

## QCM : practical exercises

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### — Problem 1 : CPT and model building

Construct the cluster and model files representing a one-dimensional Hubbard model with dimerization : The hopping terms are between nearest neighbors only, but they alternate between the value  $t$  and the value  $t'$ . Plot the spectral function for the case  $t = 1.1$  and  $t' = 0.9$ , at  $U = 0$  and then  $U = 4$ , at half-filling. Compare the  $U = 0$  case with the analytic solution to the problem.

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### — Problem 2 : VCA for antiferromagnetism

**A** Find the VCA solution for Néel antiferromagnetism in the nearest-neighbor, particle-hole symmetric Hubbard model at half-filling, using **two**  $3 \times 3$  clusters embedded in a two-dimensional square lattice. Use only one variational parameter : the Weiss field  $M$ . Plot the order parameter  $\langle M \rangle$  as a function of  $U$ , from  $U = 10$  to  $U = 0$ , in steps of  $\Delta U = 0.5$ .

**B** Refine your solution by adding to the list of variational parameters the hopping  $t$  on the cluster. Use the quasi-Newton method. Just find the solution for  $U = 8$ , still at half-filling. Is the ground state energy lower than in the simpler solution above? What about the order parameter?

**C** Refine your solution further by defining an additional operator on the cluster, that describes the hopping terms along the boundary of the cluster only (i.e. over 8 links). Add this operator to the list of variational parameters. Has the ground state energy still improved? What about the order parameter?

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### — Problem 3 : CDMFT and the Mott transition in graphene

**A** Construct a model file that describes graphene, based on a single, two-site cluster with a bath of 4 sites (file L2-4b.cluster provided). Check that the correct dispersion relation is found at  $U = 0$  by plotting the spectral function.

**B** At half filling, find the CDMFT solution (normal state) as a function of  $U$ , from  $U = 10$  to  $U = 0$  in steps of  $\Delta U = 0.2$ . Can you identify the Mott transition? Stay at half-filling, **using particle-hole symmetry**.