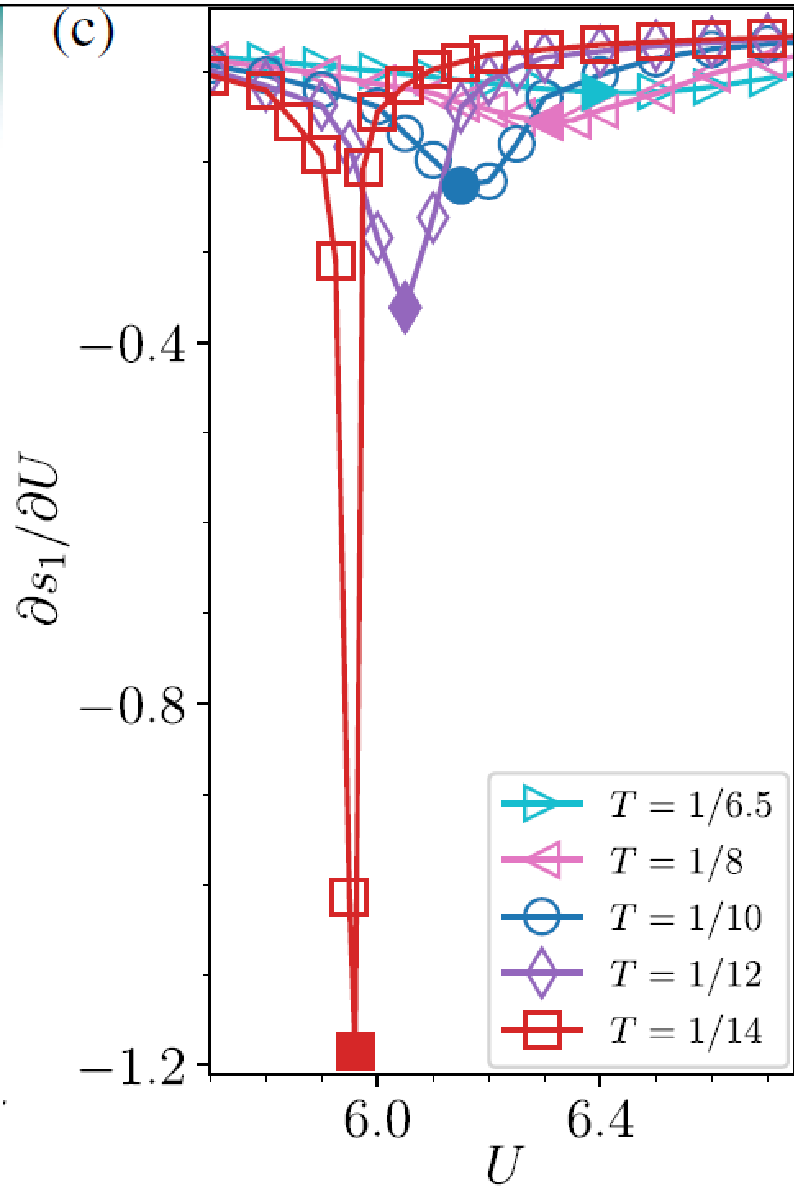


# Predictions




$$\partial s_1 / \partial T < 0$$

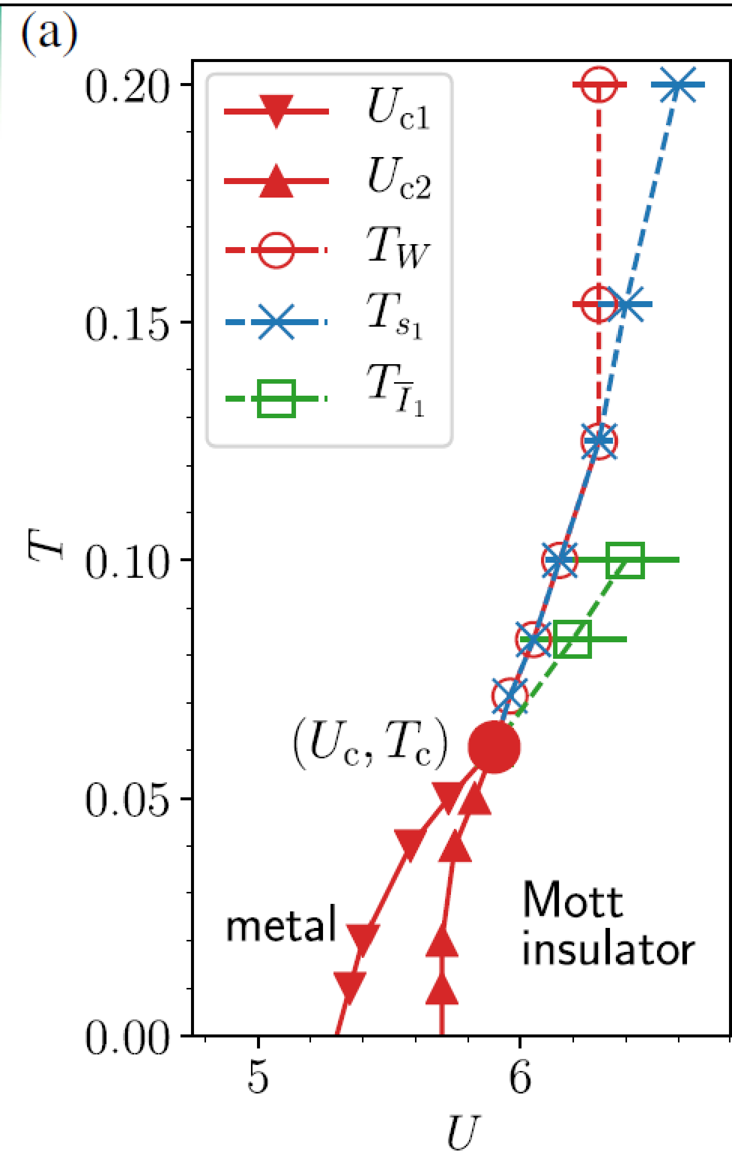
# Results




$$(\partial s_1/\partial D)(\partial D/\partial U)$$

$$\partial D/\partial U \sim -|U - U_c|^{-1+1/\delta}$$

# Transition and crossovers



# From single-site entanglement entropy

- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
  - Without knowledge of the order parameter of the transition

# Mutual information

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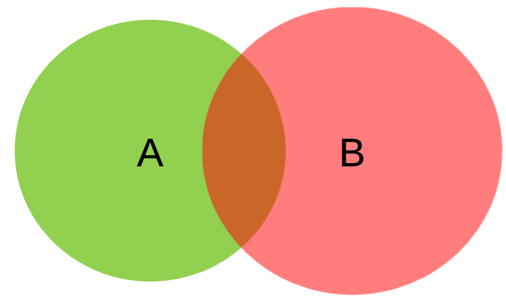
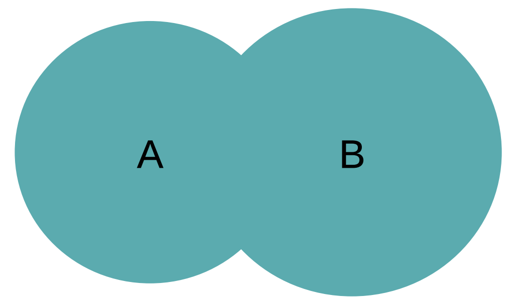
# Mutual information

$$I(A:B) = s_A + s_B - s_{AB}$$




Here we are *not* looking at the area law

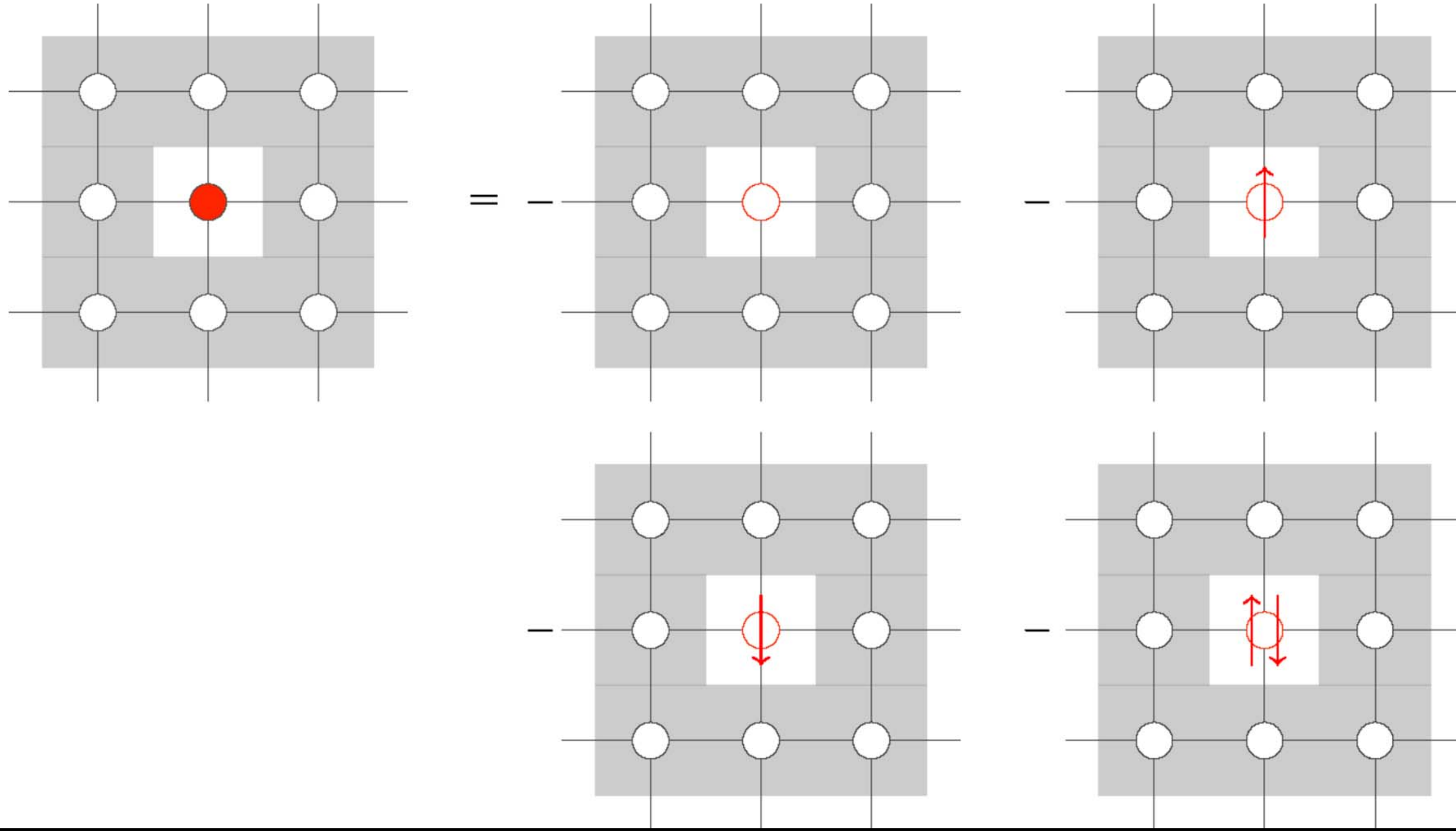
What is measured experimentally



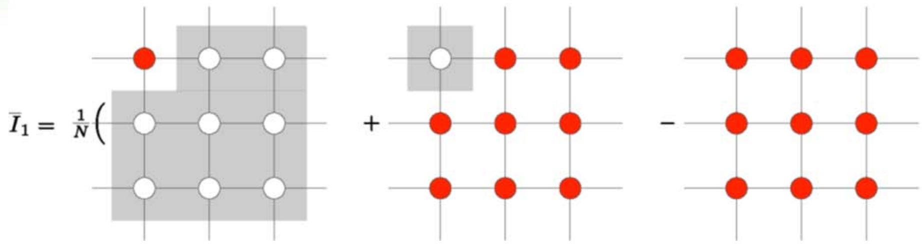

$$\bar{I}_1 = s_1 - s.$$

**Total mutual information**

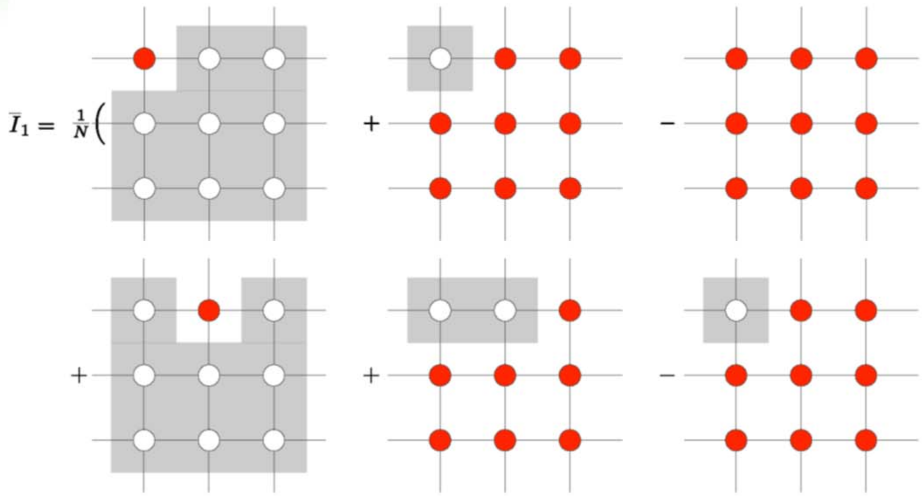
$$s_1 = \mathrm{Tr}_A(\rho_A \ln \rho_A) = -\sum_i p_i \ln p_i$$
$$= -p_0 \ln p_0 - p_\uparrow \ln p_\uparrow - p_\downarrow \ln p_\downarrow - p_{\uparrow\downarrow} \ln p_{\uparrow\downarrow}$$



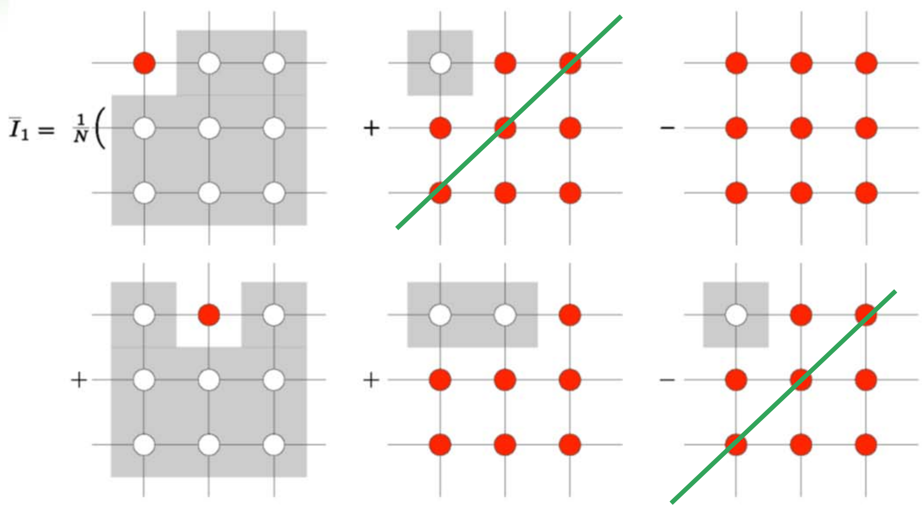
$$\overline{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i:\{>i\}) = \frac{1}{N}\sum_{i=1}^{N}\left(s_1(i) + s_{\{>i\}} - s_{\{>i-1\}}\right)$$



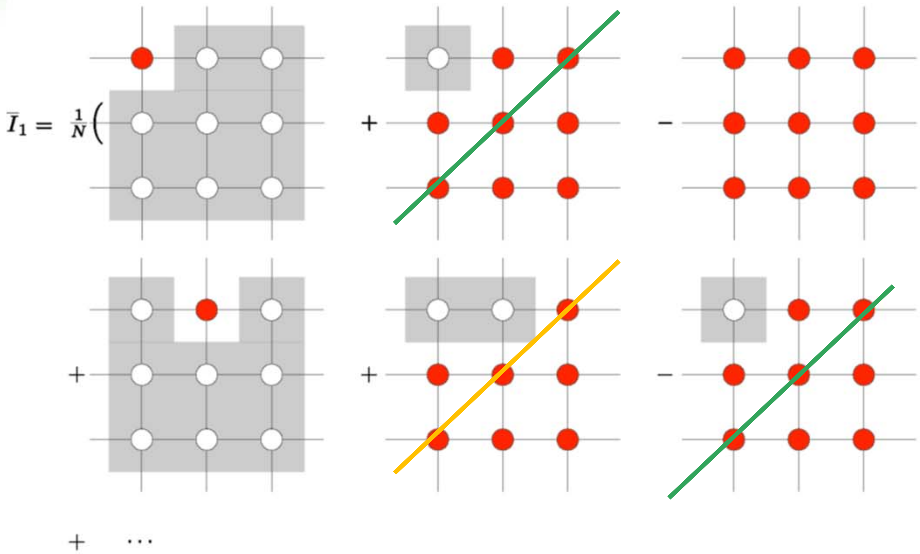
$$\bar{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i : \{> i\}) = \frac{1}{N}\sum_{i=1}^{N} \left(s_1(i) + s_{\{>i\}} - s_{\{>i-1\}}\right)$$



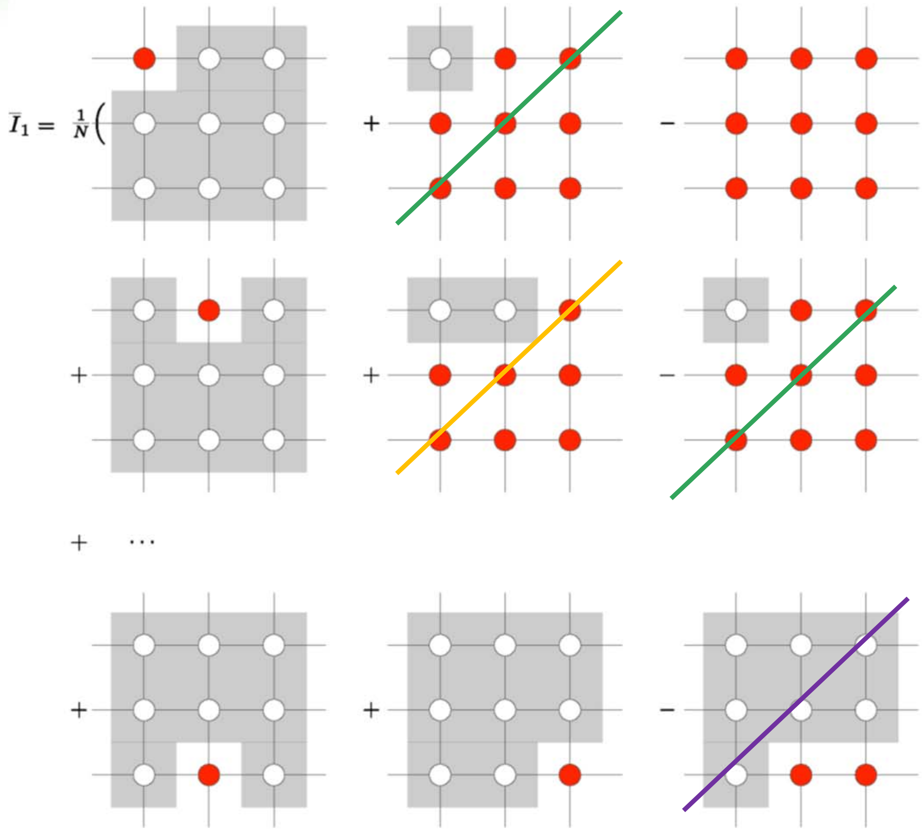
$$\bar{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i:\{>i\}) = \frac{1}{N}\sum_{i=1}^{N}\left(s_1(i)+s_{\{>i\}}-s_{\{>i-1\}}\right)$$



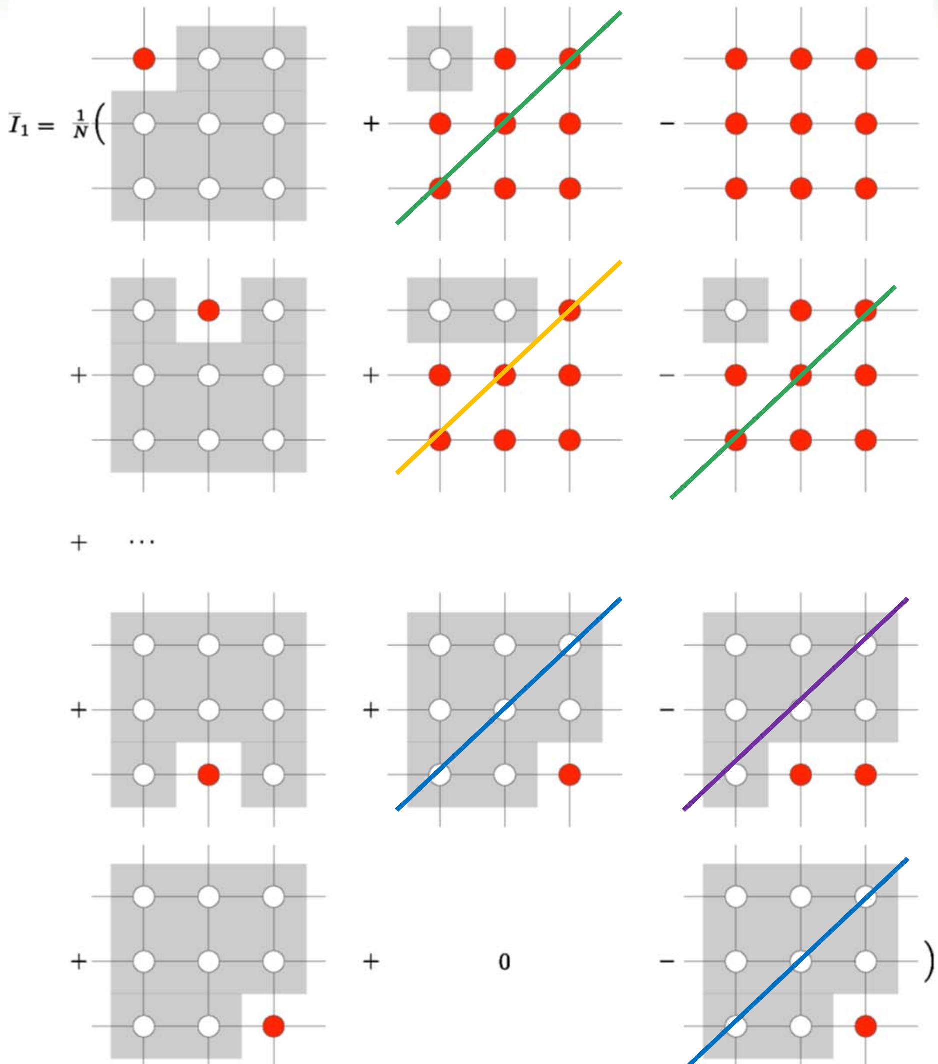
$$\bar{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i:\{>i\}) = \frac{1}{N}\sum_{i=1}^{N}\left(s_1(i) + s_{\{>i\}} - s_{\{>i-1\}}\right)$$



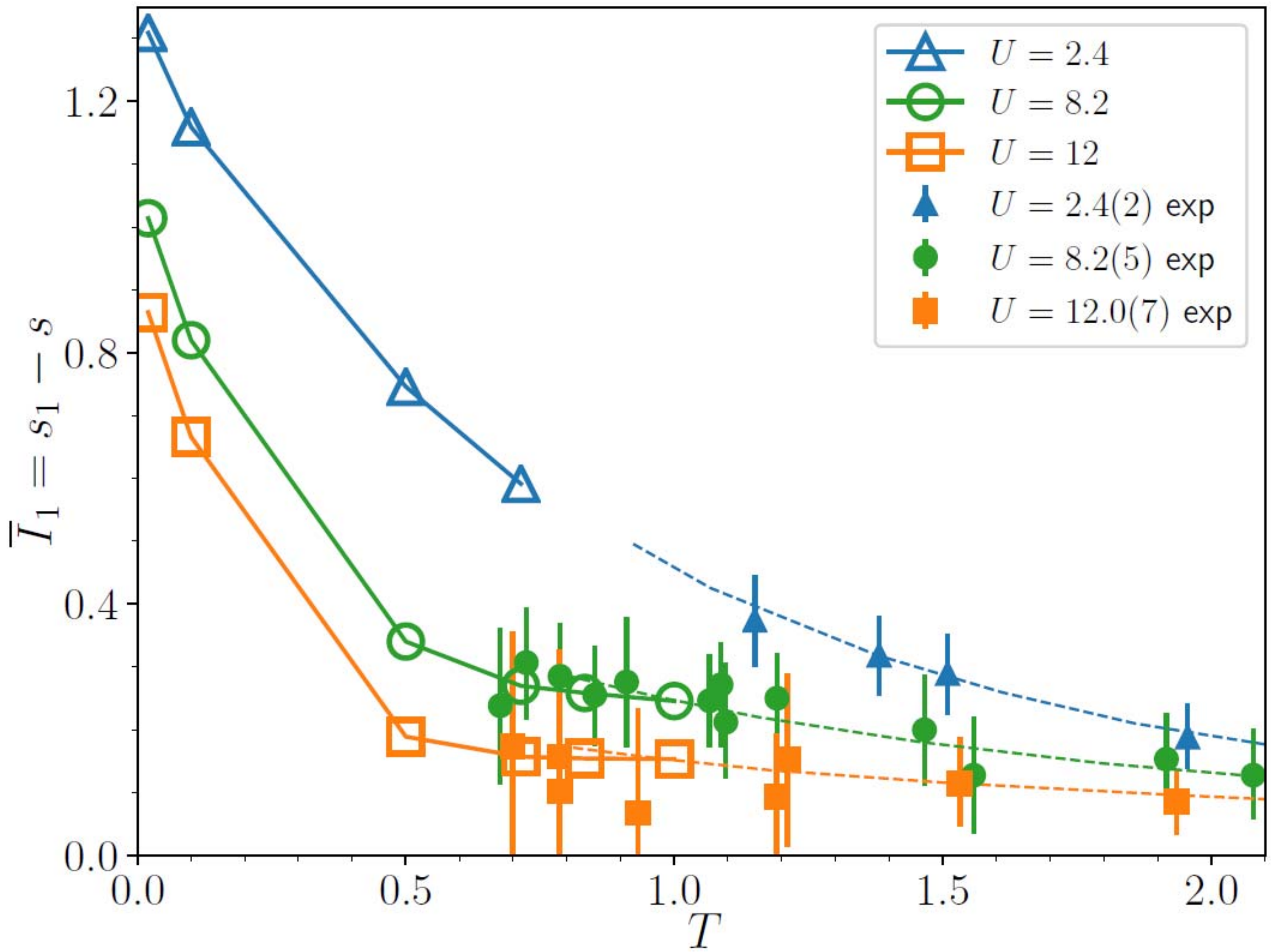
$$\bar{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i:\{>i\}) = \frac{1}{N}\sum_{i=1}^{N}\left(s_1(i) + s_{\{>i\}} - s_{\{>i-1\}}\right)$$



$$\bar{I}_1 = \frac{1}{N}\sum_{i=1}^{N} I(i:\{>i\}) = \frac{1}{N}\sum_{i=1}^{N}\left(s_1(i) + s_{\{>i\}} - s_{\{>i-1\}}\right)$$



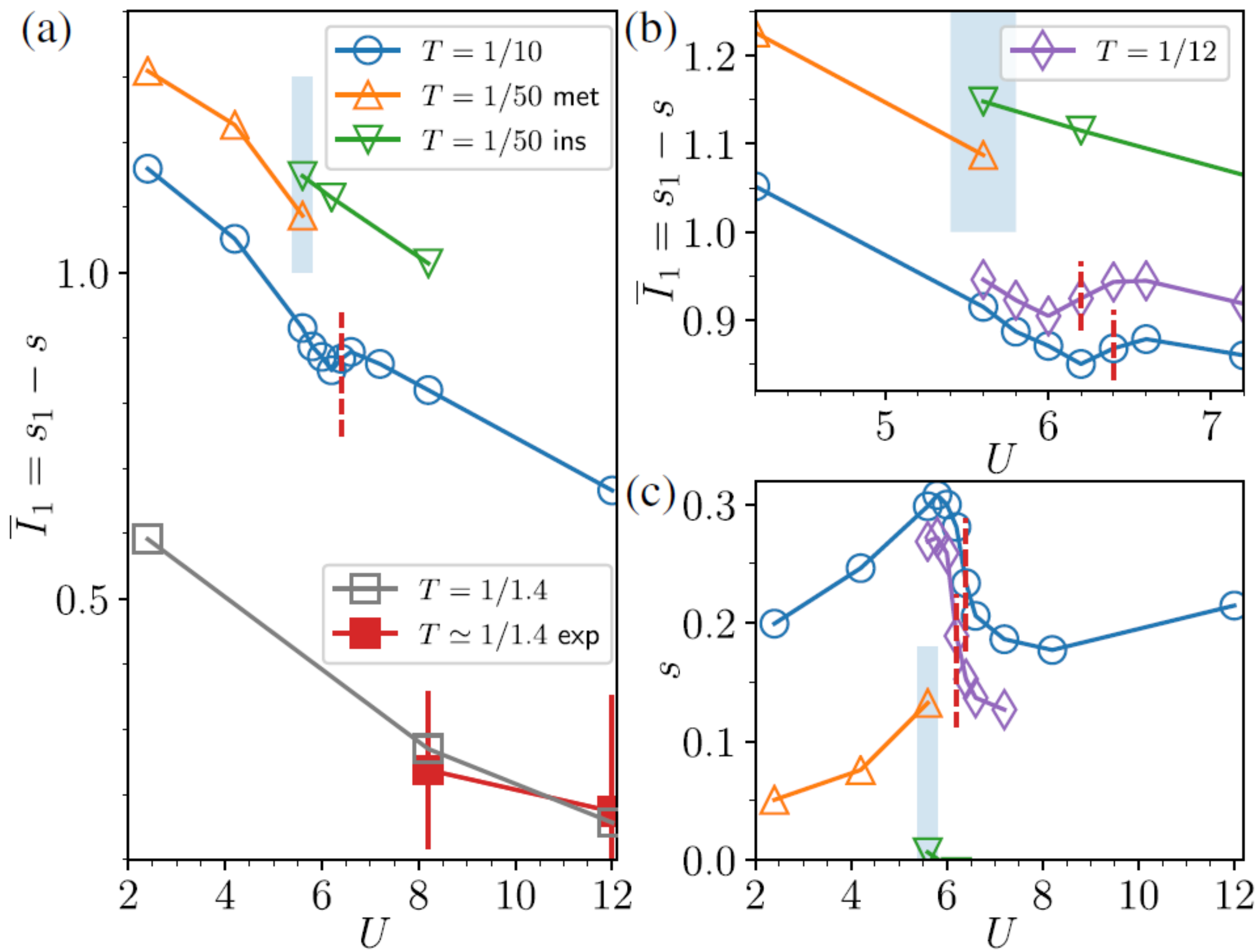
# Agreement with experiment



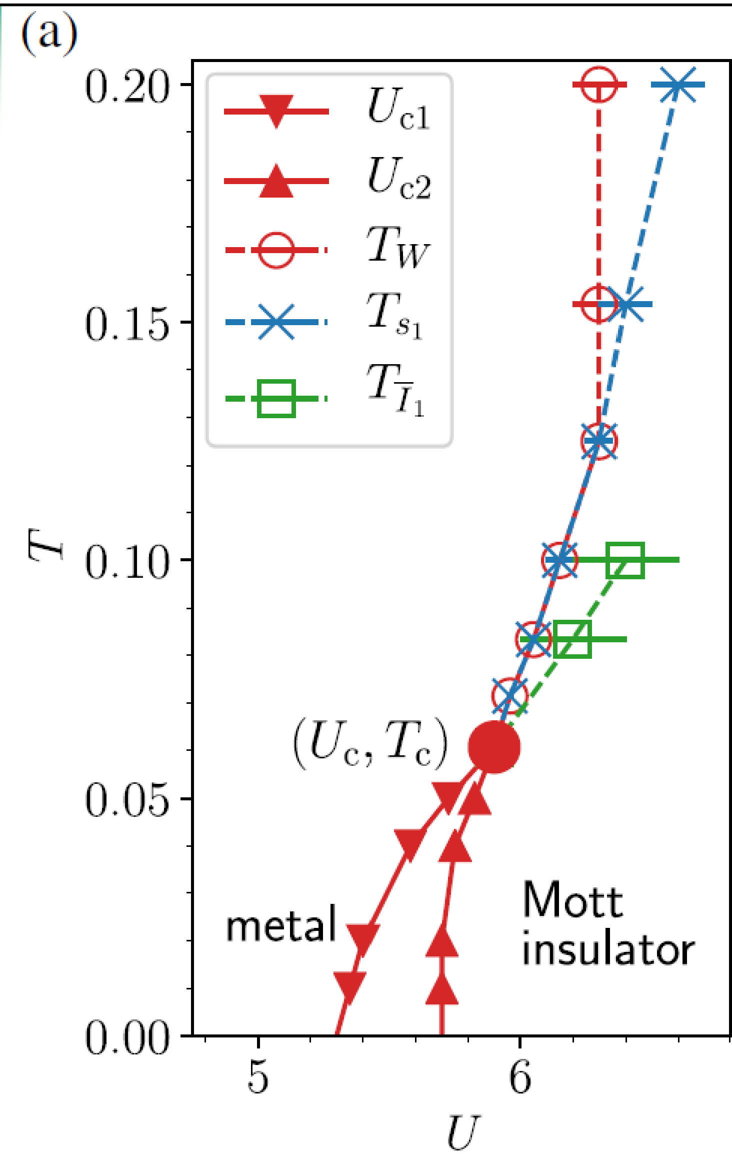
# Results

$$\bar{I}_1 \sim \mathrm{sgn}(U - U_c)|U - U_c|^{1/\delta}$$

$$J = \frac{4t^2}{U}$$



# Transition and crossovers



# From average mutual information

- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
  - Without knowledge of the order parameter of the transition


David Poulin

In memoriam

Merci
Thank you

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